ANALOG AND DIGITAL COMMUNICATION

EC8394 III Semester for CSE & IT

Problems:

1) Express a wavelength of 2.4 m in term of frequency.

Ans: To find: Frequency

Formula: $f = c/\lambda = \frac{3 \times 10^8}{2.4} = 125 MHz$

Answer $f = 125 MHz$

2) Find the wavelength of a signal at each of the following frequencies

a) 950 MHz b) 2.9 GHz c) 38 GHz d) 27 KHz

Solution:

To find: wavelength

$\lambda = c/f$ => Formula to find wavelength

a) $\lambda = \frac{3 \times 10^8}{950 MHz} = \frac{3 \times 10^8}{950 \times 10^6} = \frac{300 \times 10^6}{950 \times 10^6}$

$= 0.3157 = 0.316 m$

b) $\lambda = \frac{3 \times 10^8}{2.9 GHz} = \frac{3 \times 10^8}{2.9 \times 10^9} = \frac{3 \times 10^8}{29 \times 10^8} = \frac{3}{29}$

$= 0.1034 = 0.1 m$

c) $\lambda = \frac{3 \times 10^8}{38\ GHz} = \frac{3 \times 10^8}{38 \times 10^9} = \frac{3}{380}$

$= 7.89\ mm$

d) $\lambda = \frac{3 \times 10^8}{27\ kHz} = \frac{3 \times 10^8}{27 \times 10^3} = 11.11\ km$

$= 11.11\ km$

3) A 400 Watt Carrier is modulated to a depth of 75 percent. Calculate the total Power in the modulated wave?

Solution:

Carrier power $P_c = 400W$

Depth of modulation (or) modulation index

$m_a = 75\%$.

To find: Total power, P_T

$$\boxed{P_T = P_c\left[1 + \frac{m_a^2}{2}\right]} \Rightarrow \text{formula to find } P_T$$

$$P_T = 400\left[1 + \frac{(0.75)^2}{2}\right]$$

$$= 400\left[1 + \frac{0.562}{2}\right] = 400\left[1 + 0.281\right]$$

$$P_T = 400\left[1.281\right] = 512.5W$$

Ans: Total Power $P_T = 512.5\ W$

4) A broadcast radio transmitter radiates 10 kilowatts when the modulation percentage is 60. calculate the carrier power.

Solution:

Total power $P_T = 10kW = 10 \times 10^3 W$

modulation percentage $(m_a) = 60\% = 0.6$

To find: Carrier power.

$$P_T = P_c \left[1 + \frac{m_a^2}{2}\right]$$

$$P_c = \frac{P_T}{\left[1 + \frac{m_a^2}{2}\right]} = \left[\frac{10 \times 10^3}{1 + \frac{(0.6)^2}{2}}\right]$$

$$= \frac{10 \times 10^3}{1.18} = 8474.5 = 8.47 \, kW$$

Ans: Carrier power $P_c = 8.47 \, kW$

5) A transmitter radiates 10.125 kW with the unmodulated carrier of 9kW. calculate the modulation index and percent of modulation. If another sine wave, corresponding to 40 percent modulation is transmitted simultaneously. Determine the total radiated power?

Solution: $P_c = 9kW$, $P_T = 10.125 kW$ $m_b = 40\% = 0.4$

To find: modulation index (m_a), % of modulation

total radiated power

$$P_T = P_c\left[1 + \frac{m_a^2}{2}\right]$$

$$P_T/P_c = 1 + \frac{m_a^2}{2} \Rightarrow P_T/P_c - 1 = \frac{m_a^2}{2}$$

$$\frac{10.125}{9} - 1 = \frac{m_a^2}{2}$$

$$1.125 - 1 = m_a^2/2$$

$$0.125 = m_a^2/2$$

$$m_a^2 = 0.25$$

$$\boxed{m_a = 0.50}$$

Total modulation $(m_T) = \sqrt{m_a^2 + m_b^2}$

$$= \sqrt{(0.5)^2 + (0.4)^2}$$

$$= \sqrt{0.25 + 0.16}$$

$$= \sqrt{0.41} = 0.64$$

$$m_T = 0.64$$

Total radiated Power

$$P_T = P_c\left[1 + \frac{m_T^2}{2}\right]$$

$$= 9\left[1 + \frac{(0.64)^2}{2}\right]$$

$$= 9[1 + 0.204] = 9[1.204]$$

$$= 10.836 = 10.84\,W$$

6 A 1 MHz Carrier with an amplitude of 1 volt Peak is modulated by a 1 KHz signal with $m_a = 0.5$. Calculate upper side band, lower side band and side band voltage.

Solution:

Carrier frequency $f_c = 1 MHz$

Amplitude of carrier / Carrier voltage $A_c = 1$ volt

modulating frequency $f_m = 1 KHz$

modulation index $m_a = 0.5$

Upperside band (USB)

$$f_{USB} = f_c + f_m = 1 MHz + 1 KHz$$
$$= 1 + 0.001$$
$$= 1.001 MHz$$

lowerside band (LSB)

$$f_{LSB} = f_c - f_m = 1 MHz - 1 KHz$$
$$= 1 - 0.001$$
$$= 0.999 MHz$$

Side band Voltage

$$V = \frac{m_a A_c}{2} = \frac{0.5 \times 1}{2} = \frac{0.5}{2}$$
$$= 0.25 V$$

Problem 7

AM radio channel bandwidth is 10 kHz

what is the maximum modulating frequency?

given:

Bandwidth $B = 10 kHz$

To find : f_m

$B = 2f_m$

$f_m = B/2 = 10/2 = 5 kHz$

$\boxed{f_m = 5 kHz}$

8) A modulating signal $20 \sin(2\pi \times 10^3 t)$ is used to modulate a carrier signal $40 \sin(2\pi \times 10^4 t)$. Find out

i) Modulation index ii) % modulation

iii) frequencies of sideband components and their amplitudes

iv) Bandwidth of the modulating signal &

v) Also draw the spectrum of the AM wave.

Solution:

modulating signal $A_m(t) = 20 \sin(2\pi \times 10^3 t)$

Carrier signal $A_c(t) = 40 \sin(2\pi \times 10^4 t)$

General eauation bor modulating Signal

$$= A_m \sin(2\pi \times b_m t)$$

$$= 20 \sin(2\pi \times 10^3 t)$$

$A_m = 20$ $\quad b_m = 10^3 = 1\,KHz$

General eauation bor modulating Signal

$$= A_c \sin(2\pi \times b_c t)$$

$$= 40 \sin(2\pi \times \overset{10^4}{\cancel{a}}\, t)$$

$A_c = 40$ $\quad b_c = 10^4 = 10 \times 10^3 = 10\,KHz$

i) modulation index

$$m_a = \frac{A_m}{A_c} = \frac{20}{40} = 0.5$$

ii) % of modulation $\Rightarrow 0.5 \times 100 = 50\%$

iii) sidebands

$$b_{USB} = b_c + b_m = 10 + 1 = \cancel{9}\ 11$$

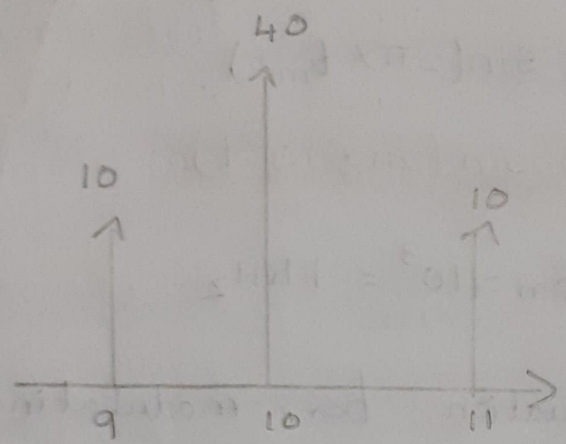
$$b_{LSB} = b_c - b_m = 10 - 1 = 9$$

iv) Voltage of sidebands

$$= \frac{m_a A_c}{2} = \frac{A_m}{2} = \frac{20}{2} = 10\,V$$

v) Bandwidth $\quad B = 2 b_m = 2 \times 1 = 2\,KHz$

Frequency Spectrum



9) For an AM DSB-FC wave with peak unmodulated carrier voltage $V_c = 10V$, a load resistance $R_L = 10\Omega$ and a modulation coefficient $m_a = 1$ determine.

i) Power of the carrier and the upper sidebands.

ii) Total sideband power

iii) Total power of the modulated wave, and

iv) Draw the power spectrum.

Given:

Carrier Voltage $V_c = 10V$

load resistance $R_L = 10\Omega$

modulation coefficient $m_a = 1$

Solution:

i) Carrier power $= P_c = \frac{V_c^2}{2R} = \frac{10^2}{2 \times 10} = 5W$

Power of USB:

$= \frac{m_a^2}{4} P_c = \frac{1}{4} \times 5 = 1.25W$

ii) Total sideband power

$$P_{TSB} = P_{LSB} + P_{USB}$$

$$= \frac{m_a^2}{4} P_c + \frac{m_a^2}{4} P_c = \frac{\cancel{2} m_a^2}{\cancel{4}\,2} P_c$$

$$= \frac{m_a^2}{2} P_c = 5/2 = 2.5\,W$$

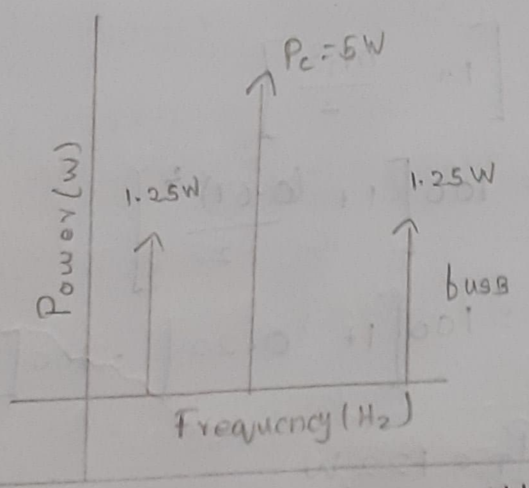
$$\boxed{P_{TSB} = 2.5\,W}$$

iii) Total power in the modulated wave

$$P_T = P_c\left(1 + m_a^2/2\right)$$

$$= 5\left[1 + 1/2\right] = 5\left[3/2\right] = 15/2 = 7.5\,W$$

iv) Diagram for power spectrum



10) For an AM DSB-Fc transmitter with an Unmodulated Carrier power $P_c = 100\,W$ and is modulated simultaneously by three modulating signals with coefficients of modulation $m_1 = 0.2$, $m_2 = 0.4$ and $m_3 = 0.5$ determine.

i) Total modulation, ii) USB and LSB Power iii) P_T

Solution:

i) Total modulation

$$m_T = \sqrt{m_1^2 + m_2^2 + m_3^2}$$

$$= \sqrt{(0.2)^2 + (0.4)^2 + (0.5)^2}$$

$$= \sqrt{0.45} = 0.67$$

$$\boxed{m_T = 0.67}$$

ii) USB and LSB power

$$P_{TSB} = \frac{m_T^2 P_c}{2} = \frac{(0.67)^2 100}{2}$$

$$\boxed{P_{TSB} = 22.45\ W}$$

iii) Total transmitted power

$$P_T = P_c\left[1 + \frac{m_T^2}{2}\right]$$

$$= 100\left[1 + \frac{(0.67)^2}{2}\right]$$

$$= 100\left[1 + 0.22\right] = 100\left[1.22\right]$$

$$\boxed{P_T = 122\ W}$$

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ii) How many AM Broadcast station can be accommodated in 101kHz bandwidth if the highest frequency modulating a carrier is 5kHz?

Solution:

$B = 10 kHz \quad b_m = 5 kHz$

No. of station $= B/b_m$

$= 10/5 = 2$ Stations

12) The tuned-Circuit of the Oscillator in an Am transmitter uses a 50 μH Coil and a 1nF Capacitor. Now, if the Oscillator output is modulated by audio frequencies upto 8 kHz, then find the frequency range occupied by the side bands.

Solution:

Carrier frequency $f_c = \dfrac{1}{2\pi\sqrt{LC}}$

$L = 50 \mu H = 50 \times 10^{-6} H$

$C = 1nF = 1 \times 10^{-9} F$

$$= \frac{1}{2\pi\sqrt{50 \times 10^{-6} \times 1 \times 10^{-9}}}$$

$$= \frac{1}{2\pi\sqrt{50 \times 10^{-15}}} = \frac{1}{2\pi\sqrt{5 \times 10^{-14}}}$$

$$= \frac{1}{2\pi \times 10^{-7} \times \sqrt{5}} = \frac{1 \times 10^{7}}{2 \times 3.14 \times \sqrt{5}}$$

$$= \frac{10^{7}}{14.0} = 714 kHz$$

frequency of sidebands

$$f_c = 714 \qquad f_m = 8$$

$$f_{USB} = f_c + f_m = 714 + 8 = 722 \text{ kHz}$$

$$f_{LSB} = f_c - f_m = 714 - 8 = 706 \text{ kHz}$$

13) i) The peak frequency deviation (Δf) and modulation index (m_f) for an FM modulator with a deviation Sensitivity $k_f = 5 \text{ kHz/V}$ and a modulating Signal. $V_m(t) = 2\cos(2\pi \times 2000t)$.

ii) The peak phase deviation (m_p) for a PM modulator with a deviation sensitivity $k_p = 2.5 \text{ rad/V}$ and a modulating signal $V_m(t) = 2\cos(2\pi \times 2000t)$.

Given:

Deviation Sensitivity $k_f = 5 \text{ kHz/V}$

$k_p = 2.5 \text{ rad/v}$

modulating Signal

$$V_m(t) = 2\cos(2\pi \times 2000t)$$

Soln

General form:

$$V_m(t) = V_m \cos(2\pi f_m t)$$

$$V_m = 2V \qquad f_m = 2000 \text{Hz} = 2 \text{ kHz}$$

To find : Peak frequency deviation (Δf)
modulation index (m_f) FM and PM

i) Peak frequency deviation (Δf)

$$\Delta f = k_f V_m$$

$$= 5 \text{ kHz}/V \times 2V$$

$$\Delta f = 10 \text{ kHz}$$

ii) Modulation index FM

$$m_f = \frac{\Delta f}{f_m} = \frac{10}{2} = 5$$

iii) " " PM

$$m_p = k_p V_m$$

$$= 2.5 \text{ rad}/V \times 2$$

$$m_p = 5 \text{ rad}$$

14) An FM wave with a frequency deviation of 10 kHz and maximum deviation allowed is 25 kHz. Find out the percentage modulation?

Given: $\Delta f (act) = 10 \text{ kHz}$

$\Delta f (max) = 25 \text{ kHz}$

$$\% \text{ of modulation} = \frac{\text{Actuall frequency deviation}}{\text{Maximum allowed deviation}}$$

$$= \Delta f(act) / \Delta f(max) = 10/25 = 0.4$$

$$= 40\%$$

15) An FM wave is represented by the Voltage equation

$V_{FM}(t) = 10\cos(8\times10^{6}t + 2\sin 3\times10^{4}t)$. Calculate

a) modulating frequency b) Carrier frequency

c) modulation index d) Frequency deviation

Given:

$$V_{FM}(t) = 10\cos(8\times10^{6}t + 2\sin 3\times10^{4}t)$$

Soln:

$$= V_c\cos(\omega_c t + m_f \sin\omega_m t)$$

$V_c = 10$ $\omega_c = 8\times10^{6}$ c) $\boxed{m_f = 2}$ $\omega_m = 3\times10^{4}$

a) $f_m = \dfrac{\omega_m}{2\pi} = \dfrac{3\times10^{4}}{2\pi} = 4.77 KHz$

b) $f_c = \dfrac{\omega_c}{2\pi} = \dfrac{8\times10^{6}}{2\pi} = 1.27 MHz$

d) $\Delta f = m_f \times f_m = 2\times4.77 = 9.54 KHz$

16) A 107.6 MHz Carrier Signal is frequency modulated by a 7 kHz sine wave. The resultant FM signal has a frequency deviation of 50 kHz. Determine the following.

i) the Carrier Swing of the FM Signal

ii) the highest and the lowest frequencies attained by the modulated signal, and

iii) the modulation index of the FM wave.

Given: $f_c = 107.6\ MHz$; $f_m = 7\ kHz$

$\Delta f = 50\ kHz$

Solution:

i) Carrier Swing $= 2 \times \Delta f = 2 \times 50 = 100\ kHz$

i) highest frequency

$f_H = f_c + \Delta f = 107.6\ MHz + 50\ kHz$

$= (107.6 \times 10^6) + (50 \times 10^3)$

$= 107600 \times 10^3 + 50 \times 10^3$

$= 107650 \times 10^3\ Hz$

$= 107.65\ MHz$

$f_L = f_c - \Delta f = 107.6\ MHz - 50\ kHz$

$= 107600 \times 10^3 - 50 \times 10^3$

$= 107550 \times 10^3$

$= 107.55\ MHz$

iii) modulation index

$m_f = \frac{\Delta f}{f_m} = \frac{50}{7} = 7.143$

17 Determine

a) the deviation ratio and bandwidth for the worst-case (widest Bandwidth) modulation index for an FM broadcast-band transmitter with a maximum frequency deviation of 75 KHz and a maximum modulating Signal frequency of 15 KHz.

b) the deviation ratio and maximum band width for an equal modulation index with only half the peak frequency deviation and modulating Signal frequency.

Given:

$$\Delta f = 75 kHz \qquad f_m = 15 kHz$$

a) $$DR = \frac{\Delta f}{f_m} = \frac{75}{15} = 5$$

A modulation index of 5 produces eight Significant sidebands

$$B = 2(n \times f_m) Hz$$

$$= 2(8 \times 15 kHz) Hz$$

$$= 2(8 \times 15000)$$

$$= 240000$$

$$B = 240 kHz$$

b) $\Delta f = 75/2 = 37.5 kHz$ $\qquad f_m = 15/2 = 7.5 kHz$

$$m_f = \frac{\Delta f}{f_m} = \frac{37.5}{7.5} = 5 \qquad m_f = 5$$

$$B = 2(n \times f_m)$$
$$= 2(8 \times 7500)$$
$$= 120000 = 120\ KHz$$

18) In an FM System, if the maximum value of deviation is 75KHz and the maximum modulating frequency is 10KHz. Calculate the deviation ratio and bandwidth of the System Using Carson's rule?

Given: $\Delta f(max) = 75KHz$ $f_m = 10KHz$

$$DR = \Delta f / f_m = 75/10 = 7.5$$

ii) System Bandwidth $B = 2(\Delta f(max) + f_m(max))$

$$= 2(75 + 10) = 2(85) = 170\ KHz$$

19) Determine the peak phase deviation for a PM modulator with a deviation sensitivity of 3.5 rad/volt and the modulating signal $V_m(t) = 2.5 \cos(2\pi 1800t)$.

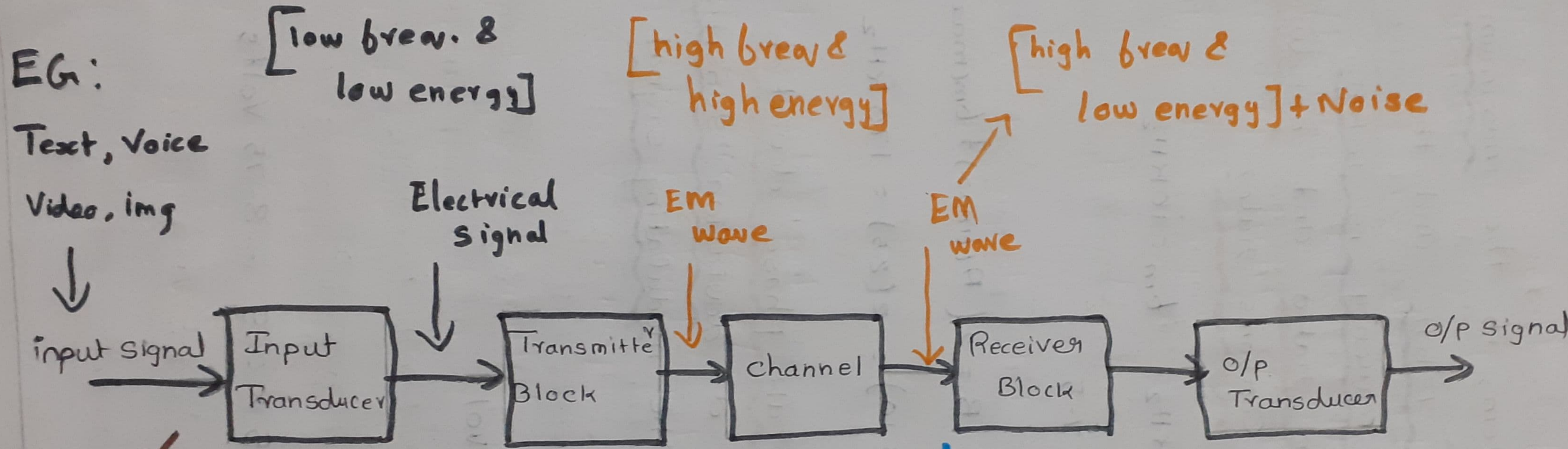
Given:

$K_p = 3.5$ rad/volt

$V_m = 2.5$

Solution:

$$m_p = K_p V_m = 3.5 \times 2.5 = 8.75\ \text{Volts}$$



-> It converts non electrical signal into electrical signal.

-> for voice, microphone

-> Amplifer
-> modulator
-> Filter
-> mixer
-> Antenna

-> wireless
wired

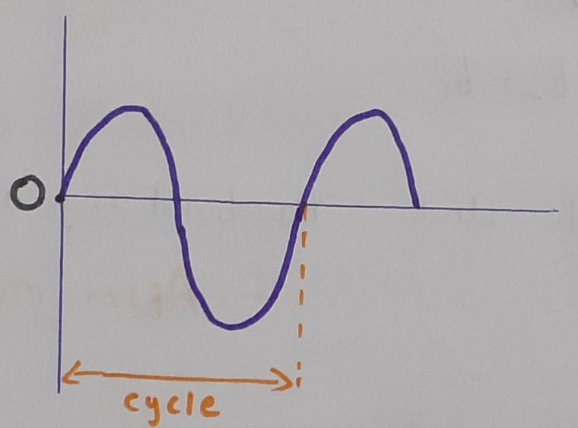
wired:
- optical
- copper

wireless:
Radio

-> Demodulator
-> Amplifer
-> Filter
-> Antenna

Frequency:

Frequency (f) is simply the no. of times a particular phenomenon occurs in a given period of time. It is also called as the no. of cycles of a waveform per second and is nothing but number of cycles per second (CPS)



$1\,KHz = 10^3\,Hz$

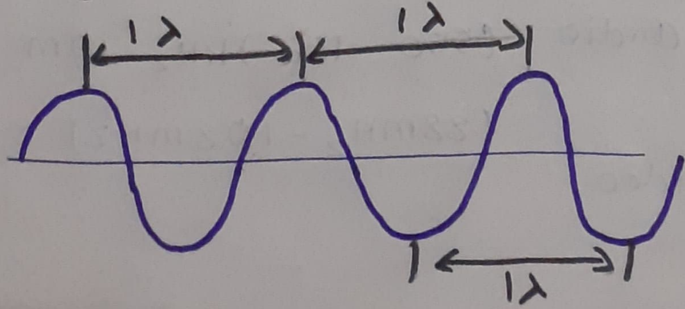
$1\,MHz = 10^6\,Hz$

$1\,GHz = 10^9\,Hz$

Wavelength:

Wavelength (λ) is defined as the ~~difference~~ distance between two points of similar cycles of a periodic wave.

It is also defined as the distance traveled by an EM wave during the time of one cycle.



$$\lambda = \frac{\text{Speed of light}}{\text{frequency}}$$

$$\lambda = c/f$$

$$\lambda = \frac{3 \times 10^8\,m/s}{f\,(Hz)}$$

Bandwidth (B)

Bandwidth is defined as "the frequency range over which information signals is transmitted".

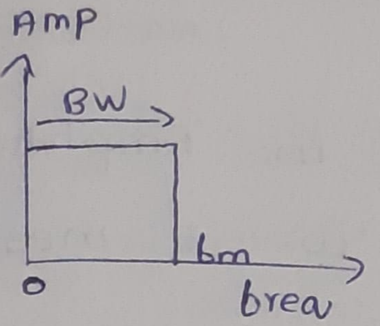
It is also defined as "the difference between the upper and lower frequency limit of the channel that will allow the signal to pass through it.

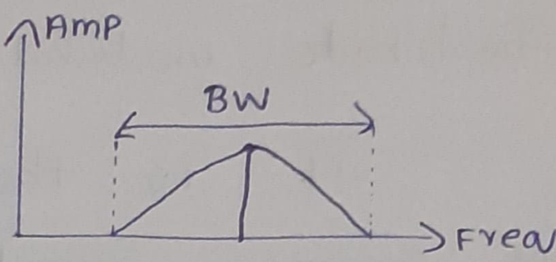
$$BW = f_2 - f_1$$

Baseband Signal and Passband Signal

↳ Pure info ↳ After modulation.

Baseband Signal	Passband Signal
All Source of info. Generates baseband Signal eg: audio, video, Image	Baseband Signal transmitted at high frequency modulated Signal Eg: AM, FM, PM
Signal are transmitted without modulation Eg: Landline	It is high frequency modulated carrier signal Eg: satellite
(0 to 20) KHz - audio signal (0 to 55) MHz - video Signal	(550 - 1650) KHz AM (88 MHz - 108 MHz) FM





Modulation:

It is a process of modification of carrier signal with respect to modulating message signal.

Need of Modulation:

i) height of Antenna:

$f_m = 3KHz \Rightarrow \lambda = c/f = \frac{3\times10^8}{3\times10^3} = 10^5 m$

$= 100000m = 100km$

$L = \lambda/4 = 100/4 = 25km$

ii) Radiated power by antenna:

$f_c = 1GHz \quad \lambda = c/f_c = \frac{3\times10^8}{10^9} = 0.3m$

$L = \lambda/4 = 0.3/4 = 0.075m$

$= 7.5cm$

iii) Multiplexing

$f_1 - 3KHz - 10KHz \longrightarrow f_{c_1} = 10.1GHz$

$f_2 - 3KHz - 10KHz \longrightarrow f_{c_2} = 10.2GHz$

$f_3 - 3KHz - 10KHz \longrightarrow f_{c_3} = 10.3GHz$

Amplitude modulation

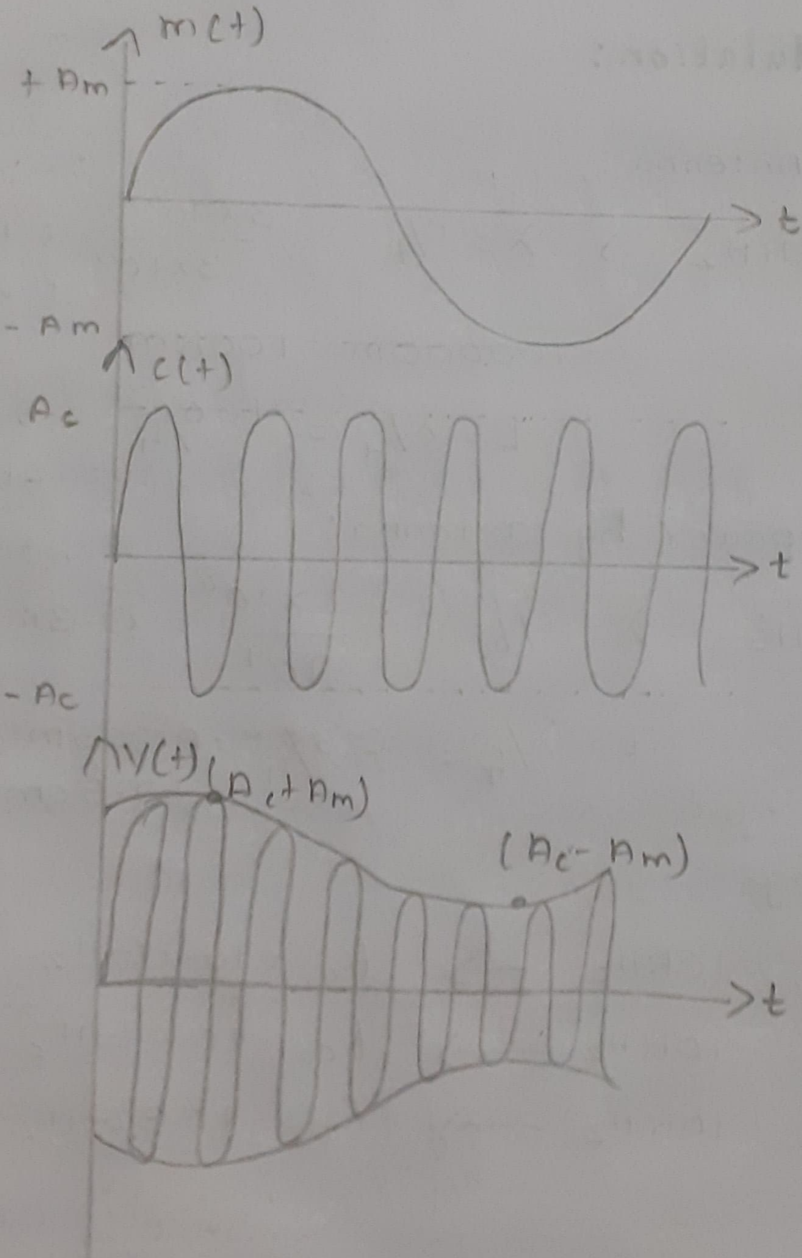
It is the process in Amplitude of carrier signal changes w.r.t message (modulating) signal.

$m(t)$ = modulating signal

$= A_m \sin \omega_m t$

$c(t)$ = Carrier signal

$= A_c \sin \omega_c t$



Derivation :

$$y(t) = A' \sin \omega_c t$$
$$= (A_c + m(t)) \sin \omega_c t$$
$$= (A_c + A_m \sin \omega_m t) \sin \omega_c t$$
$$= A_c\left(1 + A_m/A_c \sin \omega_m t\right) \sin \omega_c t$$

Here $A_m/A_c = \mu \Rightarrow$ modulation index
$\hookrightarrow m_a$

$$= A_c\left(1 + \mu \sin \omega_m t\right) \sin \omega_c t \quad \text{(or)}$$
$$A_c\left(1 + m_a \sin \omega_m t\right) \sin \omega_c t$$

$$= A_c \sin \omega_c t + A_c \mu \sin \omega_m t \sin \omega_c t$$

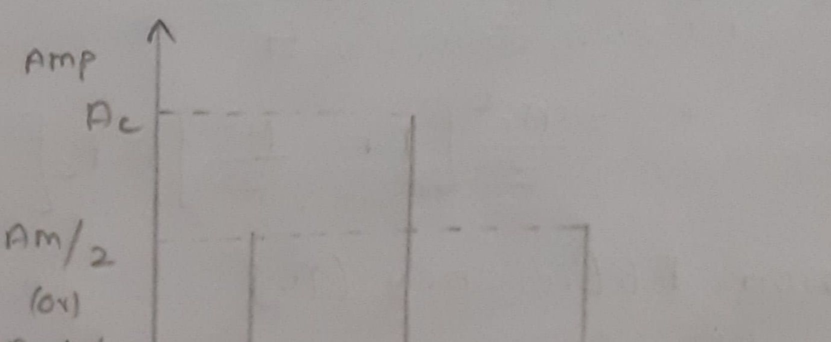
$$y(t) = A_c \sin \omega_c t + \frac{A_c \mu}{2} \cos(\omega_c - \omega_m)t + \frac{A_c \mu}{2} \cos(\omega_c + \omega_m)t$$

This Signal having three frea Component

ω_c, $\omega_c - \omega_m$ $\omega_c + \omega_m$

→ Sideband Amplitude

$$\frac{A_c \mu}{2} = \frac{\cancel{A_c}}{2} \times \frac{A_m}{\cancel{A_c}} = \frac{A_m}{2}$$



Am Signal

sidebands

$$y(t) = A_c \sin \omega_c t + \frac{A_c \mu}{2} \cos(\omega_c - \omega_m)t + \frac{A_c \mu}{2} \cos(\omega_c t + \omega_m)t$$

carrier signal

LSB

Lower Side Band

USB

upper sideband

Total transmitted power (P_t)

$$P_t = P_c + P_{USB} + P_{LSB} = P_c + P_s$$

Power of Carrier (P_c)

$$P_c = \frac{A_c^2}{2}$$

Power of USB and LSB

$$P_{LSB} = P_{USB} = \frac{1}{2}\left(\frac{A_c \mu}{2}\right)^2 = \frac{1}{2}\left(\frac{A_c^2 \mu^2}{4}\right) = \frac{1}{8} A_c^2 \mu^2$$

sideband Powers

$$P_s = P_{USB} + P_{LSB} = \frac{1}{8} A_c^2 \mu^2 + \frac{1}{8} A_c^2 \mu^2$$

$$= \frac{1}{4} A_c^2 \mu^2 = \frac{A_c^2 \mu^2}{4} = P_c \mu^2/2$$

Total Power

$$P_T = P_c + P_s = \frac{A_c^2}{2} + \frac{A_c^2 \mu^2}{4}$$

$$= \frac{A_c^2}{2}\left[1 + \frac{\mu^2}{2}\right] = P_c\left[1 + \frac{\mu^2}{2}\right]$$

Transmission Efficiency (η)

$$\eta = \frac{P_s}{P_t} = \frac{1/4 \, A_c^2 \mu^2}{1/2 \, A_c^2 \left[1 + \mu^2/2\right]} \Rightarrow \frac{\mu^2}{2 + \mu^2}$$

Deviation

$$D = 1 - R = 1 - \frac{\mu^2}{2+\mu^2} = \frac{2+\mu^2-\mu^2}{2+\mu^2} = \frac{2}{2+\mu^2}$$

Multiple tone Amplitude modulation:

let's have Carrier Signal

$$c(t) = A_c \sin\omega_c t$$

multiple tone modulation Signal

$$m(t) = A_1 \sin\omega_1 t + A_2 \sin\omega_2 t + A_3 \sin\omega_3 t$$

Am signal will be

$$y(t) = A' \sin\omega_c t$$

$$= (A_c + m(t)) \sin\omega_c t$$

$$= (A_c + A_1 \sin\omega_1 t + A_2 \sin\omega_2 t + A_3 \sin\omega_3 t) \sin\omega_c t$$

$$y(t) = A_c (1 + A_1/A_c \sin\omega_1 t + A_2/A_c \sin\omega_2 t + A_3/A_c \sin\omega_3 t) \sin\omega_c t$$

$$A_1/A_c = \mu_1 \quad A_2/A_c = \mu_2 \quad A_3/A_c = \mu_3$$

$$y(t) = A_c (1 + \mu_1 \sin\omega_1 t + \mu_2 \sin\omega_2 t + \mu_3 \sin\omega_3 t) \sin\omega_c t$$

Carrier power: $P_c = \frac{A_c^2}{2}$

sideband power: $P_s = 1/4\, A_c^2 \mu_1^2 + 1/4\, A_c^2 \mu_2^2 + 1/4\, A_c^2 \mu_3^2$

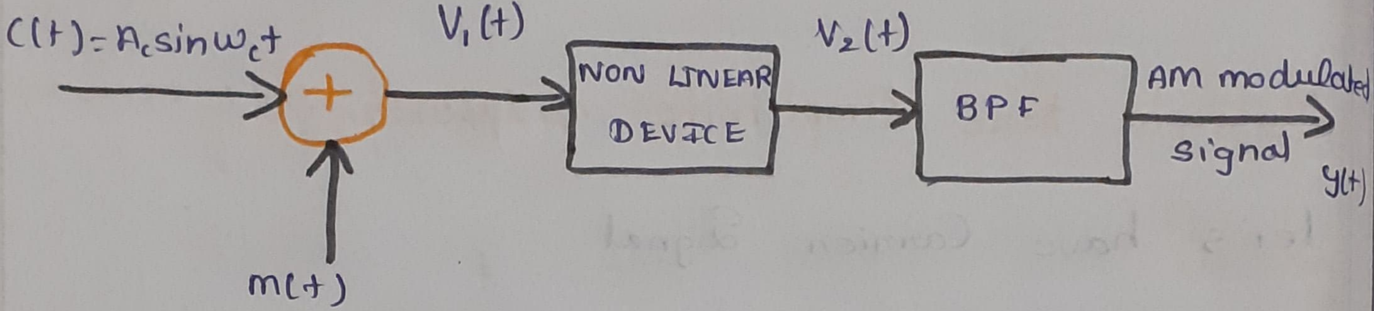
$$= 1/4\, A_c^2 [\mu_1^2 + \mu_2^2 + \mu_3^2] = 1/2\, A_c^2 [\mu]$$

$$P_s = \frac{P_c \mu^2}{2}$$

Total Power:

$$P_t = P_c + P_s = P_c + \frac{P_c \mu^2}{2} = P_c\left[1 + \frac{\mu^2}{2}\right]$$

Square Law modulator:



- Square law modulator has three major Part

1) Adder 2) Non linear device 3) Band Pass Filter (BPF)

i) After Adder:

$$v_1(t) = c(t) + m(t) = A_c \sin\omega_c t + m(t)$$

ii) After Non-linear device

$$V_2(t) = a v_1(t) + b v_1^2(t)$$

$$= a(A_c \sin\omega_c t + m(t)) + b(A_c \sin\omega_c t + m(t))^2$$

$$= aA_c \sin\omega_c t + am(t) + b[A_c^2 \sin^2\omega_c t + m^2(t) + 2A_c \sin\omega_c t\, m(t)]$$

$$= aA_c \sin\omega_c t + \cancel{am(t)} + \cancel{bA_c^2 \sin^2\omega_c t} + \cancel{bm^2(t)} + 2bA_c \sin\omega_c t\, m(t)$$

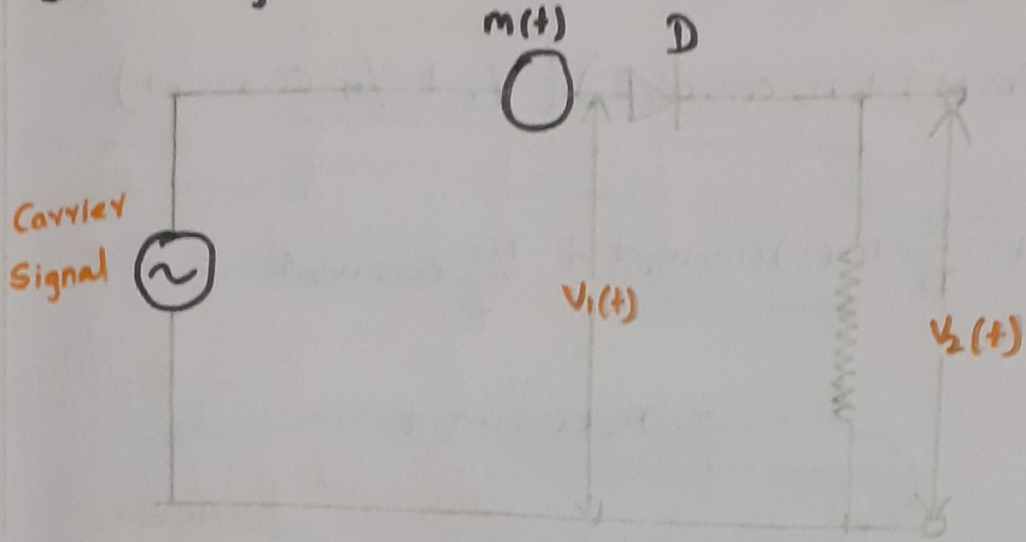
After BPF:

$$= aA_c \sin\omega_c t + 2bA_c \sin\omega_c t\, m(t)$$

$$= aA_c \sin\omega_c t \left(1 + \frac{2b}{a} \cancel{\sin\omega_c}\, m(t)\right)$$

$$= aA_c \left(1 + \frac{2b}{a} m(t)\right) \sin\omega_c t$$

Switching modulator:



→ c(t) and m(t) is connected in series

$$V_1(t) = m(t) + c(t)$$
$$= m(t) + A_c \cos \omega_c t$$

→ Amplitude of c(t) is very greater than m(t) so diode is ON and OFF by c(t).

$$V_2(t) = V_1(t) \quad c(t) > 0$$

$$V_2(t) = 0 \quad c(t) < 0$$

O/P $\quad V_2(t) = V_1(t)\, g_p(t)$

$g_p(t)$ is a periodic pulse train of duty cycle equal to half period $T_0/2$ $\quad T = 1/f_c$

The Fourier Series representation of this Periodic pulse train is

$$g_p(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n-1} \cos((2n-1)\omega_c t)$$

if n = 1

$$= \frac{1}{2} + \frac{2}{\pi} \cos \omega_c t$$

from ① ② and ③

$$V_2(t) = (m(t) + A_c \cos \omega_c t)\left(\tfrac{1}{2} + \tfrac{2}{\pi} \cos \omega_c t\right)$$

$$= \frac{m(t)}{2} + \frac{2}{\pi} m(t) \cos \omega_c t + \frac{A_c}{2} \cos \omega_c t$$

✓ Modulating Signal

$$+ \frac{2}{\pi} A_c \cos^2 \omega_c t$$

↳ 2nd harmonic

After BPF

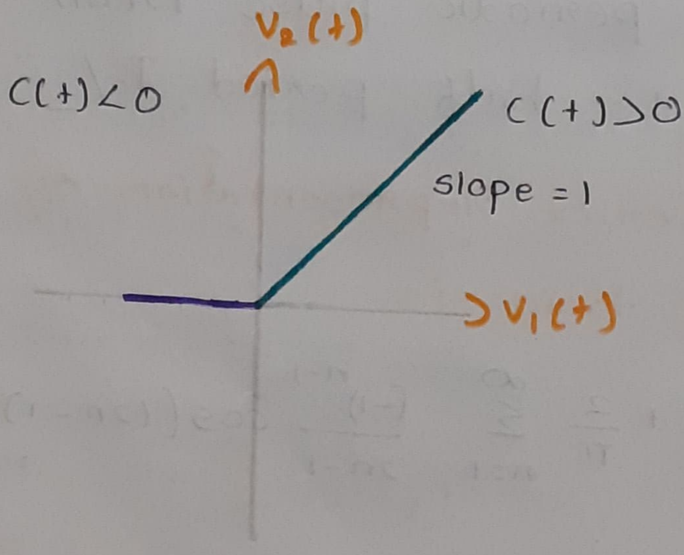
$$y(t) = \frac{2}{\pi} m(t) \cos \omega_c t + \frac{A_c}{2} \cos \omega_c t$$

$$= \frac{A_c}{2} \cos \omega_c t + \frac{2}{\pi} m(t) \cos \omega_c t$$

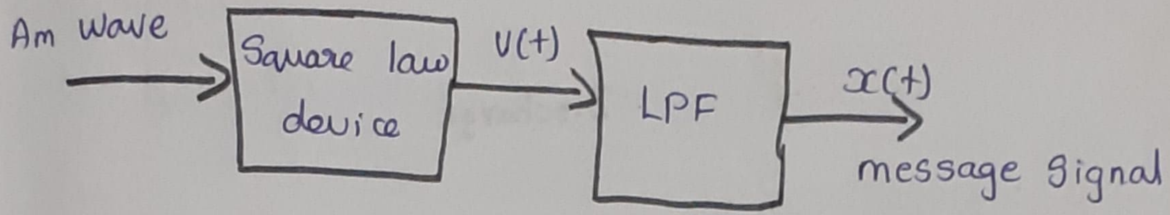
$$= \frac{A_c}{2}\left[1 + \frac{4}{\pi A_c} m(t)\right] \cos \omega_c t$$

$$= \frac{A_c}{2}\left[1 + k_a m(t)\right] \cos \omega_c t$$

↳ amplitude sensitivity



Square law Detector:



Am Signal

$$y(t) = A_c(1 + m\,x(t))\sin\omega_c t$$

V(t) is o/p of Square law device

$$V(t) = a\,y(t) + b\,y^2(t)$$

$$V(t) = a(A_c(1+mx(t))\sin\omega_c t) + b(A_c(1+mx(t))\sin\omega_c t)^2$$

$$= aA_c\sin\omega_c t + amx(t)\sin\omega_c t + bA_c^2\left[1 + m^2x^2(t) + 2mx(t)\right]\sin^2\omega_c t$$

$$= aA_c\sin\omega_c t + amx(t)\sin\omega_c t + bA_c^2\sin^2\omega_c t + bA_c^2m^2x^2(t)\sin^2\omega_c t + bA_c^2\,2mx(t)\sin^2\omega_c t$$

$$= aA_c\sin\omega_c t + amx(t)\sin\omega_c t + bA_c^2\sin^2\omega_c t + bA_c^2m^2x^2(t)\sin^2\omega_c t + \overset{b}{\cancel{b}}A_c^2\cancel{2}mx(t)\left(\frac{1-\cos 2\omega_c t}{\cancel{2}}\right)$$

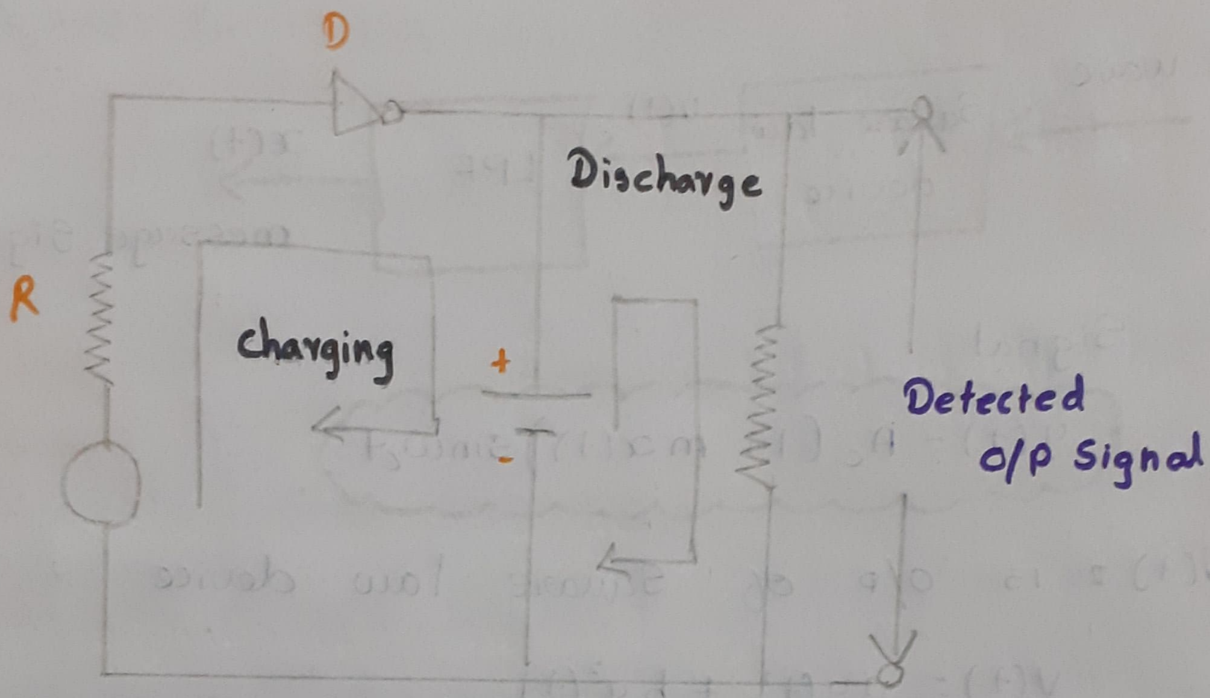
$$= aA_c\sin\omega_c t + amx(t)\sin\omega_c t + bA_c^2\sin^2\omega_c t + bA_c^2m^2x^2(t)\sin^2\omega_c t + \boxed{bA_c^2mx(t)} - bA_c^2mx(t)\cos 2\omega_c t$$

↳ message signal.

After LPF

$$= bA_c^2\,mx(t)$$

Envelope Detecton:



- Used to detect (demodulate high level AM wave.

- Envelope detector is also called as the diode detector

- input => AM wave

- capacitor charges very Quickly and discharges very slowly

DSB-SC Double sideband - Suppressed Carrier

Am Signal:

$$y(t) = A_c \cos\omega_c t + \frac{A_c \mu}{2} \cos(\omega_c + \omega_m)t$$
$$+ \frac{A_c \mu}{2} \cos(\omega_c - \omega_m)t$$

$$y(t) = A_c (1 + \mu x(t)) \cos\omega_c t$$

$$y(t) = \underbrace{A_c \cos\omega_c t}_{\text{carrier}} + \mu A_c x(t) \cos\omega_c t \rightarrow \text{Sideband}$$

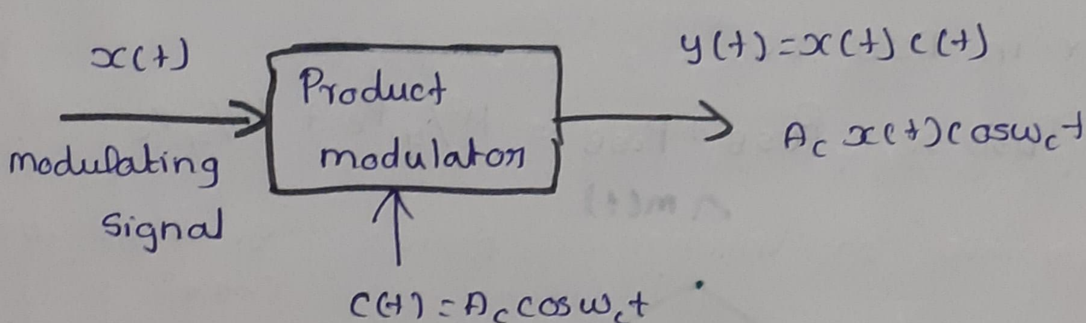
In Power transmission

$$P_t = \boxed{P_c} + P_s = P_c + \frac{\mu^2 P_c}{2}$$

for $\mu = 1$

$$P_t = P_c + \frac{1}{2} P_c = \boxed{P_c} + 0.5 P_c$$

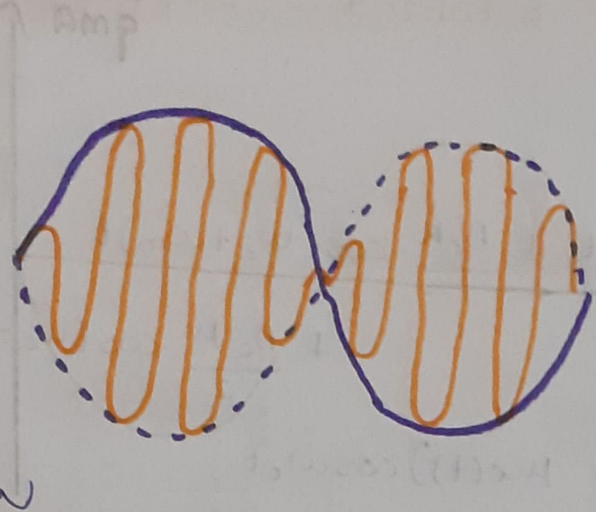
Block diagram:

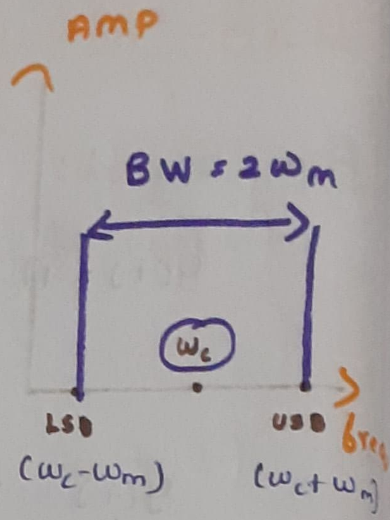


In DSB-SC

- we don't send Carrier Signal
- Only LSB & USB Signal is there
- It has 180° phase required at zero

Crossing of modulating signal





SSB-SC Single sideband Suppressed Carrier

AM signal:

$$y(t) = A_c \cos\omega_c t + \frac{A_c\mu}{2}\cos(\omega_c+\omega_m)t + \frac{A_c\mu}{2}\cos(\omega_c-\omega$$

Power in AM: => $P_t = P_c + P_s = P_c + \frac{\mu^2 P_c}{2}$

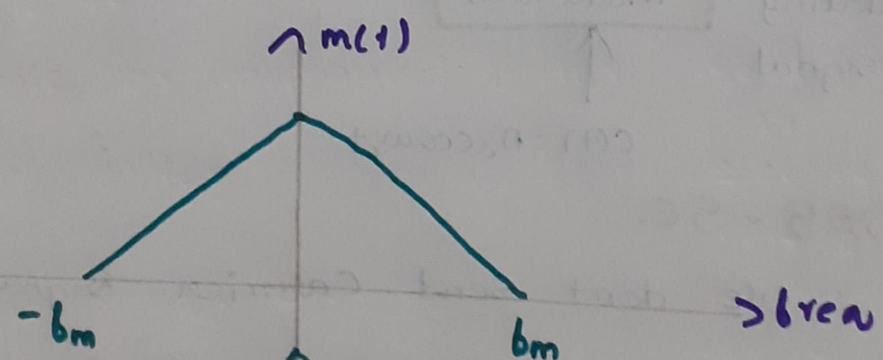
Bandwidth in AM & DSB-SC => $B = 2f_m$

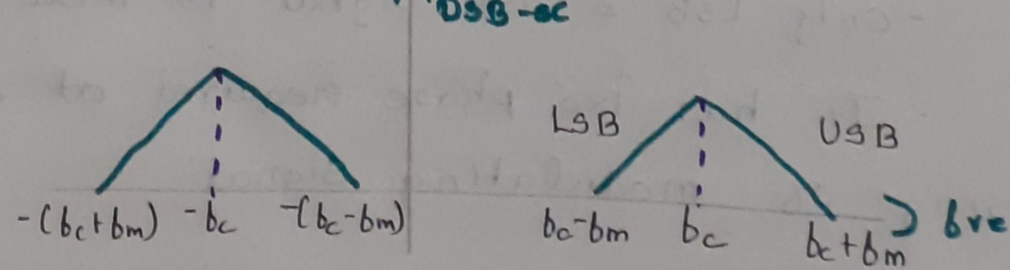
DSB-SC Signal

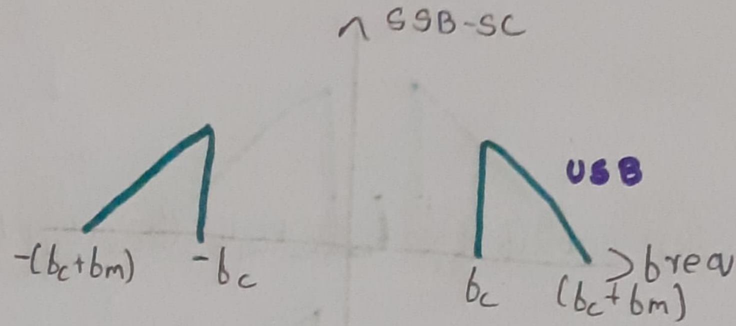
$$y(t) = \frac{\mu A_c}{2}\cos(\omega_c+\omega_m)t + \frac{\mu A_c}{2}\cos(\omega_c-\omega_m)t$$

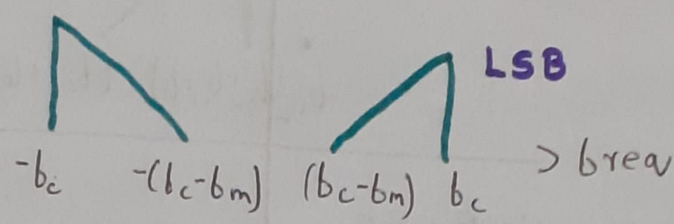
Power in DSB-SC

$$P_{DSB\text{-}SC} = P_{USB} + P_{LSB} = \mu^2/_4\, P_c + \mu^2/_4\, P_c$$

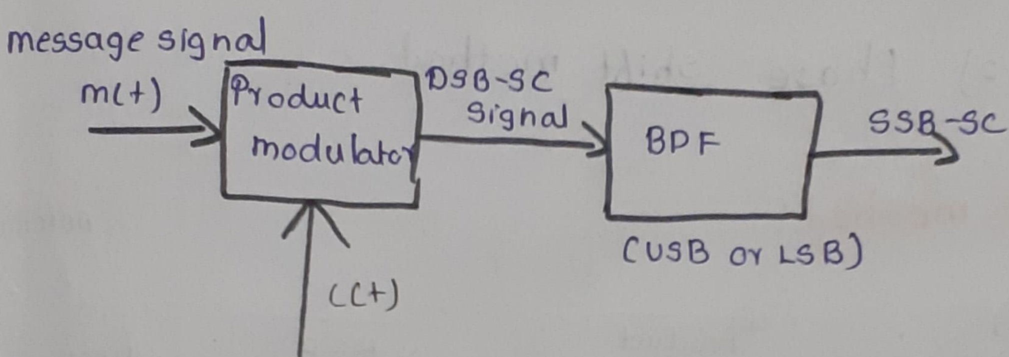


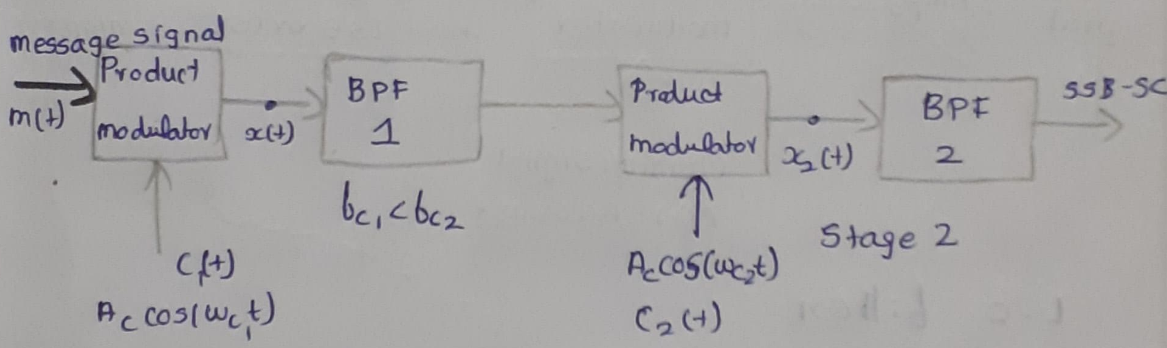


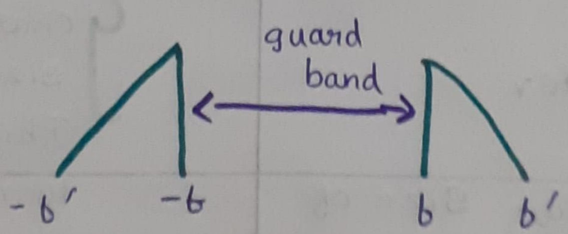


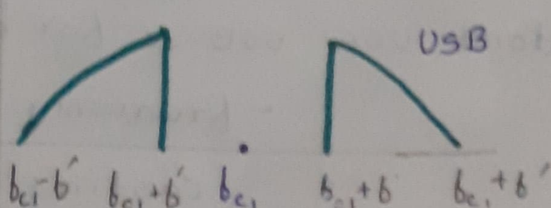


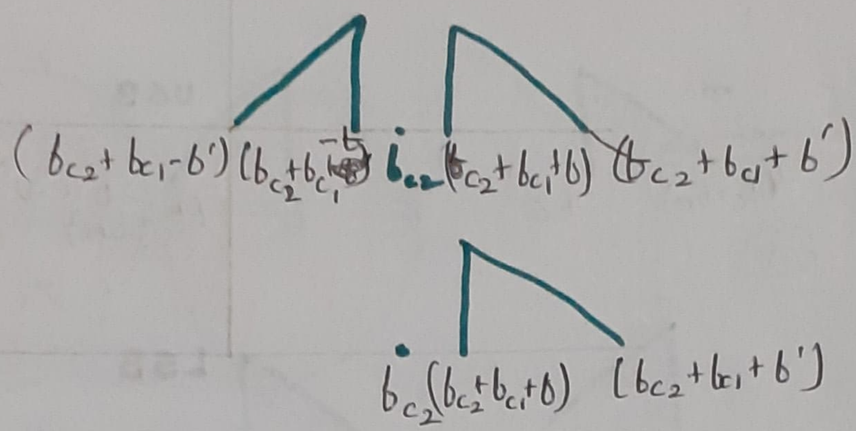
Block diagram:







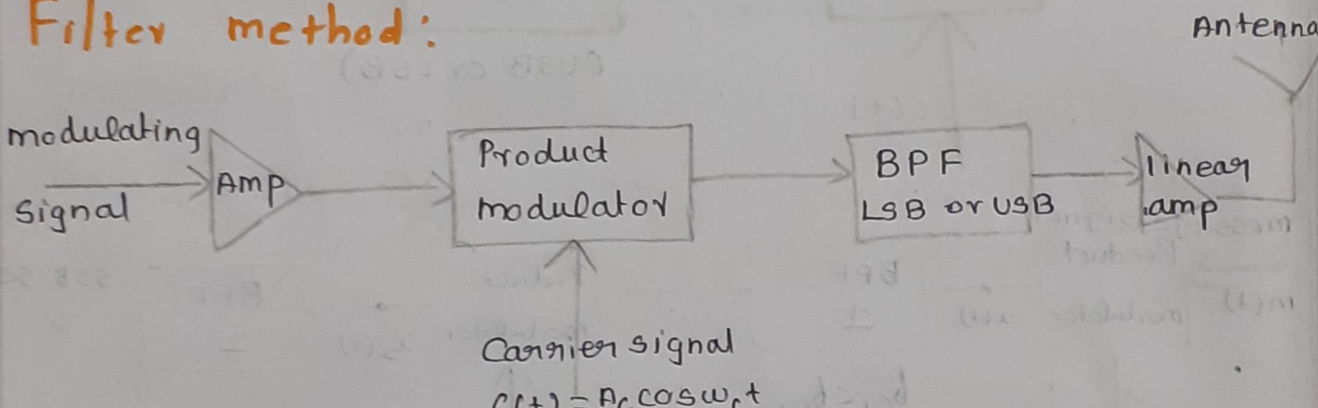




Generation of SSB-SC

1) Filter method
2) Phase shift method

Filter method:



L-C filter

Ceramic or mechanical filter } good passband char

crystal filter } size is less

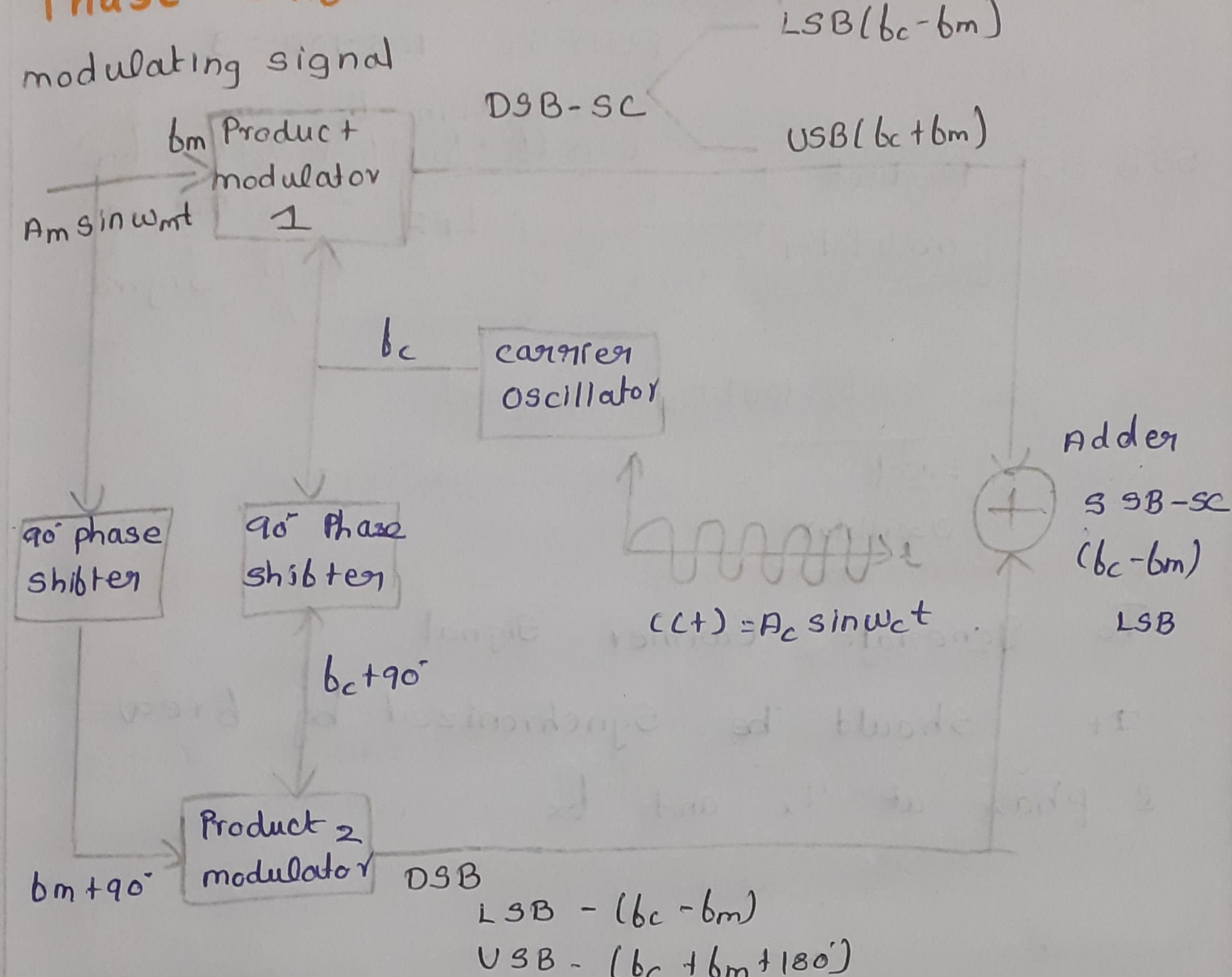
This filter works above 1MHz

For low freq. size of filter will be bulky

we don't use SSB-SC for broad casting

- frequency stability
- casting tunnable filter

Phase shift method :



Parameter	Filter method	Phase shift of 90°
Sideband cancellation	- By using filter	- By using phase shifter of 90°
SSB freq range	- Freq should be high	For any Freq
Up conversion	Needed	Not Needed
linear Amp	Needed	Needed
System design	- Filter designing - stability of freq - size	- phase shifter of 90° - product modulator

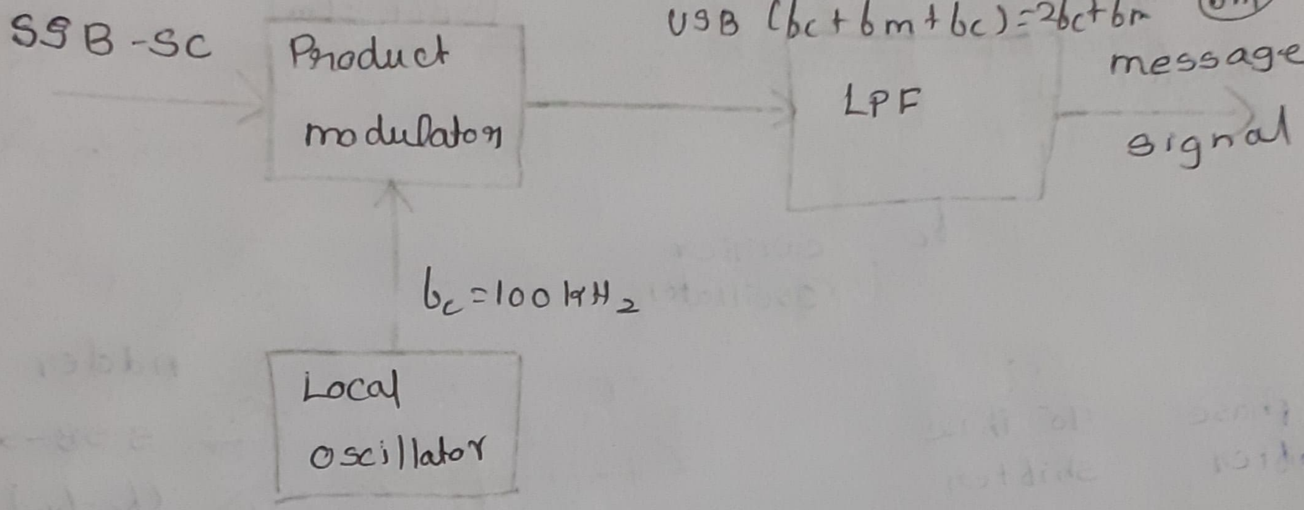
Synchronous detecton :

$(f_c + f_m)$ 103 kHz

DSB-SC 103 - 100 = 3

LSB $(f_c + f_m - f_c) = f_m$

USB $(f_c + f_m + f_c) = 2f_c + f_m$ f_m



To generate Carrier signal

It should be synchronized by frea

& phase at Tx and Rx

Angle modulation

In angle modulation, freq or phase of carrier signal changes with respect to modulating signal.

Frequency modulation

- In freq modulation freq of carrier signal changes with respect to time modulating signal

$y(t) = A_c \cos(w_c t) + \phi)$

$w(t) = F(m(t))$

Phase modulation

- In phase modulation phase of carrier signal changes with respect to modulating signal

$y(t) = A_c \cos(w_c t + \phi(t))$

$\phi(t) = F(m(t))$

Advantage

- Noise reduction
- improved system fidelity
- Efficient use of power.

Application:

Radio broadcasting

TV sound transmission

cellular radio

Frequency modulation (FM)

In frequency modulation, Frequency of Carrier Signal $c(t)$ changes with respect to modulating signal $m(t)$

- If we have Carrier Signal

$$c(t) = E_c \cos(\omega_c t + \phi)$$
$$= E_c \cos(f_c t + \phi)$$
$$= E_c \cos \theta t$$

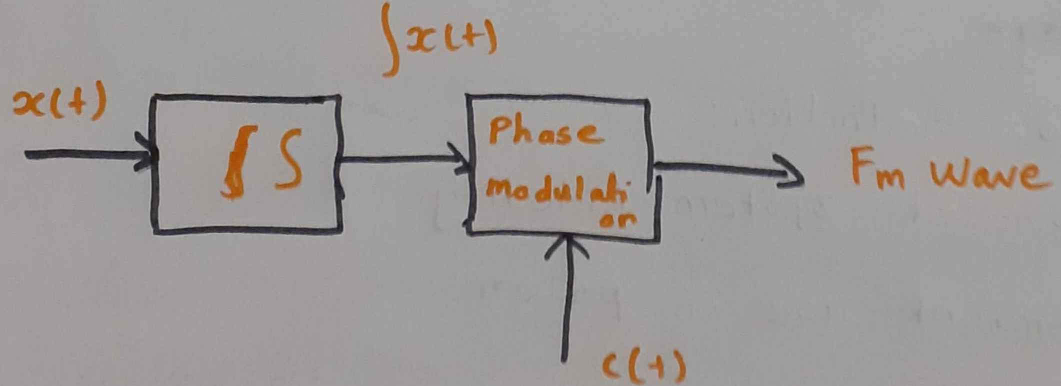
- In frequency modulation

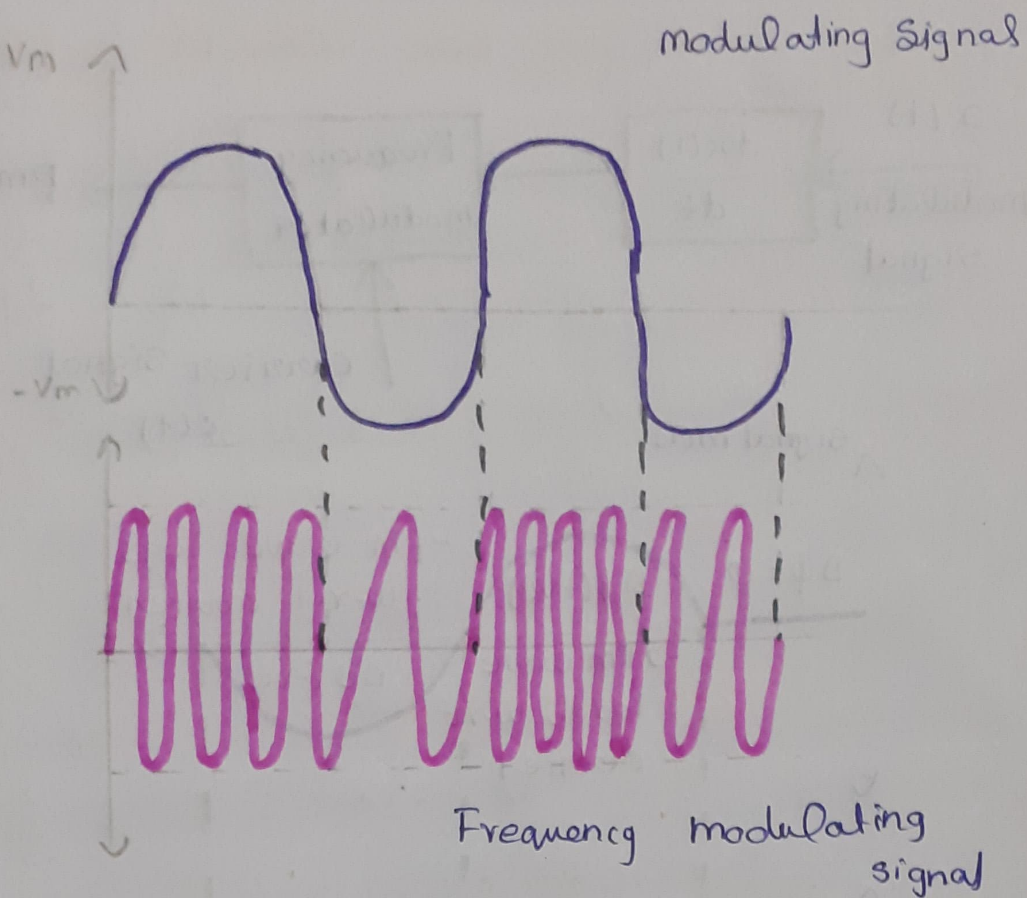
$$\theta(t) = 2\pi f_c + 2\pi k_f \int_0^t x(t)\, dt$$

modulating signal

$$y_{FM}(t) = E_c \cos\left(2\pi f_c t + 2\pi \boxed{k_f} \int_0^t x(t)\, dt\right)$$

↳ freq. Sensitivity





Phase modulation:

In phase modulation, phase of carrier signal changes with respect to modulating signal $(m(t))$.

If we have carrier signal

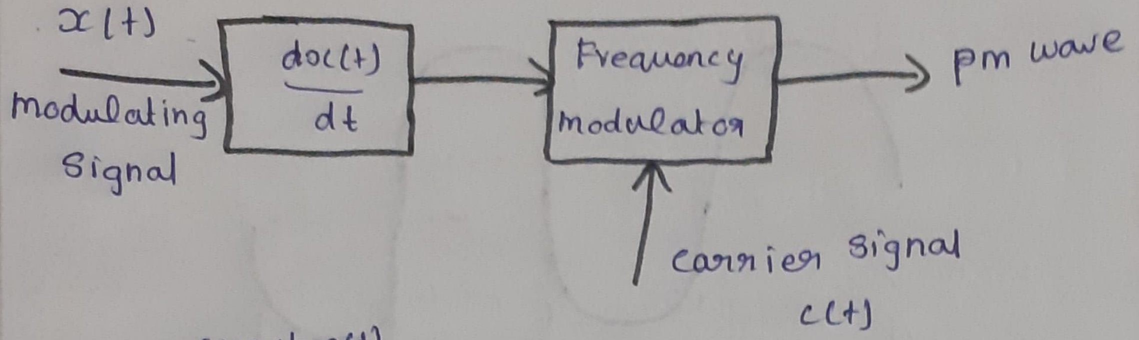
$$c(t) = E_c \cos(\omega_c t + \phi)$$
$$= E_c \cos(2\pi f_c t + \phi)$$

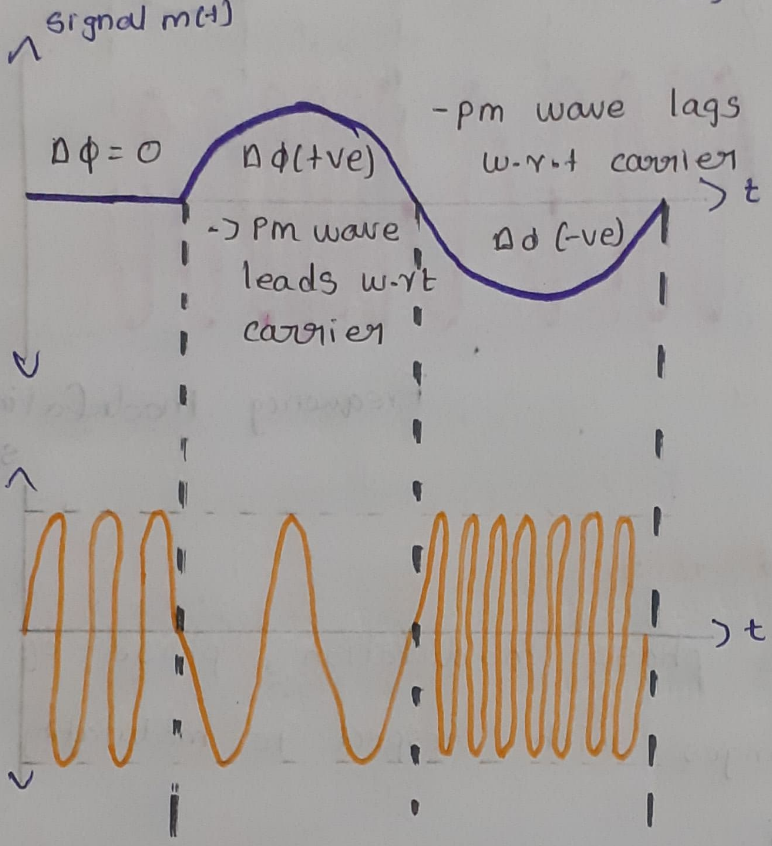
For phase modulation

$$\theta(t) = 2\pi f_c t + k_p x(t)$$

Pm wave $y_{pm}(t) = E_c \cos(2\pi f_c t + k_p x(t))$

↳ Phase Sensitivi


x(t)
modulating signal
dx(t)/dt
Frequency modulator
pm wave
carrier signal c(t)


signal m(t)
Δφ = 0
Δφ (+ve)
-> pm wave leads w.r.t carrier
- pm wave lags w.r.t carrier
Δφ (-ve)
t
t

Frequency deviation and modulating index

lets have modulating signal

$$x(t) = A_m \cos(2\pi f_m t)$$

Carrier Signal

$$e_c = A_c \sin(2\pi f_c t)$$

→ freq of f_m signal

$$F_T(t) = f_c + k_f x(t)$$

└ frequency sensitivity

$$= f_c + k_f A_m \cos(2\pi f_m t)$$

$$= f_c + \Delta f \cos(2\pi f_m t)$$

Here $\Delta f = k_f A_m$, freq deviation

max. freq. deviation = $f_c + \Delta f$

min. freq. deviation = $f_c - \Delta f$

→ FM signal

$$y_{FM}(t) = A_c \sin\left(\omega_c t + k_f 2\pi \int_0^t x(t)\, dt\right)$$

$$= A_c \sin\left(\omega_c t + k_f 2\pi \int_0^t A_m \cos(2\pi f_m t)\, dt\right.$$

$$= A_c \sin\left(\omega_c t + \frac{k_f \cancel{2\pi} A_m}{\cancel{2\pi} f_m}\left[\sin(2\pi f_m t\right]\right.$$

$$= A_c \sin\left(\omega_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t)\right)$$

└→ modulation index for fm

$$= A_c \sin\left(\omega_c t + m_f \sin(2\pi f_m t)\right)$$

Deviation ratio = $\frac{\text{max deviation}}{\text{max modulating freq}}$

$= \frac{\Delta f\,(max)}{f_m\,(max)}$

modulation index	sidebands
1	3
2	4
3	6
4	7
5	8 (max)

% of modulation of Fm = $\frac{\text{Actual freq deviation}}{\text{maximum freq deviation}}$

Spectral component of Angle modulated Signal

- In Angle modulation, there is freq & phase modulation
- spectral components are identical for freq & phase modulators
- For phase modulator

$e(t) = E_c \sin(\omega_c t + m(\cos\omega_m t))$

modulating index

$\Delta f / f_m$

Bessels function:

$$: E_c\Big[J_0 \sin \omega_c t + J_1[\sin(\omega_c t + \omega_m t) + \sin(\omega_c - \omega_m)t]$$

$$+ J_2[\sin(\omega_c + 2\omega_m)t + \sin(\omega_c - 2\omega_m)t] +$$

$$J_3[\sin(\omega_c + 3\omega_m)t + \sin(\omega_c - 3\omega_m)t] + \cdots\Big]$$

Here $J_0, J_1, J_2 \ldots$ Bessels function it's value is depending on modulating index. (m)

Angle modulation — Carrier signal amplitude $= J_0 E_c$

— Sideband amplitude $= J_1 E_c, J_2 E_c, J_3 E_c, \ldots$

Higher modulation index results into higher spectral components which increases the bandwidth of Angle modulated signal.

Bandwidth Requirement in angle modulation:

- In Angle modulation, there is Freq & phase modulation

- Spectral components are identical for freq & phase modulators

- For phase modulator

$$e(t) = E_c \sin(\omega_c t + m \cos \omega_m t)$$

=

Bandwidth Requirement in angle modulation

modulation index $m = \Delta_f / f_m$

- modulation index has three categories
  - low (less than 1)
  - medium (between 1 to 10)
  - high (greater than 10)

- minimum Bandwidth

$$B = 2f_m \quad (m<1)$$

- High modulation

$$B = 2\Delta_f \quad (m>10)$$

As a Carson rule minimum Bandwidth

$$BW = 2[\Delta_f + f_m]$$

This rule accomadates almost 98% of transmited power

For AM Signal $B = 2f_m$

For SSB-SC $B = f_m$

For VSB $B = f_v + f_m$

So we can say, angle modulation has higher bandwidth then Amplitude modulation but it has greater noise ~~in~~ community.

Average power required for Angle modulation

As per phase modulation

$$E_c\left[J_0 \sin\omega_c t + J_1\left[\sin(\omega_c+\omega_m)t + \sin(\omega_c-\omega_m)t\right]\right.$$
$$+ J_2\left[\sin(\omega_c+2\omega_m)t + \sin(\omega_c-2\omega_m)t\right] +$$
$$\left.J_3\left[\sin(\omega_c+3\omega_m)t + \sin(\omega_c-3\omega_m)t\right]\right]$$

Carrier signal has amplitude modulation $E_c J_0$

Amplitude modulation $E_c J_1$ $E_c J_2$ $E_c J_3$ - - -

For modulated spectrum $E_0 = E_c J_0$ $E_1 = E_c J_1$ $E_2 = E_c J_2$

$$P = P_0 + P_1 + P_1 + P_2 + P_2 + P_3 + P_3 + - \cdots$$

$$= P_0 + 2P_1 + 2P_2 + 2P_3 + \cdot$$

$$= \frac{E_0^2}{2R} + 2\left[\frac{E_1^2}{2R}\right] + 2\left[\frac{E_2^2}{2R}\right]$$

$$= \frac{1}{R}\left[\frac{E_0^2}{R} + E_1^2 + E_2^2 + E_3^2 + \cdots\right]$$

Types of Frequency modulation:

- Narrow Band Frequency modulation
- Wide Band Frequency modulation

Narrow Band FM:

- -> LSB is 180° out of phase w.r.t USB
- -> $m \leq 1$
- -> NBFM has only two side bands
- -> Amplitude of LSB & USB is $E_c m/2$
- -> Δf is limited have with NBFM
- -> $m = \Delta f / f_m$

Parameters	WBFM	NBFM
modulation index	$m_f >> 1$	less than 1 or slightly greater than 1
maximum deviation	75kHz	5kHz
Freq range of modulating signal	30Hz to 15kHz	30Hz to 3kHz
maximum modulation	5 to 2500	slightly greater than 1
Bandwidth	15 times then NBFM	same AM
Applications	- Entertainment broadcasting - high quality music transmission	FM mobile communication for police wireless, Ambulance for speech

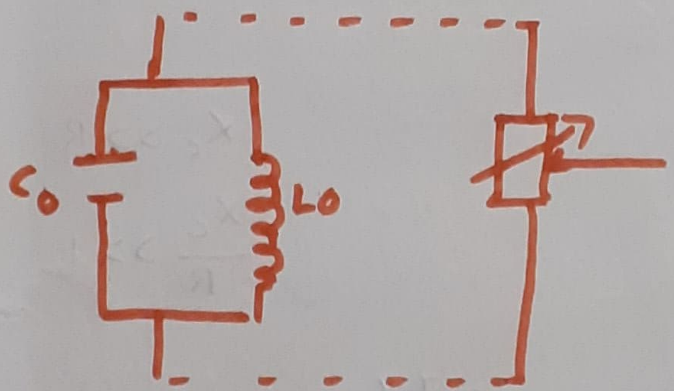
FM Generation by Direct method:

i) Varactor diode modulator

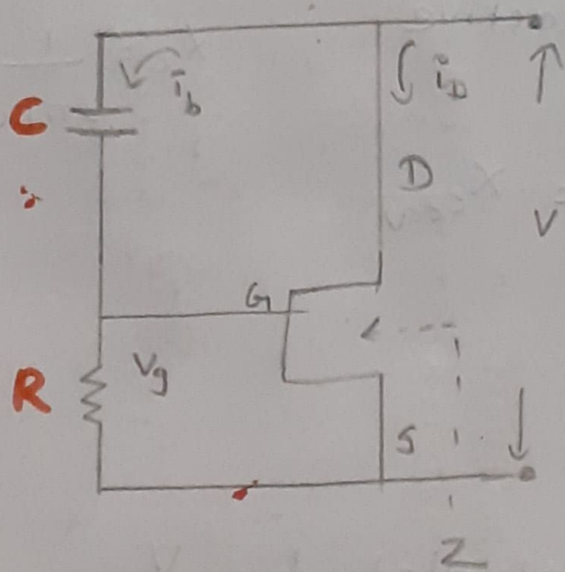
ii) Reactance tube modulator

Rectance modulator:

- direct method of FM generation
- FET Variable reactance
- '' to the oscillator tank circuit



oscillator tank circuit



Assumption

i) $i_b << i_c$

ii) $X_c >> R$

$$V_g = \frac{V \cdot R}{R - jX_c} = \frac{V}{1 - j\frac{X_c}{R}}$$

$$i_d = g_m V_g$$

Here g_m is transconductance which has relation between ratio of input voltage and output current

$$g_m = i_d / V_g \Rightarrow i_d = g_m \cdot V_g$$

$$= \frac{g_m \cdot V}{1 - j\frac{X_c}{R}}$$

$X_c >> R$

$\frac{X_c}{R} >> 1$

$$Z = \frac{V}{i_d} = \frac{1 - j\frac{X_c}{R}}{g_m} = \frac{-jX_c}{R g_m}$$

Z is purely Capactive

$X_c = \frac{1}{2\pi FC}$

$$Z = \frac{-j\,1}{2\pi b C R g_m} = X_{ceq}$$

$$C_{eq} = R C g_m$$

Conclusion:

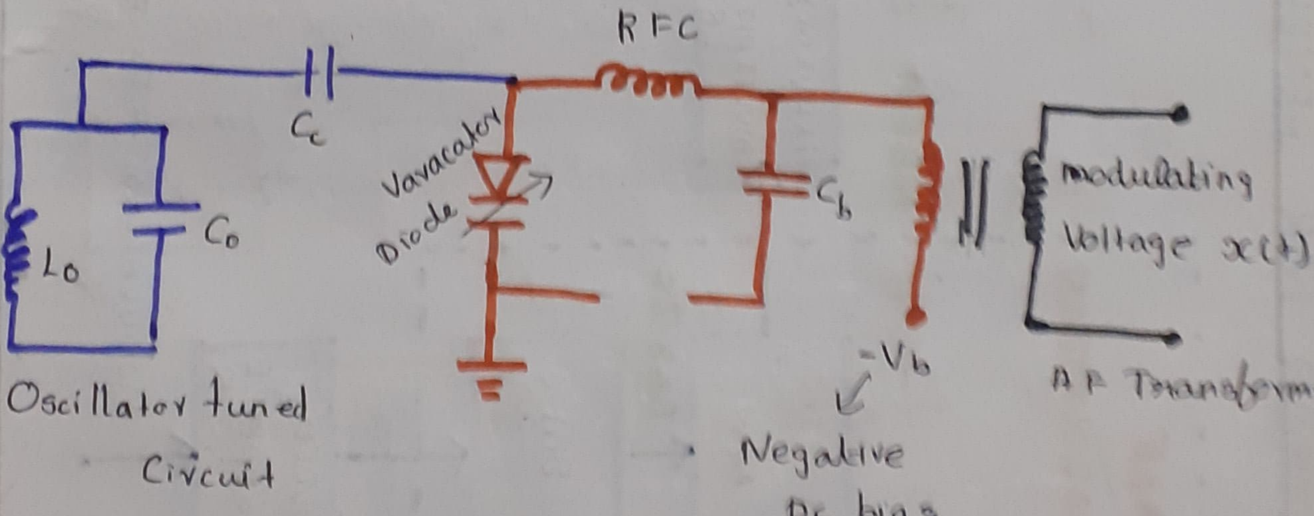
C_{eq} Can be Controlled by V_g

Varactor Diode Modulator:

- The instantaneous frequency deviation is directly proportional to the amplitude of the modulating signal. For this, a device called Voltage Controlled Oscillator (VCO) is used

- Junction Capacitance varies linearly with applied voltage.

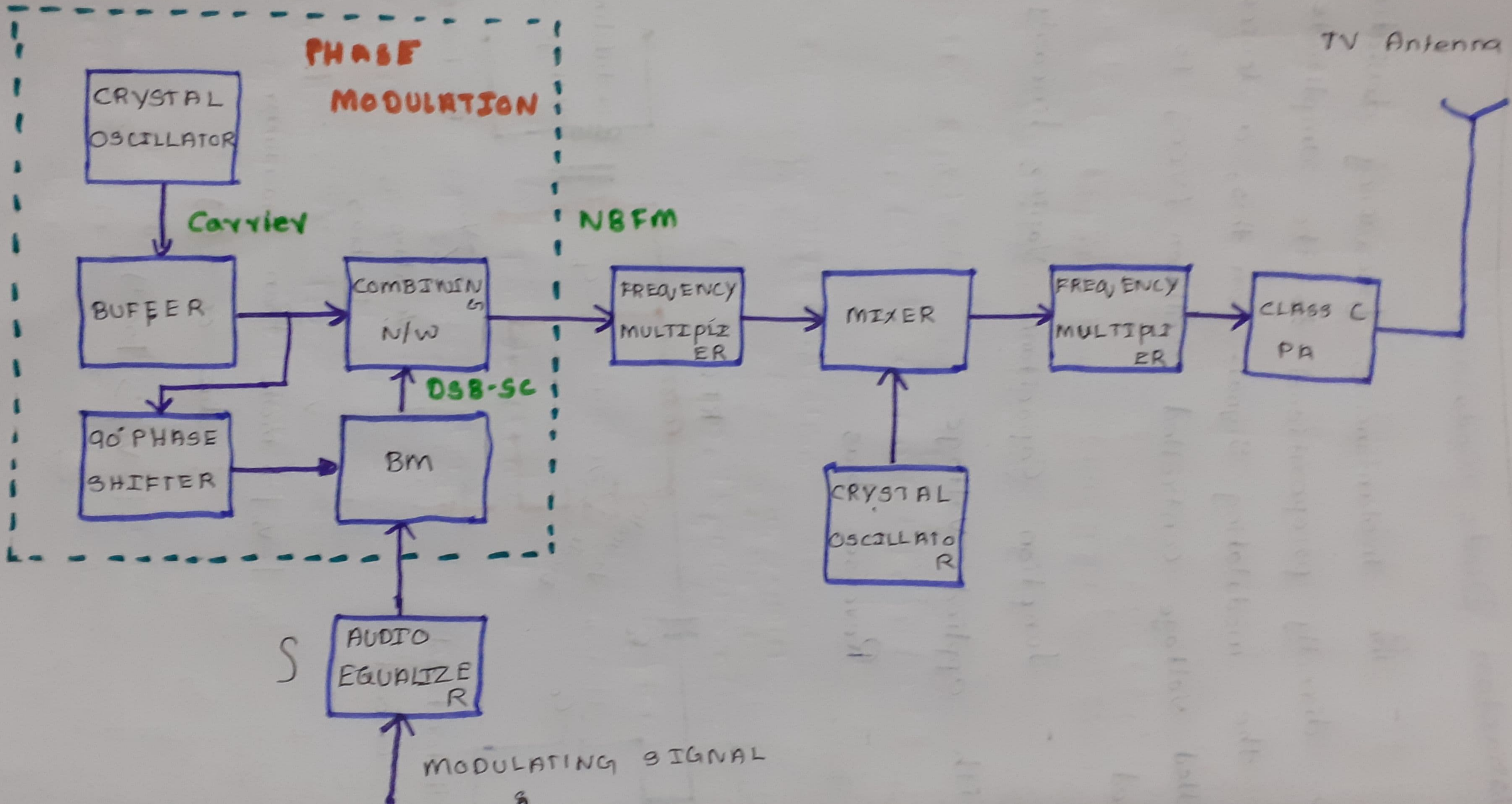
- Reverse bias



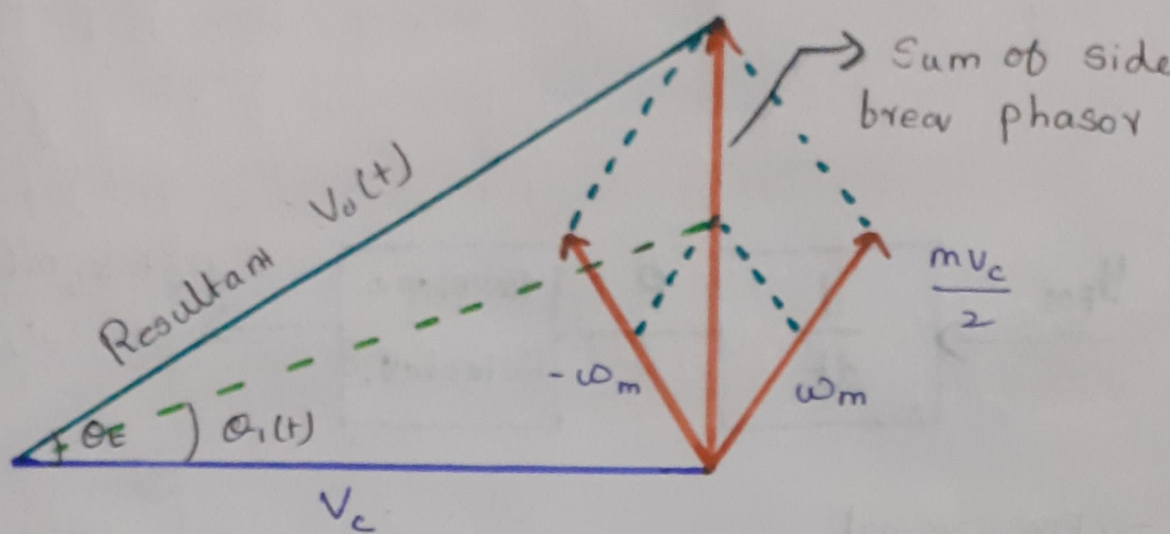
$|-V_b| \to$ more than maximum value of $x(t)$

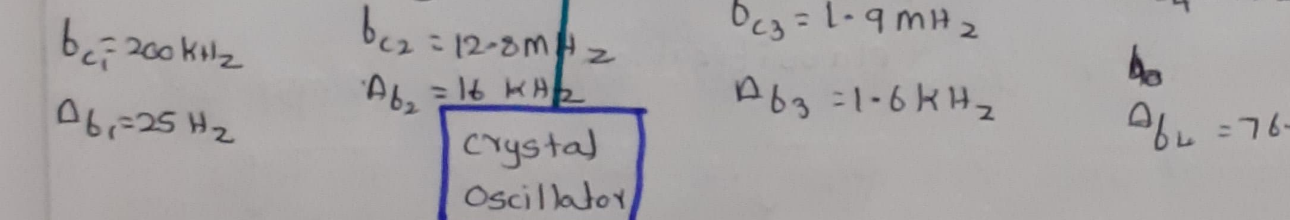
$|-V_b| > |x(t)|$

ARMSTRONG METHOD FOR GENERATION OF FM



$\theta(t) \propto m(t)$





$b_{c_1} = 200\,kHz$

$\Delta b_1 = 25\,Hz$

② $b_{c_2} = 12.8\,mHz$

$\Delta b_2 = 16\,kHz$

$b' = 10.9\,mHz$

$b_{c_3} = 1.9\,mHz$

$\Delta b_3 = 1.6\,kHz$

$b_{c_4} = 91.2\,mHz$

$\Delta b_4 = 76.8\,kHz$

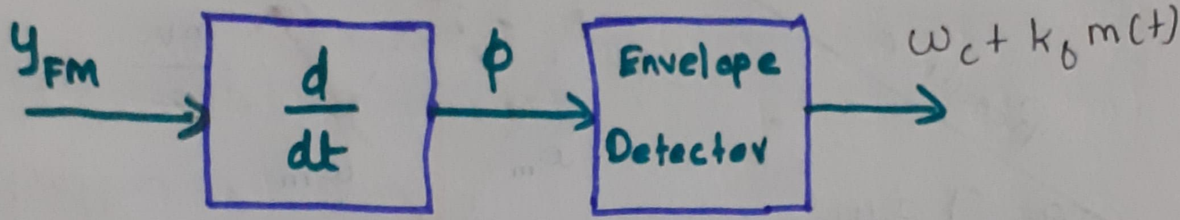
① $b_{c_2} = b_{c_1} \times 64 = 200\,kHz \times 64 = 200 \times 1000 \times 64$

$= 12800000$

$= 12.8\,mHz$

$\Delta b - \cancel{b_{c_1}} \times \Delta b_1 \times 64 = 25 \times 64 = 1600 = 1.6\,kHz$

FM demodulation & FM Classification of detection



-> FM Signal

$$y_{FM}(t) = E_c \cos(\omega_c t + k_f \int m(t)\, dt)$$

-> After ~~no~~ differentiation

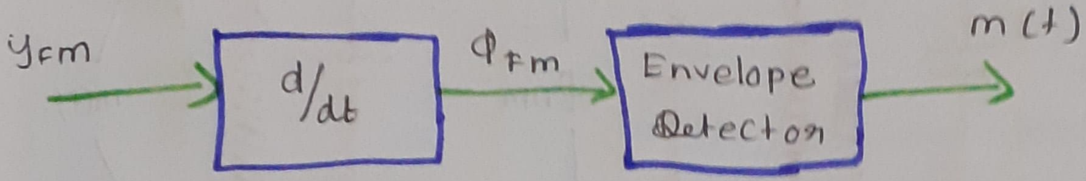
$$\omega_c > k_f m(t)$$

$$\phi = \frac{d y_{FM}(t)}{dt} = -E_c \underbrace{[\omega_c + k_f m(t)]}_{\text{Am signal}} \sin(\omega_c t + k_f \int m(t)\, dt)$$

FM detection classification

- Freq. discriminator
  - slope detector
  - Balanced slope detector
- Phase Discriminator
  - Foster seeley
  - Ratio Detector
- PLL

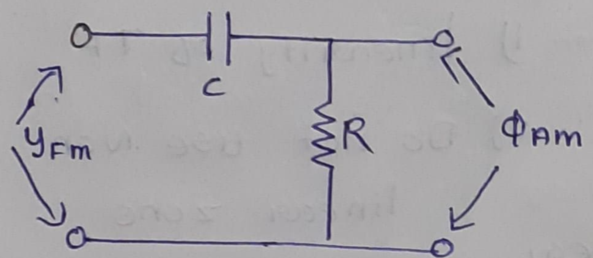
Slope method:



-> differentiator transfer function

$$H(f) \propto f$$

-> differentiator could be made by HPF



$$H(f) = \frac{R}{R + 1/j\omega_c}$$

$$= \frac{R}{\frac{Rj\omega_c + 1}{j\omega_c}}$$

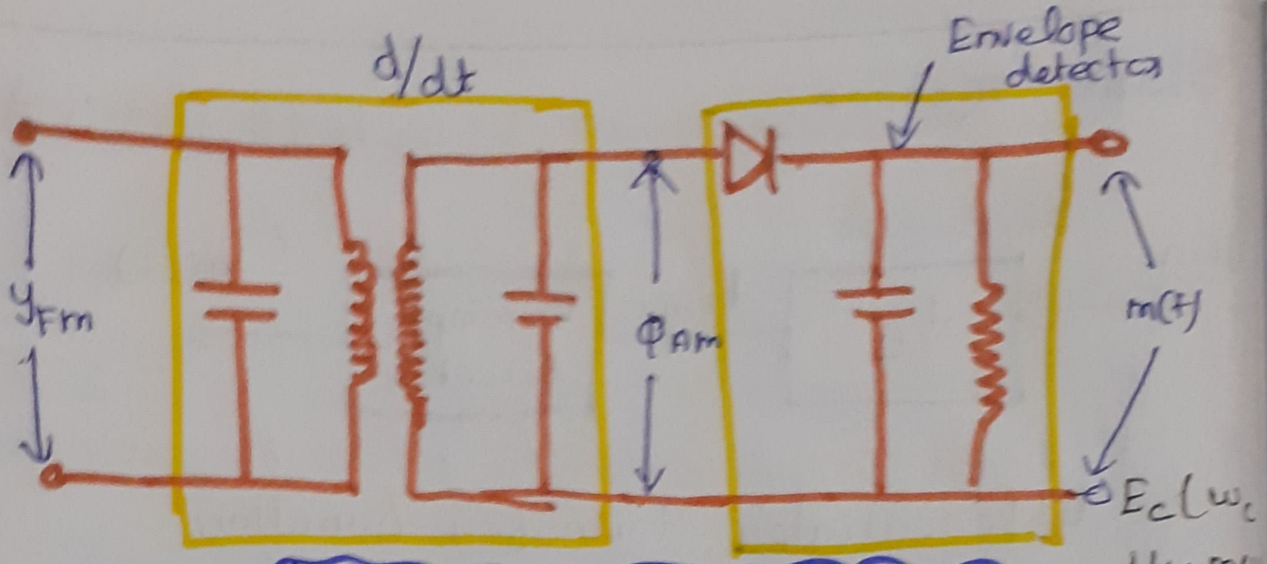
$$= \frac{Rj\omega_c}{Rj\omega_c + 1}$$

-> If consider

$$R\omega_c << 1$$

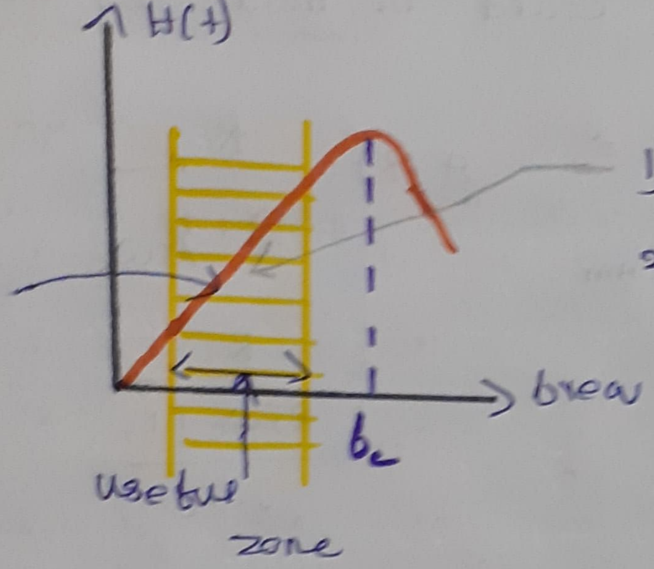
$$H(f) \approx jR\omega_c \propto \omega$$

- Here differentiator $\frac{dy_{FM}}{dt}$ = slope



Detuned amplifier circuit [differentiation)

$y_m = E_c \cos(\omega_c t + k_f \int m(t)\,dt)$



1) linearity of TF

2) Do not use Non linear zone

3) due to slope it func. as d/dt

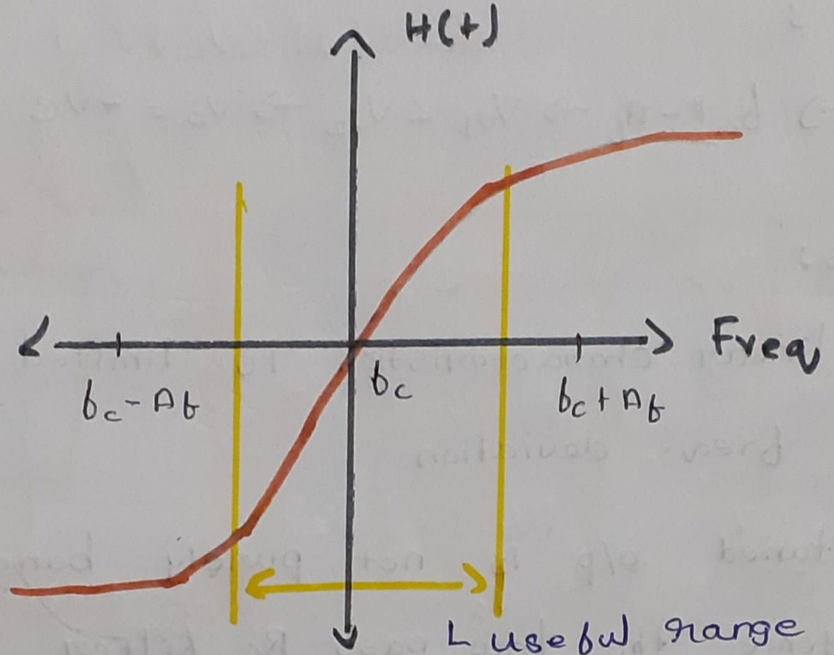
Drawbacks

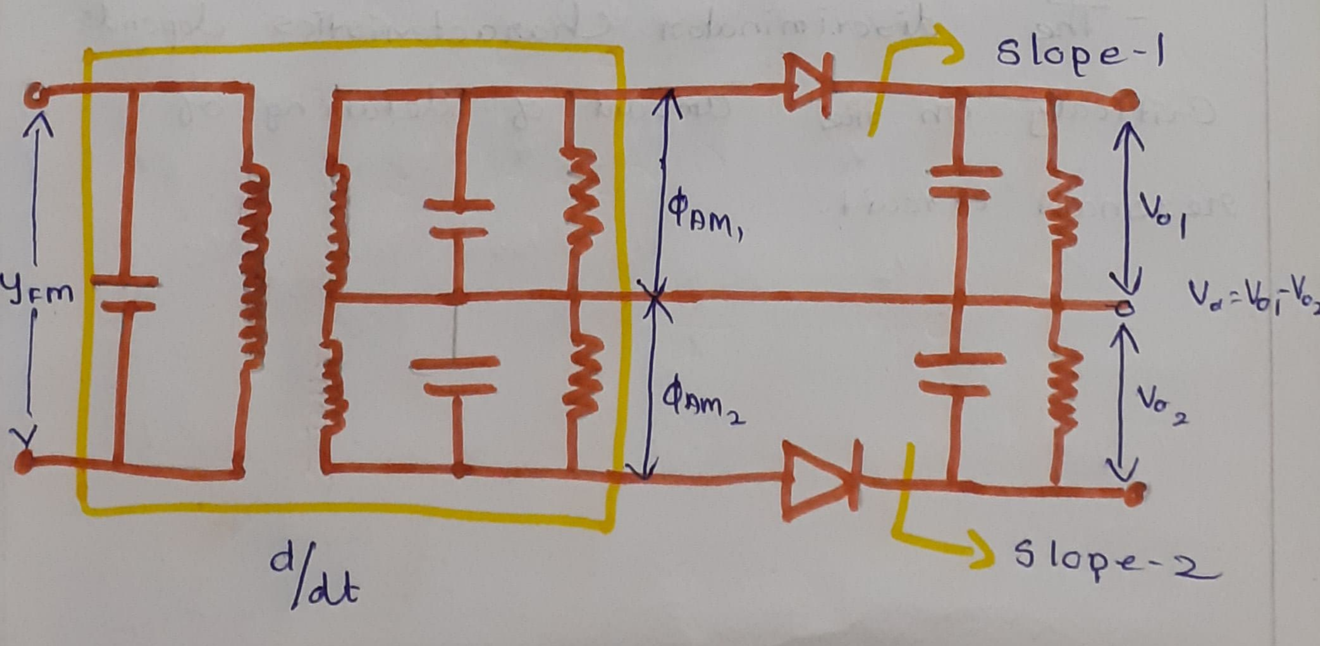
- Useful range is limited
- It does not eliminate the amplitude variation and o/p is sensitive to any amplitude variation.

Balanced Slope Detectinon:

- In single slope detection, tuning range was ~~upload~~ limited. So to increase tuning range. we provide staggered tunning.

$f_c + \Delta f \rightarrow f_c - \Delta f$





→ Case: 1

$f_{FM} \rightarrow f_c \rightarrow V_{o1} = V_{o2} \rightarrow V_o = 0$

case: 2

$f_{FM} \rightarrow f_c + \Delta f \rightarrow V_{o1} > V_{o2} \rightarrow V_o = +ve$

case: 3

$f_{FM} \rightarrow f_c - \Delta f \rightarrow V_{o1} < V_{o2} \rightarrow V_o = -ve$

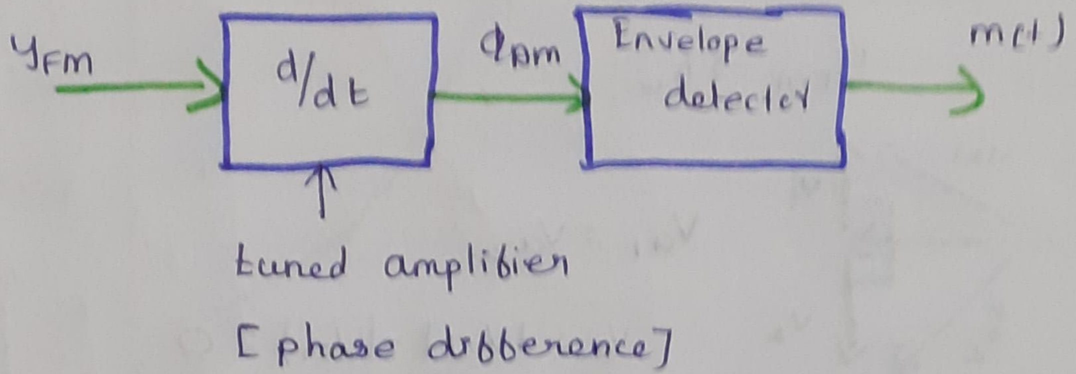
Drawbacks:

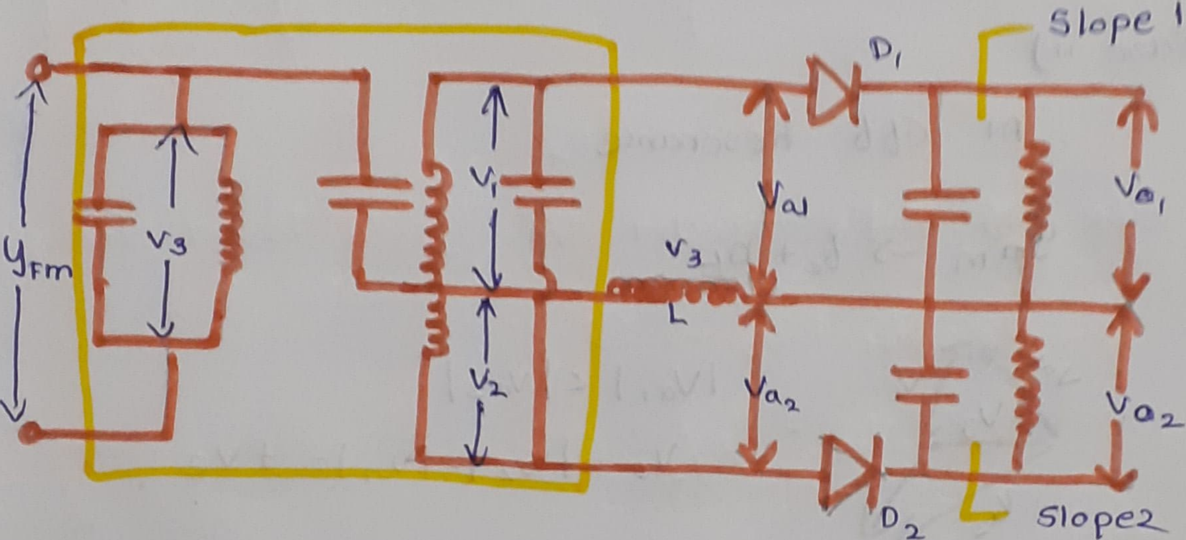
- The linear characteristics is limited to a small freq. deviation

- The tuned o/p is not purely band limited. Hence the low pass Rc filter enevelop detector introduced distortion

- The discriminator characteristics depends critically on the amount of detuning of resonant circuit.

Fosten Seeley Phase Discriminator



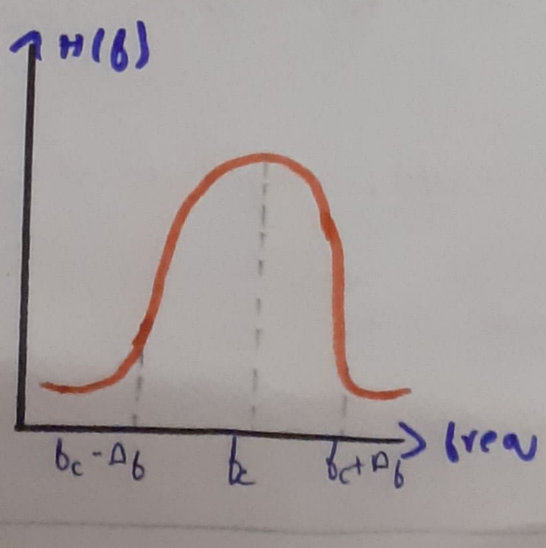


- V_1 and V_2 are out of phase to each other

- O/P voltage $V_o = |V_{o2}| - |V_{o1}|$

$V_{a1} = V_3 + V_1$

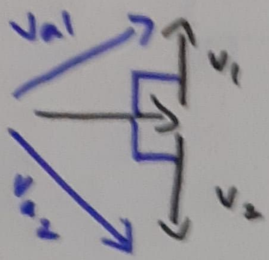
$V_{a2} = V_3 - V_2$



Case i)

At Resonance

$y_{Fm} \rightarrow f_c$



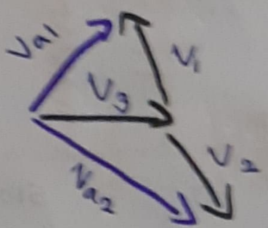
$V_{a_1} = V_{a_2}$

$V_o = |V_{o2}| - |V_{o1}| = 0$

Case ii)

At off Resonance

$y_{Fm} \rightarrow f_c + \Delta f$

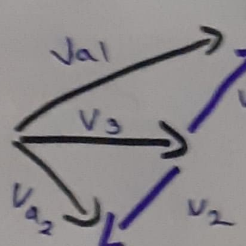


$|V_{a_1}| < |V_{a_2}|$

$V_o = |V_{o2}| - |V_{o1}| = +ve$

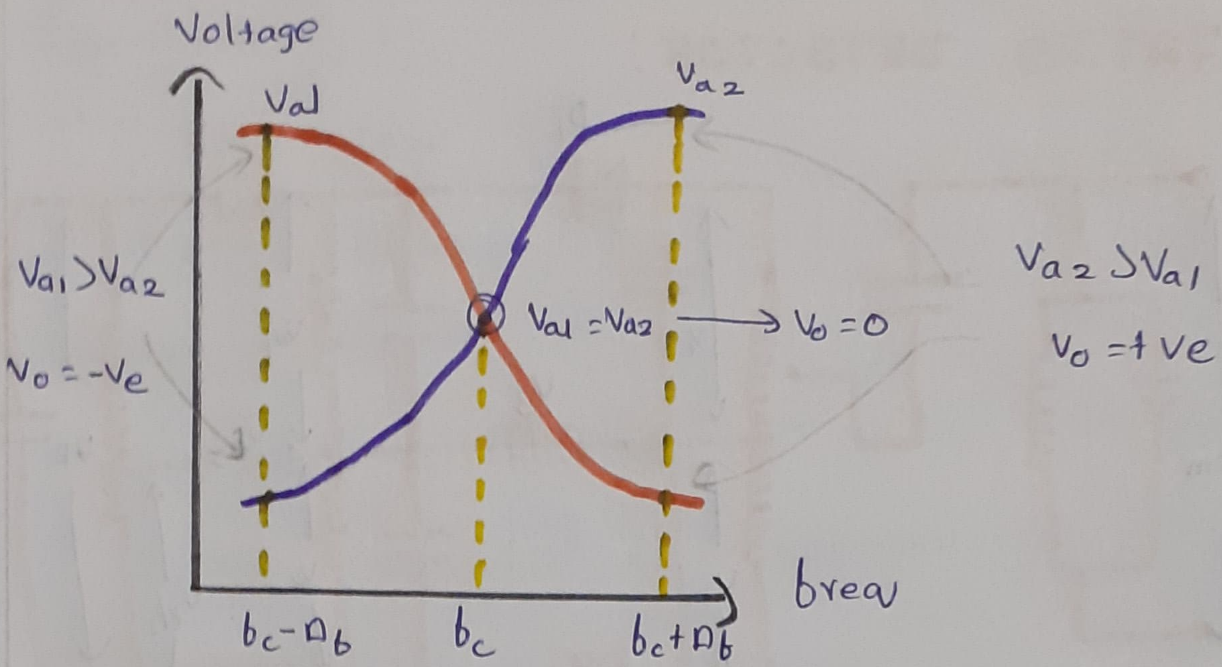
case iii)

At off Resonance



$|V_{a_2}| < |V_{a_1}|$

$V_o = |V_{o2}| - |V_{o1}| = -ve$



Advantage:

1) offers good level of performance and resonance linearity

2) Easy to Construct Using discrete Components

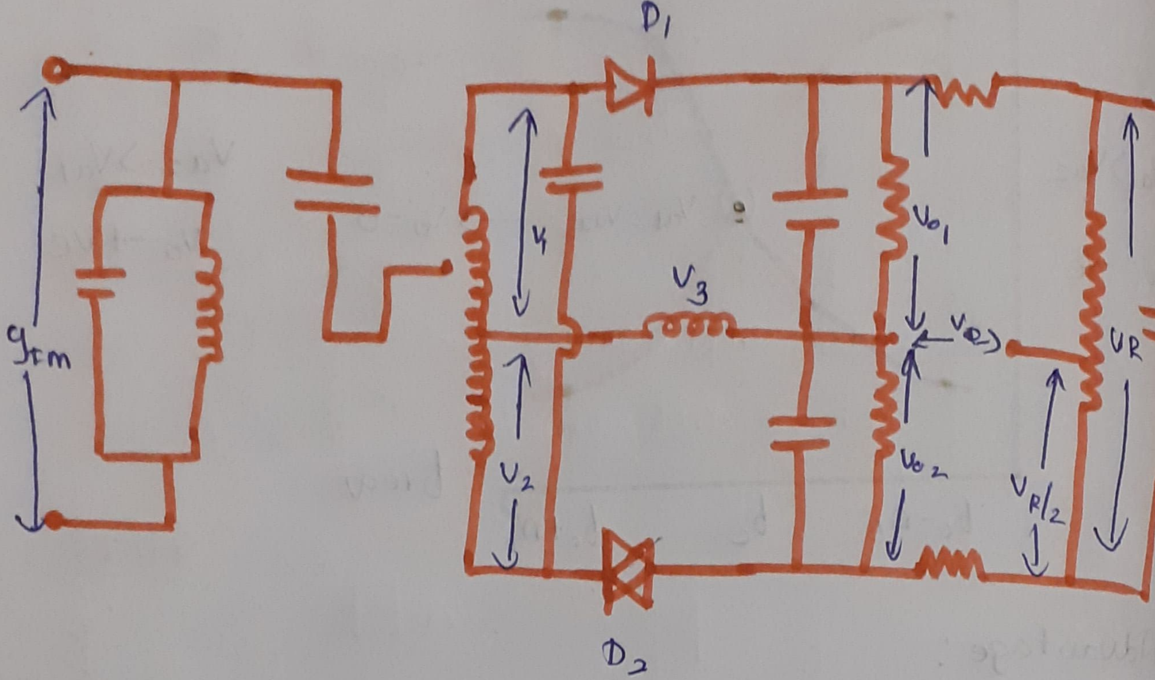
Drawbacks

1) Not Suitable to use IC technology

2) High Cost

3) Noise Immunity

RATIO DETECTOR:



O/P of Radio detector $V_o = \frac{1}{2}$ (V_o Foster Seeley)

$$V_R = V_{o1} + V_{o2} \rightarrow (1)$$

$$V_o = V_{o2} - V_R/2 \rightarrow (2)$$

FROM (1) and (2)

$$V_o = V_{o2} - \frac{V_{o1} + V_{o2}}{2} = \frac{V_{o2} - V_{o1}}{2} = \left(\frac{V_o}{2}\right. \text{ Faster selly} \left.\right)$$

Advantages:

1) Good performance and reasonable linearity

2) Better immunity against amplitude noise

3) wider Bandwidth

4) Easy to construct

Drawbacks

1) High Cost

2) o/p ebbiciency is just halb of foster seeley detector

3) Higher Distortion

4) Not suitable to implement on IC technology