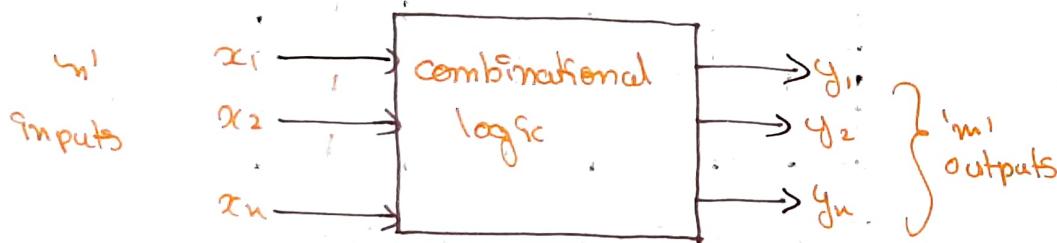


Combinational logic circuit:-

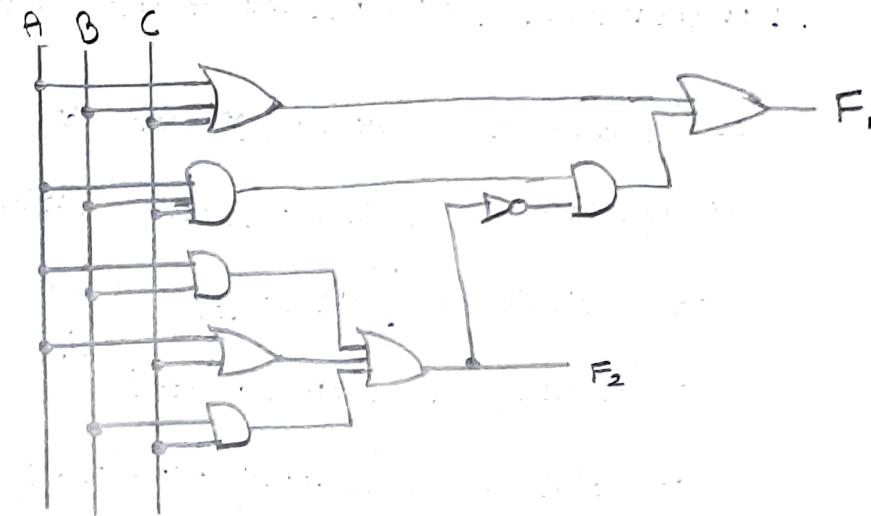
- output dependent on combination of input variables
- it doesn't use any memory.
- it has n-inputs & m-outputs.



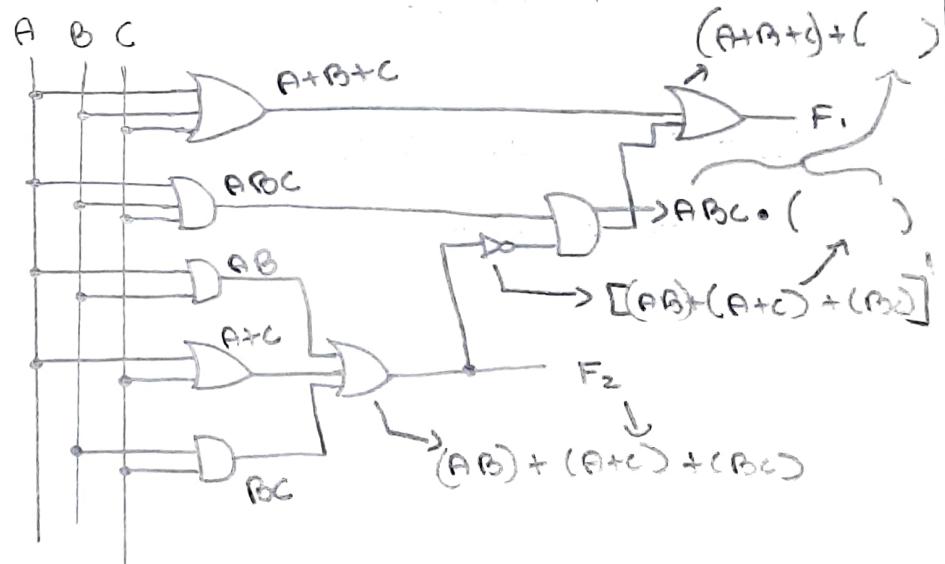
Analysis of combinational logic

- logic diagram (given in question)
- boolean expression
- truth table
- values of sop & pos.

analyze the logic diagram.



ans:-



$$\begin{aligned}
 F_1 &= A+B+C + (ABC \cdot [(AB) + (A+C) + (BC)])' \\
 &= (A+B+C) + (ABC \cdot [B(A+C) + (A+C)])' \\
 &= (A+B+C) + (ABC \cdot [(A+C)(B+1)])' \\
 &\Rightarrow (A+B+C) + [ABC \cdot (A+C)']' \\
 &= (A+B+C) + [ABC \cdot (A'C')] \\
 &= (A+B+C) + [(A \cdot A') \cdot B \cdot (C \cdot C')] \\
 &\quad \underbrace{\qquad\qquad\qquad}_{0} \quad \underbrace{\qquad\qquad\qquad}_{0} \\
 &= A+B+C \quad //.
 \end{aligned}$$

$$\begin{aligned}
 F_2 &= AB + (A+C) + (BC) \\
 &= A+C \quad //
 \end{aligned}$$

A	B	C	$A+B+C$	$A+C$
0	0	0	0	0
0	0	1	1	1
0	1	0	1	0
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

$$F_1(A, B, C) = \Sigma(1, 2, 3, 4, 5, 6, 7)$$

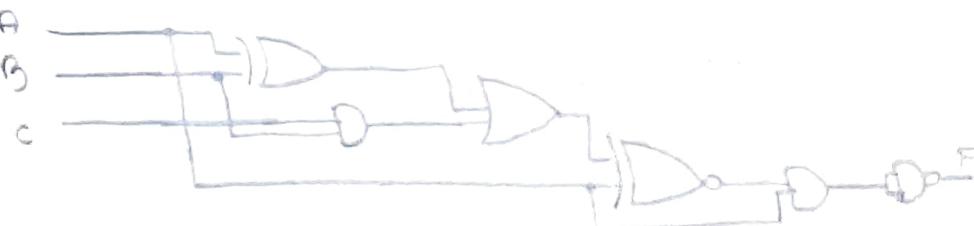
$$F_1(A, B, C) = \Pi(0)$$

$$F_2(A, B, C) = \Sigma(1, 3, 4, 5, 6, 7)$$

$$F_2(A, B, C) = \Pi(0, 2)$$

Sums to solve

i)



Solution available on github & telegram channel.

## Design of combinational circuit

→ truth table (for question)

→ do Kmap for each output

→ draw logic diagram.

Design a combinational circuit with three inputs  $x, y, z$

8/ three outputs A, B & C. when the binary input is 0, 1, 2, 3, the binary output is two greater than the input.  
when the binary input is 4, 5, 6, 7 the binary output is  
three less than the input.

Input			Output		
X	Y	Z	A	B	C
0	0	0	0	1	0
1	0	0	1	0	1
2	0	1	0	1	0
3	0	1	1	0	1
4	1	0	0	0	1
5	1	0	0	1	0
6	1	1	0	1	1
7	1	1	1	0	0

Kmap for A

X	Y	Z	Y'Z'	Y'Z	YZ	YZ'	Y'Z'
0	0	0	1	0	0	0	1
1	0	1	1	1	1	1	1
2	1	0	1	1	1	1	1
3	1	1	1	1	1	1	1

Kmap for B

X	Y	Z	Y'Z'	Y'Z	YZ	YZ'	Y'Z'
0	0	0	1	0	0	0	1
1	0	1	1	1	1	1	1
2	1	0	1	1	1	1	1
3	1	1	1	1	1	1	1

$$A = YZ + X'Y$$

$$B = X'Y' + Y'Z + X'YZ'$$

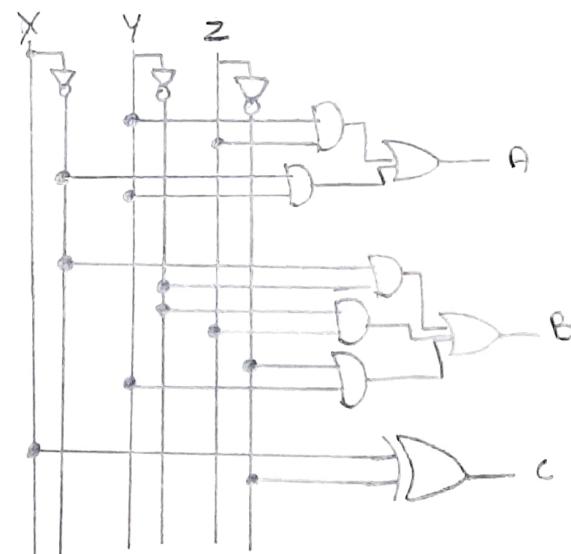
Kmap for C

X	Y	Z	Y'Z'	Y'Z	YZ	YZ'	Y'Z'
0	1	0	1	1	1	1	1
1	1	0	1	1	1	1	1
2	1	1	1	1	1	1	1

$$A'B + AB' = A \oplus B$$

$$A'B' + AB = (A \oplus B)$$

$$C = X'Z + XZ' = X \oplus Z$$



Code Conversion:

converting one set of code to another.

Convert 4bit binary to gray code.

4bit binary	Gray code
A B C D	P Q R S
0 0 0 0	0 0 0 0
0 0 0 1	0 0 0 1
0 0 1 0	0 0 1 1
0 0 1 1	0 0 1 0
0 1 0 0	0 1 1 0
0 1 0 1	0 1 1 1
0 1 1 0	0 1 0 1
0 1 1 1	0 1 0 0
1 0 0 0	1 1 0 0
1 0 0 1	1 1 0 1
1 0 1 0	1 1 1 1
1 0 1 1	1 1 1 0
1 1 0 0	1 0 1 0
1 1 0 1	1 0 1 1
1 1 1 0	1 0 0 1
1 1 1 1	1 0 0 0

Kmap for P

AB	C'D'	C'D	CD	CD'
A'B'00				
A'B'01				
A'B'11	1	1	1	1
A'B'10	1	1	1	1

$$P = A$$

Kmap for Q

AB	C'D'	C'D	CD	CD'
A'B'1				
A'B	1	1	1	1
AB				
AB'1	1	1	1	1

$$Q = A'B + A'B' \\ = A \oplus B$$

Kmap for R

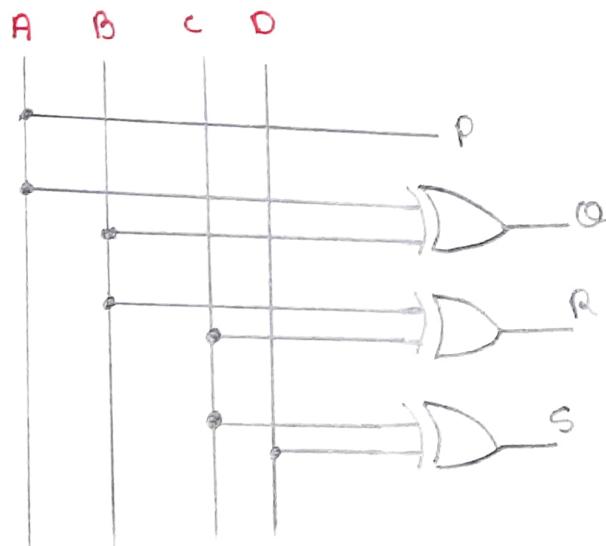
AB	C'D'	C'D	CD	CD'
A'B'1			1	1
A'B	1	1		
AB	1	1		
AB'1			1	1

$$R = B'C + B'C' \\ = B \oplus C$$

Kmap for S

	$A'B'C'D'$	$A'B'C'D$	$A'B'CD$	$A'B'CD'$
$A'B'C'D'$	X			
$A'B'C'D$		X		
$A'B'CD$			X	
$A'B'CD'$				X

$$S = c'D + D' = C \oplus D.$$



Convert Excess-3 code to BCD

Excess-3	BCD
0 0 11	0000
0 1 00	0001
0 1 01	0010
0 1 10	0011
0 1 11	0100
1 0 00	0101
1 0 01	0110
1 0 10	0111
1 0 11	1000
1 1 00	1001

Don't care input =  $2^4 = 16$  combinations

$\Rightarrow 10$  (used)

$\Rightarrow 16 - 10 = 6$  (unused)

0000 1101  
0001 1110  
0010 1111

Kmap for P

$A'B'C'D'$	$c'D'$	$c'D$	$CD$	$CD'$
$A'B'$	X	X		X
$A'B$				
$AB$	1	X	X	X
$AB'$				1

$$P = AB + A'CD$$

Kmap for Q

$AB'C'D'$	$c'D'$	$c'D$	$CD$	$CD'$
$AB'B'$	X	X		X
$AB'B$				
$AB$		X	X	X
$AB'$	1	1	1	1

$$Q = BC'D + B'D' + B'C'$$

Kmap for R

$AB'C'D'$	$00$	$01$	$11$	$10$
$00$	X	X		X
$01$		1		1
$11$		X	X	X
$10$	1			1

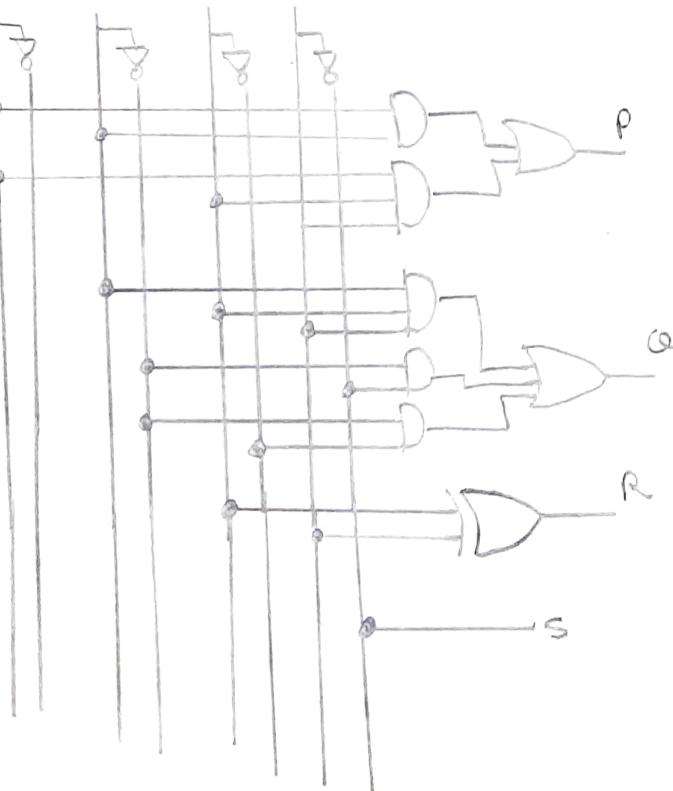
$$R = c'D + CD' = C \oplus D$$

Kmap for S

AB	CD	00	01	11	10
00	X	X		X	
01	1			1	
11	1	X	X	X	
10	1			X	1

$$S = D'$$

A B C D



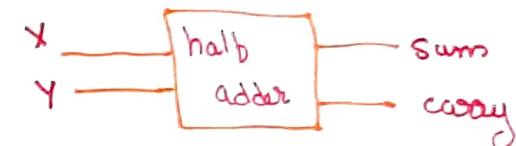
Sums to solve

2. gray to 4 bit binary
3. BCD to excess
4. Design a combinational circuit, its output is 1 when the ~~carry~~ input is even

Solution are available on our  
github & telegram channel.

Half adder

used to add 2 bits.



eg: -

$$\begin{array}{r} 0 \\ +1 \\ \hline \text{sum} \end{array}$$
$$\begin{array}{r} 1 \\ +1 \\ \hline \text{carry} \end{array}$$
$$\begin{array}{r} 1 \\ +1 \\ \hline \text{sum} \end{array}$$

X	Y	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

Kmap for sum

x	y	y1	y
x1	x		1
x	x1	1	

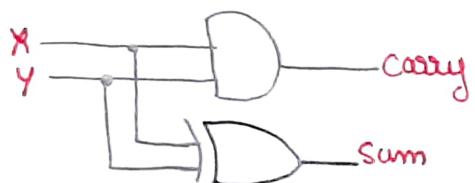
$$\begin{aligned} \text{Sum} &= x'y + xy' \\ &= x \oplus y \end{aligned}$$

Kmap for carry

x	y	y1	y
x1	x		1
x	x1	1	

$$\text{Carry} = xy$$

Logic diagram.



full adder

used for 3bit addition

$$\text{eg: } \begin{array}{r} 0 \\ +1 \\ \hline +1 \end{array} \quad \begin{array}{r} 1 \\ 1 \\ \hline +1 \end{array} \rightarrow 10+1$$

$\downarrow 10$        $\downarrow$

carry      sum      carry      sum

X	Y	Z	Sum	carry
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
0	1	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

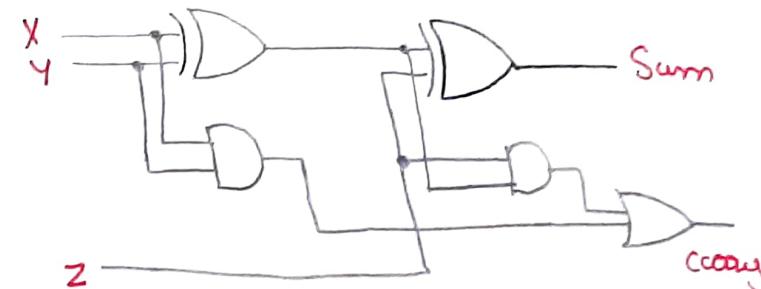
bimap for sum

$x^2$	00	01	11	10
0	1			1
1	1	1		

$$\begin{aligned}
 \text{Sum} &= x'y'z + xy'z \\
 &\quad + xyz + x'y'z \\
 \Rightarrow y'(x'z + xz') + y(xz + x'z) \\
 \Rightarrow \overline{y'} \overline{\underline{x'z}} + \overline{y} \overline{\underline{xz}} \\
 &\quad + \overline{y} \overline{\underline{xz'}} + \overline{y} \overline{\underline{x'z'}} \\
 \Rightarrow A'B + AB' &= A \oplus B \\
 \Rightarrow Y \oplus X \oplus Z \\
 \Rightarrow X \oplus Y \oplus Z
 \end{aligned}$$

binaries for carry.

$$\text{coadj} = XY + ZX + YZ$$



**NOTE:-** You can also take  $x, y, z$  as  $A, B, C_m$  (is just a variable to express)

half subtraction :-

used for 2bit subtraction.



X	Y	D	B
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

$$\begin{array}{r}
 \text{eg: } \rightarrow \begin{array}{r}
 & 1 \\
 -1 \\
 \hline
 0
 \end{array} \quad \begin{array}{r}
 \cancel{1} \quad (2) \\
 -1 \quad (1) \\
 \hline
 1
 \end{array} \quad \text{borrow (1)} \\
 \searrow \text{difference} \qquad \searrow \text{difference}
 \end{array}$$

Kmap for B

x	y	$y'$	y
x'			1
x			

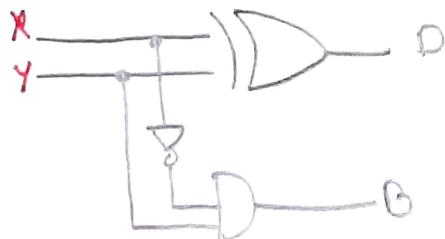
$$B = x'y$$

Kmap for D

x	y	$y'$	y
x'			1
x			

$$D = xy' + x'y \\ = x \oplus y$$

logic diagram.



full subtractor

used for 3bit subtraction.

$$\text{eg: } 0 \begin{smallmatrix} 1 \\ 0 \end{smallmatrix} - 1 \begin{smallmatrix} 0 \\ 1 \end{smallmatrix}$$

$$\begin{array}{r} 1 \\ \hline 0 \end{array} \rightarrow 1-1=0$$

$$\begin{array}{r} 1 \\ 0 \\ -1 \\ \hline 1 \end{array} \rightarrow \text{borrow}=0$$

$$\text{borrow}=1 \rightarrow \text{difference} \leftarrow 0$$

X	Y	Z	D	B
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	0
1	0	1	0	0
1	1	0	0	0
1	1	1	1	1

Kmap for D

x	$yz$	00	01	11	10
0				1	1
1		1		1	

Kmap for B

x	$yz$	00	01	11	10
0			1	1	1
1		1	1	1	*

$\Rightarrow x'y'z + x'y'z'$

$$D = x'y'z + x'y'z' + x'y'z' \\ + xyz \\ = x \oplus y \oplus z$$

$$B = x'z + x'y + yz$$

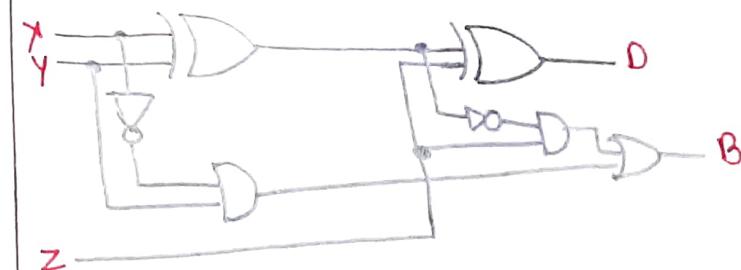
$$= x'z(y+y') + x'y(z+z') \\ + yz(x'+x)$$

$$= x'y'z + x'y'z + x'y'z + x'y'z \\ + xyz + x'y'z$$

$$= x'y'z + x'y'z + x'y'z + x'y'z$$

$$= x'y(z+z') + z(x'y' + xy)$$

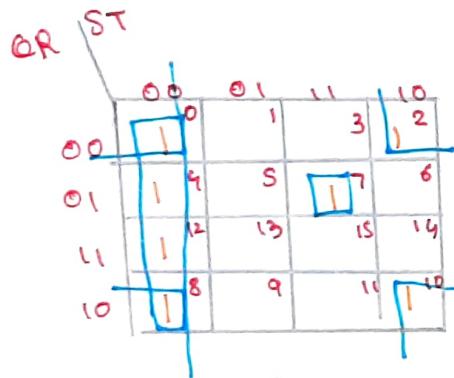
$$B = x'y + z(x \oplus y)$$



## 5 Variable Kmap

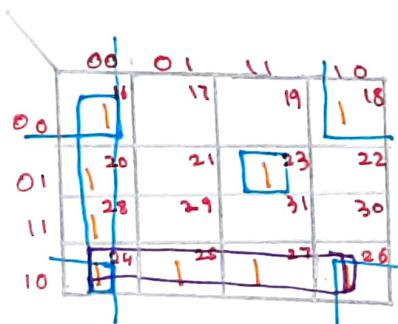
we will use 2 4 variable Kmap here

Solve  $F(PQRST) = \Sigma(0, 2, 4, 7, 8, 10, 12, 16, 18, 20, 23, 24, 25, 26, 27, 28)$



P=0

- → not overlapping
- → overlapping



P=1

$$F = S'T' + R'T' + Q'RST + PQR'$$

Sums to solve

5.  $F(A,B,C,D,E) = \Sigma(0, 2, 8, 10, 16, 18, 24, 26, 31, 30)$

6.  $F(A,B,C,D,E) = \Sigma(30, 17, 16, 24, 25, 28)$

Solution on github or telegram channel.

BCD adder:

→ addition of two BCD number

→ if the number goes beyond 9 then add 6 to it  
(0110)

Binary sum				BCD sum $\rightarrow$ carry				
K	Z <sub>8</sub>	Z <sub>4</sub>	Z <sub>2</sub>	Z <sub>1</sub>	(S <sub>8</sub> )	S <sub>4</sub>	S <sub>2</sub>	S <sub>1</sub>
0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	1
0	0	0	1	0	0	0	1	0
0	0	0	1	1	0	0	1	1
0	0	1	0	0	0	0	1	0
0	0	1	0	1	0	0	1	0
0	0	1	1	0	0	1	1	0
0	1	0	0	0	0	1	0	0
0	1	0	0	1	0	1	0	1
0	1	0	1	0	1	0	0	0
0	1	0	1	1	1	0	1	0
0	1	1	0	0	0	0	1	0
0	1	1	0	1	0	0	1	0
0	1	1	1	0	1	0	0	0
1	0	0	0	0	1	0	1	0
1	0	0	0	1	1	0	1	1
1	0	0	1	0	1	1	0	0
1	0	0	1	1	1	1	0	0

$\rightarrow$   
9+9+1=19  
(0 to 19)

↓

add 6

to all

numbers

more than

9

eg:→

(5)

$\begin{array}{r} 111 \\ 01111 \\ + 0110 \\ \hline 10101 \end{array}$

Kmap for PCD (carry)

	$Z_2 Z_1$	$Z_8 Z_4$		
0	0	1	3	2
4	4	5	7	6
12	12	13	15	14
8	8	9	11	10

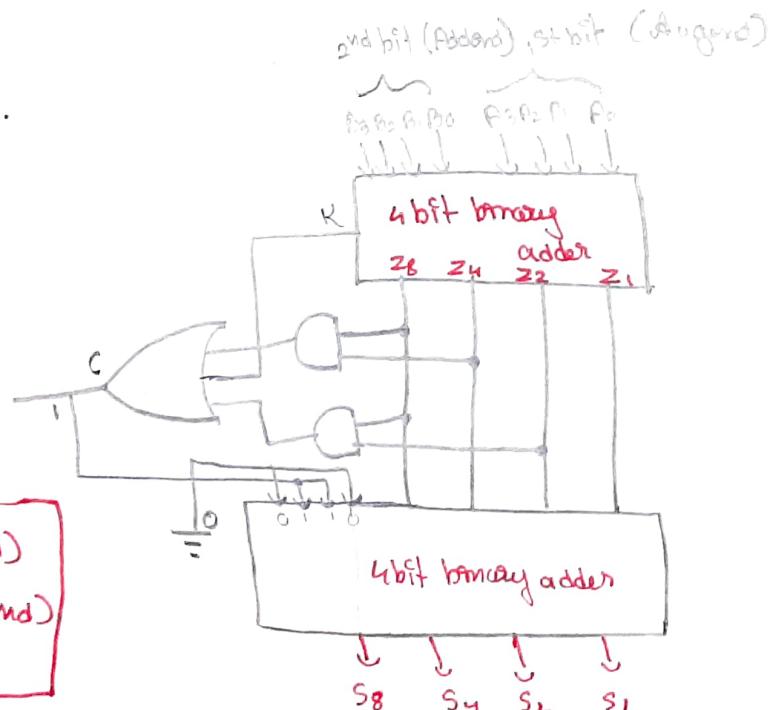
$$K = 0$$

	$Z_2 Z_1$	$Z_8 Z_4$		
16	1	1	1	1
20	X	21	X	22
28	X	29	X	31
24	X	25	X	26

$$K = 1$$

$$C = Z_8 Z_4 + Z_8 Z_2 + K$$

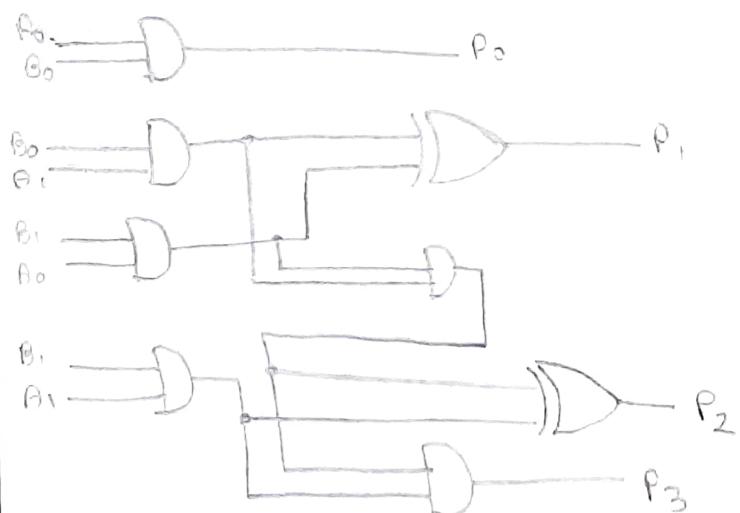
block diagram.



## 2x2 Binary Multiplier

→ used to multiply 2 bits  
→ uses combinational circuit

$A_1 \quad A_0$	$B_1 \quad B_0$	$A_1 B_0 + A_0 B_1$	$X$
$P_3$	$P_3$	$P_{14}$	$P_0$
$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
carry of $P_2$	$A_1 B_1 +$ $A_0 B_0$	$A_1 B_0 + A_0 B_1$	



# 4x4 Binary multiplier

$A_3 \ A_2 \ A_1 \ A_0$

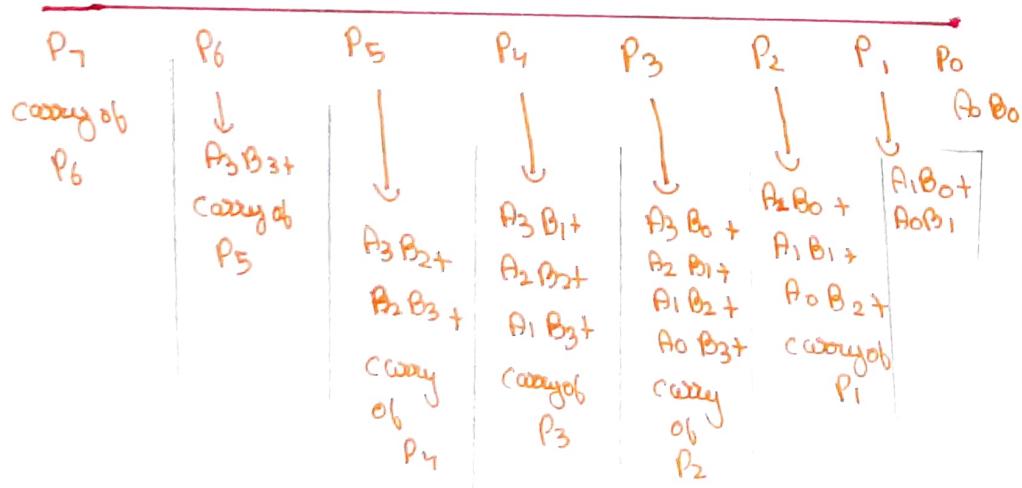
$B_3 \ B_2 \ B_1 \ B_0$

$A_3 B_0 \ A_2 B_0 \ A_1 B_0 \ A_0 B_0$

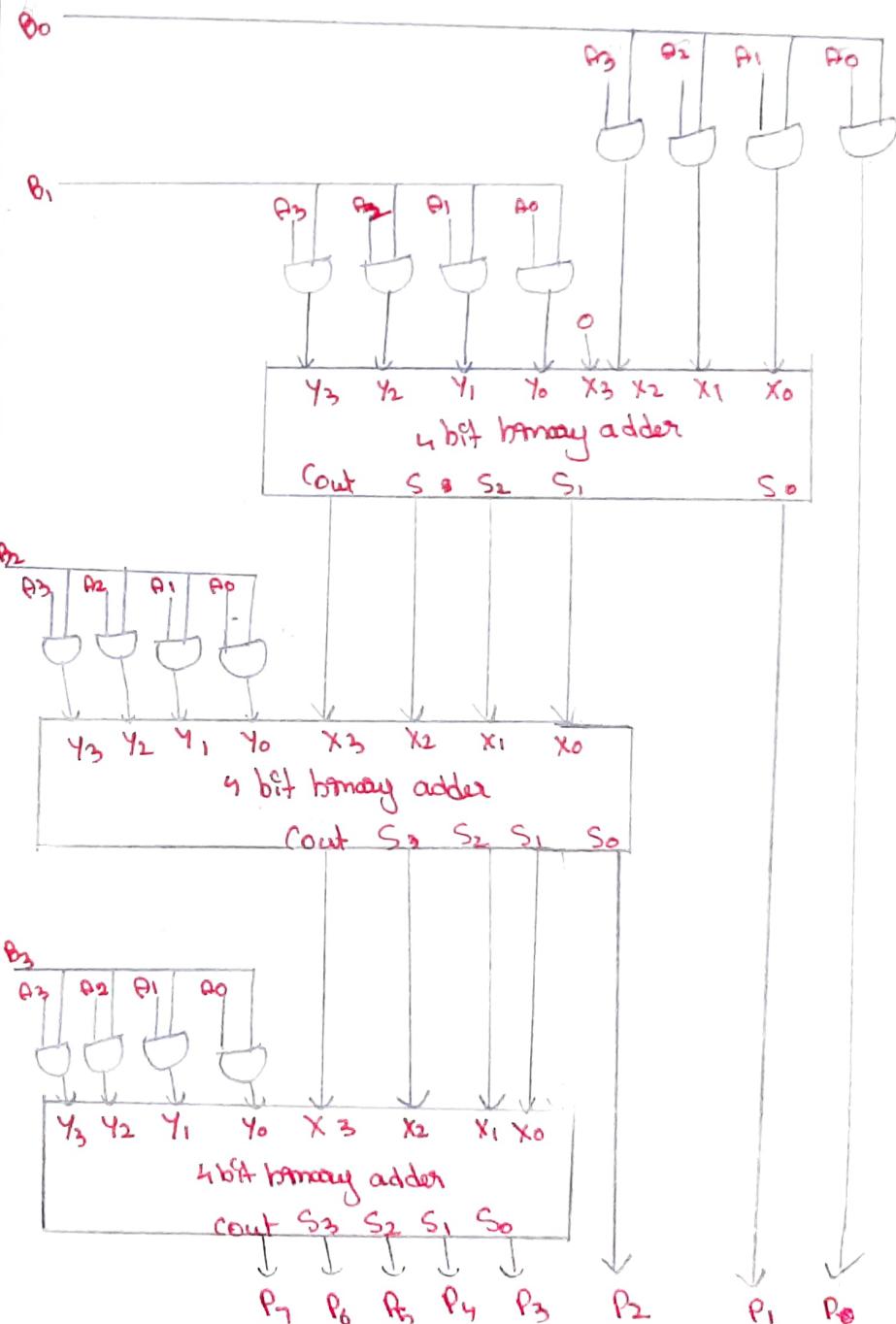
$A_3 B_1 \ A_2 B_1 \ A_1 B_1 \ A_0 B_1 \ X$

$A_3 B_2 \ A_2 B_2 \ A_1 B_2 \ A_0 B_2 \ X \ X$

$A_3 B_3 \ A_2 B_3 \ A_1 B_3 \ A_0 B_3 \ X \ X \ X$



~~\*Note: The maximum addition is 1110 & no. So there will be 2 4bit adders.~~



Magnitude comparison  $\Rightarrow$

$\rightarrow$  used to compare two binary numbers

$\rightarrow$  implemented using combinational circuit.



$$(A=B) \Rightarrow AB + A'B' = X \quad [A = A_3 A_2 A_1 A_0 \\ B = B_3 B_2 B_1 B_0]$$

$$X_0 = A_0 B_1 + A_0' B_0'$$

$$X_1 = A_1 B_1 + A_1' B_1'$$

$$X_2 = A_2 B_2 + A_2' B_2'$$

$$X_3 = A_3 B_3 + A_3' B_3'$$

$$X_0 = A_0 B_0 + A_0' B_0'$$

$$A=B \Rightarrow X_3 X_2 X_1 X_0$$

$$(A>B) \Rightarrow A_3 B_3' + X_3 A_2 B_2' + X_3 X_2 A_1 B_1' + X_3 X_2 X_1 A_0 B_0'$$

$$(A < B) \Rightarrow A_3' B_3 + X_3 A_2' B_2 + X_3 X_2 A_1' B_1 + X_3 X_2 X_1 A_0' B_0$$

Compare  $\Rightarrow A = 1011, B = 1100$

$$\begin{array}{c} A \neq / \times \neq 1 / \neq \\ B \neq / 1 \times 0 0 \end{array}$$

	A <sub>3</sub>	A <sub>2</sub>	A <sub>1</sub>	A <sub>0</sub>
A	1	0	1	1
B	1	1	0	0

	B <sub>3</sub>	B <sub>2</sub>	B <sub>1</sub>	B <sub>0</sub>
B	1	1	0	0
A	1	0	1	1

$$X_0 = (A_0 \oplus B_0)'$$

$$\Rightarrow (1 \oplus 0)' = (1)' = 0$$

$$X_1 = (1 \oplus 0)' = (1)' = 0$$

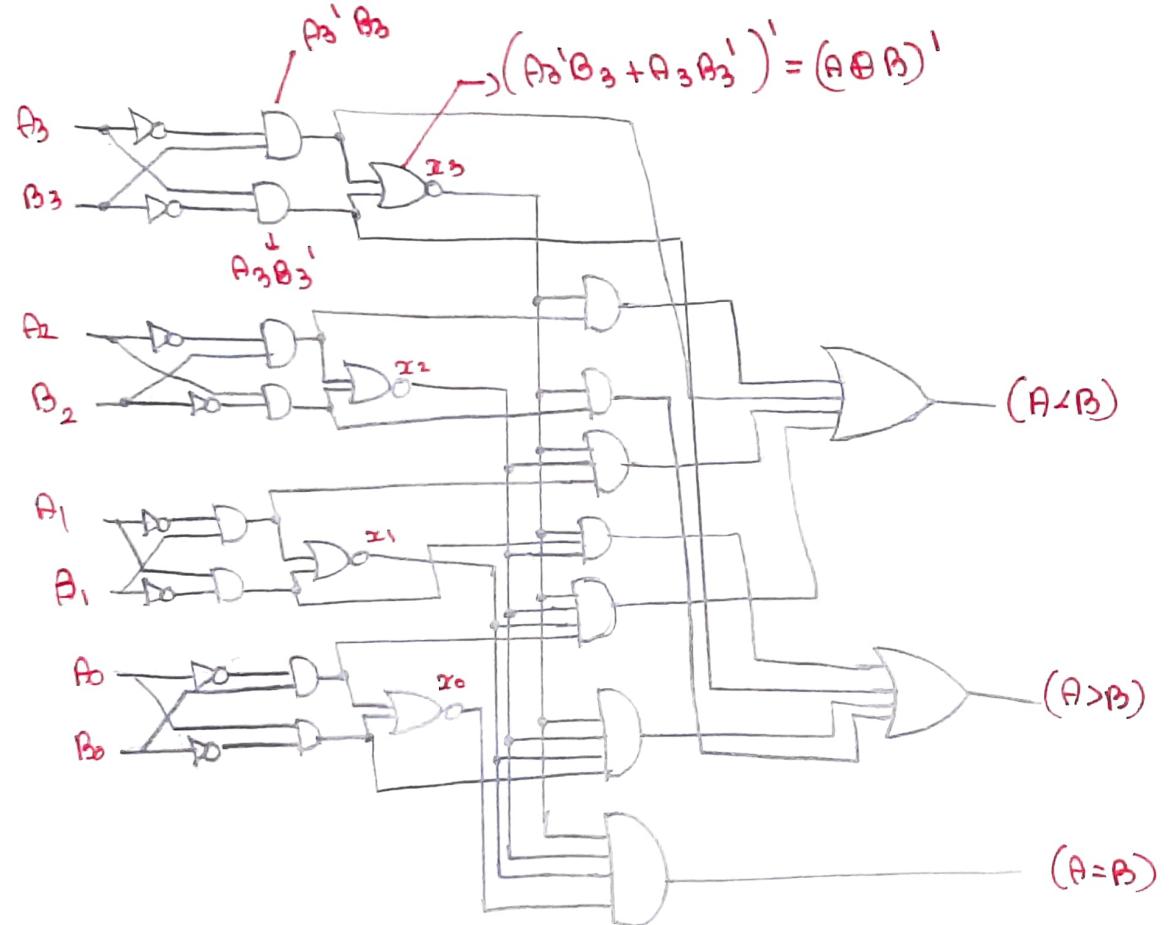
$$X_2 = (0 \oplus 1)' = 0$$

$$X_3 = (1 \oplus 1)' = (0)' = 1$$

$$A=B \Rightarrow X_3 X_2 X_1 X_0 = 1 \cdot 0 \cdot 0 \cdot 0 = 0_{11}$$

$$\begin{aligned} A > B &= 1 \cdot 0 \cdot 1' + 1 \cdot 0 \cdot (1)' + 1 \cdot 0 \cdot 1 \cdot (0)' + 1 \cdot 0 \cdot 0 \cdot 0 \cdot 1 \cdot (0)' \\ &= 1 \cdot 0 + 1 \cdot 0 \cdot 0 + 1 \cdot 0 \cdot 1 \cdot 1 + 1 \cdot 0 \cdot 0 \cdot 0 \cdot 1 \cdot 1 \\ &= 0_{11} \end{aligned}$$

$$\begin{aligned} A < B &= (1)' \cdot 1 + 1 \cdot (0)' \cdot 1 + 1 \cdot 0 \cdot 0 \cdot 1 \cdot 0 + 1 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \\ &= 0 \cdot 1 + 1 \cdot 1 \cdot 1 + 1 \cdot 0 \cdot 0 \cdot 0 + 1 \cdot 0 \cdot 0 \cdot 0 \cdot 0 \\ &= 1_{11} \end{aligned}$$

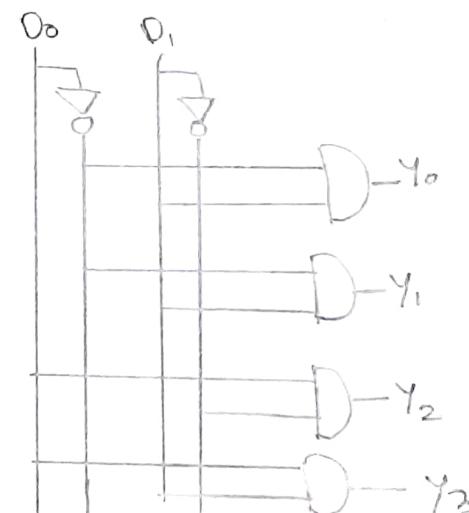


Decoder:

$\rightarrow$  generate minterm  
 $\rightarrow n \times 2^n \downarrow$  output  
 input

Truth table of 2x4 decoder.

$D_0$	$D_1$	$Y_0$	$Y_1$	$Y_2$	$Y_3$
0	0	1	0	0	0
0	1	0	1	0	0
1	0	0	0	1	0
*	*	0	0	1	1
1	1	0	0	0	1



Implementation of a boolean function using decoder

$$F(A, B, C) = \Sigma (0, 2, 3, 4, 7)$$

$$F(A, B, C) = \Sigma (0, 2, 3, 4, 7)$$

3 inputs

$$\text{decoder} = n \times 2^k = 3 \times 2^3 = 3 \times 8$$



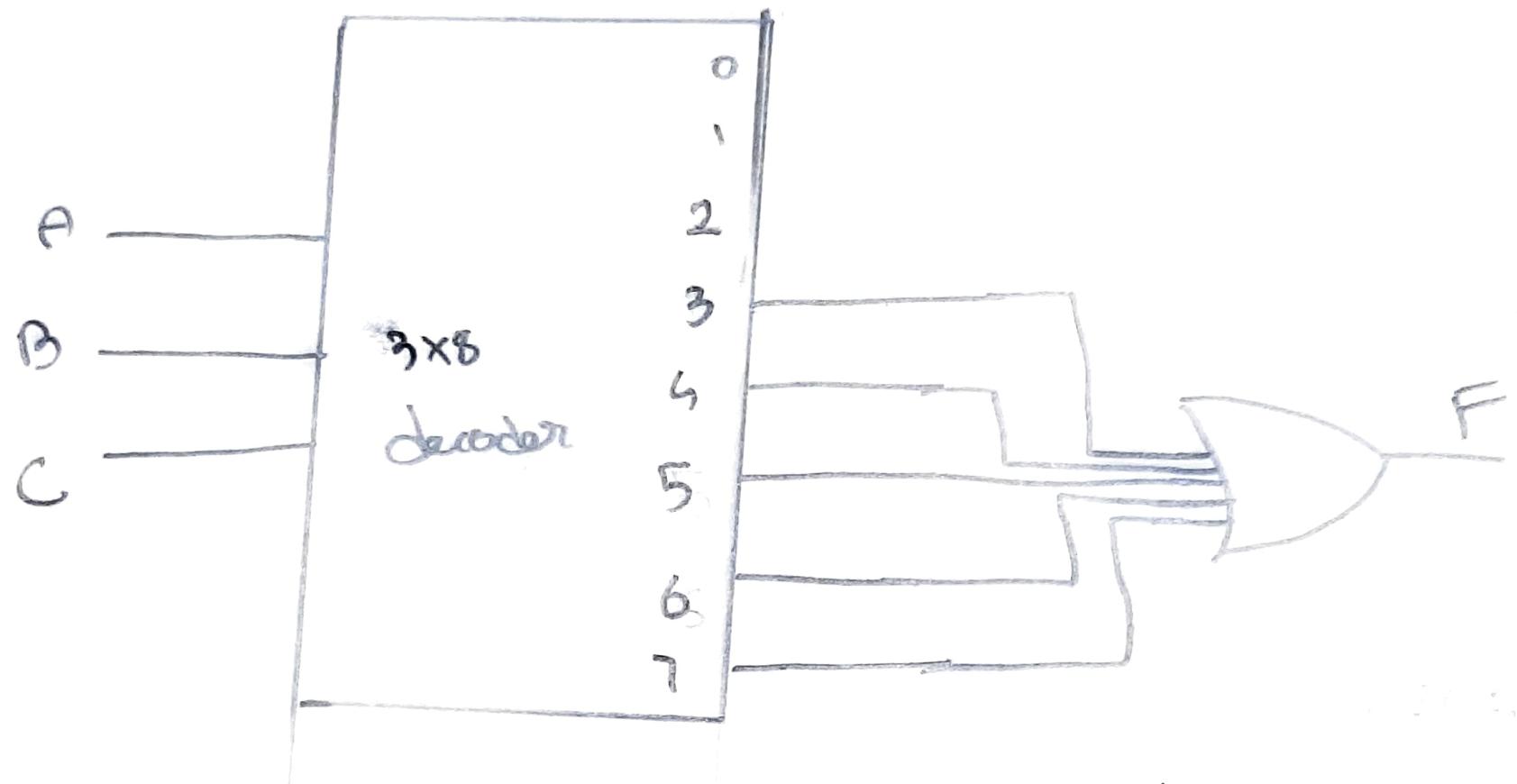
Implement the boolean function  $F = A + BC$  using decoder.

$$F = A + BC$$

$$= A(B + B') (C + C') + BC(A + A')$$

$$= \underline{AB}C + A\underline{B}C + AB\underline{C}' + AB'C' + \underline{A}BC + A'B'C$$

$$\Rightarrow \begin{array}{c} \cancel{1} \\ 7 \end{array} + \begin{array}{c} \cancel{1} \\ 5 \end{array} + \begin{array}{c} \cancel{1} \\ 6 \end{array} + \begin{array}{c} \cancel{1} \\ 4 \end{array} + \begin{array}{c} \cancel{0} \\ 3 \end{array}$$



implement a full adder using decoder.

A	B	Cin	Cout	Sum
0	0	0	0	0
0	0	1	0	1
0	1	0	0	1
0	1	1	1	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	1	1

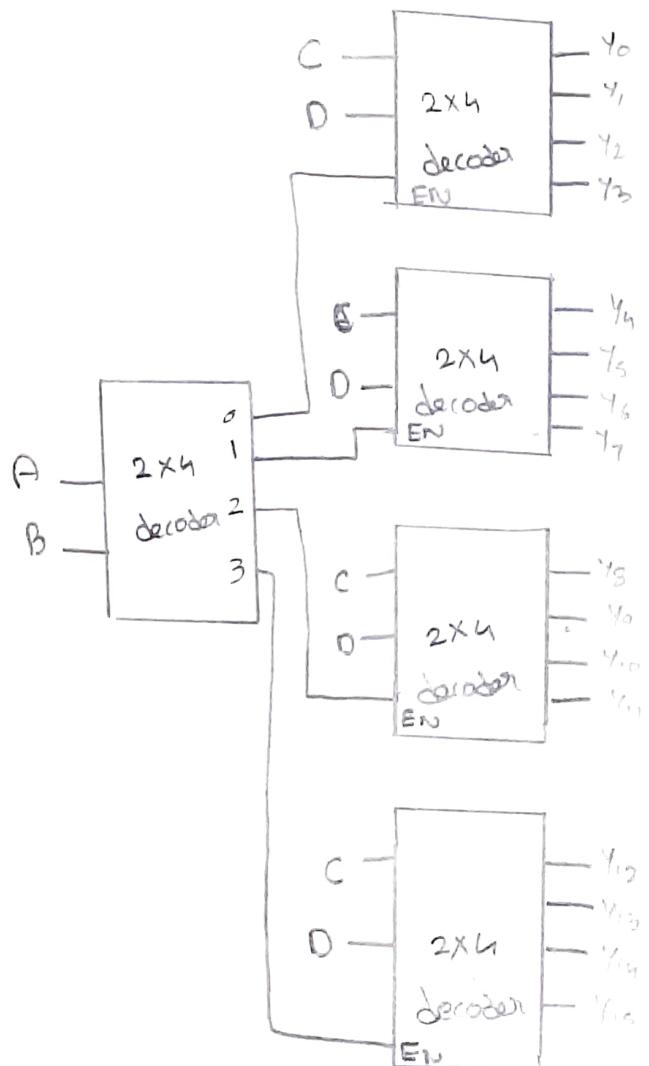
$$Cout = \Sigma(3, 5, 6, 7)$$

$$Sum = \Sigma(1, 2, 4, 7)$$



complement higher order to lower order decoder

using  
1)  $4 \times 16 \rightarrow 2 \times 4$

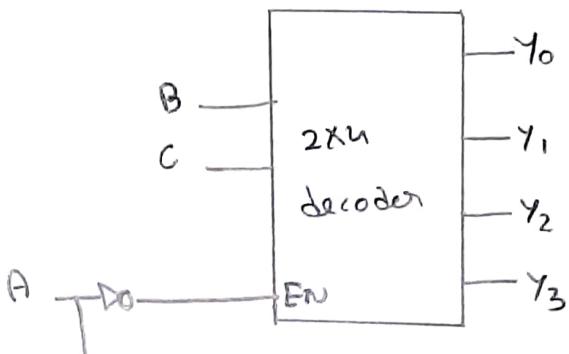


EN → enable → additional input

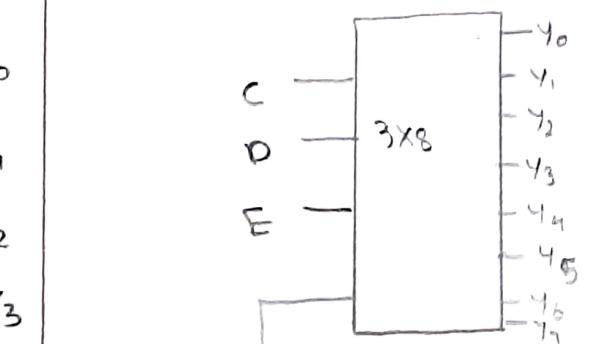
If EN=1 output = 1

If EN=0 output = 0

2)  $3 \times 8$  using  $2 \times 4$



3)  $5 \times 32$  using  $3 \times 8$

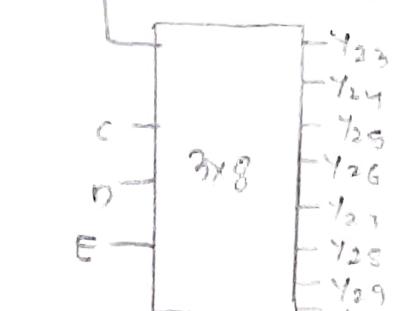


Sums to solve

\* complement 1.  $F(A, B, C, D) = \Sigma(0, 1, 2, 4)$

2.  $F = A + B D + C$

3.  $3 \times 8$  using  $1 \times 2$



Multiplexer:

it is combinational circuit that selects one of many input & direct its to the output.

$$\Rightarrow 2^n \times 1 \text{ MUX}$$

↓                  ↓  
input              output

implement the boolean function  $F = \Sigma(0, 1, 3, 7, 8, 12, 14, 15)$  using MUX

let inputs be  $A, B, C, D$

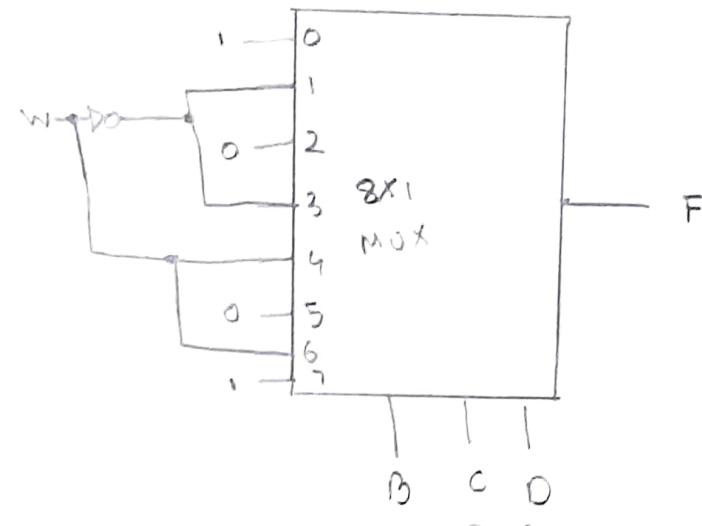
data input =  $A$

select input =  $B, C, D$

$$\Rightarrow 2^n \times 1 = 2^3 \times 1 = 8 \times \text{MUX}$$

$w'$	$I_0$	$I_1$	$I_2$	$I_3$	$I_4$	$I_5$	$I_6$	$I_7$
0	0	0	2	3	4	5	6	7
1	8	9	10	11	12	13	14	15

$w' \quad w' \quad w'$

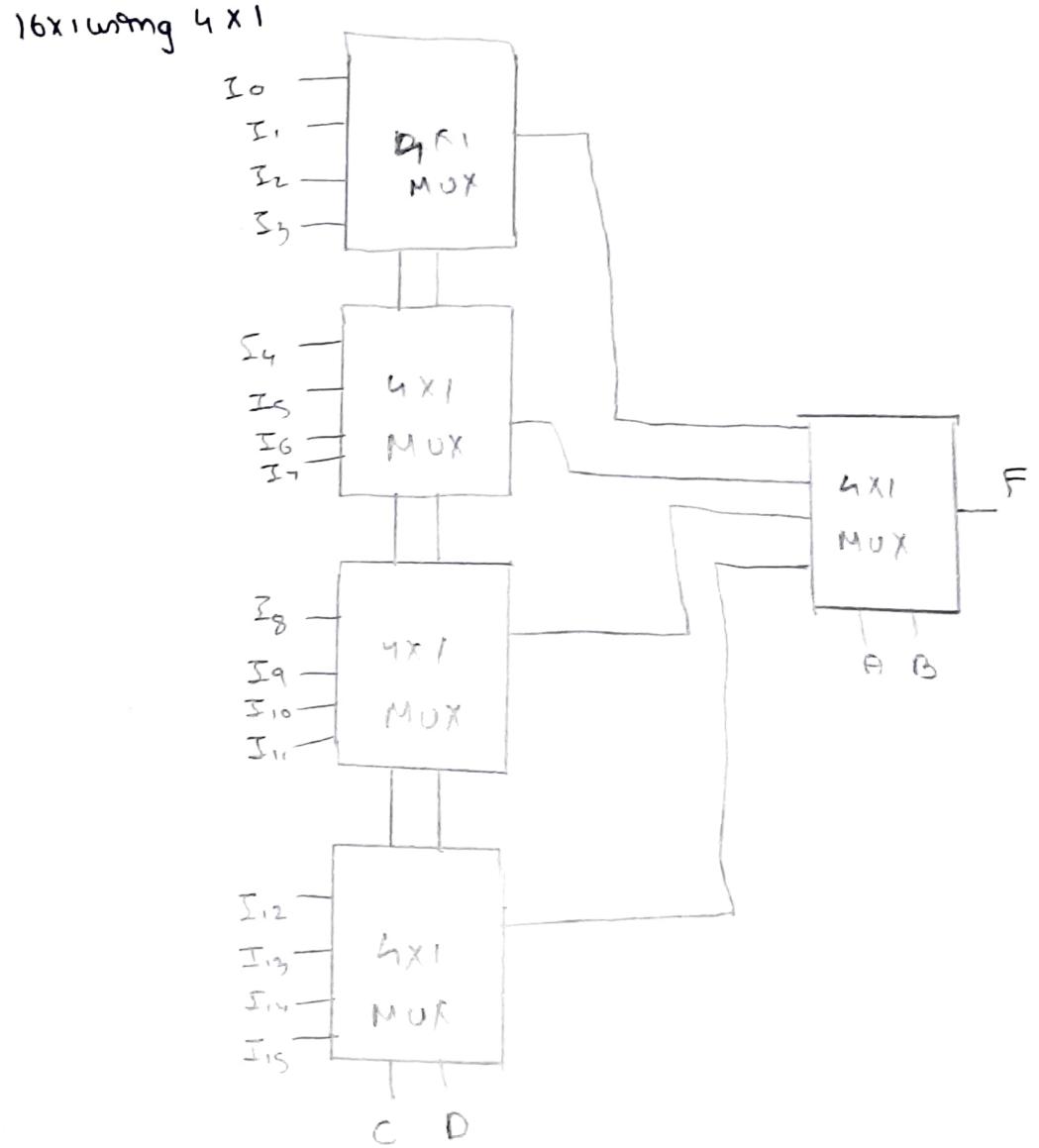
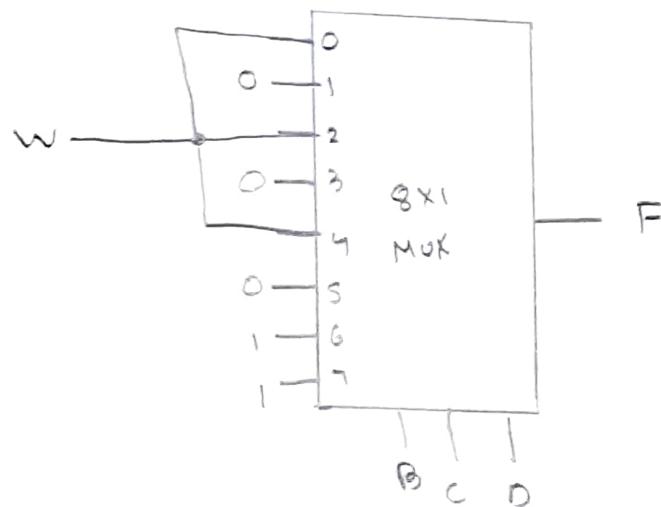


Select output (using group)

implement  $F = A'B'C + A'D' + B'CD$  using MUX

$$\begin{aligned}
 &= A'B'C(D+D') + A'D'(B+B')(C+C') \\
 &\quad + B'CD(A+A') \\
 &= \underline{A'B'C}D + \underline{A'B'C}D' + A'BCD' + AB'C'D' \\
 &\quad + AB'C'D + ABC'D' + ABCD + \underline{A'B'C}D \\
 &= A'B'CD + A'B'C'D' + A'BC'D' + A'B'C'D \\
 &\quad + AB'C'D' + ABC'D' + ABCD \\
 &= \Sigma(6, 7, 8, 10, 12, 14, 15)
 \end{aligned}$$

	$I_0$	$I_1$	$I_2$	$I_3$	$I_4$	$I_5$	$I_6$	$I_7$
$w^1$	0	1	2	3	4	5	6	7
$w$	1	(8)	9	(10)	11	(12)	13	(14)
	w	0	w	0	w	0	1	1



Sums to solve 1.  $F(A, B, C, D) = \Sigma(0, 1, 2, 4, 5)$

$$2. F = A + B + C + AB + CD$$

$$3. 32x1 wmg 8x1$$

Encoder:-

→ it is combinational circuit

→ inverse of decoder

⇒  $2^n \times n$   
↓  
output  
input

Truth table of 4x2 encoder

$D_0$	$D_1$	$D_2$	$D_3$	$Y_0$	$Y_1$
1	0	0	0	0	0
0	1	0	0	0	1
0	0	1	0	1	0
0	0	0	1	1	1

drawback:-

→ if more than one input is high

then O/P not defined

→ when all input are 0 then O/P

~~not defined~~. will be 0.

Priority encoder:-

→ uses priority function

→ if  $D_1, D_2, D_3$  are high  $D_3$  will have highest priority.

4x2 priority encoder:-

$D_0$	$D_1$	$D_2$	$D_3$	$Y_0$	$Y_1$	$V$
0	0	0	0	X	X	0
1	0	0	0	0	0	1
X	1	0	0	0	1	1
X	X	1	0	1	0	1
X	X	X	1	1	1	1

Kmaps for  $Y_0$

		00	01	11	10
		X	1	1	1
00					
01					
11					
10					

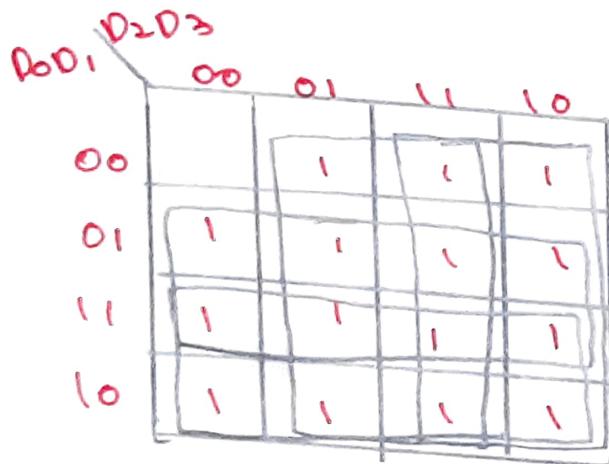
$$Y_0 = D_3 + D_2$$

Kmaps for  $Y_1$

		00	01	11	10
		X	1	1	1
00					
01					
11					
10					

$$Y_1 = D_3 + D_1 D_2'$$

Karnaugh map for V



$$V = D_0 + D_1 + D_2 + D_3$$

