

guyvitelson_mmn11_ml_latest

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1 11 - - 2025 - 203379706

##If you run this within Google Collab, Dont Worry! all the missing python files/directories/modules will be automatically feteched from my github repository

My GitHub Profile : <https://github.com/v1t3ls0n>

The Repository: https://github.com/v1t3ls0n/ml_intro_course_mmn11

Student ID: 203379706

1.1 Fetch Resources

1.1.1 External Code Imports (pip packages)

```
[1]: import os
import shutil
import sys
import logging
import numpy as np # type: ignore
import matplotlib.pyplot as plt # type: ignore
import seaborn as sns # type: ignore
import time
import pandas as pd
```

1.1.2 Fetch Missing Files For Google Colab Env

```
[2]: # %%capture run_output
# %%matplotlib inline

if sys.platform != 'win32': # check if we are running on google collab
    repo_url = "https://github.com/v1t3ls0n/ml_intro_course_mmn11"
    repo_name = "ml_intro_course_mmn11"
    from tqdm.notebook import tqdm # type: ignore

    # Clone the repository if it doesn't exist
    if not os.path.exists(repo_name):
        os.system(f"git clone {repo_url}")
```

```

# Construct the path to the repository directory
repo_path = os.path.join(os.getcwd(), repo_name)

# Add the repository directory to the Python path
if repo_path not in sys.path:
    sys.path.insert(0, repo_path)

# --- Extract 'core' and 'notebooks' directories ---
def extract_directories(source_dir, destination_dir, dir_names):
    for dir_name in dir_names:
        source_path = os.path.join(source_dir, dir_name)
        destination_path = os.path.join(destination_dir, dir_name)
        if os.path.exists(source_path):
            shutil.copytree(source_path, destination_path, dirs_exist_ok=True)

destination_path = "."
# Extract the directories
extract_directories(repo_path, destination_path, ["core"])
project_root = os.path.abspath(os.path.join(os.getcwd(), '..'))
sys.path.insert(0, project_root)
if os.path.exists("ml_intro_course_mmn11"):
    shutil.rmtree("ml_intro_course_mmn11")
if os.path.exists("sample_data"):
    shutil.rmtree("sample_data")
else:
    from tqdm import tqdm # type: ignore
    current_dir = os.getcwd() # Current working directory
    project_root = os.path.abspath(os.path.join(current_dir, '..')) # Root
    ↪directory of the project
    sys.path.insert(0, project_root)

```

1.1.3 Internal Code Imports (original code)

```

[3]: # ===== Internal Code Imports =====

#Logger
from core.logger.config import logger

# Data Preprocessing
from core.data.mnist_loader import load_mnist
from core.data.data_preprocessing import preprocess_data

# Models
from core.models.perceptron.multi_class_perceptron import MultiClassPerceptron
from core.models.logistic_regression.softmax_lregression import ↪
    ↪SoftmaxRegression
from core.models.linear_regression.linear_regression import LinearRegression

```

```

# Performance & Plotting
from core.analysis.evaluation_functions import (
    evaluate_model,
    aggregate_iteration_losses,
    aggregate_iteration_losses_softmax
)

from core.analysis.plotting import (
    plot_confusion_matrix_annotated,
    plot_error_curves,
    plot_accuracy_vs_max_iter,
    plot_runtime_vs_max_iter,
    plot_performance_summary_extended,
    plot_train_curves_three_models,
    plot_metric_vs_learning_rate,
    plot_accuracy_vs_max_iter_4models,
    plot_runtime_vs_max_iter_4models,
    plot_accuracy_vs_runtime,
    plot_performance_summary_extended_by_runtime,
    plot_performance_summary_4models_by_runtime,
    plot_accuracy_vs_runtime_4models
)

logger = logging.getLogger("MyGlobalLogger") # configured in core/logger/config.
↳py

```

2 Overview

2.1 MNIST Digit Classification Report

2.1.1 Approach

Data Preprocessing The MNIST dataset was prepared by: - Splitting into training (60,000 samples) and test sets (10,000 samples). - Normalizing pixel values to the [0,1] range. - Flattening images into vectors (784 pixels plus 1 bias term). - Encoding labels into one-hot vectors.

Model Implementation

- **Multi-Class Perceptron:**
 - One-vs-all strategy implemented with standard Perceptron and Pocket Perceptron algorithms.
- **Softmax Regression:**
 - Implemented using cross-entropy loss and adaptive learning rates (AdaGrad).
 - Included early stopping based on loss improvement.
- **Linear Regression:**
 - Utilized mean squared error loss with gradient descent.
 - AdaGrad adaptive learning rate and early stopping were applied.

2.1.2 Results

- **Accuracy:**
 - Softmax Regression achieved the highest accuracy.
 - Multi-class Pocket Perceptron showed good performance, surpassing standard Perceptron.
 - Linear Regression exhibited relatively lower accuracy due to its limitations for classification tasks.

Confusion Matrices and Metrics

- Softmax Regression demonstrated the lowest misclassification rates across digits.
- Pocket Perceptron reduced errors compared to standard Perceptron, indicating improved robustness.
- Sensitivity and accuracy clearly highlighted Softmax Regression as superior for multi-class digit classification.

2.1.3 Discussion

- Softmax Regression proved best for digit classification, providing reliable probability estimations and stable convergence.
- Pocket Perceptron algorithm offered notable improvements over standard Perceptron, highlighting its utility in non-linearly separable scenarios.
- Linear Regression's limitations in classification tasks were evident, reaffirming theoretical expectations.

2.1.4 Conclusions

- Softmax Regression is the most suitable algorithm for multi-class digit recognition problems.
- Pocket Perceptron serves as an effective alternative, offering a balance between simplicity and performance.
- Linear Regression, while straightforward, is suboptimal for classification due to its inherent limitations.

3 Choose Run Parameters (Significant Effect On Model's Runtime!)

```
[4]: #####  
# SEPARATE RUN PARAMETERS FOR PERCEPTRONS vs. REGRESSIONS  
#####  
  
# Perceptrons (Clean & Pocket) iteration-based run  
perceptron_max_iter_values = [50,100,1000] # for Clean PLA & Pocket PLA  
# Logging the run parameters  
logger.info(f"=== Perceptron Run Parameters ===")  
logger.info(f"max_iter_values = {perceptron_max_iter_values}")
```

```

# Regression (Softmax & Linear) run parameters.
learning_rates = [0.1] # for Softmax & Linear Regression
iteration_counts = [50,100,1000]
regression_run_configs = [
    {
        "label": f"LR={lr}/Iter={it}",
        "learning_rate": lr,
        "max_iter": it
    }
    for lr in learning_rates
    for it in iteration_counts
]

logger.info(f"=== Regression Run Parameters ===")
for cfg in regression_run_configs:
    logger.info(f"{cfg['label']} -> learning_rate={cfg['learning_rate']},
    ↪max_iter={cfg['max_iter']}")

```

```

INFO - === Perceptron Run Parameters ===
INFO - max_iter_values = [50, 100, 1000]
INFO - === Regression Run Parameters ===
INFO - LR=0.1/Iter=50 -> learning_rate=0.1, max_iter=50
INFO - LR=0.1/Iter=100 -> learning_rate=0.1, max_iter=100
INFO - LR=0.1/Iter=1000 -> learning_rate=0.1, max_iter=1000
INFO - max_iter_values = [50, 100, 1000]
INFO - === Regression Run Parameters ===
INFO - LR=0.1/Iter=50 -> learning_rate=0.1, max_iter=50
INFO - LR=0.1/Iter=100 -> learning_rate=0.1, max_iter=100
INFO - LR=0.1/Iter=1000 -> learning_rate=0.1, max_iter=1000

```

4 Load and Preprocess the MNIST Dataset

```

[5]: '''
    We'll load the MNIST dataset using our custom loader (`mnist_loader`) and then
    ↪apply preprocessing (`data_preprocessing`).
    The preprocessing step normalizes each image to the range [0, 1] and adds a
    ↪bias term, resulting in input samples with 785 features.
    This setup ensures that the training set contains 60,000 samples and the test
    ↪set 10,000 samples, preparing the data for the subsequent classification
    ↪tasks.
    '''

# New section
# Load raw MNIST data (X: images, y: labels)
X_raw, y_raw = load_mnist()

```

```

logger.info("Raw MNIST data shapes: X_raw: %s, y_raw: %s", X_raw.shape, y_raw.
↳shape)

# Preprocess (normalize & add bias = True)
X = preprocess_data(X_raw, add_bias=True, normalize=True)
logger.info("Preprocessed shape: %s", X.shape)

# Split into train/test manually or with 60k/10k as the task suggests
X_train, y_train = X[:60000], y_raw[:60000]
X_test, y_test = X[60000:], y_raw[60000:]

logger.info("Train set: X_train: %s, y_train: %s", X_train.shape, y_train.shape)
logger.info("Test set: X_test: %s, y_test: %s", X_test.shape, y_test.shape)

```

```

INFO - Raw MNIST data shapes: X_raw: (70000, 784), y_raw: (70000,)
INFO - Preprocessed shape: (70000, 785)
INFO - Train set: X_train: (60000, 785), y_train: (60000,)
INFO - Test set: X_test: (10000, 785), y_test: (10000,)

```

5 Train

```

[6]: # =====
# TRAINING CELL
# =====

# 1) Dictionaries to store trained models
trained_models_clean = {}
trained_models_pocket = {}
trained_models_softmax = {}
trained_models_linear = {}

# 2) Train Regression Models (Softmax & Linear)
logger.info("== TRAINING REGRESSION MODELS (Softmax & Linear) ==")
for cfg in tqdm(regression_run_configs, desc="Train Regressions"):
    lr_val = cfg["learning_rate"]
    max_iter_val = cfg["max_iter"]
    label = cfg["label"] # e.g. "LR=0.001/Iter=1000"

    # --- Softmax ---
    logger.info(f"--- Softmax {label} ---")
    s_model = SoftmaxRegression(
        num_classes=10,
        max_iter=max_iter_val,
        learning_rate=lr_val,
        adaptive_lr=True
    )

```

```

s_model.fit(X_train, y_train)
trained_models_softmax[(lr_val, max_iter_val)] = s_model

# --- Linear ---
logger.info(f"--- Linear Regression {label} ---")
lin_model = LinearRegression(
    num_classes=10,
    max_iter=max_iter_val,
    learning_rate=lr_val,
    adaptive_lr=True,
    early_stopping=False
)
lin_model.fit(X_train, y_train)
trained_models_linear[(lr_val, max_iter_val)] = lin_model

logger.info("Training complete for Softmax and Linear.")

# 3) Train Perceptron Models (Clean & Pocket)
logger.info("=== TRAINING PERCEPTRON MODELS (Clean & Pocket) ===")
for max_iter in tqdm(perceptron_max_iter_values, desc="Train Clean & Pocket"):
    logger.info(f"--- Clean PLA, max_iter={max_iter} ---")
    clean_perc = MultiClassPerceptron(num_classes=10, max_iter=max_iter,
    ↪use_pocket=False)
    clean_perc.fit(X_train, y_train)
    trained_models_clean[max_iter] = clean_perc

    logger.info(f"--- Pocket PLA, max_iter={max_iter} ---")
    pocket_perc = MultiClassPerceptron(num_classes=10, max_iter=max_iter,
    ↪use_pocket=True)
    pocket_perc.fit(X_train, y_train)
    trained_models_pocket[max_iter] = pocket_perc

logger.info("Training complete for Clean PLA and Pocket PLA.")
logger.info("=== ALL TRAINING COMPLETE ===")

```

```

INFO - === TRAINING REGRESSION MODELS (Softmax & Linear) ===
Train Regressions:  0%|          | 0/3 [00:00<?, ?it/s]INFO - --- Softmax
LR=0.1/Iter=50 ---
Train Regressions:  0%|          | 0/3 [00:00<?, ?it/s]INFO - --- Softmax
LR=0.1/Iter=50 ---
INFO - Iter 1/50, Loss: 2.3410, Avg Adaptive LR: 14.388479
INFO - Iter 11/50, Loss: 0.4330, Avg Adaptive LR: 3.270055
INFO - Iter 21/50, Loss: 0.3701, Avg Adaptive LR: 3.260155
INFO - Iter 31/50, Loss: 0.3467, Avg Adaptive LR: 3.258578
INFO - Iter 41/50, Loss: 0.3320, Avg Adaptive LR: 3.257611
INFO - SoftmaxRegression training completed in 2.00 seconds.
INFO - --- Linear Regression LR=0.1/Iter=50 ---

```

```

INFO - LinearRegressionClassifier training completed in 1.53 seconds.
Train Regressions: 33%|          | 1/3 [00:03<00:07, 3.54s/it]INFO - ---
Softmax LR=0.1/Iter=100 ---
INFO - Iter 1/100, Loss: 2.3143, Avg Adaptive LR: 13.916900
INFO - Iter 11/100, Loss: 0.4305, Avg Adaptive LR: 3.267324
INFO - Iter 21/100, Loss: 0.3740, Avg Adaptive LR: 3.263789
INFO - Iter 31/100, Loss: 0.3487, Avg Adaptive LR: 3.262145
INFO - Iter 41/100, Loss: 0.3330, Avg Adaptive LR: 3.261111
INFO - Iter 51/100, Loss: 0.3219, Avg Adaptive LR: 3.260377
INFO - Iter 61/100, Loss: 0.3135, Avg Adaptive LR: 3.259820
INFO - Iter 71/100, Loss: 0.3069, Avg Adaptive LR: 3.259377
INFO - Iter 81/100, Loss: 0.3014, Avg Adaptive LR: 3.259013
INFO - Iter 91/100, Loss: 0.2968, Avg Adaptive LR: 3.258706
INFO - SoftmaxRegression training completed in 3.92 seconds.
INFO - --- Linear Regression LR=0.1/Iter=100 ---
INFO - Iter 100/100, Loss: 0.4740, Gradient Norm: 12.8816, Avg Adaptive LR:
1.39800421679503
INFO - LinearRegressionClassifier training completed in 3.05 seconds.
Train Regressions: 67%|          | 2/3 [00:10<00:05, 5.56s/it]INFO - ---
Softmax LR=0.1/Iter=1000 ---
INFO - Iter 1/1000, Loss: 2.3519, Avg Adaptive LR: 14.777751
INFO - Iter 11/1000, Loss: 0.4406, Avg Adaptive LR: 2.878976
INFO - Iter 21/1000, Loss: 0.3809, Avg Adaptive LR: 2.853751
INFO - Iter 31/1000, Loss: 0.3530, Avg Adaptive LR: 2.851568
INFO - Iter 41/1000, Loss: 0.3375, Avg Adaptive LR: 2.850786
INFO - Iter 51/1000, Loss: 0.3265, Avg Adaptive LR: 2.850226
INFO - Iter 61/1000, Loss: 0.3180, Avg Adaptive LR: 2.849797
INFO - Iter 71/1000, Loss: 0.3113, Avg Adaptive LR: 2.849453
INFO - Iter 81/1000, Loss: 0.3057, Avg Adaptive LR: 2.849169
INFO - Iter 91/1000, Loss: 0.3010, Avg Adaptive LR: 2.848929
INFO - Iter 101/1000, Loss: 0.2969, Avg Adaptive LR: 2.848721
INFO - Iter 111/1000, Loss: 0.2934, Avg Adaptive LR: 2.848540
INFO - Iter 121/1000, Loss: 0.2903, Avg Adaptive LR: 2.848379
INFO - Iter 131/1000, Loss: 0.2875, Avg Adaptive LR: 2.848236
INFO - Iter 141/1000, Loss: 0.2849, Avg Adaptive LR: 2.848106
INFO - Iter 151/1000, Loss: 0.2826, Avg Adaptive LR: 2.847988
INFO - Iter 161/1000, Loss: 0.2805, Avg Adaptive LR: 2.847881
INFO - Iter 171/1000, Loss: 0.2786, Avg Adaptive LR: 2.847781
INFO - Iter 181/1000, Loss: 0.2768, Avg Adaptive LR: 2.847690
INFO - Iter 191/1000, Loss: 0.2752, Avg Adaptive LR: 2.847605
INFO - Iter 201/1000, Loss: 0.2736, Avg Adaptive LR: 2.847526
INFO - Iter 211/1000, Loss: 0.2722, Avg Adaptive LR: 2.847452
INFO - Iter 221/1000, Loss: 0.2709, Avg Adaptive LR: 2.847383
INFO - Iter 231/1000, Loss: 0.2696, Avg Adaptive LR: 2.847317
INFO - Iter 241/1000, Loss: 0.2684, Avg Adaptive LR: 2.847256
INFO - Iter 251/1000, Loss: 0.2673, Avg Adaptive LR: 2.847198
INFO - Iter 261/1000, Loss: 0.2662, Avg Adaptive LR: 2.847142
INFO - Iter 271/1000, Loss: 0.2652, Avg Adaptive LR: 2.847090

```


INFO - Iter 281/1000, Loss: 0.2642, Avg Adaptive LR: 2.847040
INFO - Iter 291/1000, Loss: 0.2633, Avg Adaptive LR: 2.846993
INFO - Iter 301/1000, Loss: 0.2624, Avg Adaptive LR: 2.846948
INFO - Iter 311/1000, Loss: 0.2616, Avg Adaptive LR: 2.846904
INFO - Iter 321/1000, Loss: 0.2608, Avg Adaptive LR: 2.846863
INFO - Iter 331/1000, Loss: 0.2600, Avg Adaptive LR: 2.846823
INFO - Iter 341/1000, Loss: 0.2592, Avg Adaptive LR: 2.846785
INFO - Iter 351/1000, Loss: 0.2585, Avg Adaptive LR: 2.846749
INFO - Iter 361/1000, Loss: 0.2578, Avg Adaptive LR: 2.846713
INFO - Iter 371/1000, Loss: 0.2572, Avg Adaptive LR: 2.846680
INFO - Iter 381/1000, Loss: 0.2565, Avg Adaptive LR: 2.846647
INFO - Iter 391/1000, Loss: 0.2559, Avg Adaptive LR: 2.846615
INFO - Iter 401/1000, Loss: 0.2553, Avg Adaptive LR: 2.846585
INFO - Iter 411/1000, Loss: 0.2548, Avg Adaptive LR: 2.846555
INFO - Iter 421/1000, Loss: 0.2542, Avg Adaptive LR: 2.846527
INFO - Iter 431/1000, Loss: 0.2537, Avg Adaptive LR: 2.846499
INFO - Iter 441/1000, Loss: 0.2532, Avg Adaptive LR: 2.846473
INFO - Iter 451/1000, Loss: 0.2527, Avg Adaptive LR: 2.846447
INFO - Iter 461/1000, Loss: 0.2522, Avg Adaptive LR: 2.846421
INFO - Iter 471/1000, Loss: 0.2517, Avg Adaptive LR: 2.846397
INFO - Iter 481/1000, Loss: 0.2512, Avg Adaptive LR: 2.846373
INFO - Iter 491/1000, Loss: 0.2508, Avg Adaptive LR: 2.846350
INFO - Iter 501/1000, Loss: 0.2504, Avg Adaptive LR: 2.846328
INFO - Iter 511/1000, Loss: 0.2499, Avg Adaptive LR: 2.846306
INFO - Iter 521/1000, Loss: 0.2495, Avg Adaptive LR: 2.846285
INFO - Iter 531/1000, Loss: 0.2491, Avg Adaptive LR: 2.846264
INFO - Iter 541/1000, Loss: 0.2487, Avg Adaptive LR: 2.846244
INFO - Iter 551/1000, Loss: 0.2483, Avg Adaptive LR: 2.846224
INFO - Iter 561/1000, Loss: 0.2480, Avg Adaptive LR: 2.846205
INFO - Iter 571/1000, Loss: 0.2476, Avg Adaptive LR: 2.846186
INFO - Iter 581/1000, Loss: 0.2472, Avg Adaptive LR: 2.846168
INFO - Iter 591/1000, Loss: 0.2469, Avg Adaptive LR: 2.846150
INFO - Iter 601/1000, Loss: 0.2466, Avg Adaptive LR: 2.846132
INFO - Iter 611/1000, Loss: 0.2462, Avg Adaptive LR: 2.846115
INFO - Iter 621/1000, Loss: 0.2459, Avg Adaptive LR: 2.846098
INFO - Iter 631/1000, Loss: 0.2456, Avg Adaptive LR: 2.846082
INFO - Iter 641/1000, Loss: 0.2453, Avg Adaptive LR: 2.846066
INFO - Iter 651/1000, Loss: 0.2450, Avg Adaptive LR: 2.846050
INFO - Iter 661/1000, Loss: 0.2447, Avg Adaptive LR: 2.846035
INFO - Iter 671/1000, Loss: 0.2444, Avg Adaptive LR: 2.846020
INFO - Iter 681/1000, Loss: 0.2441, Avg Adaptive LR: 2.846005
INFO - Iter 691/1000, Loss: 0.2438, Avg Adaptive LR: 2.845990
INFO - Iter 701/1000, Loss: 0.2435, Avg Adaptive LR: 2.845976
INFO - Iter 711/1000, Loss: 0.2433, Avg Adaptive LR: 2.845962
INFO - Iter 721/1000, Loss: 0.2430, Avg Adaptive LR: 2.845949
INFO - Iter 731/1000, Loss: 0.2427, Avg Adaptive LR: 2.845935
INFO - Iter 741/1000, Loss: 0.2425, Avg Adaptive LR: 2.845922
INFO - Iter 751/1000, Loss: 0.2422, Avg Adaptive LR: 2.845909

```

INFO - Iter 761/1000, Loss: 0.2420, Avg Adaptive LR: 2.845896
INFO - Iter 771/1000, Loss: 0.2417, Avg Adaptive LR: 2.845884
INFO - Iter 781/1000, Loss: 0.2415, Avg Adaptive LR: 2.845871
INFO - Iter 791/1000, Loss: 0.2413, Avg Adaptive LR: 2.845859
INFO - Iter 801/1000, Loss: 0.2410, Avg Adaptive LR: 2.845848
INFO - Iter 811/1000, Loss: 0.2408, Avg Adaptive LR: 2.845836
INFO - Iter 821/1000, Loss: 0.2406, Avg Adaptive LR: 2.845824
INFO - Iter 831/1000, Loss: 0.2404, Avg Adaptive LR: 2.845813
INFO - Iter 841/1000, Loss: 0.2401, Avg Adaptive LR: 2.845802
INFO - Iter 851/1000, Loss: 0.2399, Avg Adaptive LR: 2.845791
INFO - Iter 861/1000, Loss: 0.2397, Avg Adaptive LR: 2.845780
INFO - Iter 871/1000, Loss: 0.2395, Avg Adaptive LR: 2.845770
INFO - Iter 881/1000, Loss: 0.2393, Avg Adaptive LR: 2.845759
INFO - Iter 891/1000, Loss: 0.2391, Avg Adaptive LR: 2.845749
INFO - Iter 901/1000, Loss: 0.2389, Avg Adaptive LR: 2.845739
INFO - Iter 911/1000, Loss: 0.2387, Avg Adaptive LR: 2.845729
INFO - Iter 921/1000, Loss: 0.2385, Avg Adaptive LR: 2.845719
INFO - Iter 931/1000, Loss: 0.2383, Avg Adaptive LR: 2.845709
INFO - Iter 941/1000, Loss: 0.2382, Avg Adaptive LR: 2.845700
INFO - Iter 951/1000, Loss: 0.2380, Avg Adaptive LR: 2.845690
INFO - Iter 961/1000, Loss: 0.2378, Avg Adaptive LR: 2.845681
INFO - Iter 971/1000, Loss: 0.2376, Avg Adaptive LR: 2.845672
INFO - Iter 981/1000, Loss: 0.2374, Avg Adaptive LR: 2.845663
INFO - Iter 991/1000, Loss: 0.2373, Avg Adaptive LR: 2.845654
INFO - SoftmaxRegression training completed in 39.38 seconds.
INFO - --- Linear Regression LR=0.1/Iter=1000 ---
INFO - Iter 100/1000, Loss: 0.7044, Gradient Norm: 15.9647, Avg Adaptive LR:
1.3976809293777233
INFO - Iter 200/1000, Loss: 0.3742, Gradient Norm: 11.3253, Avg Adaptive LR:
0.9920285500646571
INFO - Iter 300/1000, Loss: 0.2559, Gradient Norm: 9.1024, Avg Adaptive LR:
0.8113817577426795
INFO - Iter 400/1000, Loss: 0.1938, Gradient Norm: 7.6836, Avg Adaptive LR:
0.7033840911723017
INFO - Iter 500/1000, Loss: 0.1584, Gradient Norm: 6.7434, Avg Adaptive LR:
0.6295677445844906
INFO - Iter 600/1000, Loss: 0.1345, Gradient Norm: 6.0242, Avg Adaptive LR:
0.5750117000881021
INFO - Iter 700/1000, Loss: 0.1193, Gradient Norm: 5.5215, Avg Adaptive LR:
0.5325510575431905
INFO - Iter 800/1000, Loss: 0.1073, Gradient Norm: 5.0894, Avg Adaptive LR:
0.4983286812515176
INFO - Iter 900/1000, Loss: 0.0983, Gradient Norm: 4.7382, Avg Adaptive LR:
0.46996251675315753
INFO - Iter 1000/1000, Loss: 0.0917, Gradient Norm: 4.4641, Avg Adaptive LR:
0.44592064247846064
INFO - LinearRegressionClassifier training completed in 31.66 seconds.
Train Regressions: 100%|          | 3/3 [01:21<00:00, 27.19s/it]

```

```

INFO - Training complete for Softmax and Linear.
INFO - === TRAINING PERCEPTRON MODELS (Clean & Pocket) ===
Train Clean & Pocket:  0%|          | 0/3 [00:00<?, ?it/s]INFO - --- Clean PLA,
max_iter=50 ---
INFO - Training for digit 0...
INFO - Training for digit 1...
INFO - Training for digit 2...
INFO - Training for digit 3...
INFO - Training for digit 4...
INFO - Training for digit 5...
INFO - Training for digit 6...
INFO - Training for digit 7...
INFO - Training for digit 8...
INFO - Training for digit 9...
INFO - --- Pocket PLA, max_iter=50 ---
INFO - Training for digit 0...
INFO - Training for digit 1...
INFO - Training for digit 2...
INFO - Training for digit 3...
INFO - Training for digit 4...
INFO - Training for digit 5...
INFO - Training for digit 6...
INFO - Training for digit 7...
INFO - Training for digit 8...
INFO - Training for digit 9...
Train Clean & Pocket:  33%|          | 1/3 [00:47<01:35, 47.62s/it]INFO - ---
Clean PLA, max_iter=100 ---
INFO - Training for digit 0...
INFO - Training for digit 1...
INFO - Training for digit 2...
INFO - Training for digit 3...
INFO - Training for digit 4...
INFO - Training for digit 5...
INFO - Training for digit 6...
INFO - Training for digit 7...
INFO - Training for digit 8...
INFO - Training for digit 9...
INFO - --- Pocket PLA, max_iter=100 ---
INFO - Training for digit 0...
INFO - Training for digit 1...
INFO - Training for digit 2...
INFO - Training for digit 3...
INFO - Training for digit 4...
INFO - Training for digit 5...
INFO - Training for digit 6...
INFO - Training for digit 7...
INFO - Training for digit 8...
INFO - Training for digit 9...

```

```

Train Clean & Pocket: 67%|          | 2/3 [02:11<01:08, 68.77s/it] INFO - ---
Clean PLA, max_iter=1000 ---
INFO - Training for digit 0...
INFO - Training for digit 1...
INFO - Training for digit 2...
INFO - Training for digit 3...
INFO - Training for digit 4...
INFO - Training for digit 5...
INFO - Training for digit 6...
INFO - Training for digit 7...
INFO - Training for digit 8...
INFO - Training for digit 9...
INFO - --- Pocket PLA, max_iter=1000 ---
INFO - Training for digit 0...
INFO - Training for digit 1...
INFO - Training for digit 2...
INFO - Training for digit 3...
INFO - Training for digit 4...
INFO - Training for digit 5...
INFO - Training for digit 6...
INFO - Training for digit 7...
INFO - Training for digit 8...
INFO - Training for digit 9...
Train Clean & Pocket: 100%|          | 3/3 [13:31<00:00, 270.60s/it]
INFO - Training complete for Clean PLA and Pocket PLA.
INFO - === ALL TRAINING COMPLETE ===

```

6 Evaluate

```

[7]: #####
# EVALUATION CELL (with pandas DataFrame)
#####

# 1) Evaluate Perceptrons: Clean & Pocket
accuracies_clean, accuracies_pocket = [], []
runtimes_clean, runtimes_pocket = [], []
sensitivities_clean, sensitivities_pocket = [], []
selectivities_clean, selectivities_pocket = [], []

conf_clean, conf_pocket = [], []
meta_clean, meta_pocket = [], []

for max_iter in tqdm(perceptron_max_iter_values, desc="Evaluate Clean &
↳Pocket"):
    # === Evaluate Clean PLA ===
    c_model = trained_models_clean[max_iter]

```

```

cm_c, acc_c, s_c, sp_c, rt_c, ex_c = evaluate_model(
    c_model, X_test, y_test, classes=range(10), model_name="Clean PLA"
)
accuracies_clean.append(acc_c)
runtimes_clean.append(rt_c)
sensitivities_clean.append(np.mean(s_c))
selectivities_clean.append(np.mean(sp_c))
conf_clean.append(cm_c)

cdict = {
    "max_iter": max_iter,
    "accuracy": acc_c,
    "runtime": rt_c,
    "avg_sensitivity": np.mean(s_c),
    "avg_selectivity": np.mean(sp_c),
    "method": "Clean PLA"
}
cdict.update(ex_c)
meta_clean.append(cdict)

# === Evaluate Pocket PLA ===
p_model = trained_models_pocket[max_iter]
cm_p, acc_p, s_p, sp_p, rt_p, ex_p = evaluate_model(
    p_model, X_test, y_test, classes=range(10), model_name="Pocket PLA"
)
accuracies_pocket.append(acc_p)
runtimes_pocket.append(rt_p)
sensitivities_pocket.append(np.mean(s_p))
selectivities_pocket.append(np.mean(sp_p))
conf_pocket.append(cm_p)

pdict = {
    "max_iter": max_iter,
    "accuracy": acc_p,
    "runtime": rt_p,
    "avg_sensitivity": np.mean(s_p),
    "avg_selectivity": np.mean(sp_p),
    "method": "Pocket PLA"
}
pdict.update(ex_p)
meta_pocket.append(pdict)

# Aggregated iteration-level training curves for Perceptrons
clean_train_curve = aggregate_iteration_losses(
    [trained_models_clean[m] for m in perceptron_max_iter_values]
)
pocket_train_curve = aggregate_iteration_losses(

```

```

    [trained_models_pocket[m] for m in perceptron_max_iter_values]
)

# 2) Evaluate Regression Models: Softmax & Linear
accuracies_softmax = []
runtimes_softmax = []
sensitivities_soft = []
selectivities_soft = []
conf_soft = []
meta_soft = []

accuracies_linear = []
runtimes_linear = []
sensitivities_lin = []
selectivities_lin = []
conf_linear = []
meta_linear = []

for cfg in tqdm(regression_run_configs, desc="Evaluate Regressions"):
    lr_val = cfg["learning_rate"]
    max_iter_val = cfg["max_iter"]
    label = cfg["label"]

    # === Evaluate Softmax ===
    s_model = trained_models_softmax[(lr_val, max_iter_val)]
    cm_s, a_s, se_s, sp_s, r_s, ex_s = evaluate_model(
        s_model, X_test, y_test, classes=range(10),
        model_name=f"Softmax ({label})"
    )
    accuracies_softmax.append(a_s)
    runtimes_softmax.append(r_s)
    sensitivities_soft.append(np.mean(se_s))
    selectivities_soft.append(np.mean(sp_s))
    conf_soft.append(cm_s)

    ms = {
        "label": label,
        "learning_rate": lr_val,
        "max_iter": max_iter_val,
        "accuracy": a_s,
        "runtime": r_s,
        "avg_sensitivity": np.mean(se_s),
        "avg_selectivity": np.mean(sp_s),
        "method": "Softmax"
    }
    ms.update(ex_s)
    meta_soft.append(ms)

```

```

# === Evaluate Linear ===
lin_model = trained_models_linear[(lr_val, max_iter_val)]
cm_l, a_l, se_l, sp_l, r_l, ex_l = evaluate_model(
    lin_model, X_test, y_test, classes=range(10),
    model_name=f"Linear ({label})"
)
accuracies_linear.append(a_l)
runtimes_linear.append(r_l)
sensitivities_lin.append(np.mean(se_l))
selectivities_lin.append(np.mean(sp_l))
conf_linear.append(cm_l)

ml = {
    "label": label,
    "learning_rate": lr_val,
    "max_iter": max_iter_val,
    "accuracy": a_l,
    "runtime": r_l,
    "avg_sensitivity": np.mean(se_l),
    "avg_selectivity": np.mean(sp_l),
    "method": "Linear Regression"
}
ml.update(ex_l)
meta_linear.append(ml)

logger.info("Evaluation complete for Perceptrons & Regressions.")

```

Evaluate Clean & Pocket: 0% | 0/3 [00:00<?, ?it/s] INFO - Built-in

Confusion Matrix:

```

[[ 959   0   3   6   1   0   7   1   3   0]
 [   0 1077  18  23   1   0   4   2  10   0]
 [   9   7  912  26  16   0  20  20  19   3]
 [   6   0  20  955   1   4   4  13   6   1]
 [   1   0   8   7  933   0  14   2   4  13]
 [  33   6  15 169  42 526  27  18  45  11]
 [  19   3   9   7  11   3 905   0   1   0]
 [   6  13  44  13   9   0   2 936   0   5]
 [  18   7  39 137  18   8  18  23 704   2]
 [  13   8  23  47 174   5   0  98   3 638]]

```

INFO - Overall Accuracy: 85.45%

INFO - Class '0': TPR=0.98, TNR=0.99

INFO - Class '1': TPR=0.95, TNR=1.00

INFO - Class '2': TPR=0.88, TNR=0.98

INFO - Built-in Confusion Matrix:

```

[[ 959   0   3   6   1   0   7   1   3   0]
 [   0 1077  18  23   1   0   4   2  10   0]

```

```

[ 9 7 912 26 16 0 20 20 19 3]
[ 6 0 20 955 1 4 4 13 6 1]
[ 1 0 8 7 933 0 14 2 4 13]
[ 33 6 15 169 42 526 27 18 45 11]
[ 19 3 9 7 11 3 905 0 1 0]
[ 6 13 44 13 9 0 2 936 0 5]
[ 18 7 39 137 18 8 18 23 704 2]
[ 13 8 23 47 174 5 0 98 3 638]]
INFO - Overall Accuracy: 85.45%
INFO - Class '0': TPR=0.98, TNR=0.99
INFO - Class '1': TPR=0.95, TNR=1.00
INFO - Class '2': TPR=0.88, TNR=0.98
INFO - Class '3': TPR=0.95, TNR=0.95
INFO - Class '4': TPR=0.95, TNR=0.97
INFO - Class '5': TPR=0.59, TNR=1.00
INFO - Class '6': TPR=0.94, TNR=0.99
INFO - Class '7': TPR=0.91, TNR=0.98
INFO - Class '8': TPR=0.72, TNR=0.99
INFO - Class '9': TPR=0.63, TNR=1.00
Evaluating class metrics: 100%| 10/10 [00:00<00:00, 2890.43it/s]
INFO - Built-in Confusion Matrix:
[[ 956 0 3 3 1 0 6 1 10 0]
 [ 0 1075 14 18 1 2 4 1 20 0]
 [ 9 5 901 21 15 0 18 18 42 3]
 [ 6 0 20 928 1 17 4 12 21 1]
 [ 1 0 8 6 924 1 13 2 13 14]
 [ 20 4 9 88 20 653 20 10 64 4]
 [ 16 3 7 5 11 12 892 0 12 0]
 [ 5 12 44 11 8 0 2 934 5 7]
 [ 11 3 19 58 7 16 9 14 837 0]
 [ 12 7 21 31 149 17 0 83 36 653]]
INFO - Overall Accuracy: 87.53%
INFO - Class '0': TPR=0.98, TNR=0.99
INFO - Class '1': TPR=0.95, TNR=1.00
INFO - Class '2': TPR=0.87, TNR=0.98
INFO - Class '3': TPR=0.92, TNR=0.97
INFO - Class '4': TPR=0.94, TNR=0.98
INFO - Class '5': TPR=0.73, TNR=0.99
INFO - Class '6': TPR=0.93, TNR=0.99
INFO - Class '7': TPR=0.91, TNR=0.98
INFO - Class '8': TPR=0.86, TNR=0.98
INFO - Class '9': TPR=0.65, TNR=1.00
Evaluating class metrics: 100%| 10/10 [00:00<00:00, 2707.57it/s]
INFO - Built-in Confusion Matrix:
[[ 964 0 3 2 1 1 6 2 1 0]
 [ 0 1107 10 6 0 2 5 2 3 0]
 [ 18 12 914 9 13 1 23 20 15 7]
 [ 12 1 26 910 2 20 6 18 3 12]

```



```

[  2   1   5   0 930   0  11   3   2  28]
[ 25   6  13  44  33 703  30  18   8  12]
[ 12   3   5   2  10   6 920   0   0   0]
[  5   8  29   5   7   0   2 951   0  21]
[ 30  14 105  81  59  44  29  32 542  38]
[ 12   7  10  17  93   5   1  60   0 804]]
INFO - Overall Accuracy: 87.45%
INFO - Class '0': TPR=0.98, TNR=0.99
INFO - Class '1': TPR=0.98, TNR=0.99
INFO - Class '2': TPR=0.89, TNR=0.98
INFO - Class '3': TPR=0.90, TNR=0.98
INFO - Class '4': TPR=0.95, TNR=0.98
INFO - Class '5': TPR=0.79, TNR=0.99
INFO - Class '6': TPR=0.96, TNR=0.99
INFO - Class '7': TPR=0.93, TNR=0.98
INFO - Class '8': TPR=0.56, TNR=1.00
INFO - Class '9': TPR=0.80, TNR=0.99
Evaluating class metrics: 100%|          | 10/10 [00:00<00:00, 3527.00it/s]
INFO - Built-in Confusion Matrix:
[[ 963   0   3   3   1   0   5   2   3   0]
 [  0 1097   9   6   0   1   4   1  17   0]
 [  8   3  906  20  12   0  16  17  43   7]
 [  6   0  21  921   1  18   4  13  19   7]
 [  2   0   8   2  916   1   9   2  11  31]
 [ 21   4  10  65  24  664  22  14  58  10]
 [ 12   3   9   3  10   7  909   0   5   0]
 [  5   7  32   9   6   0   2  943   2  22]
 [ 13   3  24  51  14  12  14  17  821   5]
 [ 10   7  11  20  70  10   0  46  11  824]]
INFO - Overall Accuracy: 89.64%
INFO - Class '0': TPR=0.98, TNR=0.99
INFO - Class '1': TPR=0.97, TNR=1.00
INFO - Class '2': TPR=0.88, TNR=0.99
INFO - Class '3': TPR=0.91, TNR=0.98
INFO - Class '4': TPR=0.93, TNR=0.98
INFO - Class '5': TPR=0.74, TNR=0.99
INFO - Class '6': TPR=0.95, TNR=0.99
INFO - Class '7': TPR=0.92, TNR=0.99
INFO - Class '8': TPR=0.84, TNR=0.98
INFO - Class '9': TPR=0.82, TNR=0.99
Evaluating class metrics: 100%|          | 10/10 [00:00<00:00, 2939.45it/s]
INFO - Built-in Confusion Matrix:
[[ 949   0   7   3   0   5  12   3   1   0]
 [  0 1100  24   2   0   2   4   2   1   0]
 [  9   2  968  10  10   2  13   8   7   3]
 [  4   2  46  915   1  24   3  12   3   0]
 [  2   3  14   4  934   0   8   6   3   8]
 [ 15   4  22  37  12  773  13   6   9   1]

```

```

[ 12  3  23  1  4  19 894  2  0  0]
[  5  5  47  6  9  2  1 949  0  4]
[ 43 29 148 50 38 74  7 34 551  0]
[ 24 13 18 21 132 20 0 181  0 600]]
INFO - Overall Accuracy: 86.33%
INFO - Class '0': TPR=0.97, TNR=0.99
INFO - Class '1': TPR=0.97, TNR=0.99
INFO - Class '2': TPR=0.94, TNR=0.96
INFO - Class '3': TPR=0.91, TNR=0.99
INFO - Class '4': TPR=0.95, TNR=0.98
INFO - Class '5': TPR=0.87, TNR=0.98
INFO - Class '6': TPR=0.93, TNR=0.99
INFO - Class '7': TPR=0.92, TNR=0.97
INFO - Class '8': TPR=0.57, TNR=1.00
INFO - Class '9': TPR=0.59, TNR=1.00
Evaluating class metrics: 100%|          | 10/10 [00:00<00:00, 3530.26it/s]
INFO - Built-in Confusion Matrix:
[[ 961  0  0  2  0  4  7  3  3  0]
 [  0 1110  3  2  0  3  5  2 10  0]
 [ 12  5 926 18  7  6 17 14 22  5]
 [  6  2 21 914  2 29  6 11 12  7]
 [  3  2  7  4 908  2  9  6  5 36]
 [ 14  3  5 33  9 776 19  4 23  6]
 [ 13  3  6  1  9 18 906  1  1  0]
 [  5  8 24  8  7  3  1 943  1 28]
 [ 22 21 14 35 30 49 17 26 754  6]
 [ 19 10  1 14 48 14  0 54  2 847]]
INFO - Overall Accuracy: 90.45%
INFO - Class '0': TPR=0.98, TNR=0.99
INFO - Class '1': TPR=0.98, TNR=0.99
INFO - Class '2': TPR=0.90, TNR=0.99
INFO - Class '3': TPR=0.90, TNR=0.99
INFO - Class '4': TPR=0.92, TNR=0.99
INFO - Class '5': TPR=0.87, TNR=0.99
INFO - Class '6': TPR=0.95, TNR=0.99
INFO - Class '7': TPR=0.92, TNR=0.99
INFO - Class '8': TPR=0.77, TNR=0.99
INFO - Class '9': TPR=0.84, TNR=0.99
Evaluating class metrics: 100%|          | 10/10 [00:00<00:00, 2048.10it/s]
Evaluate Clean & Pocket: 100%|          | 3/3 [00:00<00:00, 53.50it/s]
Aggregating train losses across Perceptron models: 100%|          | 3/3
[00:00<00:00, 873.93it/s]
Aggregating train losses across Perceptron models: 100%|          | 3/3
[00:00<00:00, 1116.30it/s]
Evaluate Regressions: 0%|          | 0/3 [00:00<?, ?it/s]
INFO - Built-in
Confusion Matrix:
[[ 954  0  2  3  0  4  8  2  7  0]
 [  0 1109  2  3  1  2  4  0 14  0]

```

```

[ 14  11 904  17  11  2  12  16  43  2]
[  4  2  18 908  1  30  4  11  23  9]
[  1  0  5  1 913  0  14  3  7  38]
[ 13  4  4  35  11 751  16  8  40  10]
[ 16  3  10  1  8  13 899  2  6  0]
[  2  12 23  7  7  0  1 928  3  45]
[  6  7  8  27  10 24  10 15 854 13]
[ 13  5  3  14 36  8  0 25  5 900]]
INFO - Overall Accuracy: 91.20%
INFO - Class '0': TPR=0.97, TNR=0.99
INFO - Class '1': TPR=0.98, TNR=1.00
INFO - Class '2': TPR=0.88, TNR=0.99
INFO - Class '3': TPR=0.90, TNR=0.99
INFO - Class '4': TPR=0.93, TNR=0.99
INFO - Class '5': TPR=0.84, TNR=0.99
INFO - Class '6': TPR=0.94, TNR=0.99
INFO - Class '7': TPR=0.90, TNR=0.99
INFO - Class '8': TPR=0.88, TNR=0.98
INFO - Class '9': TPR=0.89, TNR=0.99
Evaluating class metrics: 100%|          | 10/10 [00:00<00:00, 3234.35it/s]
INFO - Built-in Confusion Matrix:
[[ 973  0  0  0  0  0  5  0  2  0]
 [  2 1101  0  2  0  0  4  0  26  0]
 [ 479 183 85 38  0  0 116  1 130  0]
 [ 194  40  1 704  0  1  18  2  49  1]
 [ 433 126  0 35 18  9  96  0 261  4]
 [ 386  25  0 63  0 252 31  0 135  0]
 [ 137  14  0  0  0  2 802  0  3  0]
 [ 412 137  0 25  0  0  22 375 56  1]
 [ 125  77  0  9  0  0  21  0 742  0]
 [ 586  88  0 25  0  8  30  0 224 48]]
INFO - Overall Accuracy: 51.00%
INFO - Class '0': TPR=0.99, TNR=0.69
INFO - Class '1': TPR=0.97, TNR=0.92
INFO - Class '2': TPR=0.08, TNR=1.00
INFO - Class '3': TPR=0.70, TNR=0.98
INFO - Class '4': TPR=0.02, TNR=1.00
INFO - Class '5': TPR=0.28, TNR=1.00
INFO - Class '6': TPR=0.84, TNR=0.96
INFO - Class '7': TPR=0.36, TNR=1.00
INFO - Class '8': TPR=0.76, TNR=0.90
INFO - Class '9': TPR=0.05, TNR=1.00
Evaluating class metrics: 100%|          | 10/10 [00:00<00:00, 3968.87it/s]
INFO - Built-in Confusion Matrix:
[[ 958  0  0  2  0  3  8  2  6  1]
 [  0 1112  2  3  1  2  4  0 11  0]
 [  9  9 912 18 13  1 11 13 43  3]
 [  3  1 17 915  0 26  2 12 24 10]

```

```

[ 1 1 7 0 921 1 11 2 8 30]
[ 7 2 3 31 11 764 20 8 36 10]
[ 13 3 9 0 7 10 911 2 3 0]
[ 1 10 23 6 8 0 0 940 3 37]
[ 8 5 8 24 12 25 8 15 856 13]
[ 11 5 1 13 36 9 0 23 7 904]]
INFO - Overall Accuracy: 91.93%
INFO - Class '0': TPR=0.98, TNR=0.99
INFO - Class '1': TPR=0.98, TNR=1.00
INFO - Class '2': TPR=0.88, TNR=0.99
INFO - Class '3': TPR=0.91, TNR=0.99
INFO - Class '4': TPR=0.94, TNR=0.99
INFO - Class '5': TPR=0.86, TNR=0.99
INFO - Class '6': TPR=0.95, TNR=0.99
INFO - Class '7': TPR=0.91, TNR=0.99
INFO - Class '8': TPR=0.88, TNR=0.98
INFO - Class '9': TPR=0.90, TNR=0.99
Evaluating class metrics: 100%| 10/10 [00:00<00:00, 2751.27it/s]
INFO - Built-in Confusion Matrix:
[[ 2 251 7 0 9 1 438 0 272 0]
 [ 0 1117 0 0 0 0 5 0 13 0]
 [ 0 229 603 0 17 0 80 0 103 0]
 [ 0 608 21 0 3 0 113 0 265 0]
 [ 0 80 1 0 816 0 43 0 42 0]
 [ 0 171 2 0 30 168 145 0 376 0]
 [ 0 30 0 0 4 0 920 0 4 0]
 [ 0 542 29 0 82 0 112 60 203 0]
 [ 0 125 1 0 5 0 30 0 813 0]
 [ 0 291 3 0 363 0 90 0 262 0]]
INFO - Overall Accuracy: 44.99%
INFO - Class '0': TPR=0.00, TNR=1.00
INFO - Class '1': TPR=0.98, TNR=0.74
INFO - Class '2': TPR=0.58, TNR=0.99
INFO - Class '3': TPR=0.00, TNR=1.00
INFO - Class '4': TPR=0.83, TNR=0.94
INFO - Class '5': TPR=0.19, TNR=1.00
INFO - Class '6': TPR=0.96, TNR=0.88
INFO - Class '7': TPR=0.06, TNR=1.00
INFO - Class '8': TPR=0.83, TNR=0.83
INFO - Class '9': TPR=0.00, TNR=1.00
Evaluating class metrics: 100%| 10/10 [00:00<00:00, 3937.20it/s]
INFO - Built-in Confusion Matrix:
[[ 962 0 0 2 0 5 6 3 2 0]
 [ 0 1113 4 3 0 1 3 2 9 0]
 [ 7 9 924 20 7 3 11 9 39 3]
 [ 2 0 18 924 1 24 3 10 23 5]
 [ 1 1 4 1 916 0 13 4 10 32]
 [ 8 4 2 34 8 776 13 11 30 6]

```

```

[ 11  3  8  1  7 15 909  2  2  0]
[  1  6 24  5  6  0  0 953  4 29]
[  6  9  5 21  9 20  7 13 877  7]
[ 10  7  1  9 26  7  0 20  5 924]]
INFO - Overall Accuracy: 92.78%
INFO - Class '0': TPR=0.98, TNR=0.99
INFO - Class '1': TPR=0.98, TNR=1.00
INFO - Class '2': TPR=0.90, TNR=0.99
INFO - Class '3': TPR=0.91, TNR=0.99
INFO - Class '4': TPR=0.93, TNR=0.99
INFO - Class '5': TPR=0.87, TNR=0.99
INFO - Class '6': TPR=0.95, TNR=0.99
INFO - Class '7': TPR=0.93, TNR=0.99
INFO - Class '8': TPR=0.90, TNR=0.99
INFO - Class '9': TPR=0.92, TNR=0.99
Evaluating class metrics: 100%|      | 10/10 [00:00<00:00, 3127.04it/s]
INFO - Built-in Confusion Matrix:
[[ 958  0  1  0  0  4  7  2  8  0]
 [  0 1036  8  1  1  6  5  5 73  0]
 [ 30  6 865  7  4  0 21 44 53  2]
 [ 20  2 58 759  0 18 10 92 48  3]
 [ 10 11 18  0 769  5 24 49 45 51]
 [ 41  5 10 37  1 620 25 56 92  5]
 [ 37  3 15  0  1 14 874  5  9  0]
 [  3 18 19  4  3  0  2 964  5 10]
 [ 28  9 15  7  4 12 17 43 839  0]
 [ 38  5  8  5 11  4  2 261 37 638]]
INFO - Overall Accuracy: 83.22%
INFO - Class '0': TPR=0.98, TNR=0.98
INFO - Class '1': TPR=0.91, TNR=0.99
INFO - Class '2': TPR=0.84, TNR=0.98
INFO - Class '3': TPR=0.75, TNR=0.99
INFO - Class '4': TPR=0.78, TNR=1.00
INFO - Class '5': TPR=0.70, TNR=0.99
INFO - Class '6': TPR=0.91, TNR=0.99
INFO - Class '7': TPR=0.94, TNR=0.94
INFO - Class '8': TPR=0.86, TNR=0.96
INFO - Class '9': TPR=0.63, TNR=0.99
Evaluating class metrics: 100%|      | 10/10 [00:00<00:00, 3330.13it/s]
Evaluate Regressions: 100%|      | 3/3 [00:00<00:00, 53.73it/s]
INFO - Evaluation complete for Perceptrons & Regressions.

```

7 Visualize (Generate Plots, Confusion Matrices, etc.)

```
[8]: #####
# 1) CREATE A SINGLE PANDAS DATAFRAME FOR ALL RESULTS
#####
all_rows = []

# (A) Clean PLA
for i, max_iter in tqdm(
    enumerate(perceptron_max_iter_values),
    desc="Collecting Clean PLA",
    total=len(perceptron_max_iter_values)
):
    all_rows.append({
        'model': 'Clean PLA',
        'max_iter': max_iter,
        'runtime': runtimes_clean[i],
        'accuracy': accuracies_clean[i],
        'sensitivity': sensitivities_clean[i],
        'selectivity': selectivities_clean[i]
    })

# (B) Pocket PLA
for i, max_iter in tqdm(
    enumerate(perceptron_max_iter_values),
    desc="Collecting Pocket PLA",
    total=len(perceptron_max_iter_values)
):
    all_rows.append({
        'model': 'Pocket PLA',
        'max_iter': max_iter,
        'runtime': runtimes_pocket[i],
        'accuracy': accuracies_pocket[i],
        'sensitivity': sensitivities_pocket[i],
        'selectivity': selectivities_pocket[i]
    })

# (C) Softmax
for i, row_meta in tqdm(
    enumerate(meta_soft),
    desc="Collecting Softmax",
    total=len(meta_soft)
):
    all_rows.append({
        'model': 'Softmax',
        'max_iter': row_meta['max_iter'],
        'runtime': runtimes_softmax[i],
```

```

        'accuracy': accuracies_softmax[i],
        'sensitivity': sensitivities_soft[i],
        'selectivity': selectivities_soft[i]
    })

# (D) Linear
for i, row_meta in tqdm(
    enumerate(meta_linear),
    desc="Collecting Linear",
    total=len(meta_linear)
):
    all_rows.append({
        'model': 'Linear',
        'max_iter': row_meta['max_iter'],
        'runtime': runtimes_linear[i],
        'accuracy': accuracies_linear[i],
        'sensitivity': sensitivities_lin[i],
        'selectivity': selectivities_lin[i]
    })

df_results = pd.DataFrame(all_rows)
logger.info("Combined Results DataFrame:\n%s", df_results)
display(df_results.head(20))

#####
# 2) CONFUSION MATRICES FOR ALL MODELS (GROUPED BY PLOT TYPE)
#####

logger.info("=== Plotting ALL Confusion Matrices ===")

# 2A) Perceptron: Clean
for idx, meta in tqdm(enumerate(meta_clean), total=len(meta_clean),
    desc="Confusions: Clean PLA"):
    title = f"Clean PLA (max_iter={meta['max_iter']}), Acc={meta['accuracy']*100:
    .2f}%"
    plot_confusion_matrix_annotated(
        conf_clean[idx],
        classes=range(10),
        title=title,
        method=meta["method"],
        max_iter=meta["max_iter"]
    )

# 2B) Perceptron: Pocket
for idx, meta in tqdm(enumerate(meta_pocket), total=len(meta_pocket),
    desc="Confusions: Pocket PLA"):

```

```

        title = f"Pocket PLA (max_iter={meta['max_iter']}),  

↳ Acc={meta['accuracy']*100:.2f}%"
        plot_confusion_matrix_annotated(
            conf_pocket[idx],
            classes=range(10),
            title=title,
            method=meta["method"],
            max_iter=meta["max_iter"]
        )

# 2C) Softmax
for idx, meta in tqdm(enumerate(meta_soft), total=len(meta_soft),
↳ desc="Confusions: Softmax"):
    title = f"Softmax ({meta['label']}, Acc={meta['accuracy']*100:.2f}%"
    plot_confusion_matrix_annotated(
        conf_soft[idx],
        classes=range(10),
        title=title,
        method=meta["method"],
        max_iter=meta["max_iter"]
    )

# 2D) Linear
for idx, meta in tqdm(enumerate(meta_linear), total=len(meta_linear),
↳ desc="Confusions: Linear"):
    title = f"Linear ({meta['label']}, Acc={meta['accuracy']*100:.2f}%"
    plot_confusion_matrix_annotated(
        conf_linear[idx],
        classes=range(10),
        title=title,
        method=meta["method"],
        max_iter=meta["max_iter"]
    )

#####
# 3) ITERATION-LEVEL PLOTS (ALL MODELS)
#####

logger.info("=== Iteration-Level Visualization (All Models) ===")

# 3A) Perceptron: Clean & Pocket
for max_iter, c_model in trained_models_clean.items():
    df_iter = c_model.get_iteration_df()
    if not df_iter.empty and "train_error" in df_iter.columns:
        title = f"Clean PLA max_iter={max_iter}: Train Error vs. Iteration"

```



```

        df_iter.plot(x="iteration", y="train_error", marker='o', figsize=(8,5),
↳title=title)
        plt.grid(True, linestyle='--', alpha=0.7)
        plt.show()

for max_iter, p_model in trained_models_pocket.items():
    df_iter = p_model.get_iteration_df()
    if not df_iter.empty and "train_error" in df_iter.columns:
        title = f"Pocket PLA max_iter={max_iter}: Train Error vs. Iteration"
        df_iter.plot(x="iteration", y="train_error", marker='o', figsize=(8,5),
↳title=title)
        plt.grid(True, linestyle='--', alpha=0.7)
        plt.show()

# 3B) Softmax
for (lr_val, max_iter_val), s_model in trained_models_softmax.items():
    df_iter = s_model.get_iteration_df() # Must be implemented in your
↳SoftmaxRegression
    if not df_iter.empty:
        title = f"Softmax LR={lr_val}, max_iter={max_iter_val}: Train Loss vs.
↳Iteration"
        df_iter.plot(x="iteration", y="train_loss", marker='o', figsize=(8,5),
↳title=title)
        plt.grid(True, linestyle='--', alpha=0.7)
        plt.show()

        if "test_loss" in df_iter.columns:
            title = f"Softmax LR={lr_val}, max_iter={max_iter_val}: Train &
↳Test Loss"
            df_iter.plot(x="iteration", y=["train_loss", "test_loss"],
↳marker='o', figsize=(8,5), title=title)
            plt.grid(True, linestyle='--', alpha=0.7)
            plt.show()

            if "avg_adaptive_lr" in df_iter.columns:
                title = f"Softmax LR={lr_val}, max_iter={max_iter_val}: Avg
↳Adaptive LR vs. Iteration"
                df_iter.plot(x="iteration", y="avg_adaptive_lr", marker='x',
↳figsize=(8,5), title=title)
                plt.grid(True, linestyle='--', alpha=0.7)
                plt.show()

# 3C) Linear
for (lr_val, max_iter_val), lin_model in trained_models_linear.items():
    df_iter = lin_model.get_iteration_df() # Must be implemented in your
↳LinearRegression

```

```

    if not df_iter.empty:
        title = f"Linear LR={lr_val}, max_iter={max_iter_val}: Train Loss vs. Iteration"
        df_iter.plot(x="iteration", y="train_loss", marker='o', figsize=(8,5), title=title)
        plt.grid(True, linestyle='--', alpha=0.7)
        plt.show()

    if "test_loss" in df_iter.columns:
        title = f"Linear LR={lr_val}, max_iter={max_iter_val}: Train & Test Loss"
        df_iter.plot(x="iteration", y=["train_loss", "test_loss"], marker='o', figsize=(8,5), title=title)
        plt.grid(True, linestyle='--', alpha=0.7)
        plt.show()

    if "avg_adaptive_lr" in df_iter.columns:
        title = f"Linear LR={lr_val}, max_iter={max_iter_val}: Avg Adaptive LR vs. Iteration"
        df_iter.plot(x="iteration", y="avg_adaptive_lr", marker='x', figsize=(8,5), title=title)
        plt.grid(True, linestyle='--', alpha=0.7)
        plt.show()

#####
# 4) PANDAS + SEABORN PLOTS
#####

logger.info("=== Pandas + Seaborn Plots ===")

# 4A) LINE PLOT: Accuracy vs. max_iter (Perceptrons Only)
df_perc = df_results[df_results['model'].isin(['Clean PLA', 'Pocket PLA'])].copy()
df_perc.sort_values(['model', 'max_iter'], inplace=True)

plt.figure(figsize=(6,4))
sns.lineplot(
    data=df_perc,
    x='max_iter', y='accuracy',
    hue='model', marker='o'
)
plt.title("Perceptrons: Accuracy vs. max_iter (Pandas/Seaborn)")
plt.grid(True, linestyle='--', alpha=0.7)
plt.show()

```

```

# 4B) BAR CHART: Average Accuracy by Model
df_mean = df_results.groupby('model', as_index=False)['accuracy'].mean()

plt.figure(figsize=(6,4))
sns.barplot(data=df_mean, x='model', y='accuracy')
plt.title("Average Accuracy by Model (Pandas/Seaborn)")
plt.ylim(0.7, 1.0)
plt.grid(True, axis='y', linestyle='--', alpha=0.7)
plt.show()

# 4C) SCATTER PLOT: Accuracy vs. Runtime, colored by model
plt.figure(figsize=(6,4))
sns.scatterplot(
    data=df_results,
    x='runtime', y='accuracy',
    hue='model', style='model',
    s=100
)
plt.title("Accuracy vs. Runtime (All Models) (Pandas/Seaborn)")
plt.grid(True, linestyle='--', alpha=0.7)
plt.show()

#####
# 5) CUSTOM SUMMARY PLOTS (AGGREGATED CURVES, ETC.)
#####

logger.info("== Custom Summaries (Aggregated Curves, etc.) ==")

# 5A) Aggregated Perceptron Curves
plot_train_curves_three_models(
    clean_train_curve=clean_train_curve,
    pocket_train_curve=pocket_train_curve,
    softmax_train_curve=None, # no Softmax aggregator
    title="Aggregated Perceptron Train Curves (Clean vs. Pocket)",
    max_iter=perceptron_max_iter_values[-1]
)

# 5B) Summaries for Perceptron
plot_accuracy_vs_max_iter(
    max_iter_values=perceptron_max_iter_values,
    accuracies_clean=accuracies_clean,
    accuracies_pocket=accuracies_pocket,
    accuracies_softmax=None
)

plot_runtime_vs_max_iter(

```

```

max_iter_values=perceptron_max_iter_values,
runtimes_clean=runtimes_clean,
runtimes_pocket=runtimes_pocket,
runtimes_softmax=None
)

plot_accuracy_vs_runtime(
    runtimes_clean=runtimes_clean,
    accuracies_clean=accuracies_clean,
    runtimes_pocket=runtimes_pocket,
    accuracies_pocket=accuracies_pocket,
    title="Perceptrons: Accuracy vs. Runtime"
)

plot_performance_summary_extended_by_runtime(
    runtimes_clean=runtimes_clean,
    accuracies_clean=accuracies_clean,
    sensitivities_clean=sensitivities_clean,
    selectivities_clean=selectivities_clean,
    runtimes_pocket=runtimes_pocket,
    accuracies_pocket=accuracies_pocket,
    sensitivities_pocket=sensitivities_pocket,
    selectivities_pocket=selectivities_pocket,
    title="Perceptrons: Performance vs. Runtime"
)

# 5C) Summaries for Softmax & Linear
plot_accuracy_vs_runtime(
    runtimes_clean=runtimes_softmax,
    accuracies_clean=accuracies_softmax,
    title="Softmax: Accuracy vs. Runtime"
)
plot_accuracy_vs_runtime(
    runtimes_clean=runtimes_linear,
    accuracies_clean=accuracies_linear,
    title="Linear: Accuracy vs. Runtime"
)
plot_accuracy_vs_runtime(
    runtimes_clean=runtimes_softmax,
    accuracies_clean=accuracies_softmax,
    runtimes_pocket=runtimes_linear,
    accuracies_pocket=accuracies_linear,
    title="Softmax vs. Linear: Accuracy vs. Runtime"
)
plot_performance_summary_extended_by_runtime(
    runtimes_clean=runtimes_softmax,
    accuracies_clean=accuracies_softmax,

```

```

sensitivities_clean=sensitivities_soft,
selectivities_clean=selectivities_soft,
runtimes_pocket=runtimes_linear,
accuracies_pocket=accuracies_linear,
sensitivities_pocket=sensitivities_lin,
selectivities_pocket=selectivities_lin,
title="Softmax vs. Linear: TPR/TNR vs. Runtime"
)

# 5D) 4-Model Comparison
plot_performance_summary_4models_by_runtime(
    runtimes_clean, accuracies_clean, sensitivities_clean, selectivities_clean,
    runtimes_pocket, accuracies_pocket, sensitivities_pocket,↵
    ↪selectivities_pocket,
    runtimes_softmax, accuracies_softmax, sensitivities_softmax,↵
    ↪selectivities_softmax,
    runtimes_linear, accuracies_linear, sensitivities_linear, selectivities_linear,
    title="Performance vs. Runtime (4-Model Comparison)"
)

plot_accuracy_vs_runtime_4models(
    rt_clean=runtimes_clean,
    acc_clean=accuracies_clean,
    rt_pocket=runtimes_pocket,
    acc_pocket=accuracies_pocket,
    rt_softmax=runtimes_softmax,
    acc_softmax=accuracies_softmax,
    rt_linear=runtimes_linear,
    acc_linear=accuracies_linear,
    title="Accuracy vs. Runtime (4 Models)"
)

logger.info("=== All Visualizations Complete ===")

```

```

Collecting Clean PLA: 100%|      | 3/3 [00:00<00:00, 21509.25it/s]
Collecting Pocket PLA: 100%|     | 3/3 [00:00<00:00, 71089.90it/s]
Collecting Softmax: 100%|       | 3/3 [00:00<00:00, 45425.68it/s]
Collecting Linear: 100%|        | 3/3 [00:00<00:00, 32263.88it/s]
INFO - Combined Results DataFrame:

```

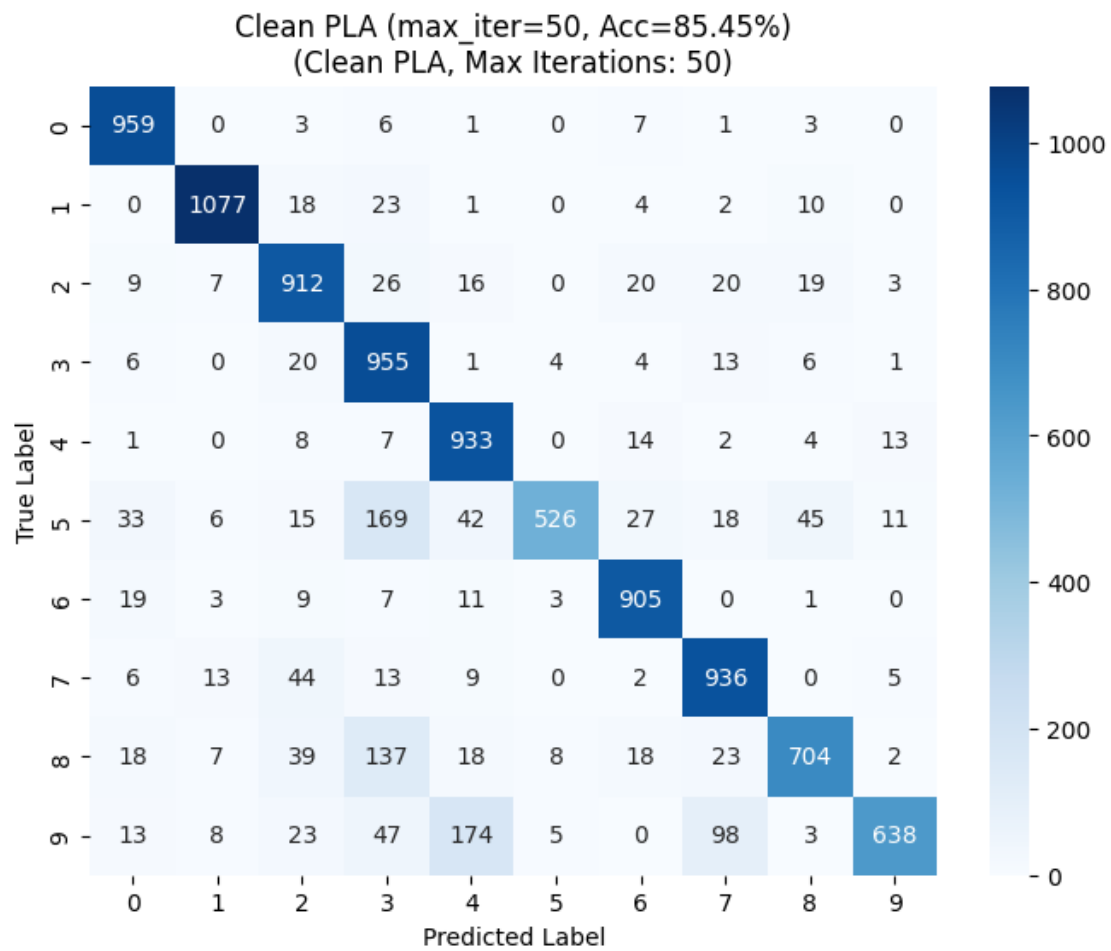
	model	max_iter	runtime	accuracy	sensitivity	selectivity
0	Clean PLA	50	23.716434	0.8545	0.850681	0.983826
1	Clean PLA	100	41.754540	0.8745	0.871954	0.986055
2	Clean PLA	1000	340.150336	0.8633	0.861587	0.984807
3	Pocket PLA	50	23.907245	0.8753	0.873371	0.986151
4	Pocket PLA	100	41.822874	0.8964	0.894188	0.988493
5	Pocket PLA	1000	340.440259	0.9045	0.903203	0.989398
6	Softmax	50	2.004833	0.9120	0.910711	0.990229

7	Softmax	100	3.922128	0.9193	0.918146	0.991041
8	Softmax	1000	39.382281	0.9278	0.926726	0.991983
9	Linear	50	1.531054	0.5100	0.505446	0.945553
10	Linear	100	3.053406	0.4499	0.444318	0.938597
11	Linear	1000	31.663074	0.8322	0.830192	0.981350

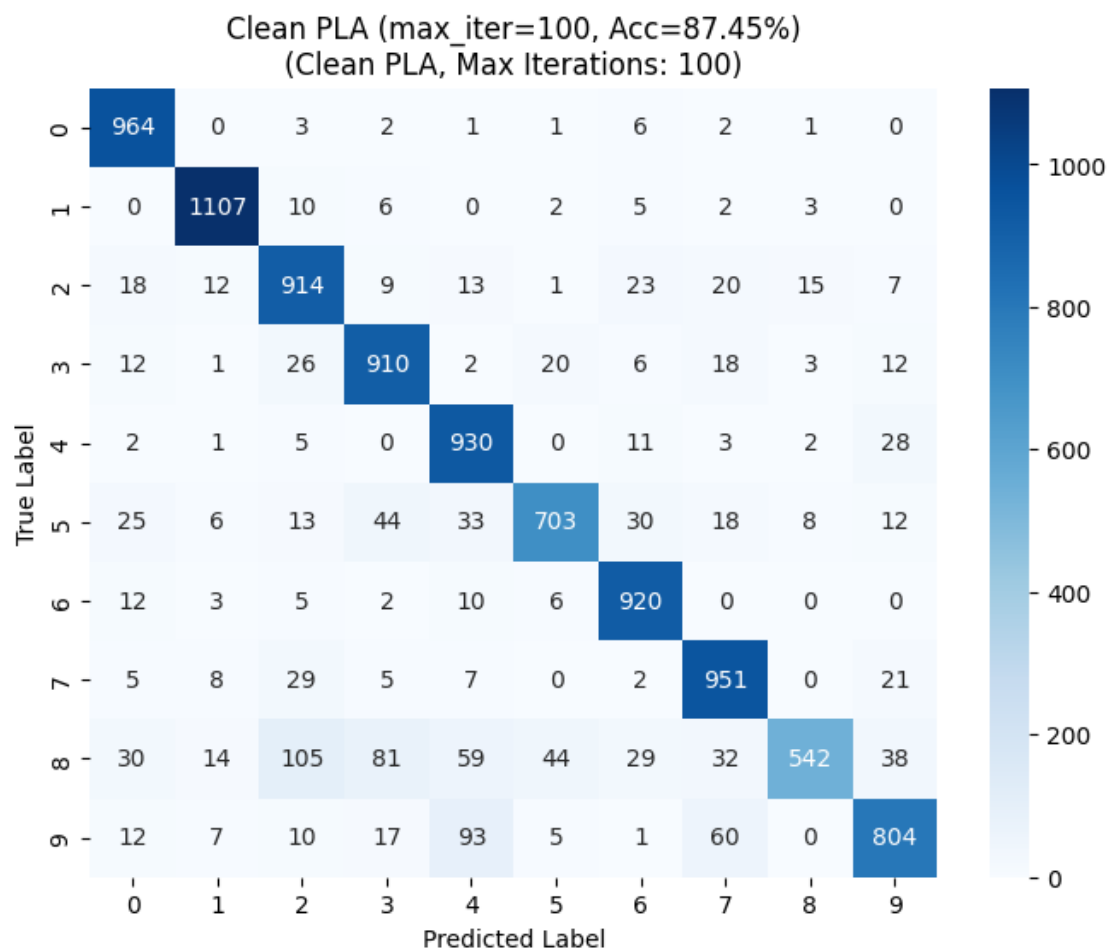
	model	max_iter	runtime	accuracy	sensitivity	selectivity
0	Clean PLA	50	23.716434	0.8545	0.850681	0.983826
1	Clean PLA	100	41.754540	0.8745	0.871954	0.986055
2	Clean PLA	1000	340.150336	0.8633	0.861587	0.984807
3	Pocket PLA	50	23.907245	0.8753	0.873371	0.986151
4	Pocket PLA	100	41.822874	0.8964	0.894188	0.988493
5	Pocket PLA	1000	340.440259	0.9045	0.903203	0.989398
6	Softmax	50	2.004833	0.9120	0.910711	0.990229
7	Softmax	100	3.922128	0.9193	0.918146	0.991041
8	Softmax	1000	39.382281	0.9278	0.926726	0.991983
9	Linear	50	1.531054	0.5100	0.505446	0.945553
10	Linear	100	3.053406	0.4499	0.444318	0.938597
11	Linear	1000	31.663074	0.8322	0.830192	0.981350

INFO - === Plotting ALL Confusion Matrices ===

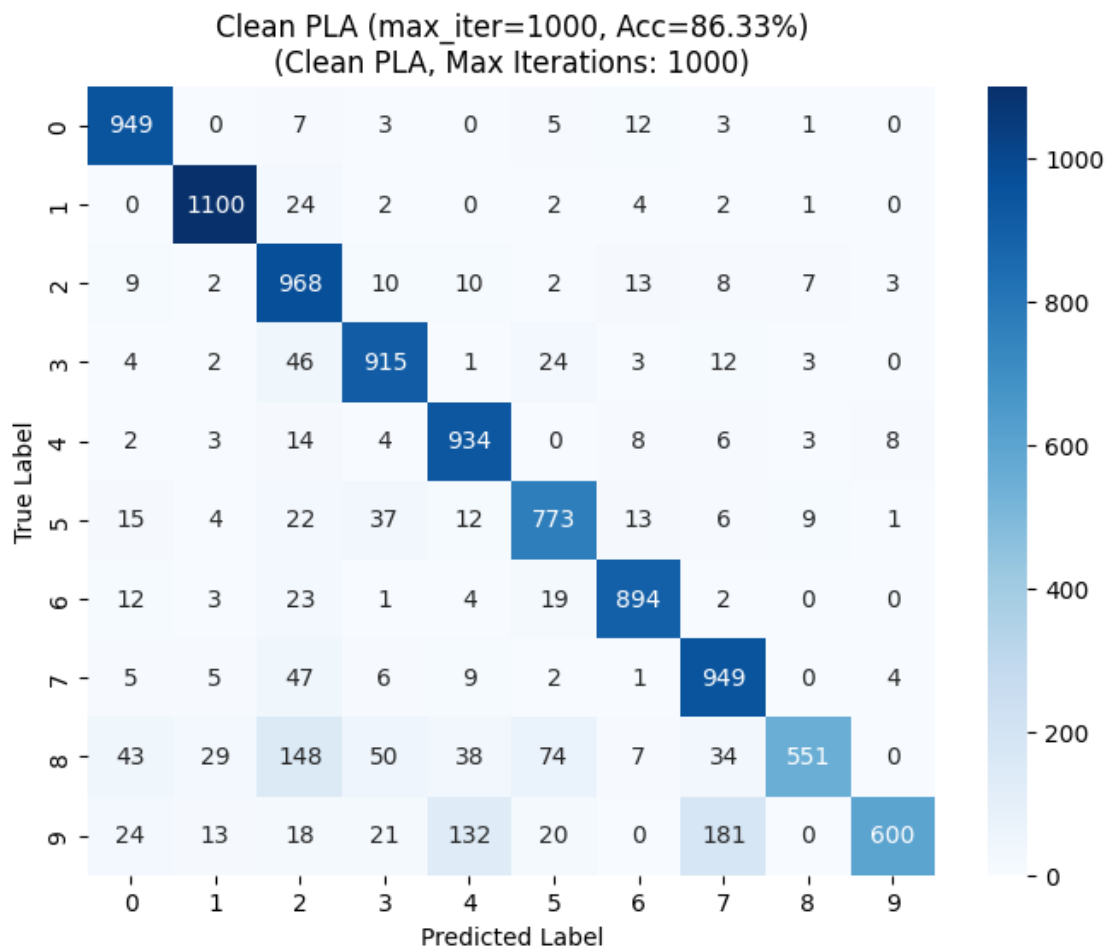
Confusions: Clean PLA: 0%| | 0/3 [00:00<?, ?it/s]



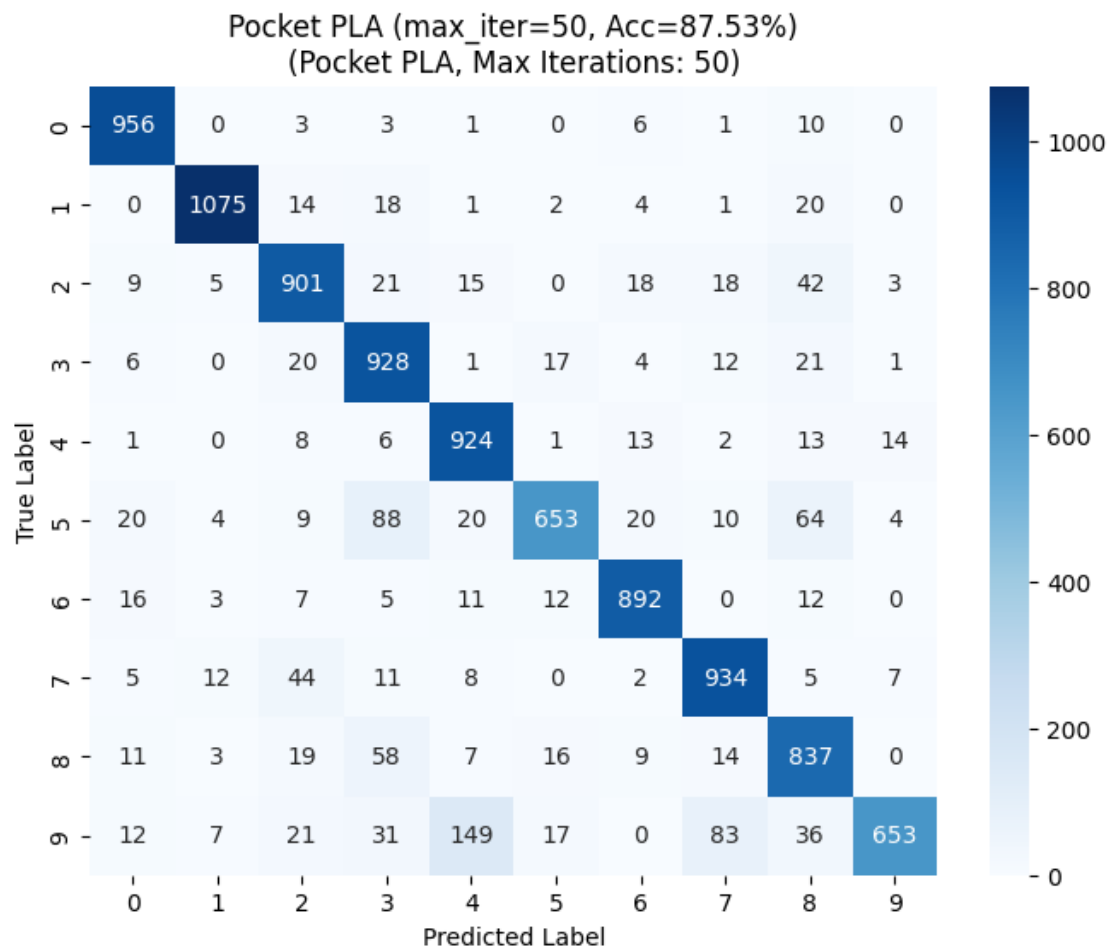
Confusions: Clean PLA: 33% | 1/3 [00:00<00:00, 7.64it/s]



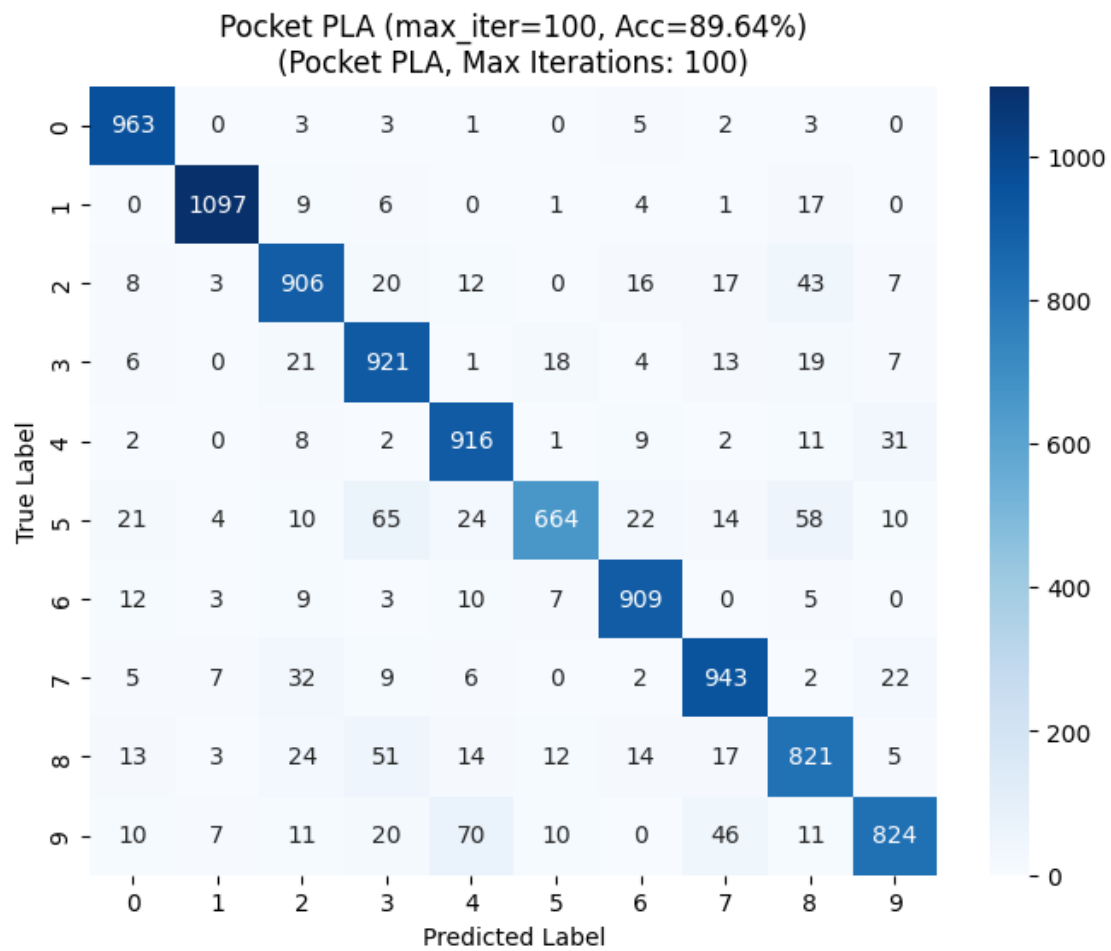
Confusions: Clean PLA: 67% | 2/3 [00:00<00:00, 8.27it/s]



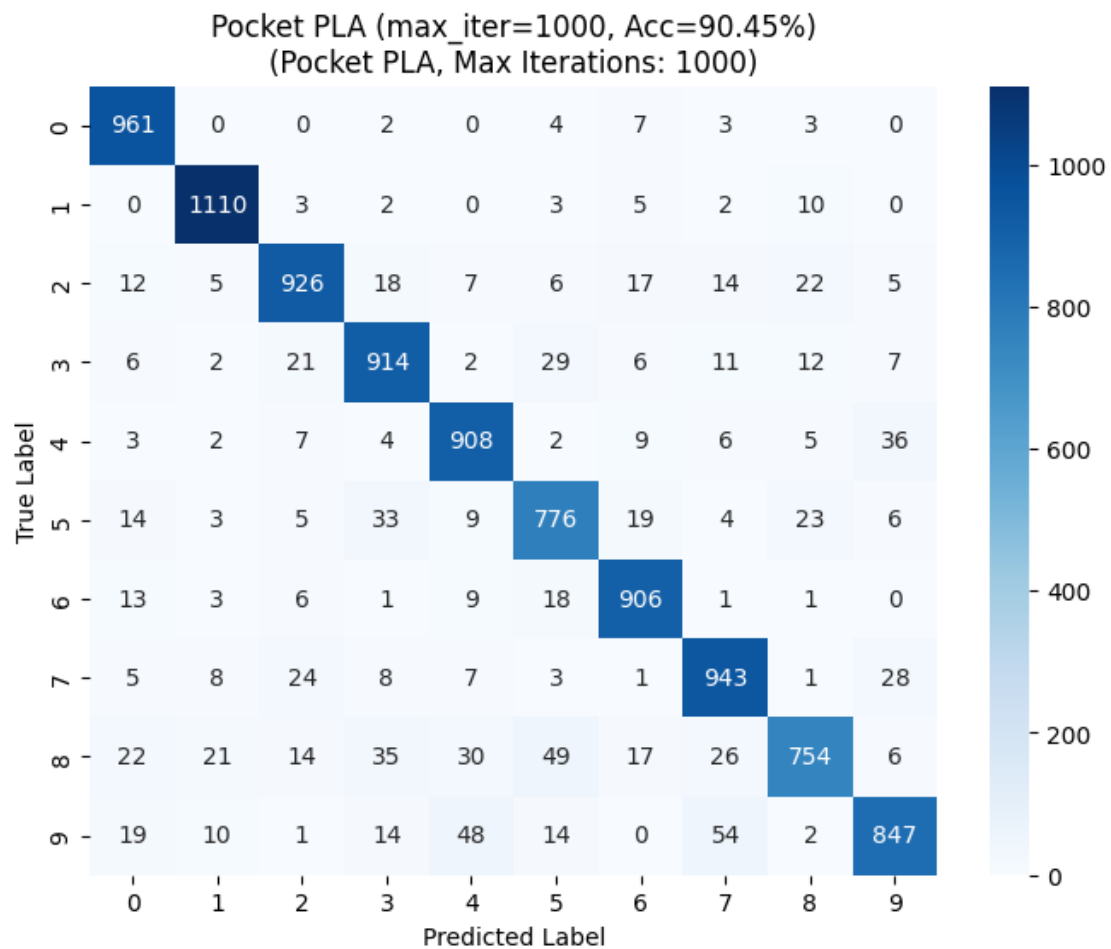
Confusions: Clean PLA: 100% | 3/3 [00:00<00:00, 8.52it/s]
 Confusions: Pocket PLA: 0% | 0/3 [00:00<?, ?it/s]



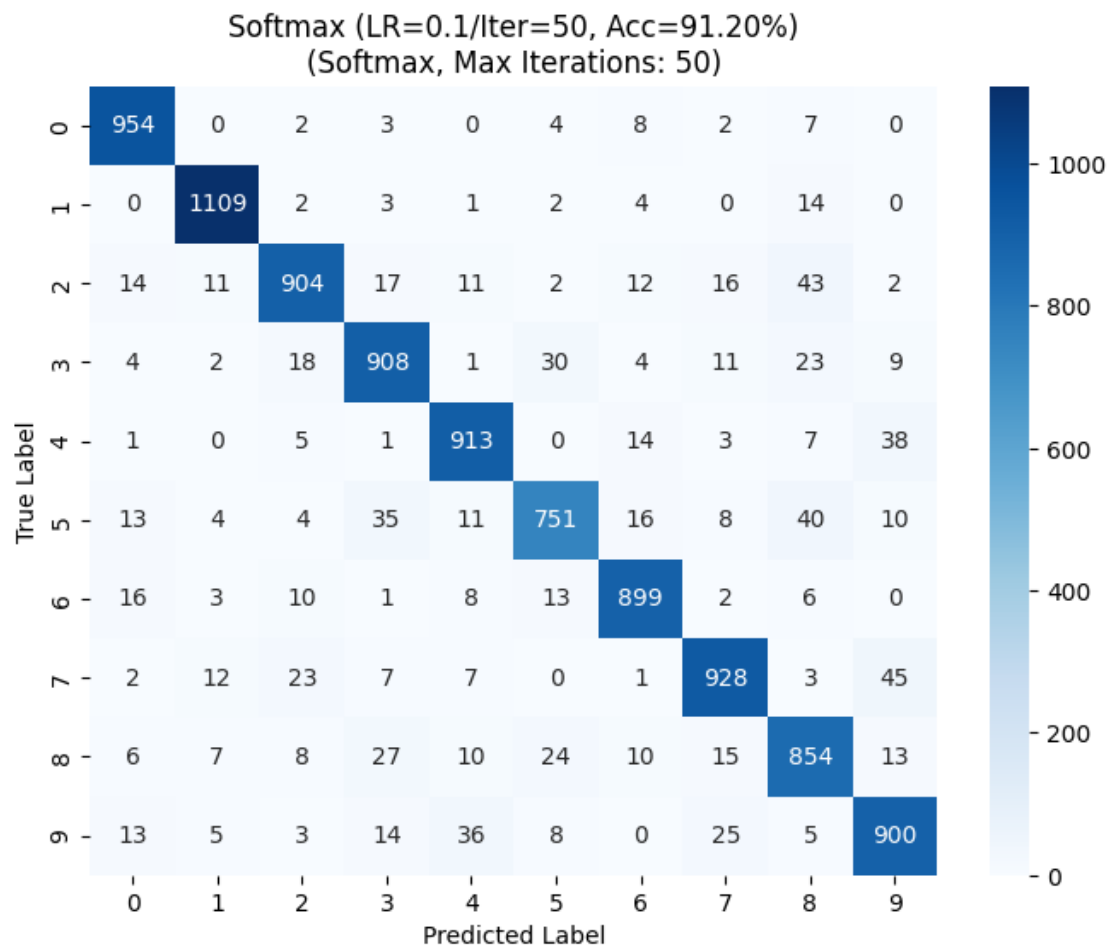
Confusions: Pocket PLA: 33% | 1/3 [00:00<00:00, 9.44it/s]



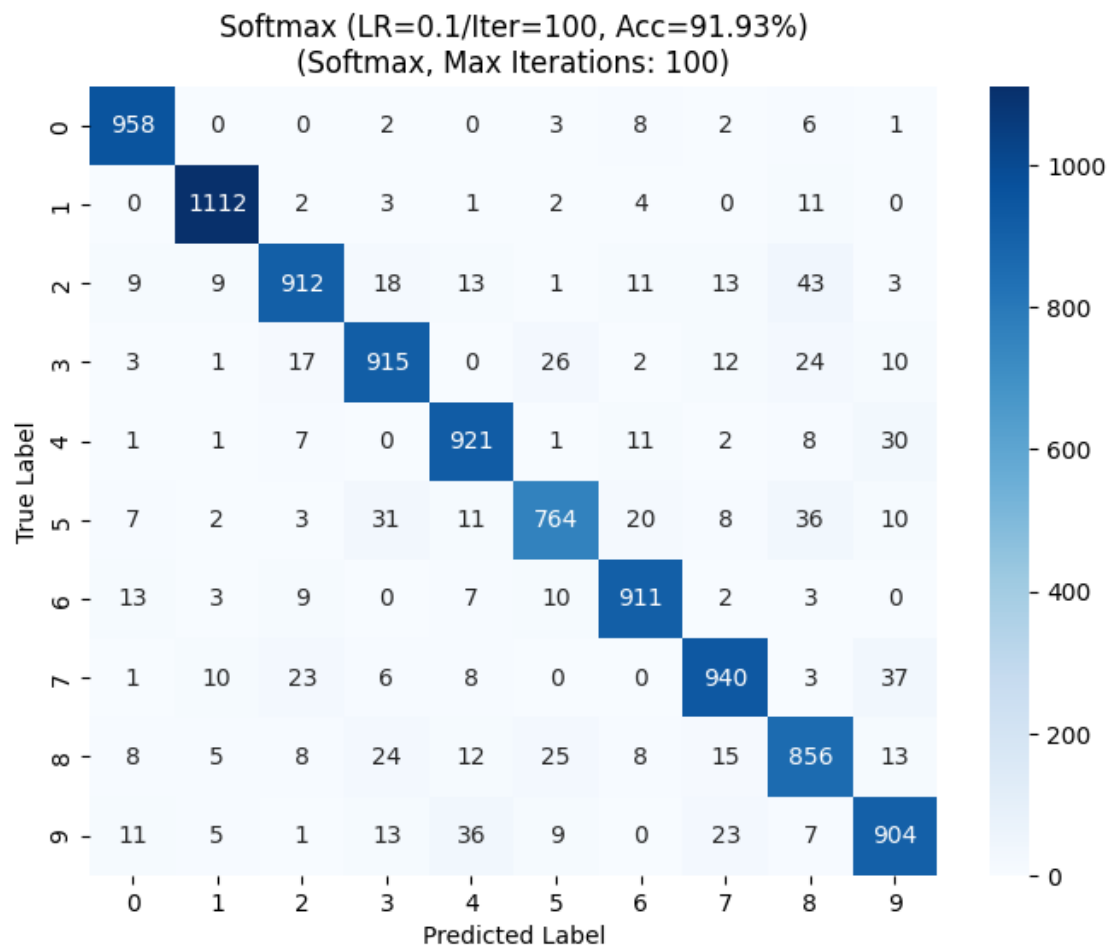
Confusions: Pocket PLA: 67% | 2/3 [00:00<00:00, 9.42it/s]



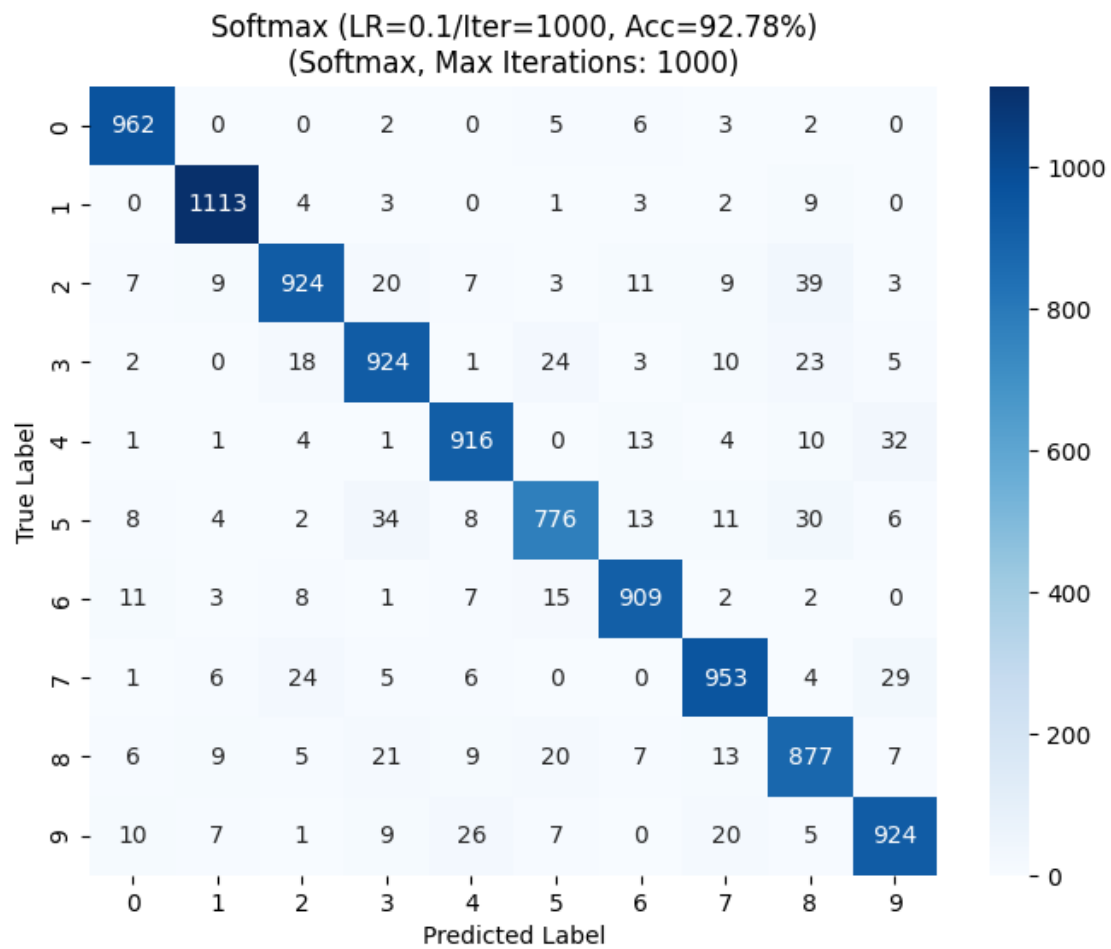
Confusions: Pocket PLA: 100% | 3/3 [00:00<00:00, 9.30it/s]
Confusions: Softmax: 0% | 0/3 [00:00<?, ?it/s]



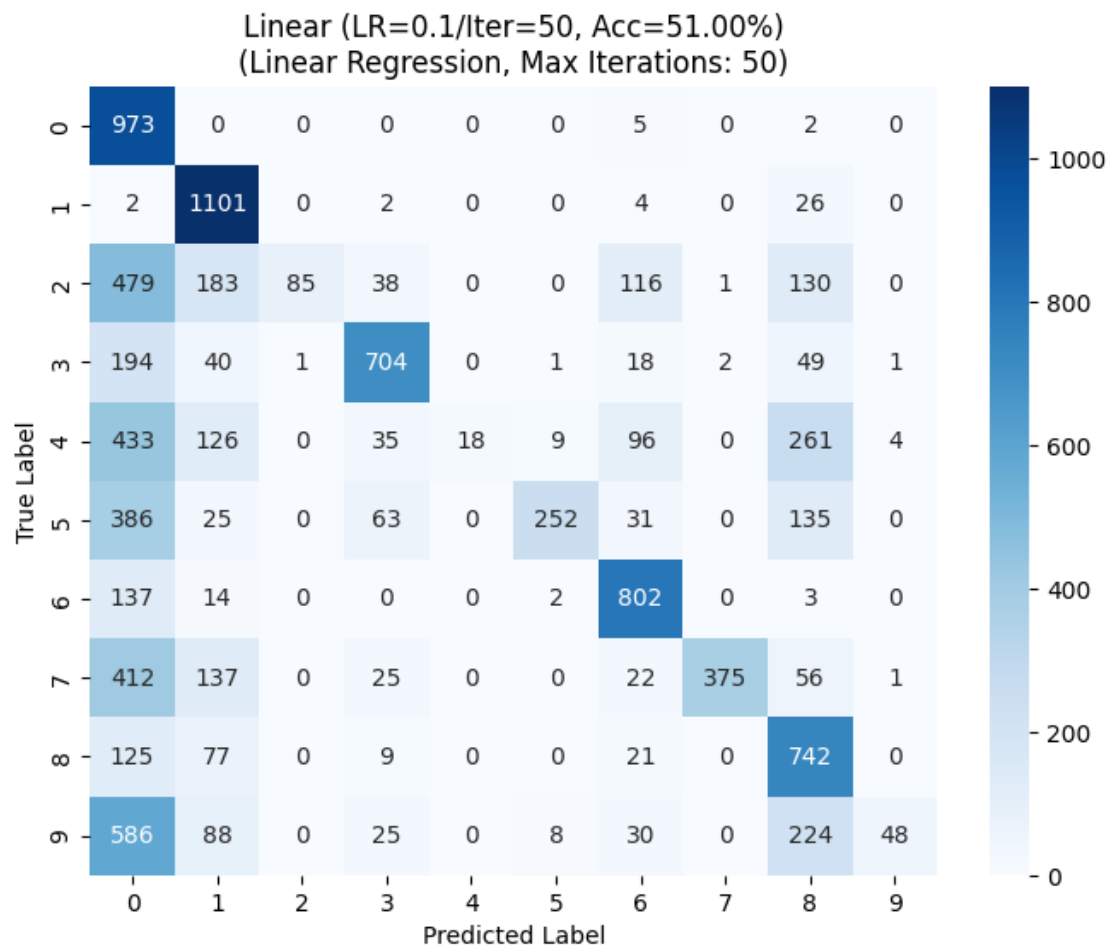
Confusions: Softmax: 33% | 1/3 [00:00<00:00, 9.46it/s]



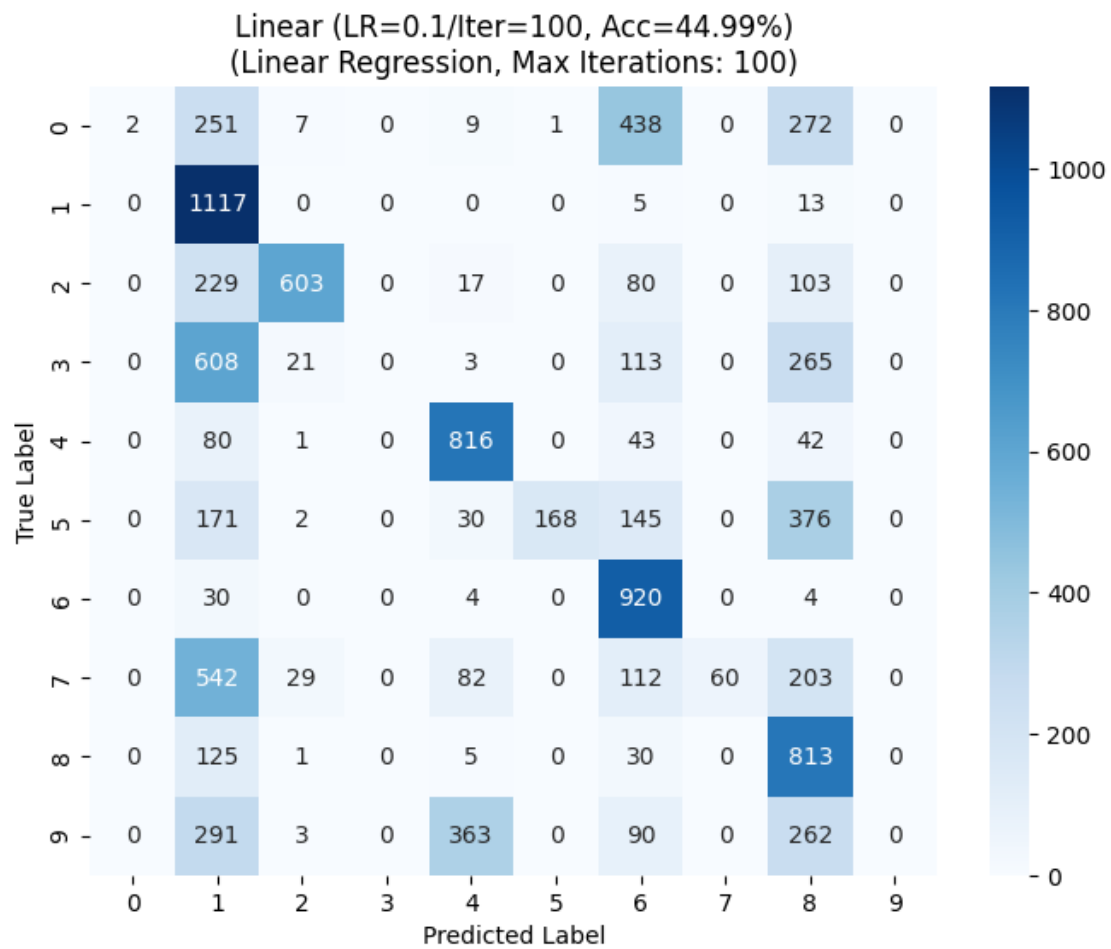
Confusions: Softmax: 67% | 2/3 [00:00<00:00, 9.38it/s]



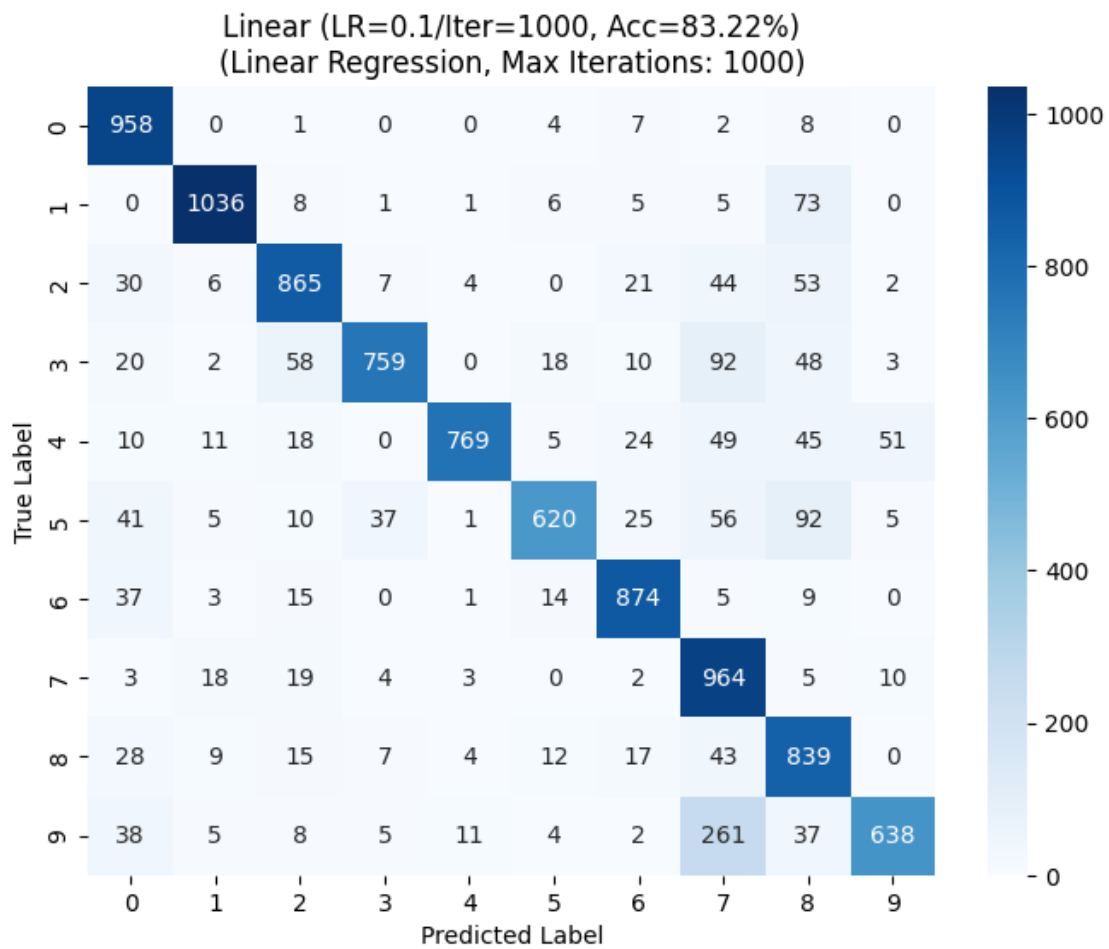
Confusions: Softmax: 100%| | 3/3 [00:00<00:00, 9.31it/s]
 Confusions: Linear: 0%| | 0/3 [00:00<?, ?it/s]



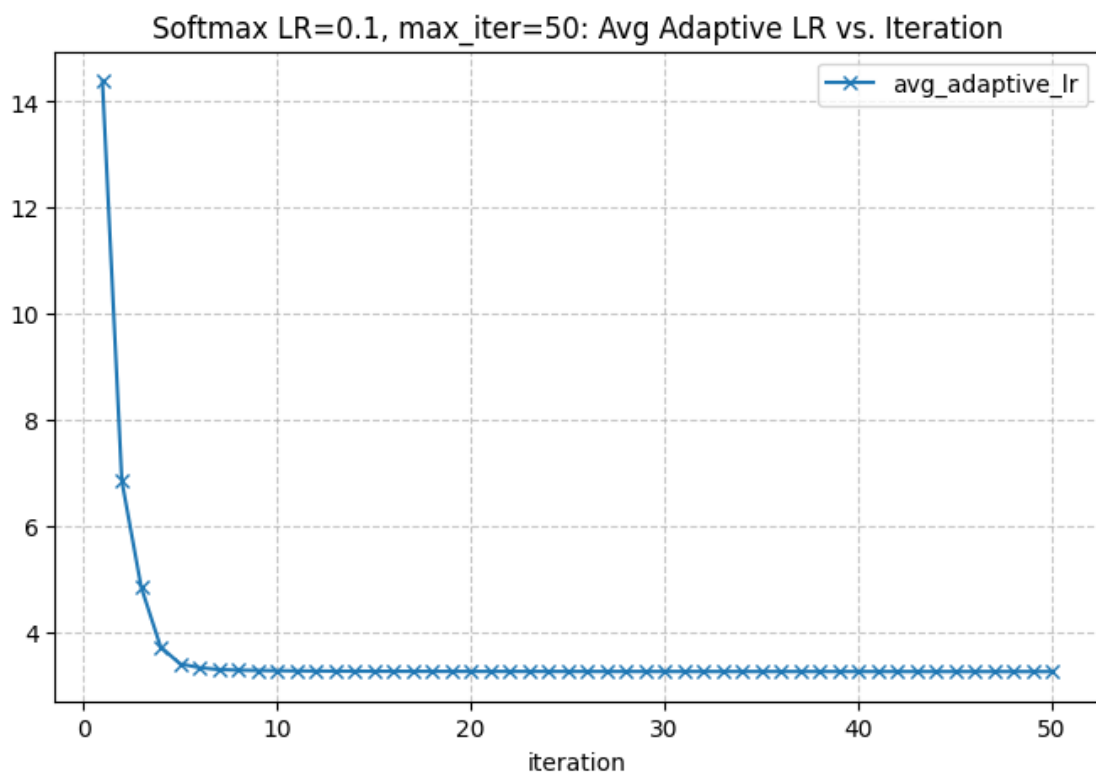
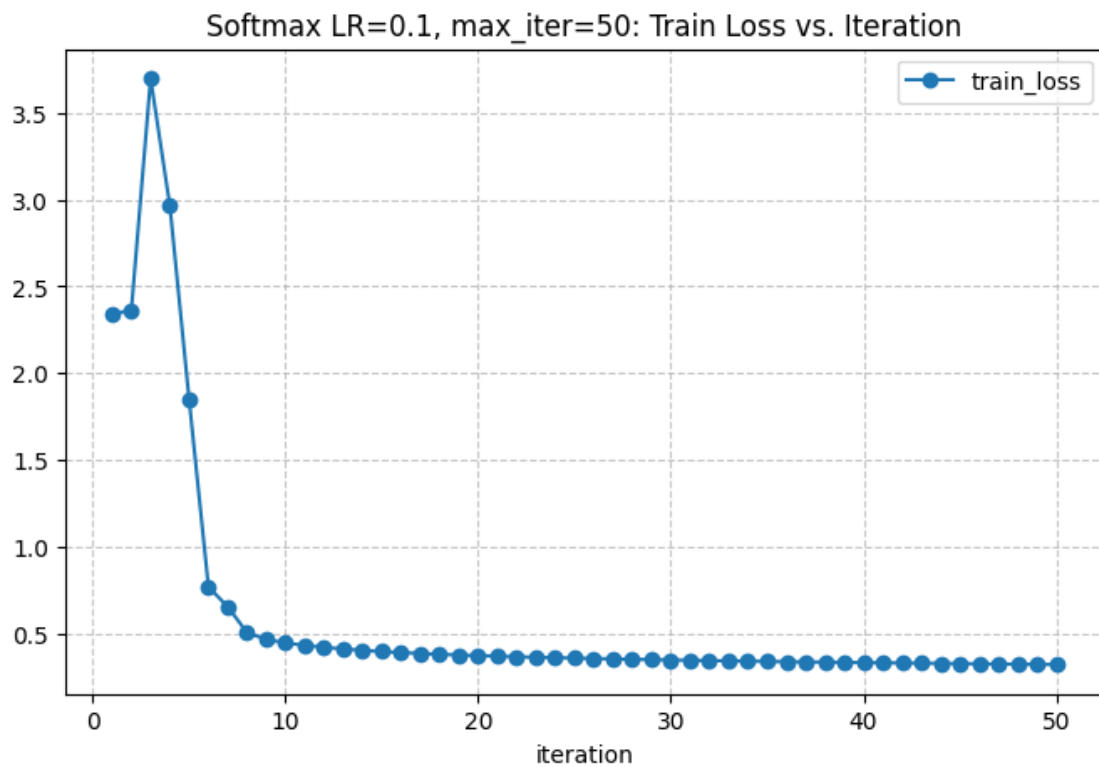
Confusions: Linear: 33% | 1/3 [00:00<00:00, 5.92it/s]

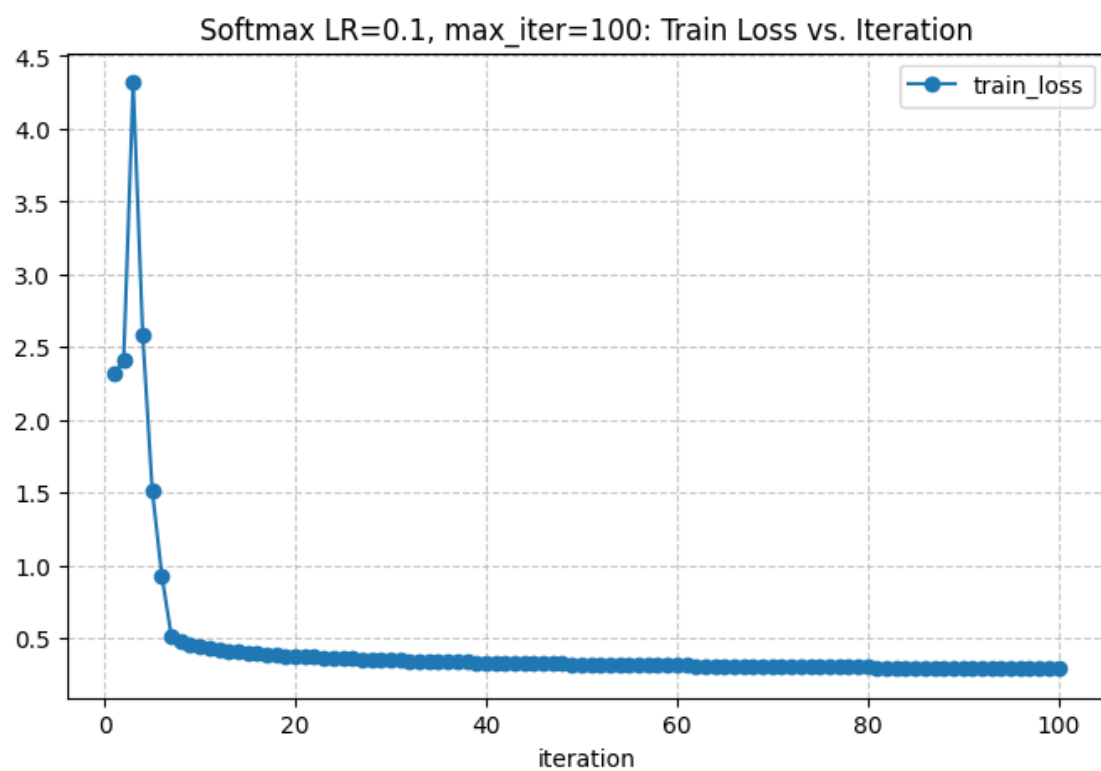


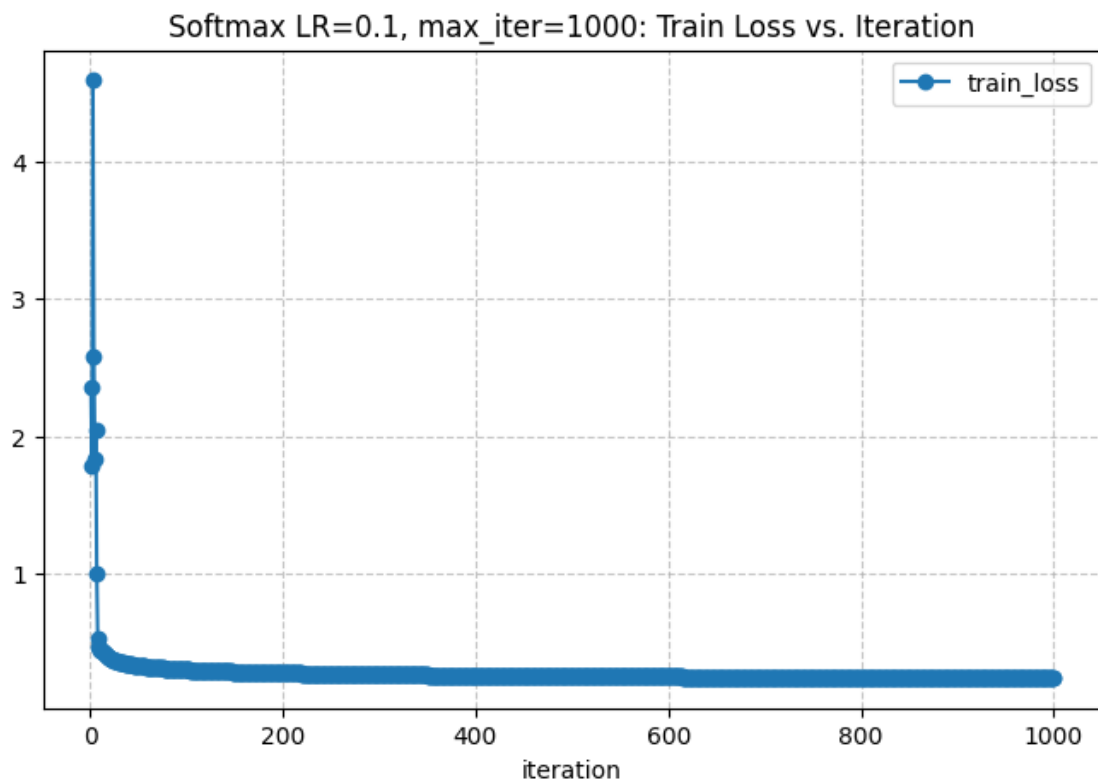
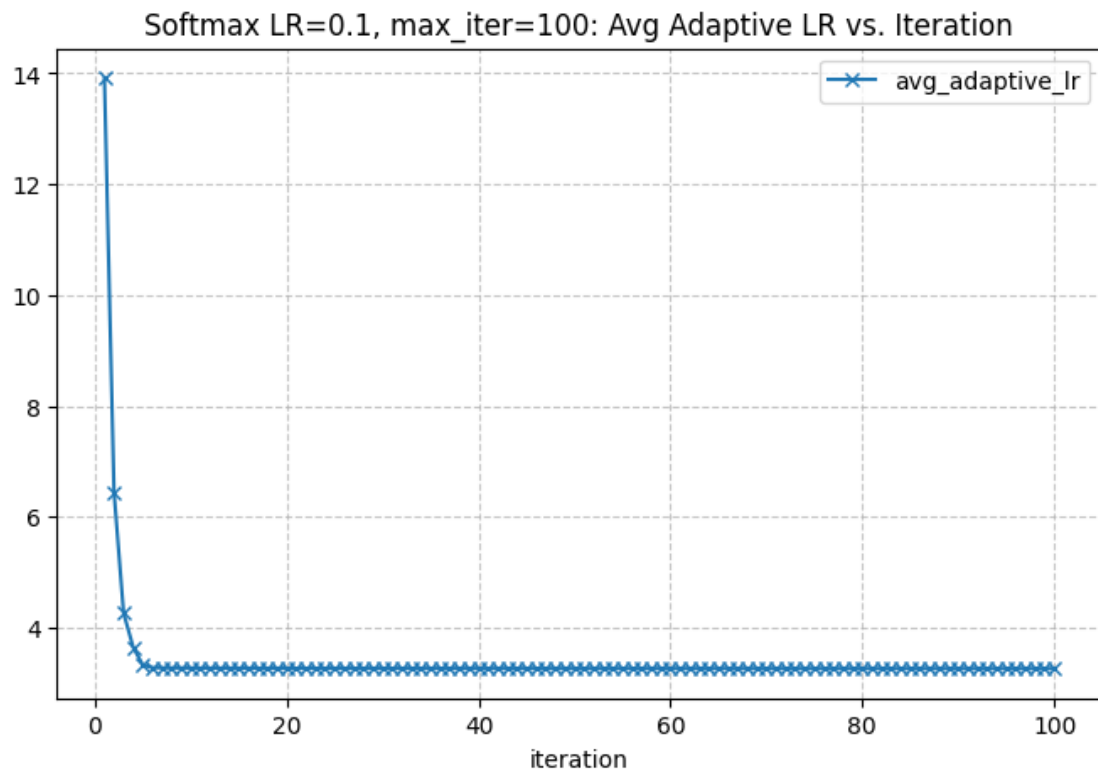
Confusions: Linear: 67% | 2/3 [00:00<00:00, 7.47it/s]

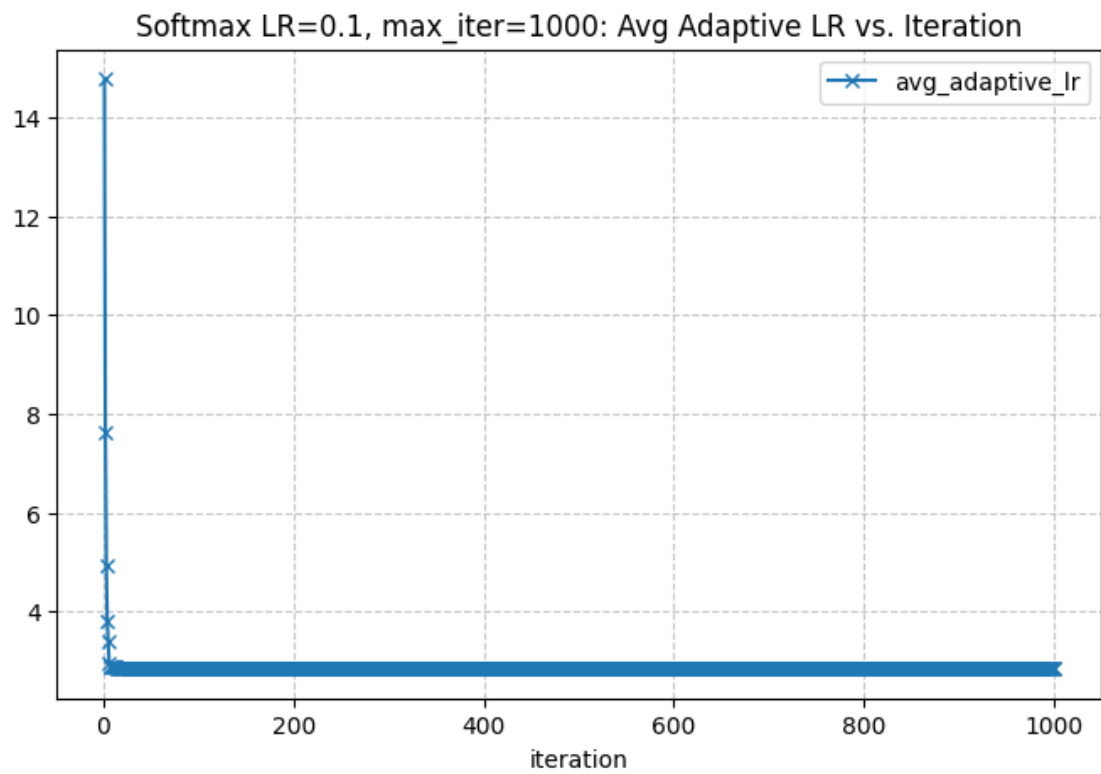


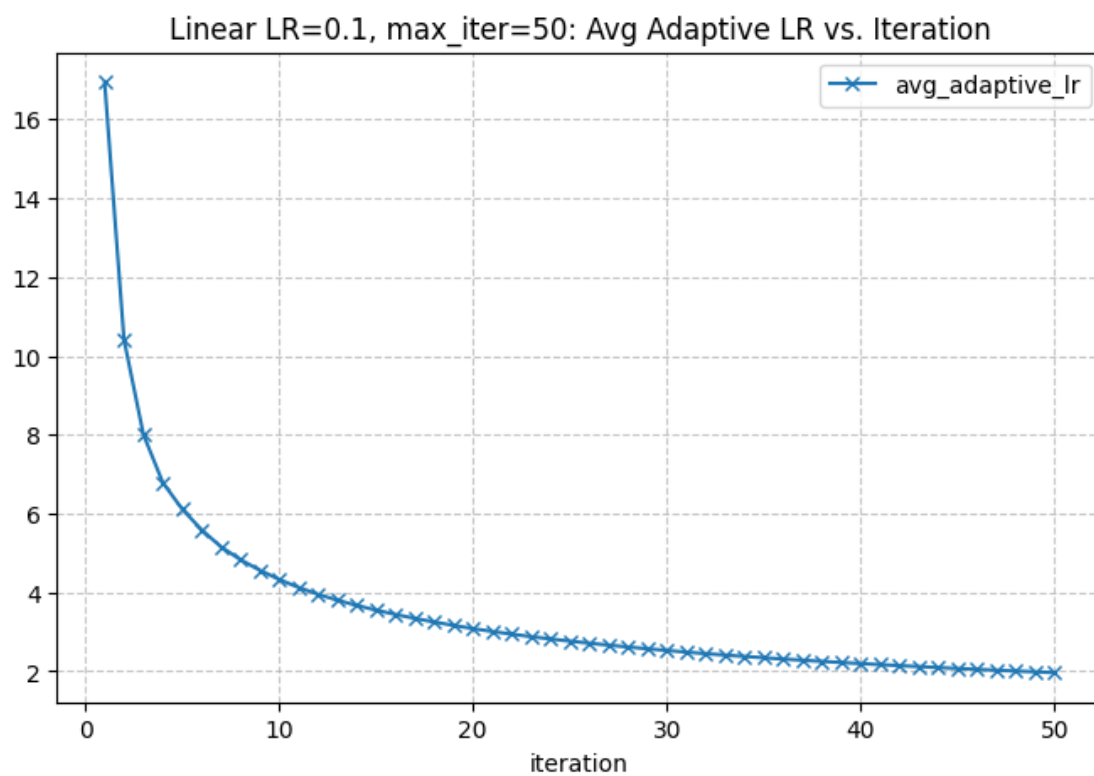
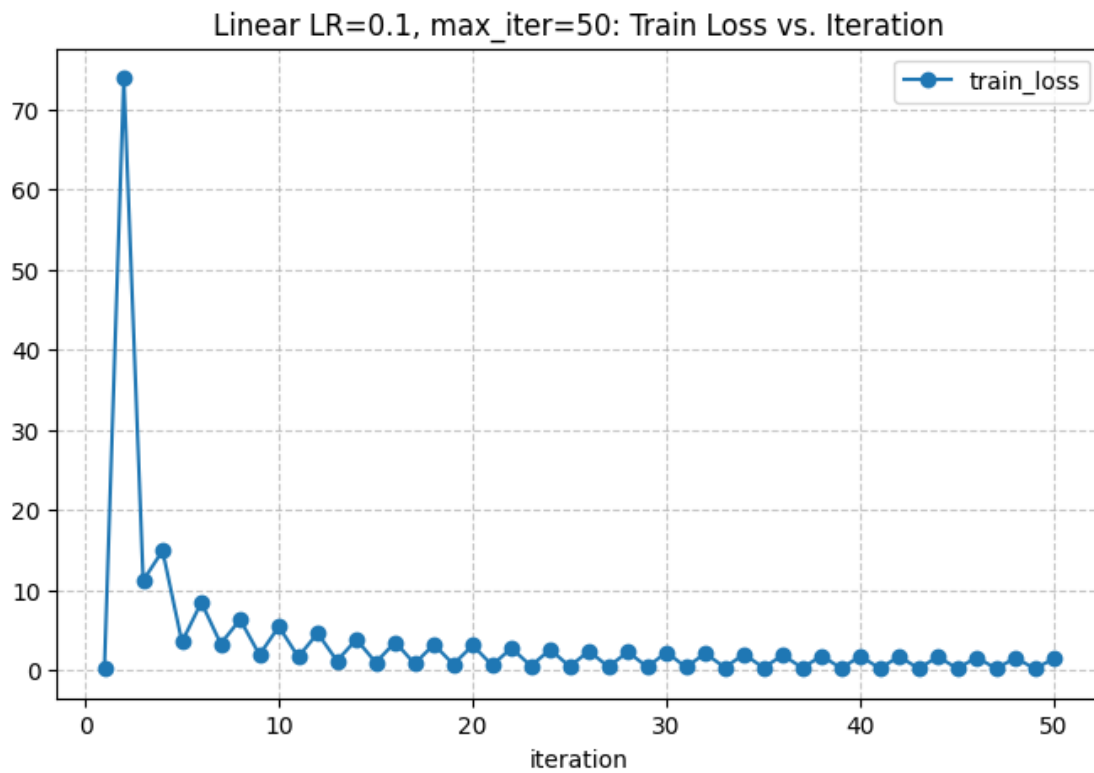
Confusions: Linear: 100%| 3/3 [00:00<00:00, 7.74it/s]
INFO - === Iteration-Level Visualization (All Models) ===

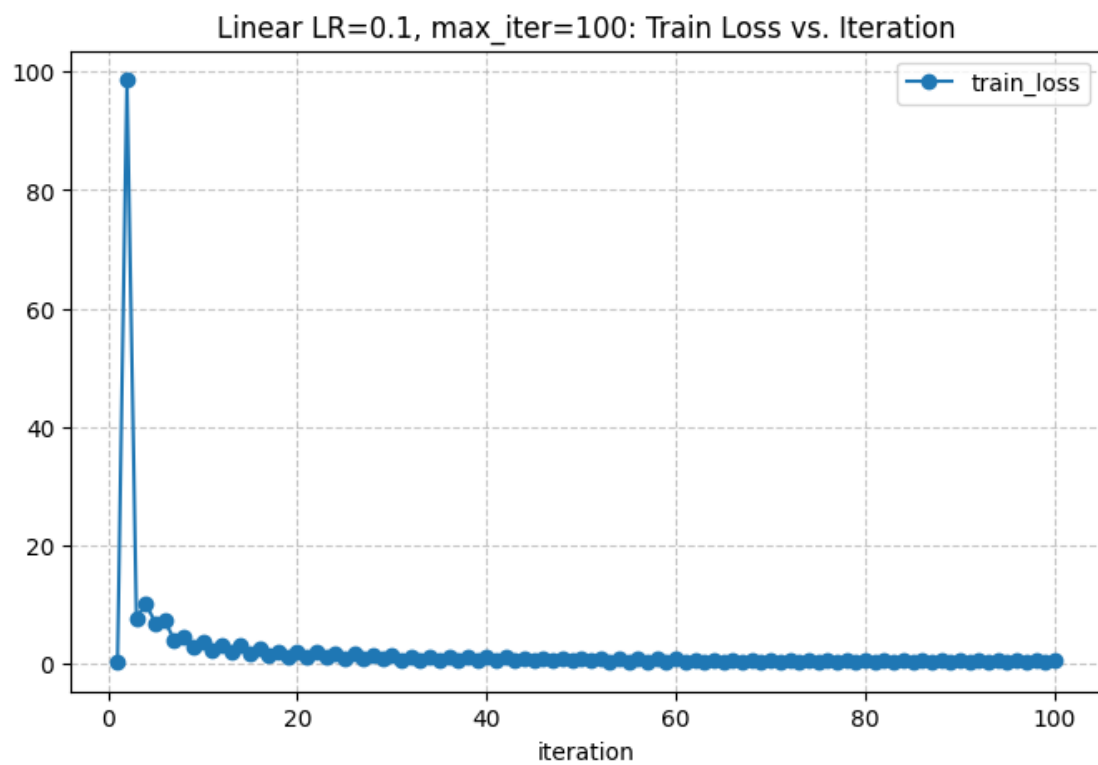


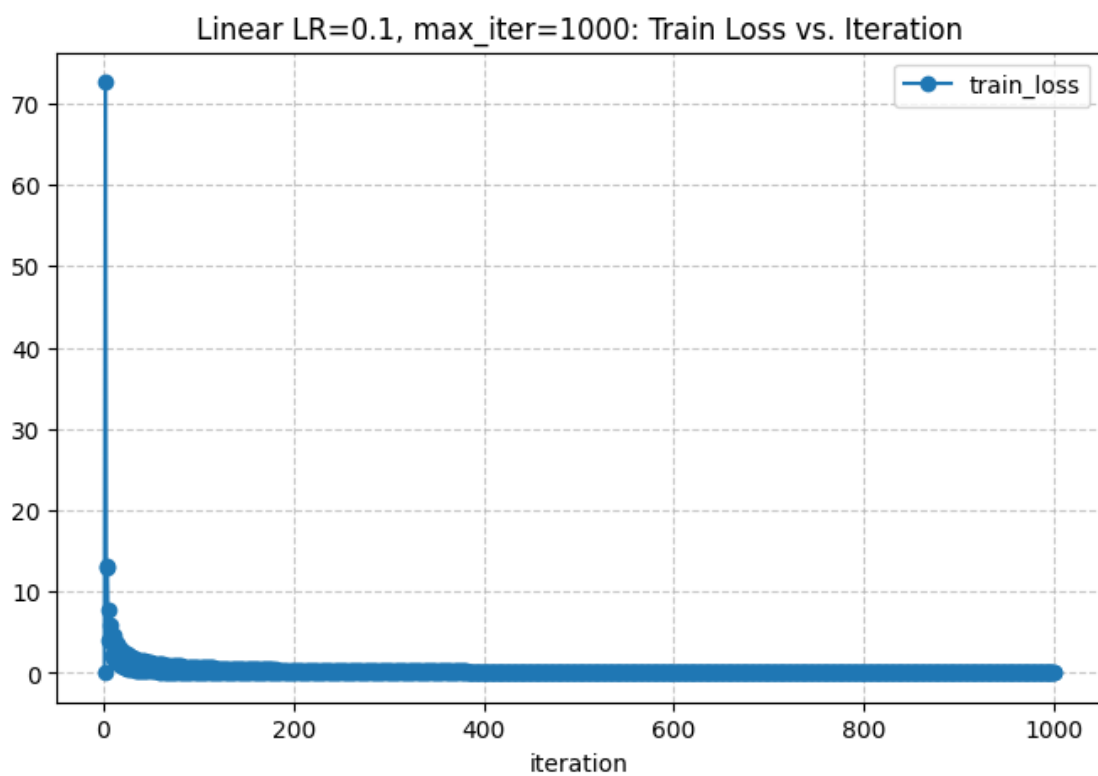
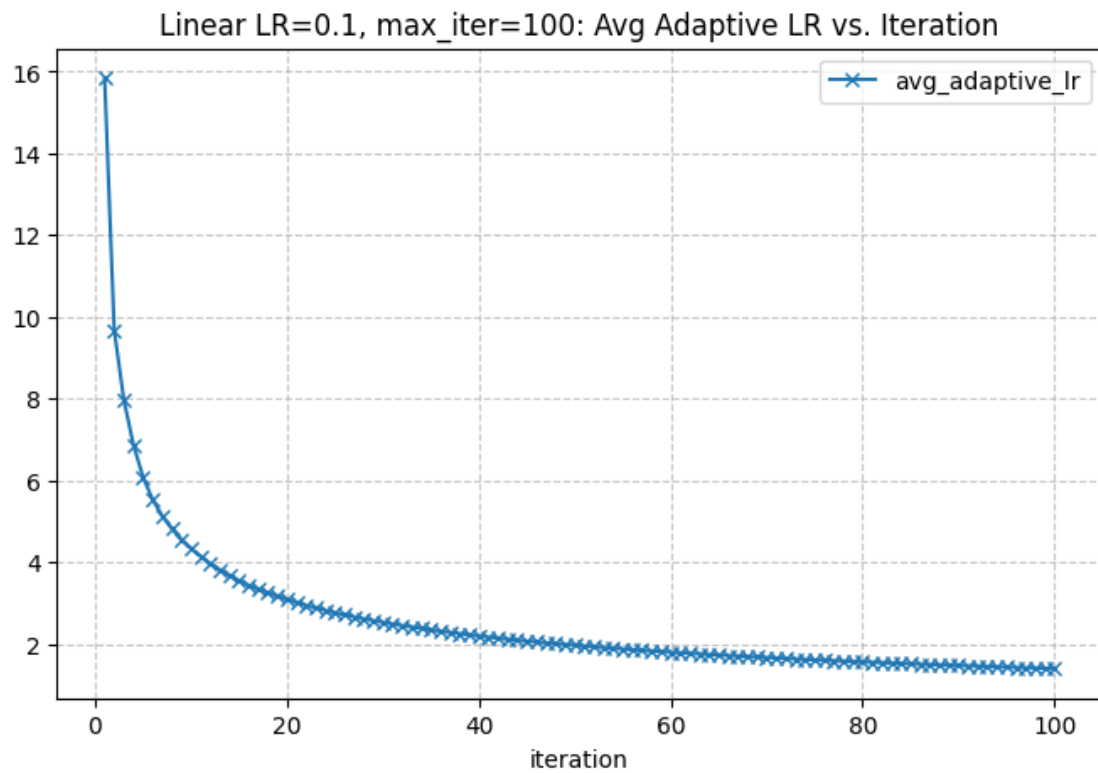


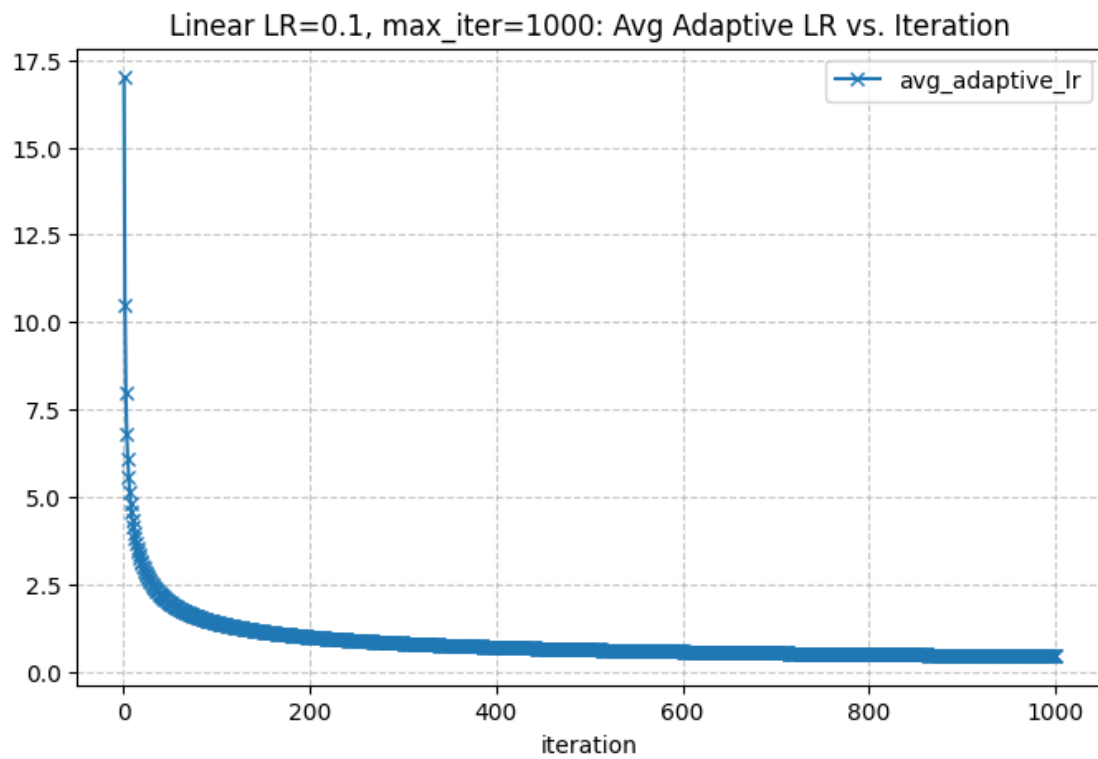




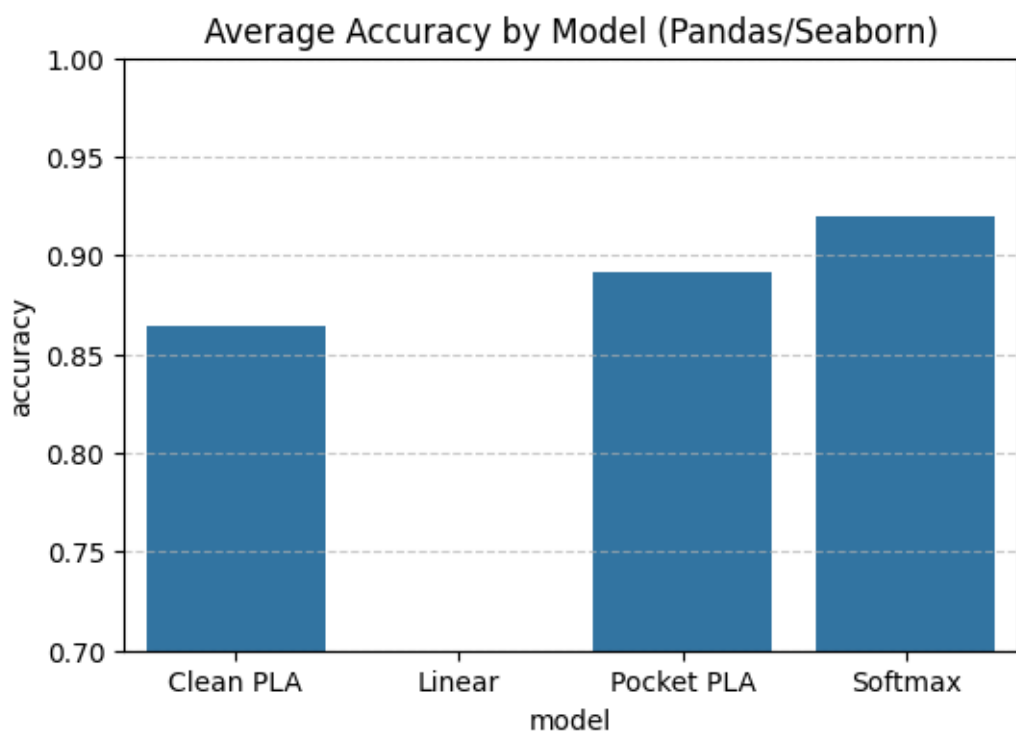
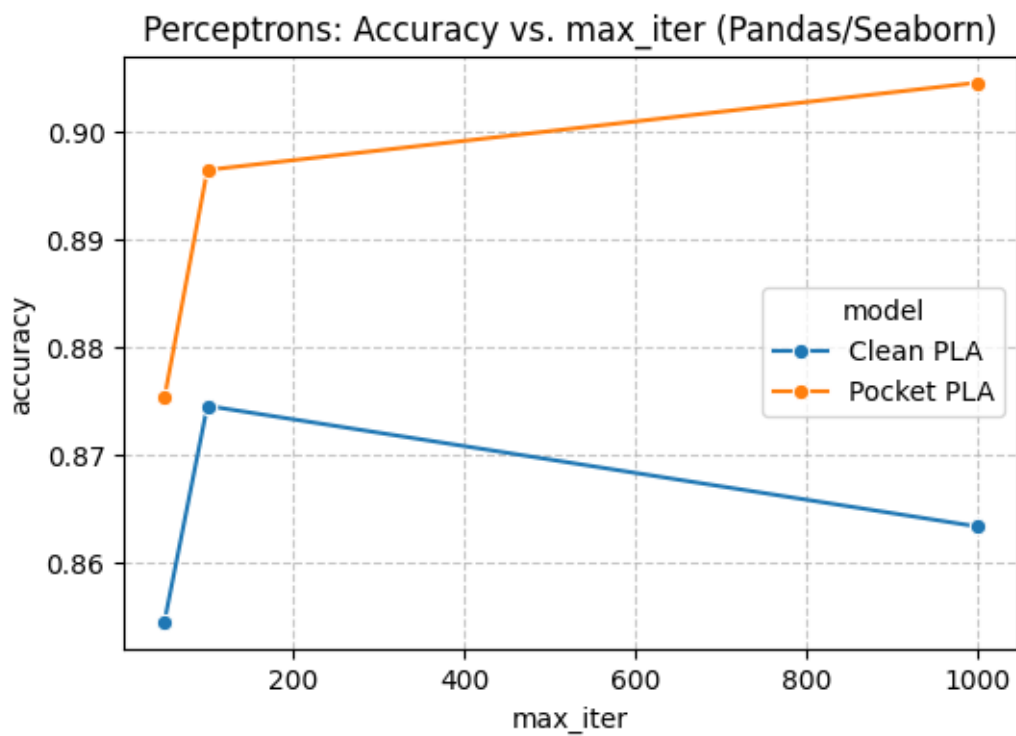


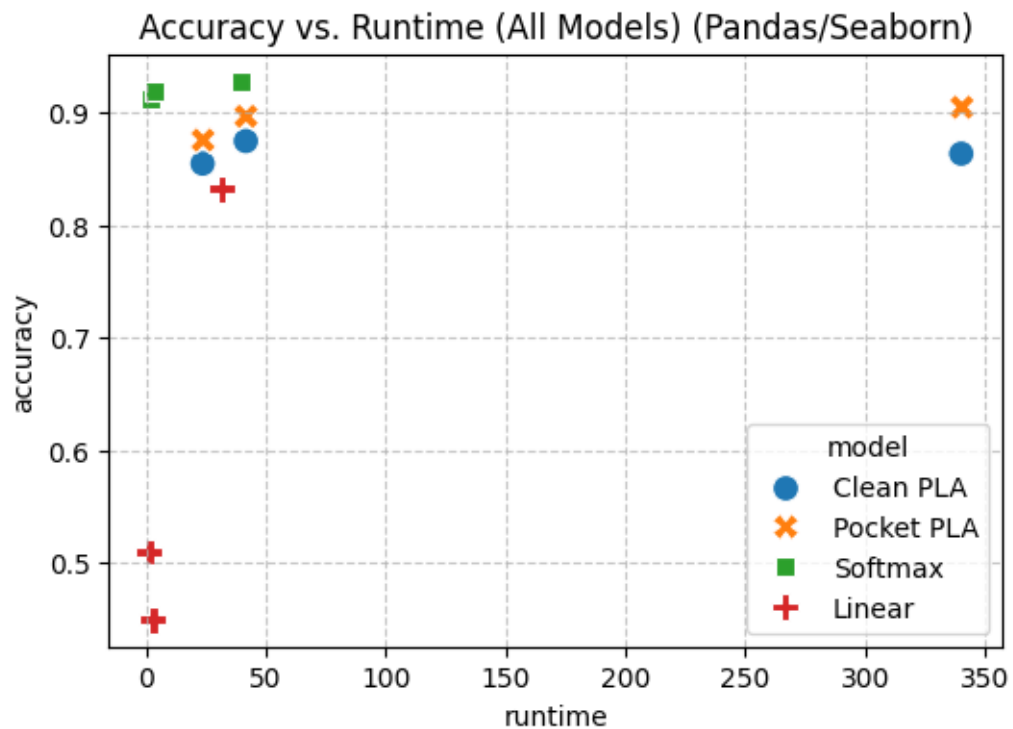




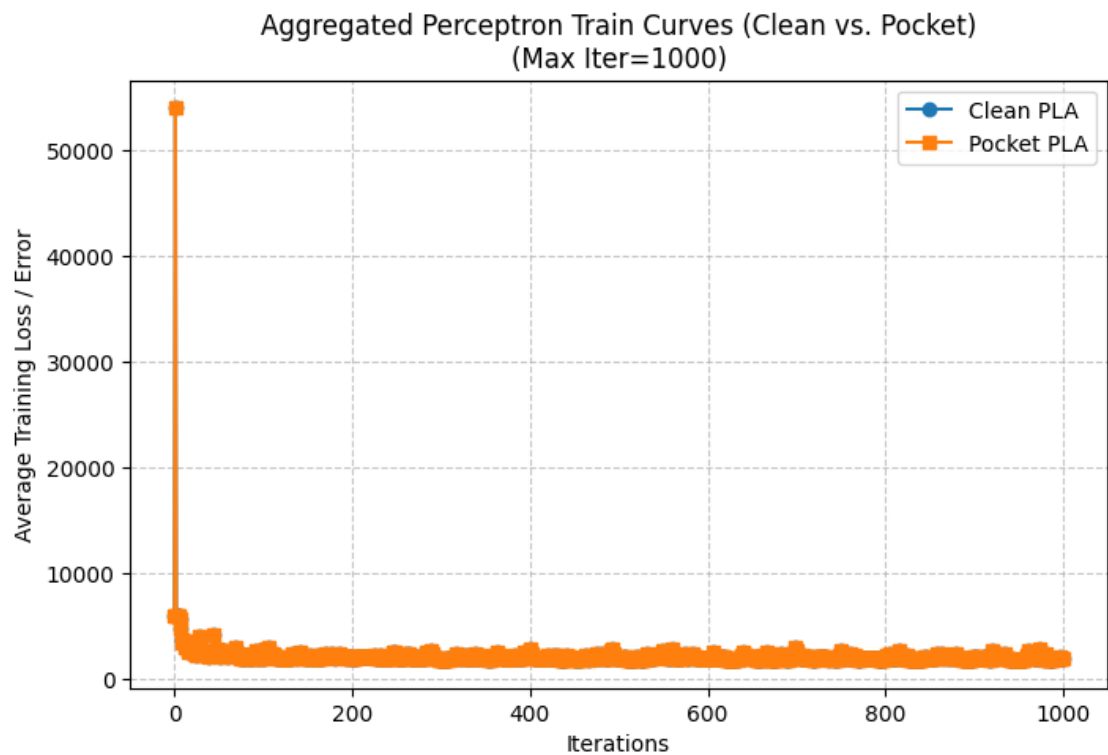


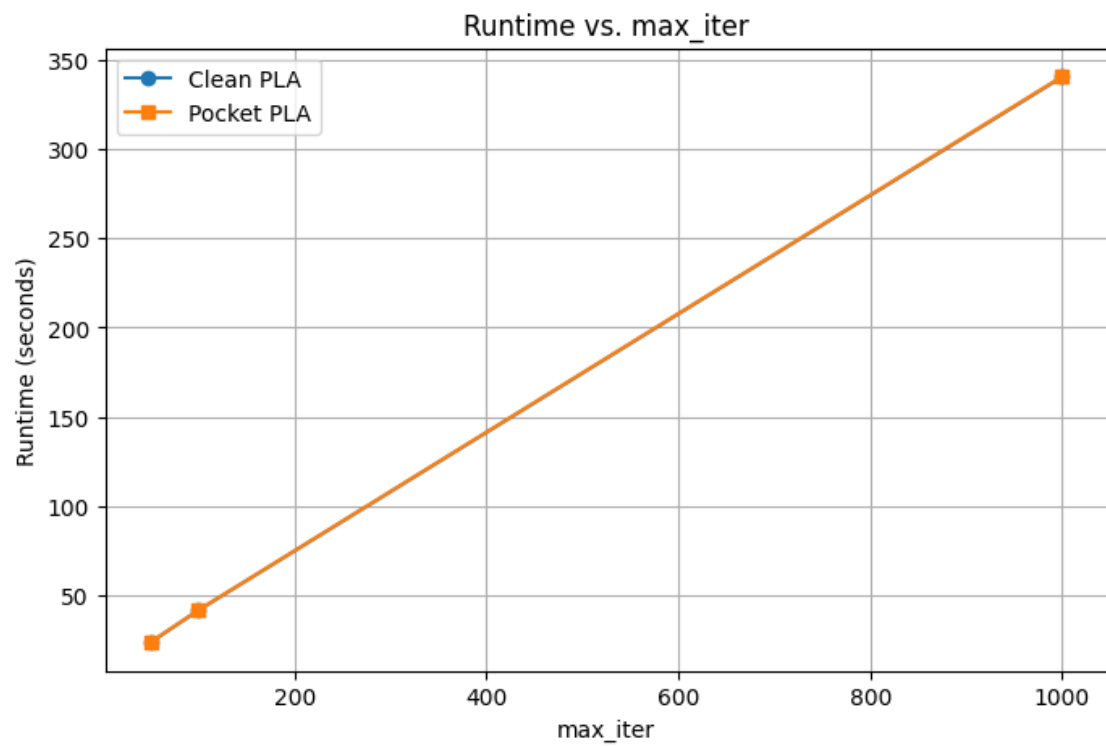
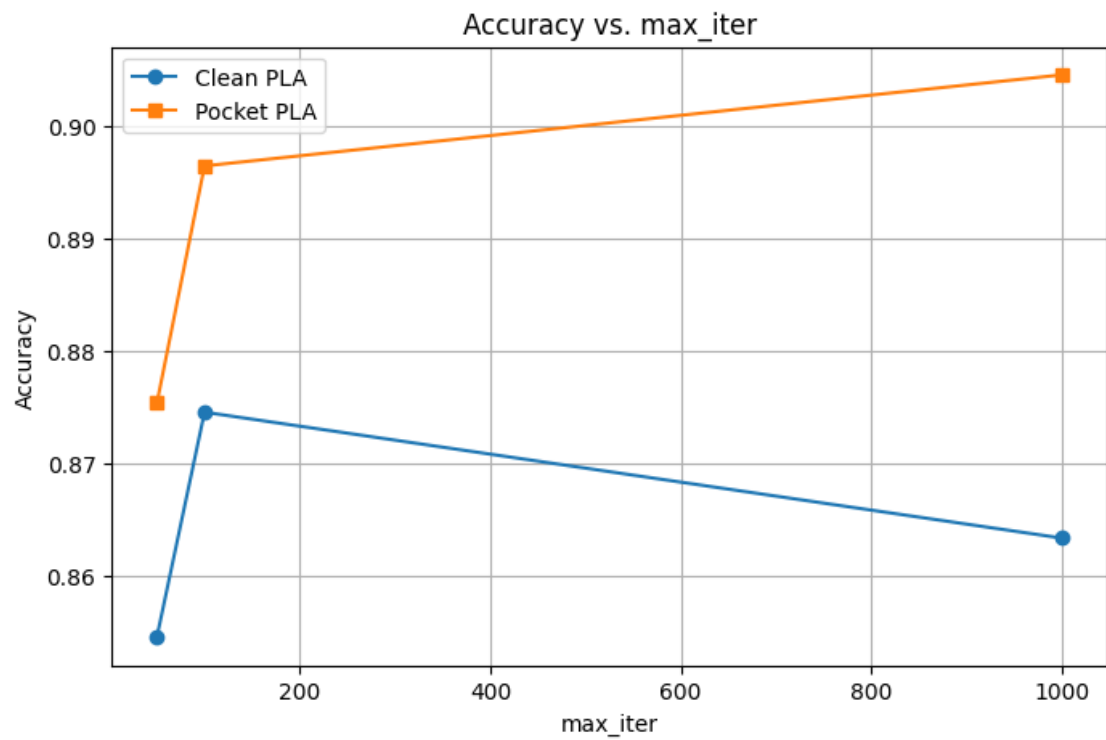
INFO - === Pandas + Seaborn Plots ===

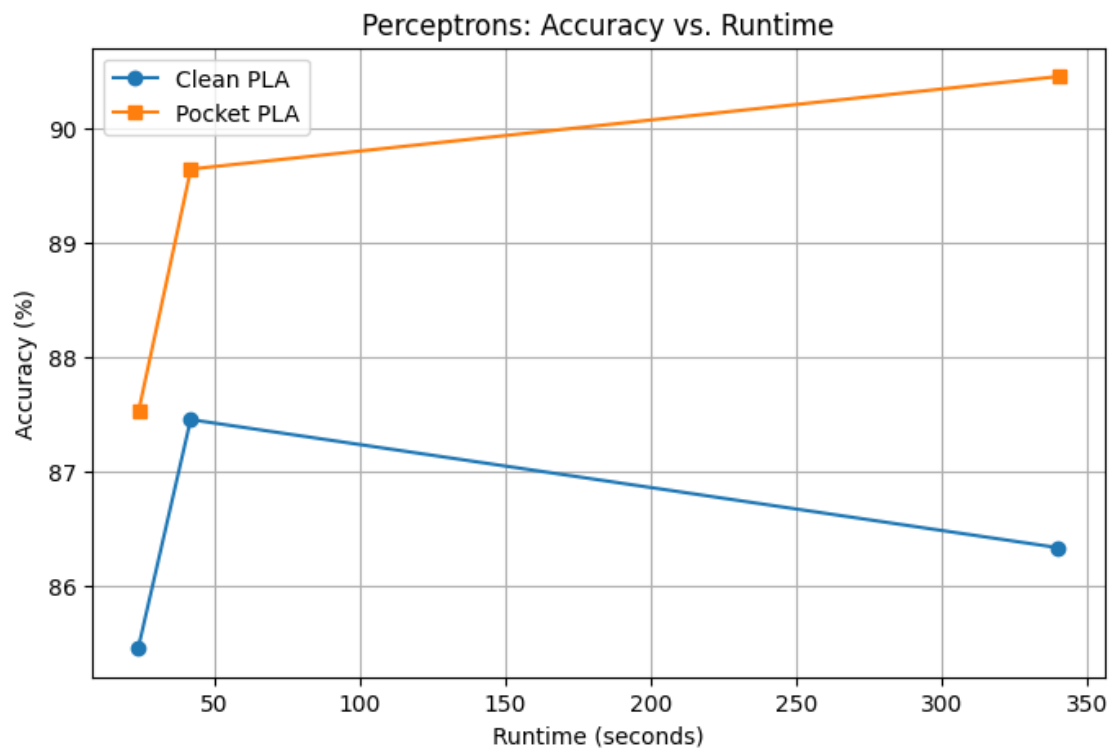




INFO - === Custom Summaries (Aggregated Curves, etc.) ===

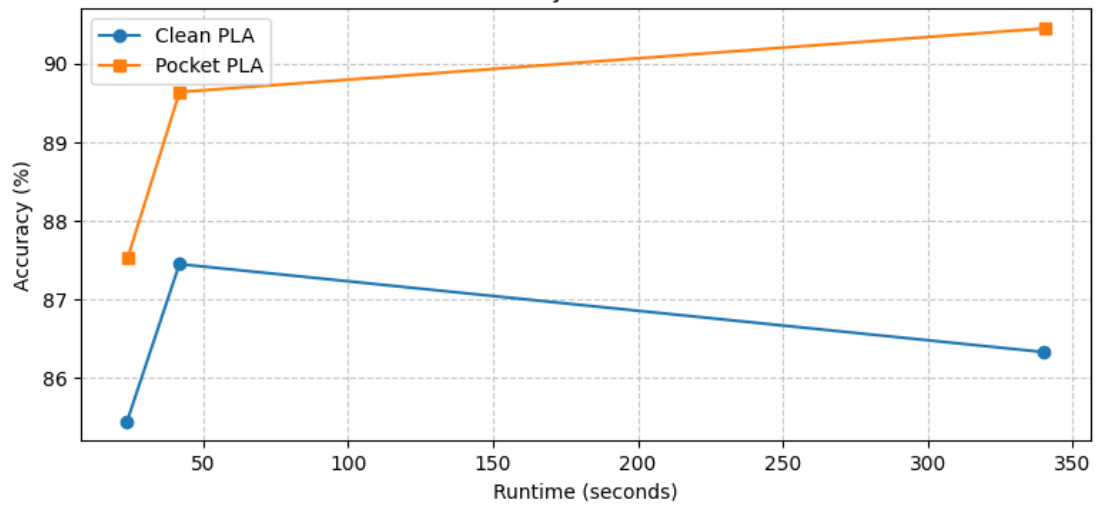




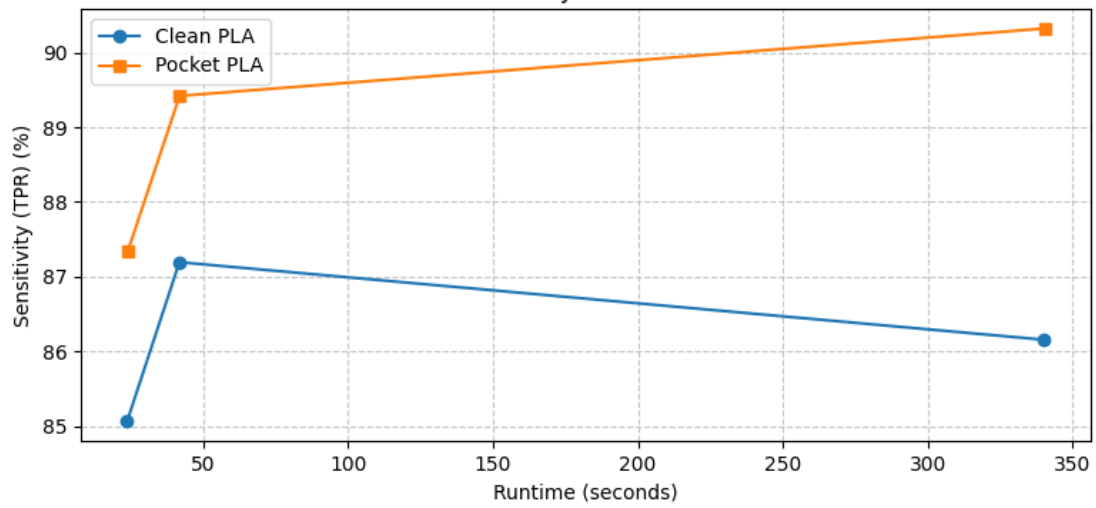


Perceptrons: Performance vs. Runtime

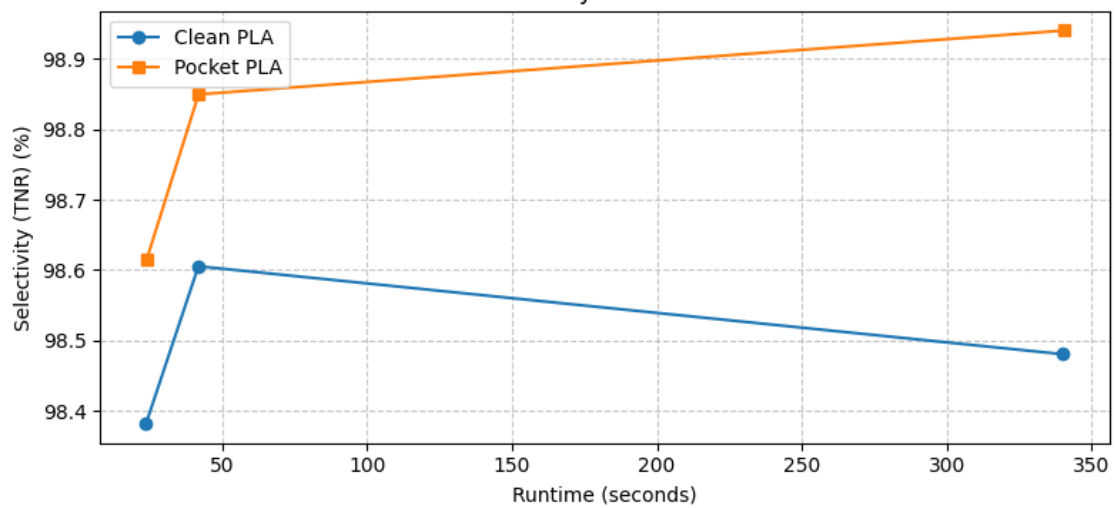
Accuracy vs. Runtime

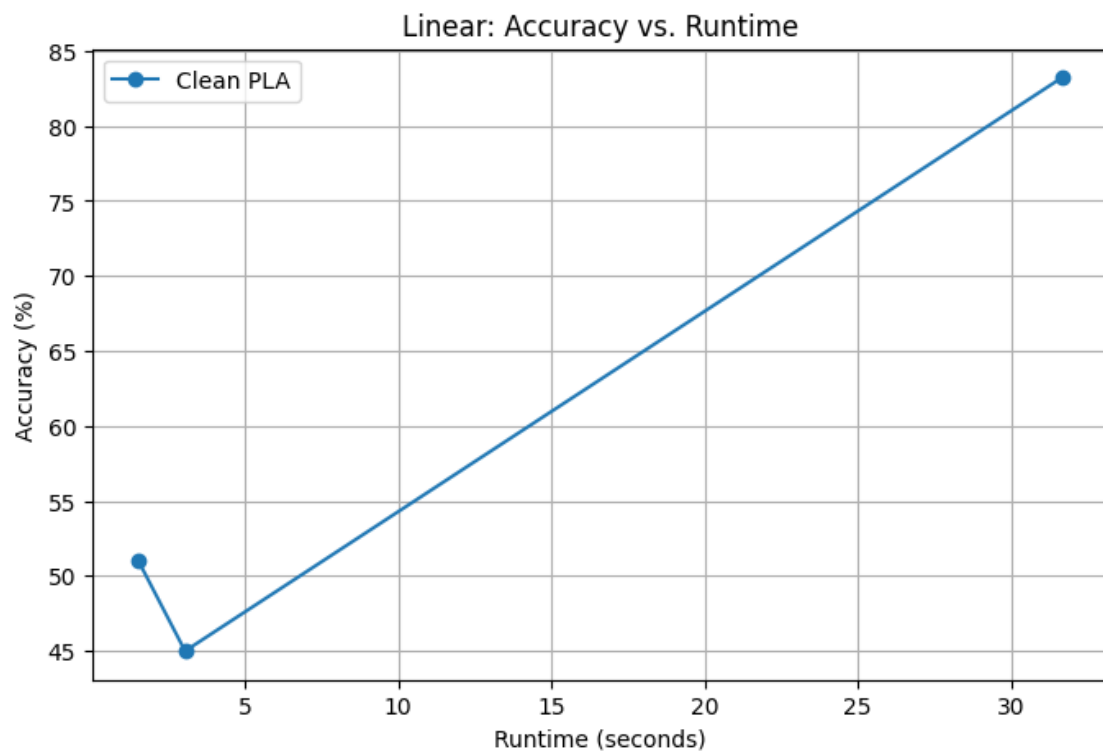
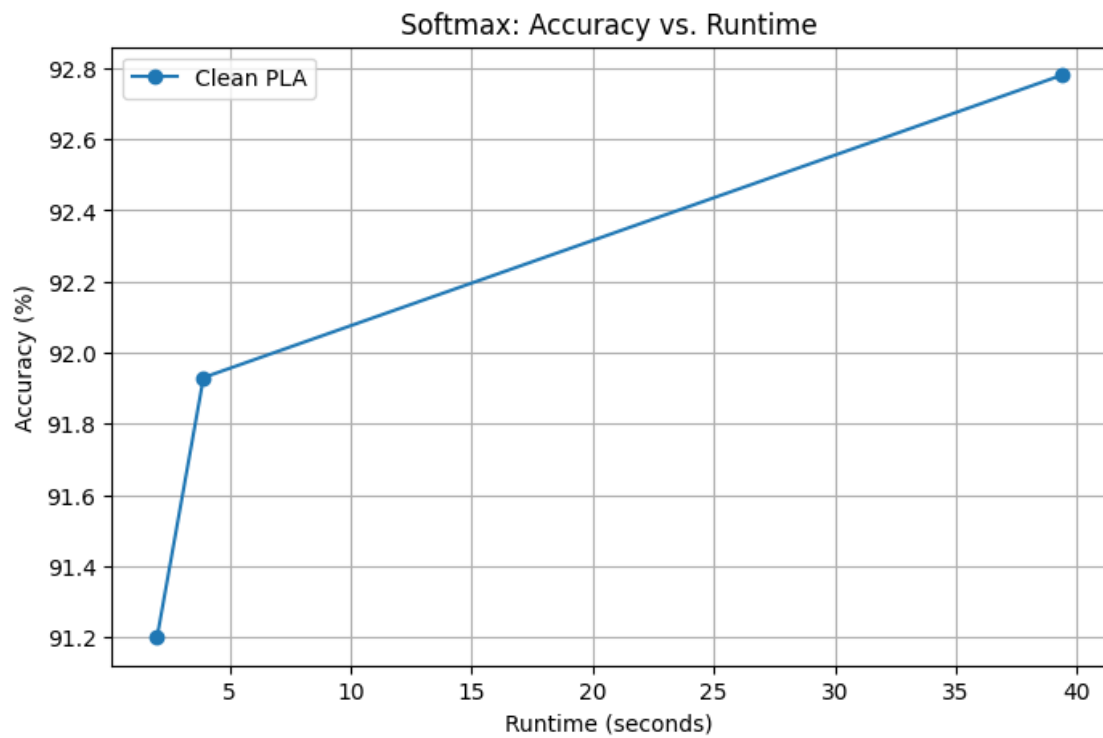


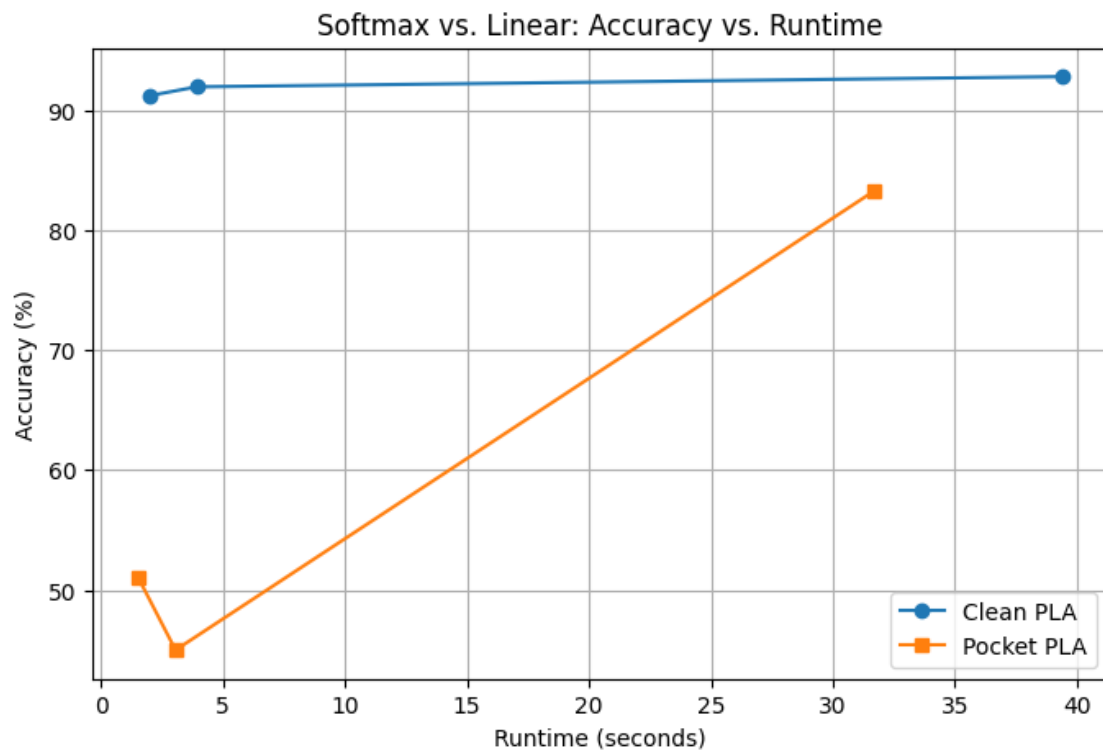
Sensitivity vs. Runtime



Selectivity vs. Runtime

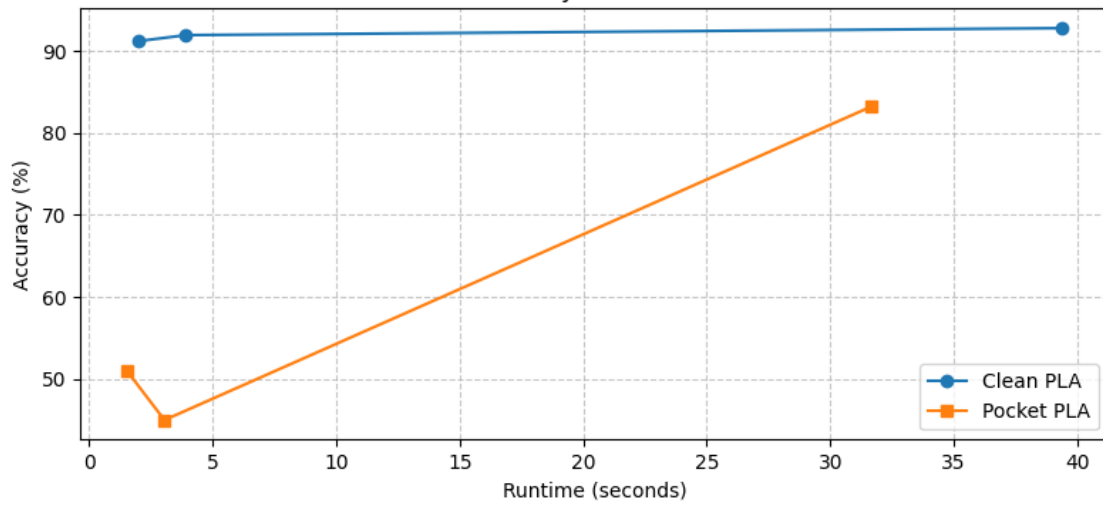




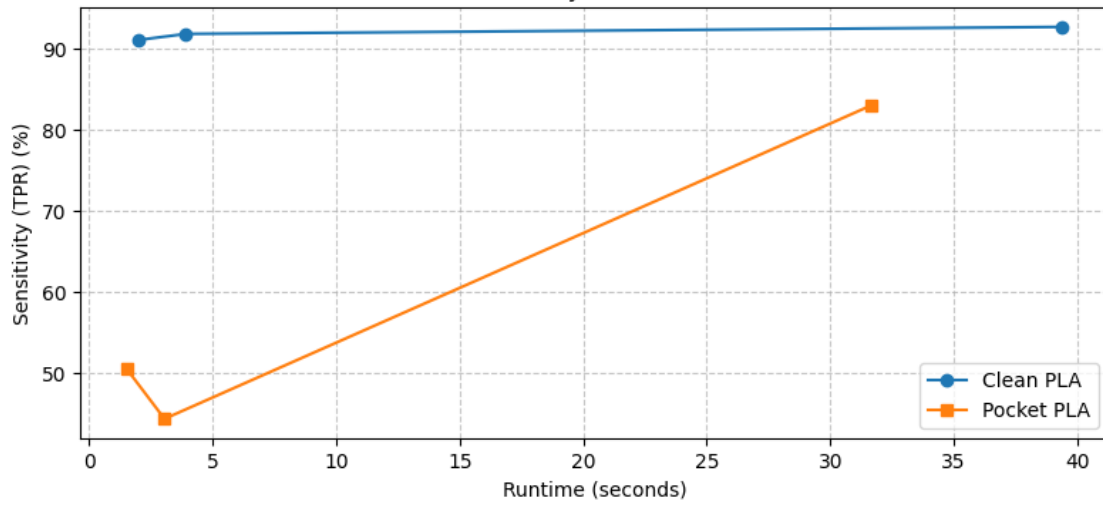


Softmax vs. Linear: TPR/TNR vs. Runtime

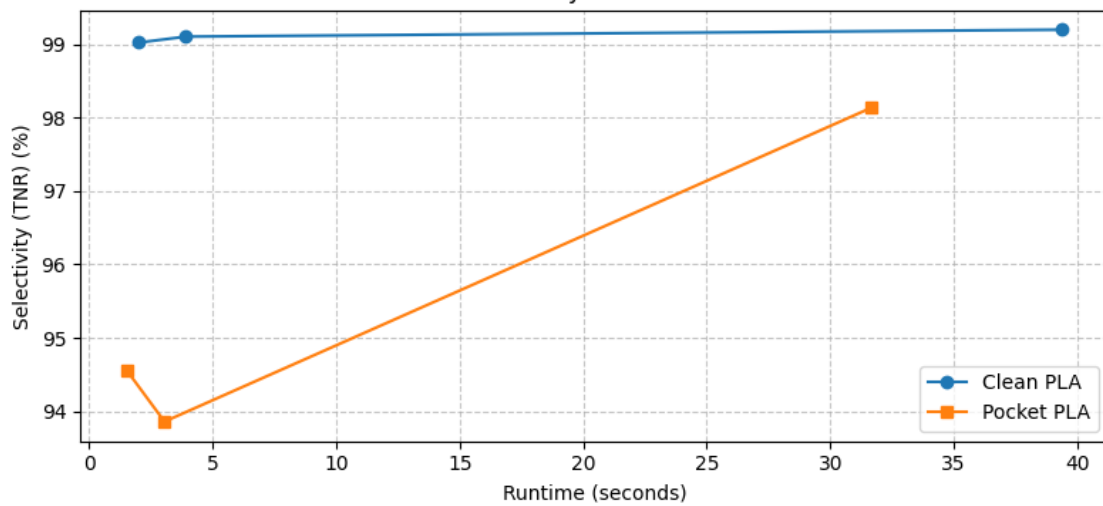
Accuracy vs. Runtime



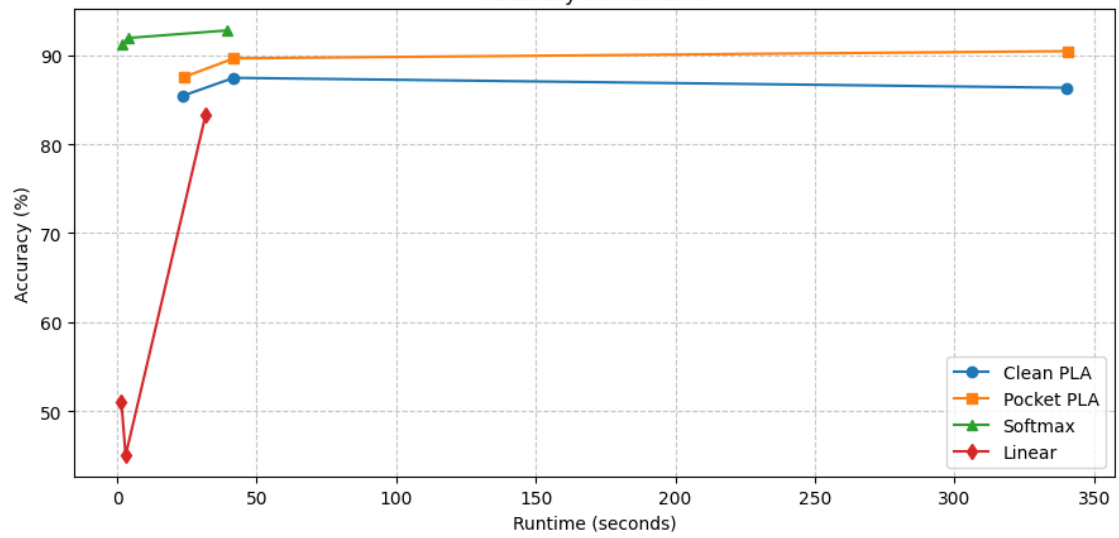
Sensitivity vs. Runtime



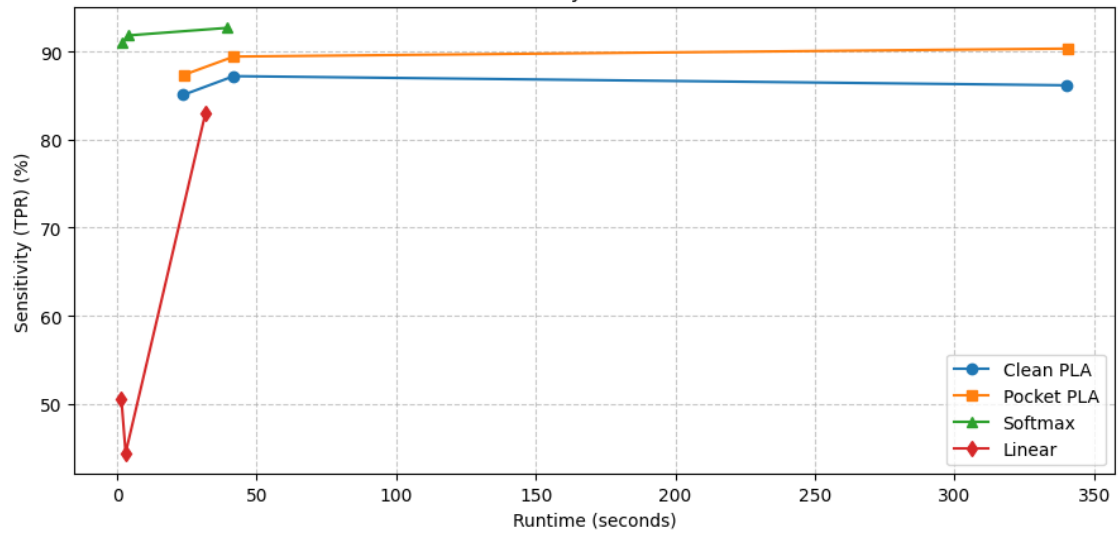
Selectivity vs. Runtime



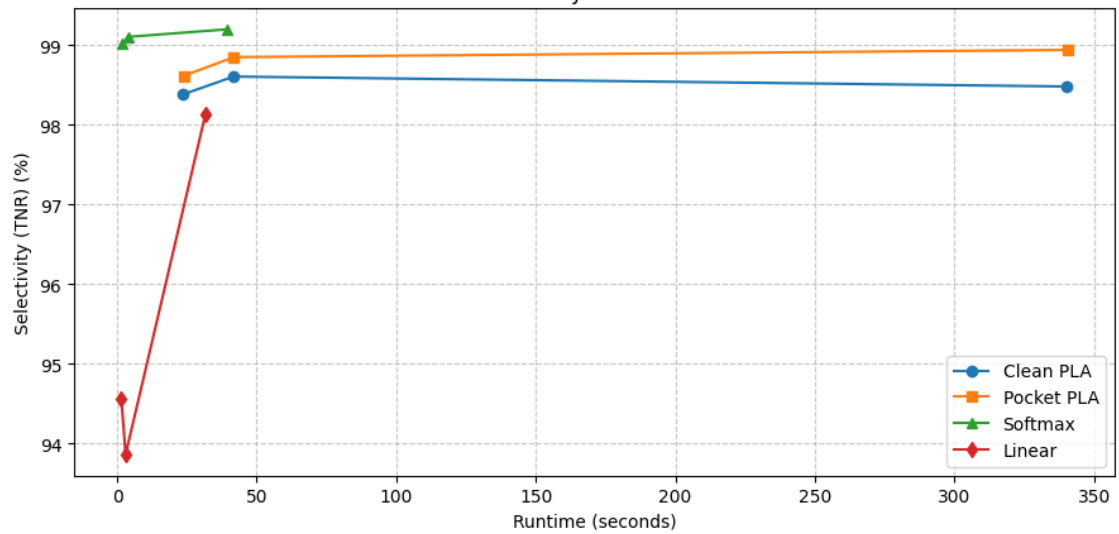
Performance vs. Runtime (4-Model Comparison)
Accuracy vs. Runtime

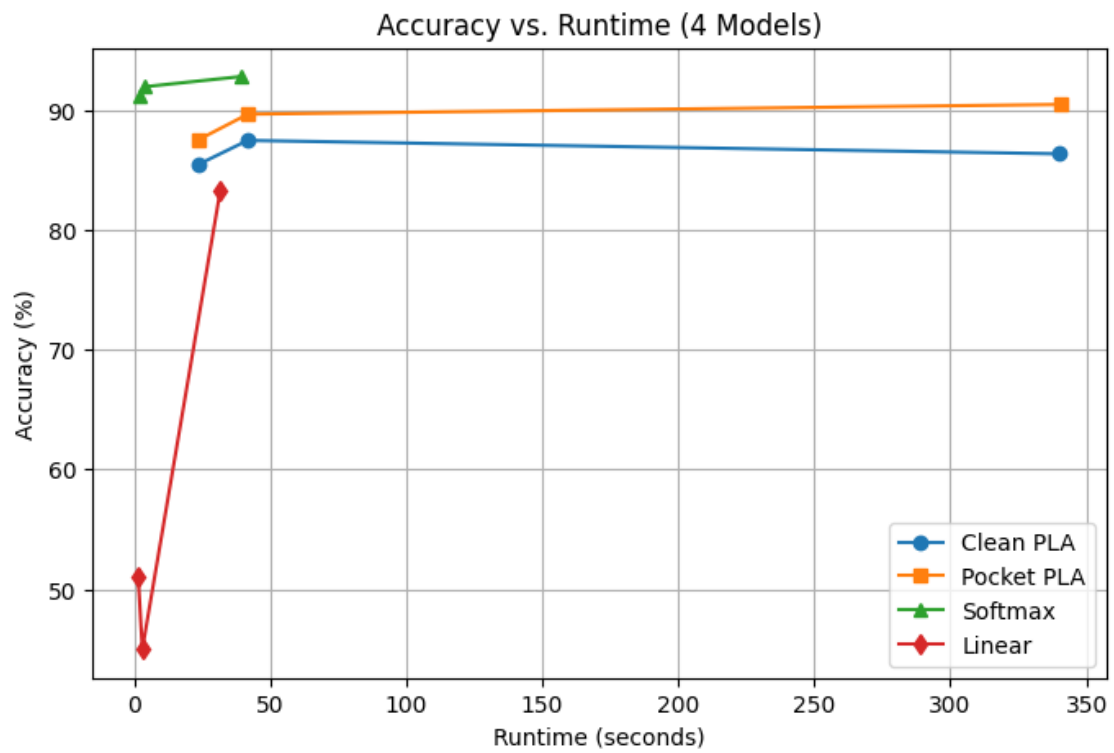


Sensitivity vs. Runtime



Selectivity vs. Runtime





INFO - === All Visualizations Complete ===