Agnesi Function Based Ionic Particle System

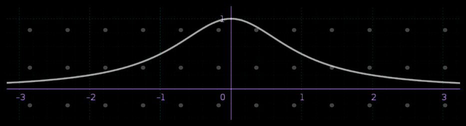
This system presents the following properties:

- Function based
- Emergent behavior from the math properties themselves
- Conceptually applicable to physical analog replicas
- Simple (without excesive mathematical complexity)
- Inefficient for a conventional digital computing system

$\mathbb{R}^1$

Particle Representation Using Agnesi Function

$$f(x) = \frac{1}{x^2+1}$$



I chose this function because it's like a spike of energy that influences the entire field around it. There's no distance where the other particles can't sense that energy. That's caused by the horizontal asymptote of the function.

Horizontal Asymptote

$$\lim_{x\to\infty} \frac{1}{x^2+1} = 0 \qquad f(x)>0 \ \forall \ x\in\mathbb{R}$$

Original	Variable Particle Position	Variable Mass Coefficent
$\frac{1}{x^2+1}$ →	$\frac{1}{(x-\alpha)^2+1}$ →	$\frac{\mu}{((x-\alpha)/\mu)^2+1}$

Variable Particle Charge

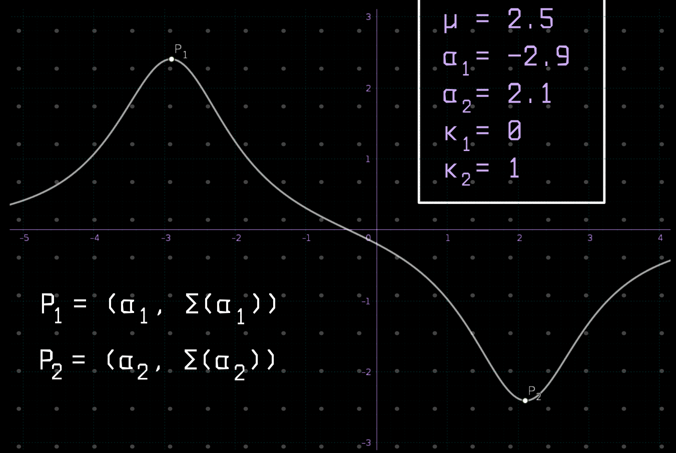
$$\rightarrow \frac{(-1)^{\kappa}\mu}{((x-\alpha)/\mu)^2+1}$$

The mass coefficient (μ) is a global parameter. The particle charge ($(-1)^{\kappa}$) allows the mass to be negative or positive

Particle

$$p_i(x) = \frac{(-1)^{\kappa_i}\mu}{((x-\alpha_i)/\mu)^2+1} \qquad \alpha_i, \mu \in\mathbb{R},\ \kappa_i\in\mathbb{N}$$

Example field



$n = 2$
$\mu = 2.5$
$\alpha_1 = -2.9$
$\alpha_2 = 2.1$
$\kappa_1 = 0$
$\kappa_2 = 1$

$P_1 = (\alpha_1, \Sigma(\alpha_1))$

$P_2 = (\alpha_2, \Sigma(\alpha_2))$

Field

$$\Sigma(x) = \sum_{i=1}^{n} p_i(x)$$

The field represents the sum of each particle function

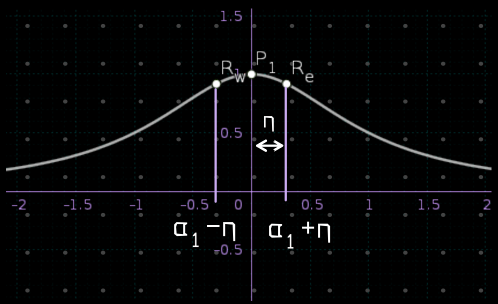
n = Number of particles

The field is the function.
The energy is the value of the field at a specific x position

Interactions

In order to move, the only information the particle needs to know is the energy at certain short distance in each side of its position. I named that distance η

$n = 1$
$\mu = 1$
$\eta = 0.3$
$\alpha_1 = 0$
$\kappa_1 = 0$



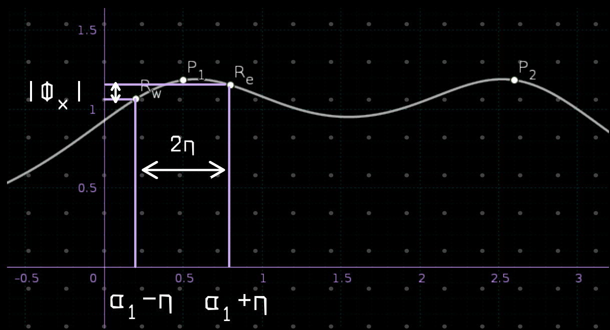
In $\mathbb{R}^1$ we got just 2 reference values per particle: $\alpha_i-\eta$ and $\alpha_i+\eta$

Force

The force is the difference between the energy in the left reference value and the right one

$$\Phi_x = \Sigma(\alpha_i-\eta)-\Sigma(\alpha_i+\eta)$$

$n = 2$
$\mu = 1$
$\eta = 0.3$
$\alpha_1 = 0.5$
$\alpha_2 = 2.6$
$\kappa_1 = 0$
$\kappa_2 = 0$



When there's more energy at the right side, the force is negative. If it's at the left side, the force is positive. That force value will increase as the particle moves towards the energy source

Translating to $\mathbb{R}^2$ (2D)

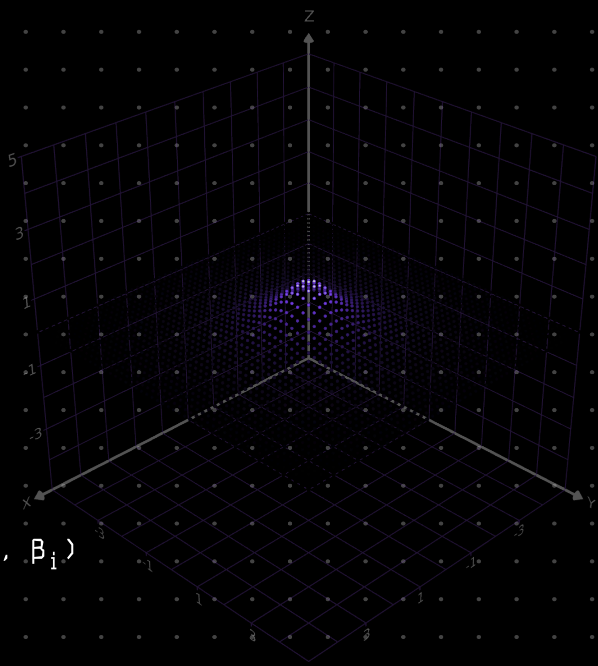
Original

$$f(x) = \frac{1}{x^2+1}$$

Radial Symmetry

$$g(x,y) = f(\sqrt{(x^2+y^2)})$$

$$\frac{1}{(\sqrt{(x^2+y^2)})^2+1} = \frac{1}{x^2+y^2+1}$$



Particle

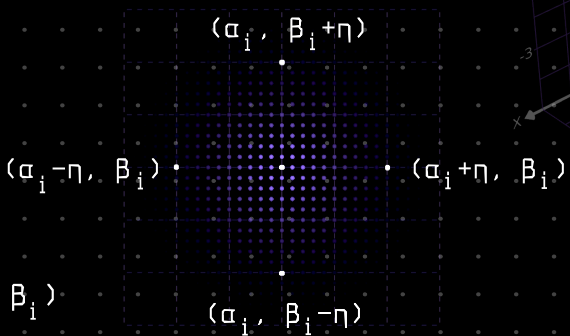
$$\rho_i(x) = \frac{(-1)^{\kappa_i}\mu}{((x-\alpha_i)/\mu)^2+((y-\beta_i)/\mu)^2+1}$$

$\alpha_i, \mu \in \mathbb{R}$, $\kappa_i \in \mathbb{N}$

Field

$$\Sigma(x,y) = \sum_{i=1}^{n} \rho_i(x,y)$$

Reference Values



Forces

$$\Phi_x = \Sigma(\alpha_i-\eta, \beta_i)-\Sigma(\alpha_i+\eta, \beta_i)$$

$$\Phi_y = \Sigma(\alpha_i, \beta_i-\eta)-\Sigma(\alpha_i, \beta_i+\eta)$$

With this parameters, now the particles will follow the lowest energy path. In consequence the particles of opposite charge will attract each other and the ones with same charge are going to repel each other. Everything is going to converge into energetically balanced structures

Position Update

Now with everything in place. The next step is to update the positions using the force and charge values

$$\alpha_i = \alpha_i+(-1)^{\kappa_i}\Phi_x$$

$$\beta_i = \beta_i+(-1)^{\kappa_i}\Phi_y$$