Lab_1_2020

February 27, 2020

1 Lab1: Simple Connectionist Models of Language

Lab design and questions: Jennifer Spenader 2019/2020, Python code version: Wietse de Vries, 2019

Revised by: Jacqueline van Arkel, Sohyung Kim, Giorgos Tziafas, 2020

Varun Ravi Varma (S3893030), *worked with Magdalena Bilska (S4086511)*

1.1 Learning Goals for this lab:

- Get used to use the Keras deep learning framework
- Build simple feed-forward connectionist networks
- Understand the usefulness of hidden layers
- Understand the features of simple recurrent networks (SRNs)
- Build a simple recurrent network by using Keras that performs the sameas Elman (1990)
- Carry out a follow-up experiment with the Elman (1990) network
- Answer questions related to Elman (1990)
- All lab reports should be handed in individually. However, most questions can be compeleted in pairs if desired. The exception is Exercise 6 at the end of the lab. Exercise 6 should be answered individually. If you work together, please clearly identify who you worked with in your homework. Also, please add your name and student number to the first page of the homework.

Note: only questions next to capital letters in `Exercise` sections need to be included in the report. Other questions are rhetorical.

2 Part 1: Learning boolean operators using a feed-forward neural networks

In this section, you will learn how to create a simple feed-forward neural network in Python by using Keras. Feed-forward neural networks are one of the simplest types, because all information is passed on in one direction (forward!), with no information being 'looped' back into the netwokr.

Keras is a high-level neural network API that runs on top of lower level neural network libraries like Tensorflow or Theano. Since the Tensorflow 2.0 official release, Keras is integrated as a back-end to the Tensorflow framework. Keras will let use easily build neural networks by simply defining what their design architecture should be.

Before we get started, make sure you have installed Keras with a backend of your choice. Follow the official instructions by Keras or get started quickly by running the following commands:

```
pip3 install tensorflow
pip3 install keras
```

This is all you need to do to get started creating your first neural network! Now we are ready to create your first neural network that simulates the boolean AND operator.

2.1 Model definition

To set up our neural network, e.g. define our 'model', we have to import some modules from Keras:

[39]:
```
seed_value = 0
# Set the global pythonhashseed variable to zero for reproducibility
import os
os.environ['PYTHONHASHSEED']=str(seed_value)
# Set the `numpy` pseudo-random generator at a fixed value
import numpy as np
np.random.seed(seed_value)
# Setting the tensorflow seed
import tensorflow as tf
tf.random.set_seed(seed_value)
```

[40]:
```
from tensorflow.keras.models import Sequential
from tensorflow.keras.layers import Dense
from tensorflow.keras.optimizers import SGD
```

Now we want to create very simple perceptron model that has two binary input nodes (that accept `0`'s or `1`'s), and a single binary output node that outputs a `1` if and only if both inputs were `1`, and otherwise will output a `0`. (Can you already guess what this network is?)

In Keras, we can create a simple feed-forward neural network by creating what's termed a `Sequential` model. Within a `Sequential` model, an arbitrary number of layers can be defined. The last layer is the output layer and any preceding layers are hidden layers. The first layer in the model requires information about the input shape, i.e. how many nodes are 'sending through' input. Note that the input layer is not explicitly defined as a layer in a `Sequential` Keras model.

The model can be defined the following way:

[41]:
```
model = Sequential([
    Dense(1, input_dim=2, activation='tanh', name='output')
])

model.summary()
```

```
Model: "sequential_7"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
```

```
output (Dense)               (None, 1)                 3
=================================================================
Total params: 3
Trainable params: 3
Non-trainable params: 0
_________________________________________________________________
```

```
    This command will create a feed-forward network (a `model`) that contains a single layer w
```

After defining the model, it has be be compiled, i.e. it has to be run, in a process where the weights will be modified depending on how well the network dos in predicting the output given some input.

[42]:
```
model.compile(loss='mean_squared_error',
              optimizer=SGD(lr=0.3, momentum=0.8),
              metrics=['accuracy'])
```

In compiling, the model will try to minimize error, also known as `loss`. We use the mean squared error loss function to cacluate loss with the stochastic gradient descent optimizer. (It is a good idea to read a bit about this and familiarize yourself with these terms). The learning rate and momentum often have to be chosen by trial an error. Frequently, getting the fastest or most accurate performance are considerations, but for our purposes, how realistic they are for simulating human learning might be a consideration. The `compile` method also allows definition of metrics, i.e. ways in which you can evaluate the quality of the results. This does not influence training, but just gives us some nice information to analyze (and hopefully graph!).

2.2 Training

Before we start training however, we need some training data. Python machine learning conventions are that input data is defined as an `X` variable and the corresponding target output as `y`. Recall that our model has only two input nodes, so this means the only possibilities (since these are binary input nodes) is one of four possible combinations of `0`s and `1`s. See below. So we will need two-dimensional input values and are trying to learn to predict the one-dimensional output given in `y` such athat only when both input values are `1` do we output `1`.

[43]:
```
import numpy as np
```

[44]:
```
X_and = np.array([(0,0), (1,0), (0,1), (1,1)])
y_and = np.array([0, 0, 0, 1])
```

Now we can train our model. We'll store the training in `history`. We'll train for 15 epochs, which means that we'll run the entire training set (shuffled) trhough the network, each time revising the weights based on the difference between our output and `y`, 15 times. We would also like to know what is going on as it's running, so we'll ask for some intermittend output information (e.g. `verbose=2`).

[45]:
```
history = model.fit(X_and, y_and, epochs=15, shuffle=True, verbose=2)
```

```
Train on 4 samples
Epoch 1/15
4/4 - 0s - loss: 1.0780 - accuracy: 0.7500
```

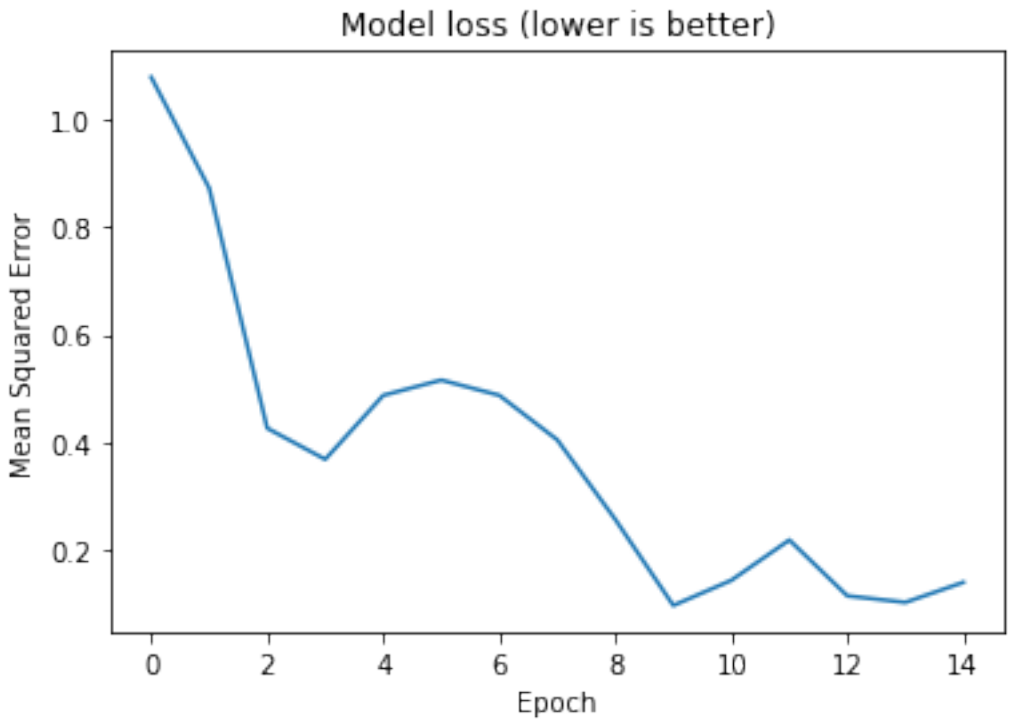
```
Epoch 2/15
4/4 - 0s - loss: 0.8720 - accuracy: 0.7500
Epoch 3/15
4/4 - 0s - loss: 0.4268 - accuracy: 0.7500
Epoch 4/15
4/4 - 0s - loss: 0.3691 - accuracy: 0.2500
Epoch 5/15
4/4 - 0s - loss: 0.4877 - accuracy: 0.2500
Epoch 6/15
4/4 - 0s - loss: 0.5161 - accuracy: 0.2500
Epoch 7/15
4/4 - 0s - loss: 0.4879 - accuracy: 0.2500
Epoch 8/15
4/4 - 0s - loss: 0.4052 - accuracy: 0.2500
Epoch 9/15
4/4 - 0s - loss: 0.2576 - accuracy: 0.5000
Epoch 10/15
4/4 - 0s - loss: 0.0980 - accuracy: 1.0000
Epoch 11/15
4/4 - 0s - loss: 0.1447 - accuracy: 0.7500
Epoch 12/15
4/4 - 0s - loss: 0.2194 - accuracy: 0.7500
Epoch 13/15
4/4 - 0s - loss: 0.1158 - accuracy: 1.0000
Epoch 14/15
4/4 - 0s - loss: 0.1037 - accuracy: 1.0000
Epoch 15/15
4/4 - 0s - loss: 0.1409 - accuracy: 1.0000
```

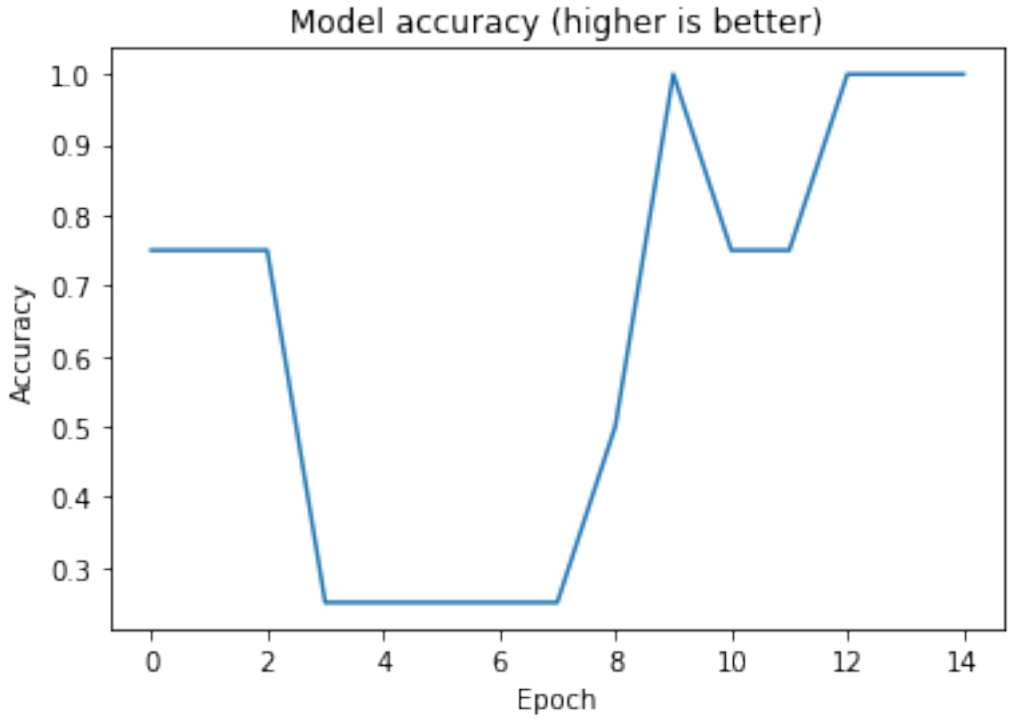
Well, an accuracy of `1.0` seems good. Lets look at our training process:

[46]:
```
import matplotlib.pyplot as plt
```

With the below code snippet, you can visualise the loss and the accuracy progression of your trained model through the `'loss'` and `'acc'` keys of the dictionary object `history.history`:

[47]:
```
plt.plot(history.history['loss'])
plt.title('Model loss (lower is better)')
plt.ylabel('Mean Squared Error')
plt.xlabel('Epoch')
plt.show()

plt.plot(history.history['accuracy'])
plt.title('Model accuracy (higher is better)')
plt.ylabel('Accuracy')
plt.xlabel('Epoch')
plt.show()
```

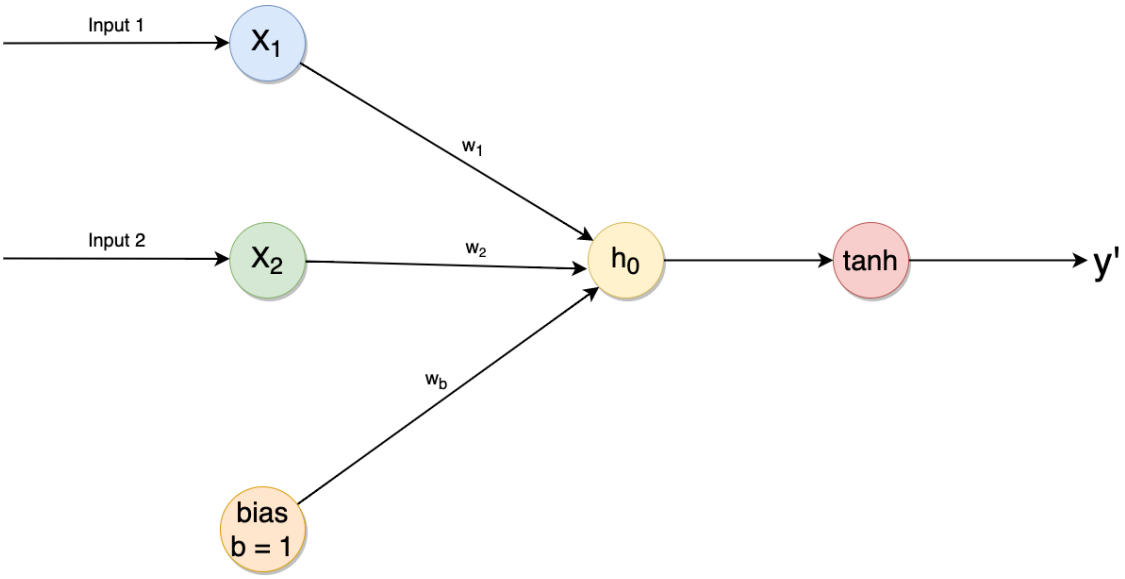

Model loss (lower is better)
Mean Squared Error
1.0
0.8
0.6
0.4
0.2
0
2
4
6
8
10
12
14
Epoch


Model accuracy (higher is better)
Accuracy
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0
2
4
6
8
10
12
14
Epoch

Note that the loss decreases nicely, but it is not yet completely zero. For deterministic problems like a simple AND operation this would be possible but in real world scenario's having a zero loss is often impossible.

2.3 Exercise 1

A) Draw the architecture diagram of the neural network with circles and arrows. How many nodes does the model have (incl. bias)?.



The model has 4 nodes (2 input nodes, 1 bias node and 1 output node).

B)What is the formula that describes the output `y'` with respect to the inputs `(x1,x2)` (Hint: use w1 and w2 for weights)?

$$y' = tanh(w_1 * x_1 + w_2 * x_2 + w_b * b)$$

C) What is the loss function used for training this model? Provide it's formula for model predictions `y'` and target outputs `y`.

The mean squared error is used for training this model. The formula is as follows:

$$MSE = \frac{1}{n}\sum_{i=0}^{n}(y_i - y_i')^2$$

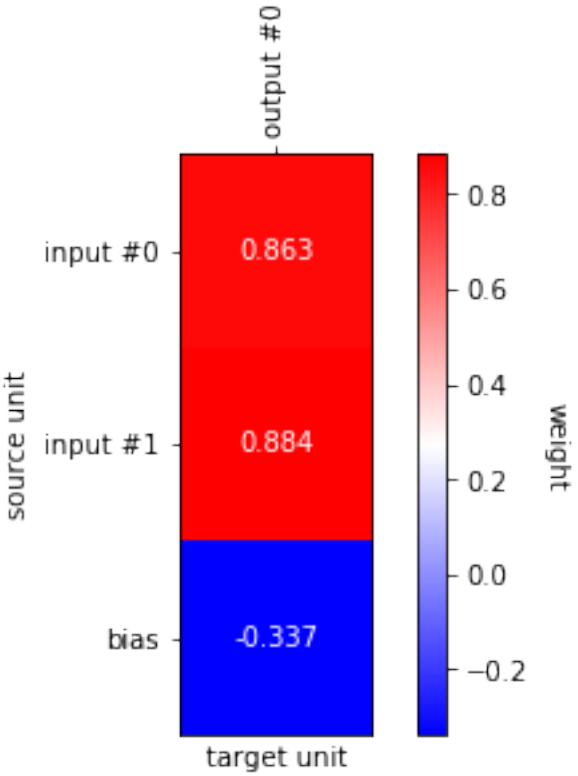
where n is the number of samples.

2.4 Network analysis

To see what the network learned, we can visualize the model weights using a special function that is defined in `plot_functions.py`. This will show us the network with its weights, showing positive weights in red, and negative weights in blue.

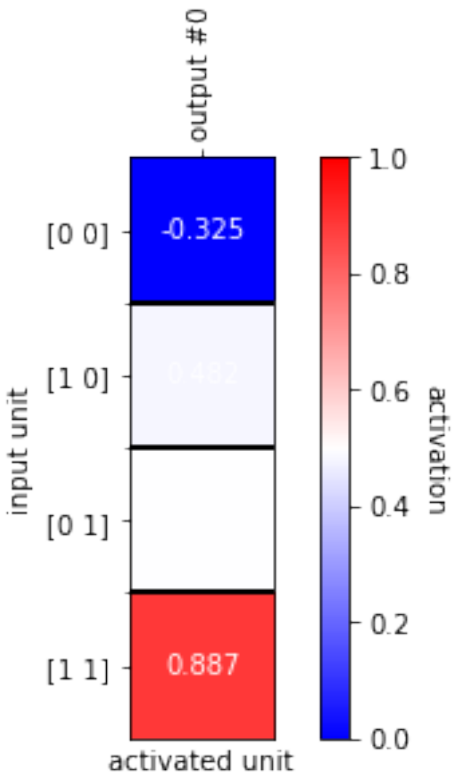
[48]:
```
from plot_functions import plot_weight_matrix, plot_activation_matrix
```

[49]:
```
plot_weight_matrix(model)
```



Now we see that the output is only above 0.5 if both input values are 1. Let's see what the exact outputs are for our four possible inputs:

[50]:
```
plot_activation_matrix(model, X_and, vmin=0, vmax=1)
```



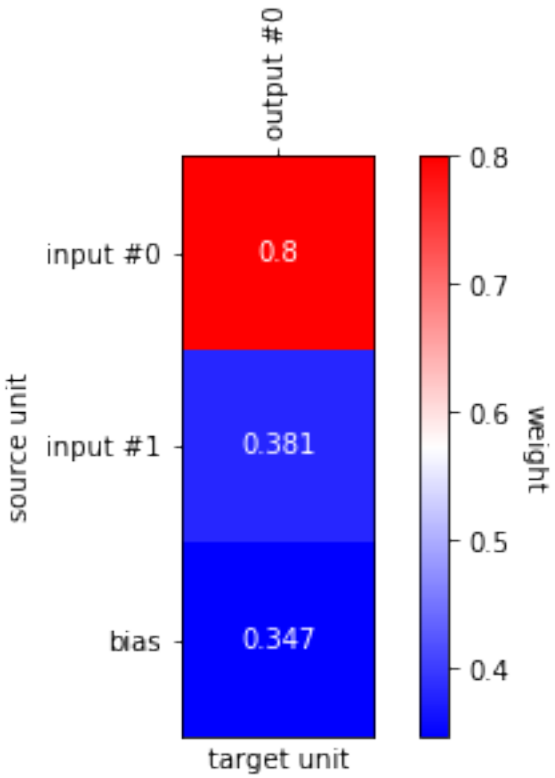
2.5 Exercise 2

A) Re-run the model, but this time only run 4 or 5 Epochs (so that you don't get 1.00 accuracy). Then plot the weight matrix for the model. Multiply the inputs by the weights to get the output. Explain how the model works and why it doesn't get everything correct.

[51]:
```
# Initializing a new model, since the old model still uses the weights from␣
 ↪previous training session
model_2 = Sequential([
    Dense(1, input_dim=2, activation='tanh', name='output')
])
model_2.compile(loss='mean_squared_error',
              optimizer=SGD(lr=0.3, momentum=0.8),
              metrics=['accuracy'])
# Training the new model for 5 epochs
model_2.fit(X_and, y_and, epochs=5, shuffle=True, verbose=2)
plot_weight_matrix(model_2)
```

```
Train on 4 samples
Epoch 1/5
4/4 - 0s - loss: 0.7218 - accuracy: 0.7500
Epoch 2/5
```

```
4/4 - 0s - loss: 0.3861 - accuracy: 0.7500
Epoch 3/5
4/4 - 0s - loss: 0.2399 - accuracy: 0.7500
Epoch 4/5
4/4 - 0s - loss: 0.3446 - accuracy: 0.2500
Epoch 5/5
4/4 - 0s - loss: 0.3604 - accuracy: 0.5000
```



The output computed by the neural network can be computed using the following equation:

$$\begin{aligned} y' &= tanh(w_1 * x_1 + w_2 * x_2 + w_b * b) \\ &= tanh(0.8 * x_1 + 0.381 * x_2 + 0.347) \end{aligned}$$

Tabulating the output $ y' $ that is calculated by the model:

$ x_{1} $	$ x_{2} $	$ y' $
0	0	0.334
1	0	0.817
0	1	0.622
1	1	0.910

The model is optimized with SGD with momentum, with a high learning rate of 0.3 and a Nesterov momentum value of 0.8. This could enable the model to skip over local minima over multiple epochs of training, but over smaller epochs, we see that the model fails to compute the optimal weights. This could be due to the fact that there is increased noise in the update of weights due to hugh values of momentum over the 5 epochs of training. Since the weights are not optimised, the model is unable to classify some inputs correctly (in this case the input $[1, 0]$ returns an output of 1).

B) Now that you know how to define and train a simple model, try to model the inclusive OR operator instead of AND using the same model. You don't have to change the model architecture: simply create a new training data and your own training data (X) and target output (y) for inclusive OR.

[52]:
```
X_or = np.array([(0,0), (1,0), (0,1), (1,1)])
y_or = np.array([0, 1, 1, 1])
# Initializing a new model, since the old model still uses the weights from␣
 ↪previous training session
model_3 = Sequential([
    Dense(1, input_dim=2, activation='tanh', name='output')
])
model_3.compile(loss='mean_squared_error',
              optimizer=SGD(lr=0.3, momentum=0.8),
              metrics=['accuracy'])
# Fitting the model on OR data
run_or = model_3.fit(X_or, y_or, epochs=30, shuffle=True, verbose=2)
```

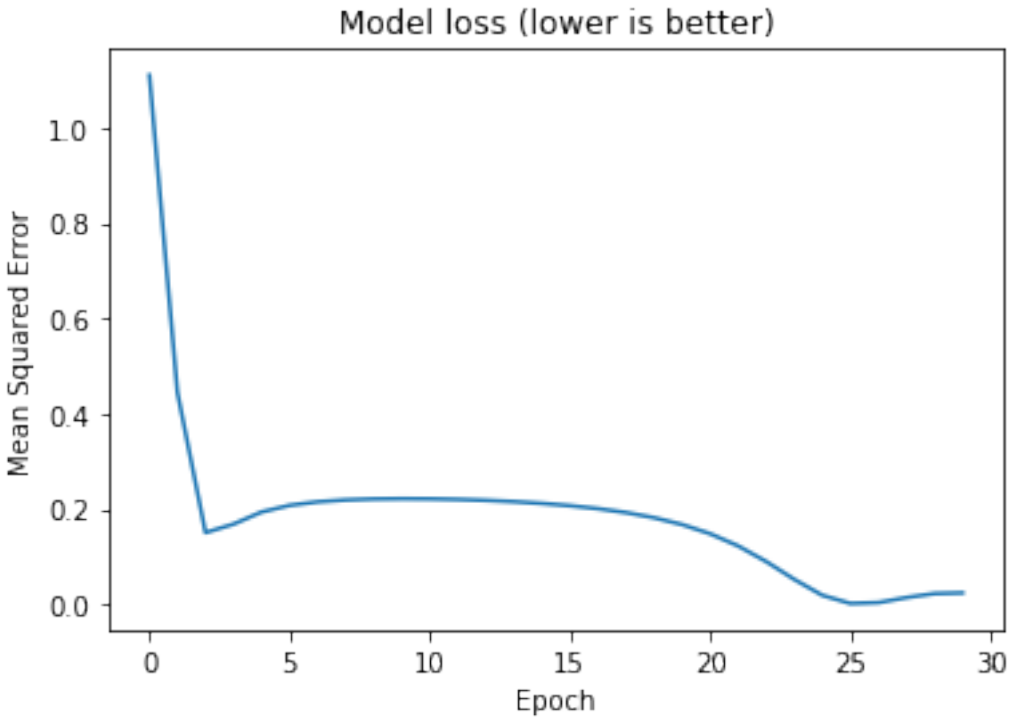
```
Train on 4 samples
Epoch 1/30
4/4 - 0s - loss: 1.1106 - accuracy: 0.5000
Epoch 2/30
4/4 - 0s - loss: 0.4447 - accuracy: 0.7500
Epoch 3/30
4/4 - 0s - loss: 0.1506 - accuracy: 0.7500
Epoch 4/30
4/4 - 0s - loss: 0.1684 - accuracy: 0.7500
Epoch 5/30
4/4 - 0s - loss: 0.1940 - accuracy: 0.7500
Epoch 6/30
4/4 - 0s - loss: 0.2078 - accuracy: 0.7500
Epoch 7/30
4/4 - 0s - loss: 0.2153 - accuracy: 0.7500
Epoch 8/30
4/4 - 0s - loss: 0.2192 - accuracy: 0.7500
Epoch 9/30
4/4 - 0s - loss: 0.2212 - accuracy: 0.7500
Epoch 10/30
4/4 - 0s - loss: 0.2218 - accuracy: 0.7500
Epoch 11/30
4/4 - 0s - loss: 0.2215 - accuracy: 0.7500
```

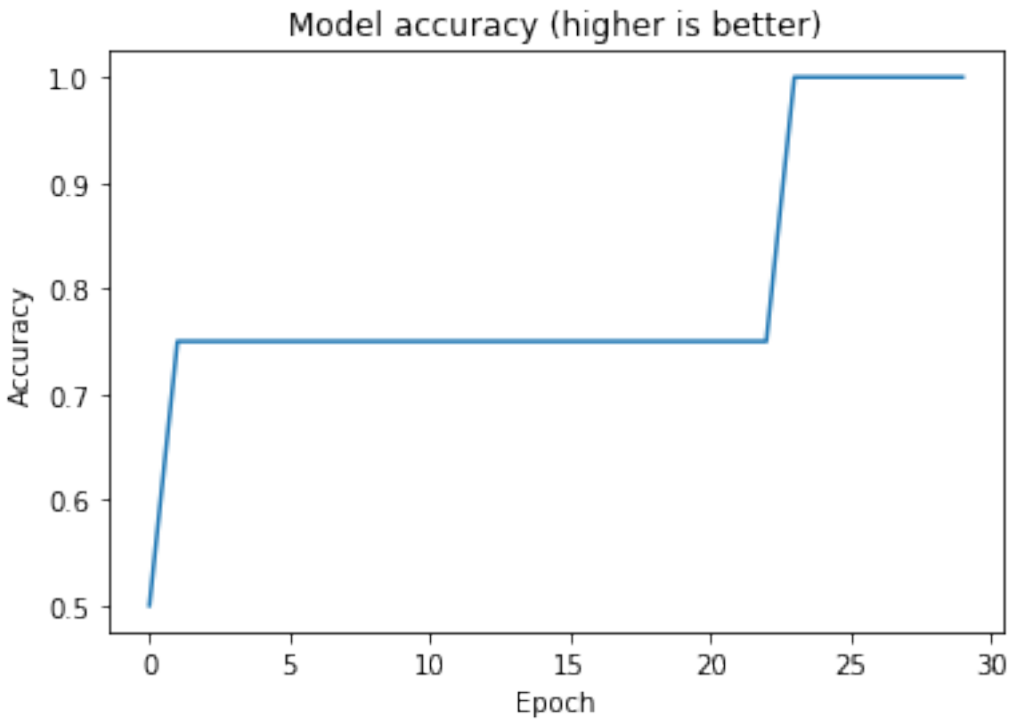
```
Epoch 12/30
4/4 - 0s - loss: 0.2203 - accuracy: 0.7500
Epoch 13/30
4/4 - 0s - loss: 0.2184 - accuracy: 0.7500
Epoch 14/30
4/4 - 0s - loss: 0.2156 - accuracy: 0.7500
Epoch 15/30
4/4 - 0s - loss: 0.2119 - accuracy: 0.7500
Epoch 16/30
4/4 - 0s - loss: 0.2071 - accuracy: 0.7500
Epoch 17/30
4/4 - 0s - loss: 0.2009 - accuracy: 0.7500
Epoch 18/30
4/4 - 0s - loss: 0.1927 - accuracy: 0.7500
Epoch 19/30
4/4 - 0s - loss: 0.1819 - accuracy: 0.7500
Epoch 20/30
4/4 - 0s - loss: 0.1675 - accuracy: 0.7500
Epoch 21/30
4/4 - 0s - loss: 0.1482 - accuracy: 0.7500
Epoch 22/30
4/4 - 0s - loss: 0.1226 - accuracy: 0.7500
Epoch 23/30
4/4 - 0s - loss: 0.0898 - accuracy: 0.7500
Epoch 24/30
4/4 - 0s - loss: 0.0524 - accuracy: 1.0000
Epoch 25/30
4/4 - 0s - loss: 0.0190 - accuracy: 1.0000
Epoch 26/30
4/4 - 0s - loss: 0.0019 - accuracy: 1.0000
Epoch 27/30
4/4 - 0s - loss: 0.0038 - accuracy: 1.0000
Epoch 28/30
4/4 - 0s - loss: 0.0144 - accuracy: 1.0000
Epoch 29/30
4/4 - 0s - loss: 0.0227 - accuracy: 1.0000
Epoch 30/30
4/4 - 0s - loss: 0.0241 - accuracy: 1.0000
```

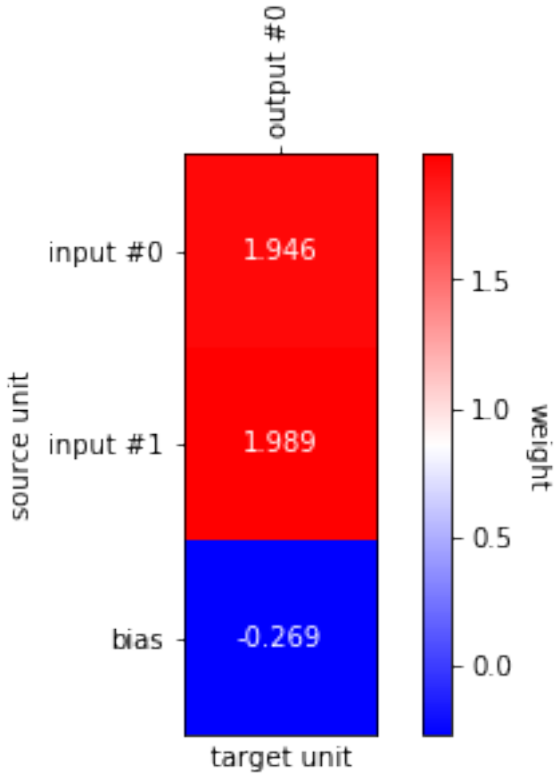
C) What is the resulting loss compared to the AND implementation? Plot the loss and accuracy and include these plots in your answer. Is the model able to deal correctly with inclusive OR ? How does the model work? Explain your answer by referring to the weights learned.

[53]:
```
plt.plot(run_or.history['loss'])
plt.title('Model loss (lower is better)')
plt.ylabel('Mean Squared Error')
plt.xlabel('Epoch')
plt.show()
```

```
plt.plot(run_or.history['accuracy'])
plt.title('Model accuracy (higher is better)')
plt.ylabel('Accuracy')
plt.xlabel('Epoch')
plt.show()

plot_weight_matrix(model_3)
```




Model accuracy (higher is better)
Accuracy
Epoch
1.0
0.9
0.8
0.7
0.6
0.5
0
5
10
15
20
25
30


output #0
input #0
input #1
bias
source unit
1.946
1.989
-0.269
target unit
1.5
1.0
0.5
0.0
weight

The model takes longer to learn the OR function as compared to the AND function. At 15 epochs the model is only able to learn to represent 3 out of 4 inputs to correct outputs. The weights from training on the OR data are significantly different as compared to the weights for the AND data. Classification is learnt as a representation in a vector space, based on the inputs and labels given to the model. This is reflected in the higher weights of the OR model as compared to the AND model. The outputs from training on the OR data are as follows:

$$y' = tanh(1.946 * x_1 + 1.989 * x_2 - 0.269 * b)$$

$ x_{1} $	$ x_{2} $	$ y' $
0	0	-0.263
1	0	0.932
0	1	0.938
1	1	0.999

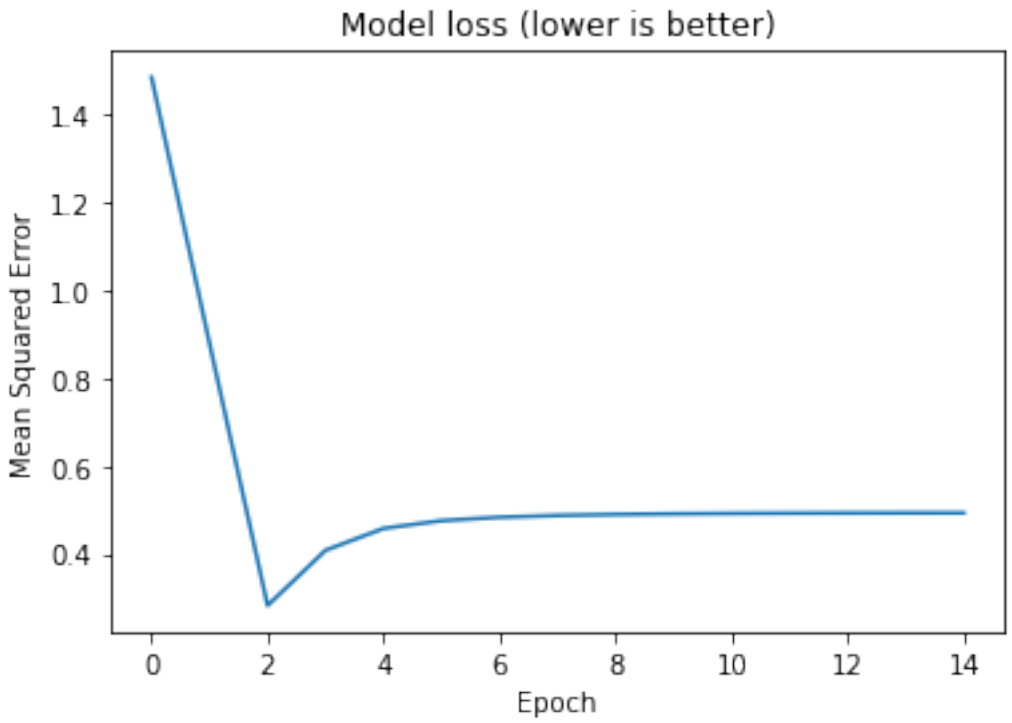
Thus the model learns to represent the OR function over longer training epochs.

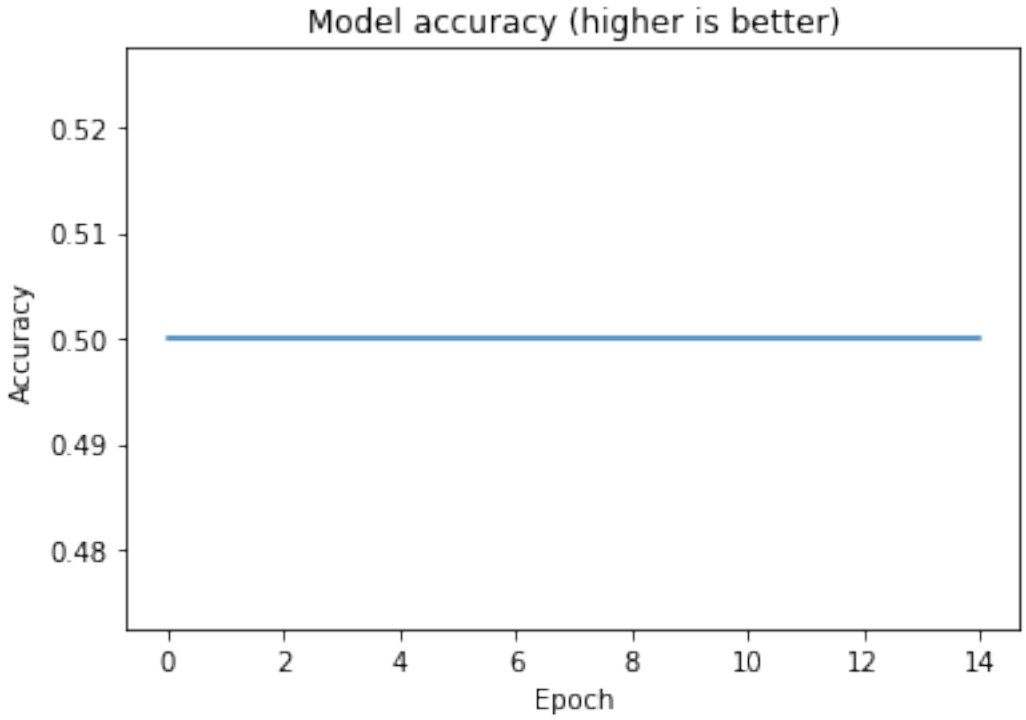
D) Try to do the same for the exclusive OR (XOR) operator. Create training and target data and use the same model to train. What do you notice? Explain what problems you encounter and why.

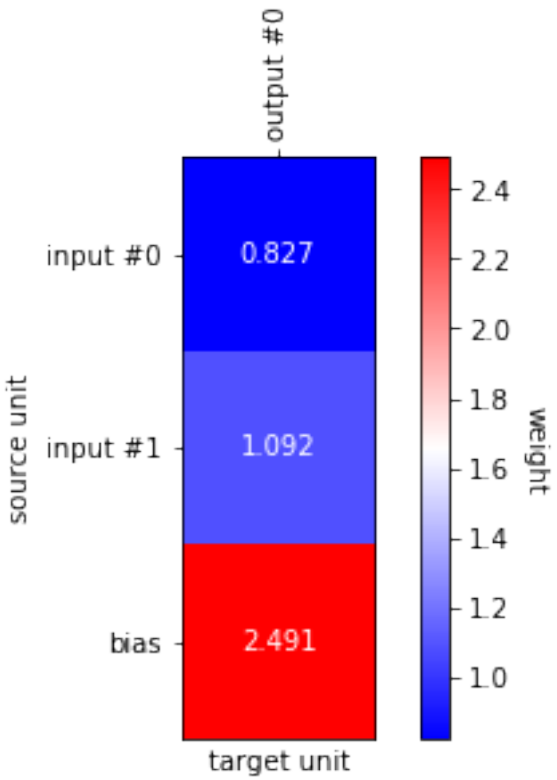
[54]:
```
X_xor = np.array([(0,0), (1,0), (0,1), (1,1)])
y_xor = np.array([0, 1, 1, 0])

# Initializing a new model, since the old model still uses the weights from␣
 ↪previous training session
model_4 = Sequential([
    Dense(1, input_dim=2, activation='tanh', name='output')
])
model_4.compile(loss='mean_squared_error',
              optimizer=SGD(lr=0.3, momentum=0.8),
              metrics=['accuracy'])

run_xor = model_4.fit(X_xor, y_xor, epochs=15, shuffle=True, verbose=2)

plt.plot(run_xor.history['loss'])
plt.title('Model loss (lower is better)')
plt.ylabel('Mean Squared Error')
plt.xlabel('Epoch')
plt.show()

plt.plot(run_xor.history['accuracy'])
plt.title('Model accuracy (higher is better)')
plt.ylabel('Accuracy')
```

```
plt.xlabel('Epoch')
plt.show()

plot_weight_matrix(model_4)
```

```
Train on 4 samples
Epoch 1/15
4/4 - 0s - loss: 1.4863 - accuracy: 0.5000
Epoch 2/15
4/4 - 0s - loss: 0.8791 - accuracy: 0.5000
Epoch 3/15
4/4 - 0s - loss: 0.2820 - accuracy: 0.5000
Epoch 4/15
4/4 - 0s - loss: 0.4078 - accuracy: 0.5000
Epoch 5/15
4/4 - 0s - loss: 0.4577 - accuracy: 0.5000
Epoch 6/15
4/4 - 0s - loss: 0.4754 - accuracy: 0.5000
Epoch 7/15
4/4 - 0s - loss: 0.4832 - accuracy: 0.5000
Epoch 8/15
4/4 - 0s - loss: 0.4872 - accuracy: 0.5000
Epoch 9/15
4/4 - 0s - loss: 0.4895 - accuracy: 0.5000
Epoch 10/15
4/4 - 0s - loss: 0.4909 - accuracy: 0.5000
Epoch 11/15
4/4 - 0s - loss: 0.4918 - accuracy: 0.5000
Epoch 12/15
4/4 - 0s - loss: 0.4924 - accuracy: 0.5000
Epoch 13/15
4/4 - 0s - loss: 0.4927 - accuracy: 0.5000
Epoch 14/15
4/4 - 0s - loss: 0.4929 - accuracy: 0.5000
Epoch 15/15
4/4 - 0s - loss: 0.4931 - accuracy: 0.5000
```


Model loss (lower is better)
Mean Squared Error
1.4
1.2
1.0
0.8
0.6
0.4
0
2
4
6
8
10
12
14
Epoch


Model accuracy (higher is better)
Accuracy
0.52
0.51
0.50
0.49
0.48
0
2
4
6
8
10
12
14
Epoch



The equation generated from training on the XOR data are as follows:

$$y' = tanh(0.827 * x_1 + 1.092 * x_2 + 2.491)$$

$ x_{1} $	$ x_{2} $	$ y' $
0	0	0.986
1	0	0.997
0	1	0.998
1	1	1.000

The network misrepresents $[0, 0]$ and $[1, 1]$ since it is not able to identify an accurate hyperplane for the given dataset. The number of training epochs would not matter as the network would only learn to accurately represent 2 of the 4 data points. The bias is significntly larger compared to the networks that learned AND and OR functions.

E) Create a new network model that does solve the XOR problem. You can do this by adding an additional `Dense` layer with two units to the network. Note: to do this, the `input_dim` parameter will need to be moved to the first layer.

[55]:
```
xor_model = Sequential()
xor_model.add(Dense(2, input_dim=2, activation='tanh', name='hidden1'))
xor_model.add(Dense(1, activation='tanh', name='output'))
xor_model.compile(loss='mean_squared_error',
                  optimizer=SGD(lr=0.3, momentum=0.8),
                  metrics=['accuracy'])

xor_model.summary()
```

```
Model: "sequential_11"
_________________________________________________________________
Layer (type)                 Output Shape              Param #
=================================================================
hidden1 (Dense)              (None, 2)                 6
_________________________________________________________________
output (Dense)               (None, 1)                 3
=================================================================
Total params: 9
Trainable params: 9
Non-trainable params: 0
_________________________________________________________________
```

F) How successful is this new model? Also, how does your network solve the problem? Look at the weight matrix of your trained model, include it in your report with a short paragraph describing in words how the model works.

[56]:
```
run_xor2 = xor_model.fit(X_xor, y_xor, epochs=100, shuffle=True, verbose=2)
plt.plot(run_xor2.history['loss'])
plt.title('Model loss (lower is better)')
plt.ylabel('Mean Squared Error')
plt.xlabel('Epoch')
plt.show()

plt.plot(run_xor2.history['accuracy'])
plt.title('Model accuracy (higher is better)')
plt.ylabel('Accuracy')
plt.xlabel('Epoch')
plt.show()

plot_weight_matrix(xor_model)
```

```
Train on 4 samples
Epoch 1/100
4/4 - 0s - loss: 0.4619 - accuracy: 0.5000
Epoch 2/100
4/4 - 0s - loss: 0.2535 - accuracy: 0.5000
Epoch 3/100
4/4 - 0s - loss: 0.2691 - accuracy: 0.5000
```

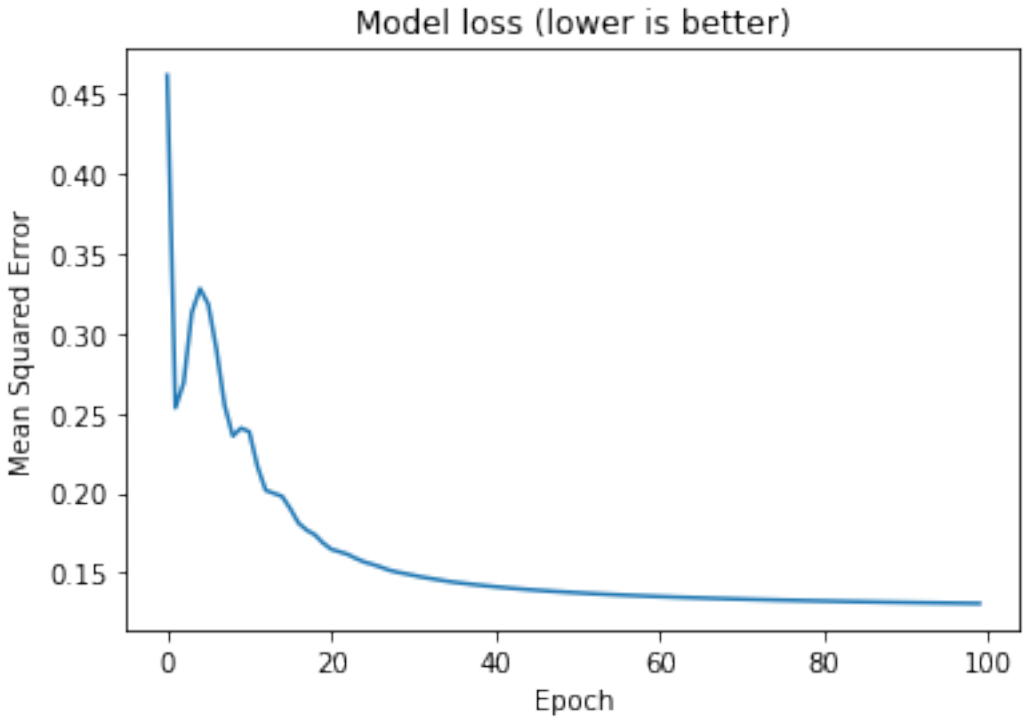
```
Epoch 4/100
4/4 - 0s - loss: 0.3132 - accuracy: 0.5000
Epoch 5/100
4/4 - 0s - loss: 0.3282 - accuracy: 0.5000
Epoch 6/100
4/4 - 0s - loss: 0.3184 - accuracy: 0.5000
Epoch 7/100
4/4 - 0s - loss: 0.2901 - accuracy: 0.5000
Epoch 8/100
4/4 - 0s - loss: 0.2549 - accuracy: 0.7500
Epoch 9/100
4/4 - 0s - loss: 0.2358 - accuracy: 0.5000
Epoch 10/100
4/4 - 0s - loss: 0.2407 - accuracy: 0.5000
Epoch 11/100
4/4 - 0s - loss: 0.2385 - accuracy: 0.2500
Epoch 12/100
4/4 - 0s - loss: 0.2168 - accuracy: 0.5000
Epoch 13/100
4/4 - 0s - loss: 0.2018 - accuracy: 0.7500
Epoch 14/100
4/4 - 0s - loss: 0.1999 - accuracy: 0.7500
Epoch 15/100
4/4 - 0s - loss: 0.1980 - accuracy: 0.7500
Epoch 16/100
4/4 - 0s - loss: 0.1903 - accuracy: 0.7500
Epoch 17/100
4/4 - 0s - loss: 0.1813 - accuracy: 0.5000
Epoch 18/100
4/4 - 0s - loss: 0.1768 - accuracy: 0.5000
Epoch 19/100
4/4 - 0s - loss: 0.1739 - accuracy: 0.5000
Epoch 20/100
4/4 - 0s - loss: 0.1688 - accuracy: 0.5000
Epoch 21/100
4/4 - 0s - loss: 0.1648 - accuracy: 0.7500
Epoch 22/100
4/4 - 0s - loss: 0.1633 - accuracy: 0.7500
Epoch 23/100
4/4 - 0s - loss: 0.1616 - accuracy: 0.7500
Epoch 24/100
4/4 - 0s - loss: 0.1591 - accuracy: 0.7500
Epoch 25/100
4/4 - 0s - loss: 0.1569 - accuracy: 0.7500
Epoch 26/100
4/4 - 0s - loss: 0.1554 - accuracy: 0.7500
Epoch 27/100
4/4 - 0s - loss: 0.1537 - accuracy: 0.7500
```

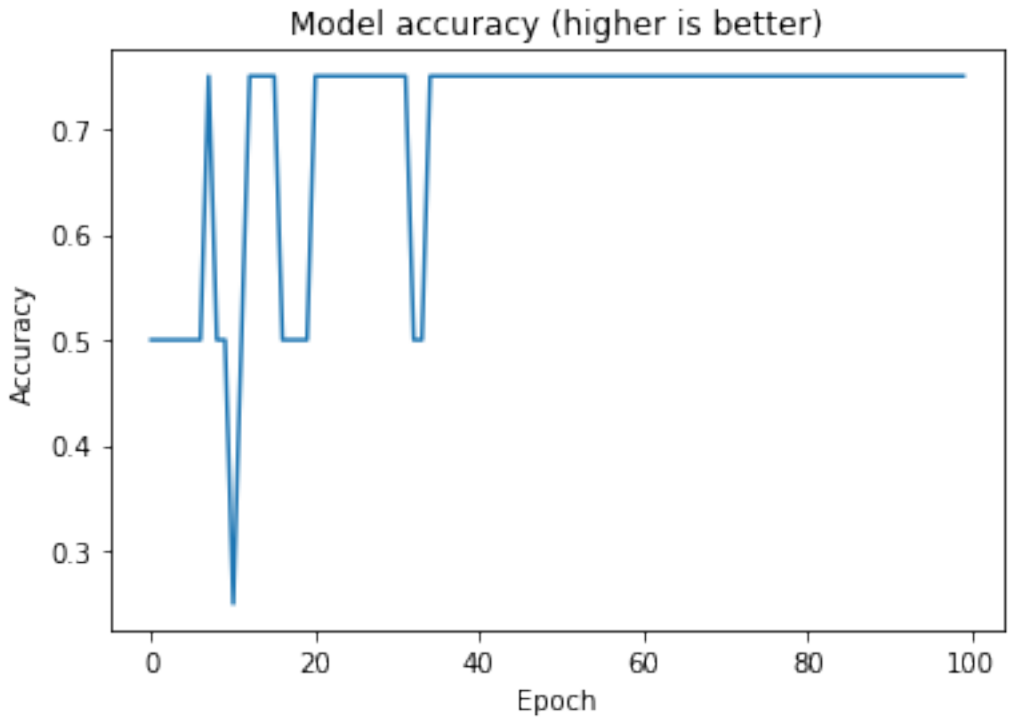
```
Epoch 28/100
4/4 - 0s - loss: 0.1518 - accuracy: 0.7500
Epoch 29/100
4/4 - 0s - loss: 0.1505 - accuracy: 0.7500
Epoch 30/100
4/4 - 0s - loss: 0.1496 - accuracy: 0.7500
Epoch 31/100
4/4 - 0s - loss: 0.1485 - accuracy: 0.7500
Epoch 32/100
4/4 - 0s - loss: 0.1474 - accuracy: 0.7500
Epoch 33/100
4/4 - 0s - loss: 0.1466 - accuracy: 0.5000
Epoch 34/100
4/4 - 0s - loss: 0.1458 - accuracy: 0.5000
Epoch 35/100
4/4 - 0s - loss: 0.1449 - accuracy: 0.7500
Epoch 36/100
4/4 - 0s - loss: 0.1442 - accuracy: 0.7500
Epoch 37/100
4/4 - 0s - loss: 0.1435 - accuracy: 0.7500
Epoch 38/100
4/4 - 0s - loss: 0.1429 - accuracy: 0.7500
Epoch 39/100
4/4 - 0s - loss: 0.1424 - accuracy: 0.7500
Epoch 40/100
4/4 - 0s - loss: 0.1418 - accuracy: 0.7500
Epoch 41/100
4/4 - 0s - loss: 0.1413 - accuracy: 0.7500
Epoch 42/100
4/4 - 0s - loss: 0.1408 - accuracy: 0.7500
Epoch 43/100
4/4 - 0s - loss: 0.1404 - accuracy: 0.7500
Epoch 44/100
4/4 - 0s - loss: 0.1400 - accuracy: 0.7500
Epoch 45/100
4/4 - 0s - loss: 0.1396 - accuracy: 0.7500
Epoch 46/100
4/4 - 0s - loss: 0.1392 - accuracy: 0.7500
Epoch 47/100
4/4 - 0s - loss: 0.1388 - accuracy: 0.7500
Epoch 48/100
4/4 - 0s - loss: 0.1385 - accuracy: 0.7500
Epoch 49/100
4/4 - 0s - loss: 0.1382 - accuracy: 0.7500
Epoch 50/100
4/4 - 0s - loss: 0.1378 - accuracy: 0.7500
Epoch 51/100
4/4 - 0s - loss: 0.1375 - accuracy: 0.7500
```

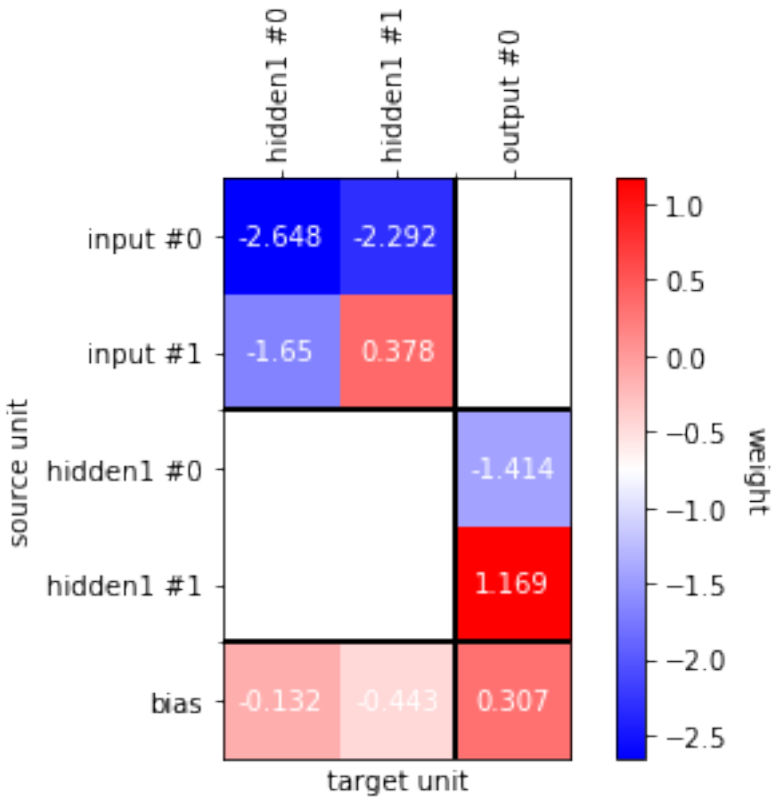
```
Epoch 52/100
4/4 - 0s - loss: 0.1373 - accuracy: 0.7500
Epoch 53/100
4/4 - 0s - loss: 0.1370 - accuracy: 0.7500
Epoch 54/100
4/4 - 0s - loss: 0.1367 - accuracy: 0.7500
Epoch 55/100
4/4 - 0s - loss: 0.1365 - accuracy: 0.7500
Epoch 56/100
4/4 - 0s - loss: 0.1362 - accuracy: 0.7500
Epoch 57/100
4/4 - 0s - loss: 0.1360 - accuracy: 0.7500
Epoch 58/100
4/4 - 0s - loss: 0.1358 - accuracy: 0.7500
Epoch 59/100
4/4 - 0s - loss: 0.1356 - accuracy: 0.7500
Epoch 60/100
4/4 - 0s - loss: 0.1354 - accuracy: 0.7500
Epoch 61/100
4/4 - 0s - loss: 0.1352 - accuracy: 0.7500
Epoch 62/100
4/4 - 0s - loss: 0.1350 - accuracy: 0.7500
Epoch 63/100
4/4 - 0s - loss: 0.1348 - accuracy: 0.7500
Epoch 64/100
4/4 - 0s - loss: 0.1346 - accuracy: 0.7500
Epoch 65/100
4/4 - 0s - loss: 0.1345 - accuracy: 0.7500
Epoch 66/100
4/4 - 0s - loss: 0.1343 - accuracy: 0.7500
Epoch 67/100
4/4 - 0s - loss: 0.1342 - accuracy: 0.7500
Epoch 68/100
4/4 - 0s - loss: 0.1340 - accuracy: 0.7500
Epoch 69/100
4/4 - 0s - loss: 0.1339 - accuracy: 0.7500
Epoch 70/100
4/4 - 0s - loss: 0.1337 - accuracy: 0.7500
Epoch 71/100
4/4 - 0s - loss: 0.1336 - accuracy: 0.7500
Epoch 72/100
4/4 - 0s - loss: 0.1334 - accuracy: 0.7500
Epoch 73/100
4/4 - 0s - loss: 0.1333 - accuracy: 0.7500
Epoch 74/100
4/4 - 0s - loss: 0.1332 - accuracy: 0.7500
Epoch 75/100
4/4 - 0s - loss: 0.1331 - accuracy: 0.7500
```

```
Epoch 76/100
4/4 - 0s - loss: 0.1329 - accuracy: 0.7500
Epoch 77/100
4/4 - 0s - loss: 0.1328 - accuracy: 0.7500
Epoch 78/100
4/4 - 0s - loss: 0.1327 - accuracy: 0.7500
Epoch 79/100
4/4 - 0s - loss: 0.1326 - accuracy: 0.7500
Epoch 80/100
4/4 - 0s - loss: 0.1325 - accuracy: 0.7500
Epoch 81/100
4/4 - 0s - loss: 0.1324 - accuracy: 0.7500
Epoch 82/100
4/4 - 0s - loss: 0.1323 - accuracy: 0.7500
Epoch 83/100
4/4 - 0s - loss: 0.1322 - accuracy: 0.7500
Epoch 84/100
4/4 - 0s - loss: 0.1321 - accuracy: 0.7500
Epoch 85/100
4/4 - 0s - loss: 0.1320 - accuracy: 0.7500
Epoch 86/100
4/4 - 0s - loss: 0.1319 - accuracy: 0.7500
Epoch 87/100
4/4 - 0s - loss: 0.1318 - accuracy: 0.7500
Epoch 88/100
4/4 - 0s - loss: 0.1317 - accuracy: 0.7500
Epoch 89/100
4/4 - 0s - loss: 0.1317 - accuracy: 0.7500
Epoch 90/100
4/4 - 0s - loss: 0.1316 - accuracy: 0.7500
Epoch 91/100
4/4 - 0s - loss: 0.1315 - accuracy: 0.7500
Epoch 92/100
4/4 - 0s - loss: 0.1314 - accuracy: 0.7500
Epoch 93/100
4/4 - 0s - loss: 0.1313 - accuracy: 0.7500
Epoch 94/100
4/4 - 0s - loss: 0.1313 - accuracy: 0.7500
Epoch 95/100
4/4 - 0s - loss: 0.1312 - accuracy: 0.7500
Epoch 96/100
4/4 - 0s - loss: 0.1311 - accuracy: 0.7500
Epoch 97/100
4/4 - 0s - loss: 0.1311 - accuracy: 0.7500
Epoch 98/100
4/4 - 0s - loss: 0.1310 - accuracy: 0.7500
Epoch 99/100
4/4 - 0s - loss: 0.1309 - accuracy: 0.7500
```

```
Epoch 100/100
4/4 - 0s - loss: 0.1309 - accuracy: 0.7500
```







The model trains for 100 epochs and reaches a training accuracy of 75%. Over more epochs, the model could learn to represent the complete XOR function.

The model can be defined using the following formula:

$$y' = tanh(-1.414 * tanh(-2.648 * x_1 - 1.65 * x_2 - 0.132 * b_1) + 1.169 * tanh(-2.292 * x_1 + 0.378 * x_2 - 0.443 * b_2)$$

---

3 Part 2: Modelling Sequential XOR with Simple Recurrent Neural Networks

If you have completed the previous exercise, you have solved the XOR problem using a feed-forward neural network in Keras. But feed-forwad networks do not have any way to keep track of sequences of words in time, that may depend on each other. But language use does rely on information about the previous word for interpreting the next. To be able to include this kind of temporal/memory information in a simple network, we need to use something more complex than a feed forward

model. In this section, we will create our first simple recurrent neural network. This network is a reproduction of the sequential XOR network by Elman (1990).

The input for our network will be a single 3,000-bit which is created by concatenating 1,000 random XOR sequences. Every first and second bit is random and the third bit is the XOR of the first two values. The network tries to predict the next bit in the sequence. For this, the network needs to remember information about previous inputs since that is the only way how every third input can be predicted correctly.

Elman, J. L. (1990). Finding structure in time. Cognitive science, 14(2), 179-211.

[57]:
```
np.random.seed(1990)  # Use random seed for reproducability
```

3.1 Model definition

Let's define a very simple network that has a single input and output unit. But we'll add more complexity by adding an additional dimension: memory for the output weights of the previous input, i.e. we'll keep some information about the step before the data we are analyzing. This will be accomplished by using what's called a `Time Distributed` unit, which will give us the memory we need. First, import necessary `keras` packages:

[58]:
```
from tensorflow.keras.layers import SimpleRNN, TimeDistributed
from tensorflow.keras.initializers import Constant
```

[59]:
```
model_rnn1 = Sequential([
    SimpleRNN(2, input_shape=(3000, 1),
              return_sequences=True,
              activation='tanh',
              recurrent_initializer=Constant(0.5),
              name='hidden'),
    TimeDistributed(Dense(1, activation='sigmoid', name='output'))
])

model_rnn1.compile(loss='mean_squared_error',
              optimizer=SGD(lr=0.4, momentum=0.9),
              metrics=['binary_accuracy'])
```

In the model above, we define a simple recurrent neural network with `SimpleRNN` with 2 units in the recurrent hidden layer. Its input shape is `(3000, 1)` which corresponds to 3000 time steps with 1 dimension each. The `return_sequences=True` argument makes sure that activation is passed on to the next layer for every time step instead of only the final time step activation. We want that the network makes a seperate prediction for every time step, so the output layer is wrapped inside of `TimeDistributed`. This wrapper makes it possible to backpropagate for every time step consequtively instead of optimizing the complete sequence at once.

3.2 Training

Now we can create our training data. This is a concatenation of 1000 random XOR problems.

[60]:
```
np.random.seed(10)
xor_data = np.array([(0,0,0), (0,1,1), (1,0,1), (1,1,0)])
xor_sequence = np.ravel(xor_data[np.random.randint(4, size=1001)])
print(xor_sequence)
X = xor_sequence[np.newaxis, :-3, np.newaxis]
y = xor_sequence[np.newaxis, 1:-2, np.newaxis]
X.shape, y.shape
```

```
[0 1 1 … 1 1 0]
```

[60]:
```
((1, 3000, 1), (1, 3000, 1))
```

The X and y shapes must be `(input_size, timesteps, input_dim)`. We only train a single sequence, it has 3000 time steps and it has 1 dimension.

Lets's look at how well the network can learn our training data:

[61]:
```
history = model_rnn1.fit(X, y, epochs=300, verbose=1)
```

```
Train on 1 samples
Epoch 1/300
1/1 [==============================] - 1s 1s/sample - loss: 0.3454 -
binary_accuracy: 0.5003
Epoch 2/300
1/1 [==============================] - 1s 777ms/sample - loss: 0.3352 -
binary_accuracy: 0.5007
Epoch 3/300
1/1 [==============================] - 1s 760ms/sample - loss: 0.3170 -
binary_accuracy: 0.5013
Epoch 4/300
1/1 [==============================] - 1s 686ms/sample - loss: 0.2953 -
binary_accuracy: 0.4807
Epoch 5/300
1/1 [==============================] - 1s 793ms/sample - loss: 0.2759 -
binary_accuracy: 0.4643
Epoch 6/300
1/1 [==============================] - 1s 683ms/sample - loss: 0.2639 -
binary_accuracy: 0.4520
Epoch 7/300
1/1 [==============================] - 1s 668ms/sample - loss: 0.2612 -
binary_accuracy: 0.5320
Epoch 8/300
1/1 [==============================] - 1s 764ms/sample - loss: 0.2651 -
binary_accuracy: 0.4997
Epoch 9/300
1/1 [==============================] - 1s 812ms/sample - loss: 0.2711 -
binary_accuracy: 0.4997
Epoch 10/300
```

```
1/1 [==============================] - 1s 687ms/sample - loss: 0.2760 -
binary_accuracy: 0.4997
Epoch 11/300
1/1 [==============================] - 1s 760ms/sample - loss: 0.2785 -
binary_accuracy: 0.4997
Epoch 12/300
1/1 [==============================] - 1s 681ms/sample - loss: 0.2781 -
binary_accuracy: 0.4997
Epoch 13/300
1/1 [==============================] - 1s 768ms/sample - loss: 0.2753 -
binary_accuracy: 0.4997
Epoch 14/300
1/1 [==============================] - 1s 679ms/sample - loss: 0.2706 -
binary_accuracy: 0.4997
Epoch 15/300
1/1 [==============================] - 1s 776ms/sample - loss: 0.2648 -
binary_accuracy: 0.4997
Epoch 16/300
1/1 [==============================] - 1s 736ms/sample - loss: 0.2591 -
binary_accuracy: 0.4997
Epoch 17/300
1/1 [==============================] - 1s 737ms/sample - loss: 0.2543 -
binary_accuracy: 0.4997
Epoch 18/300
1/1 [==============================] - 1s 695ms/sample - loss: 0.2511 -
binary_accuracy: 0.4997
Epoch 19/300
1/1 [==============================] - 1s 710ms/sample - loss: 0.2499 -
binary_accuracy: 0.4997
Epoch 20/300
1/1 [==============================] - 1s 688ms/sample - loss: 0.2504 -
binary_accuracy: 0.5003
Epoch 21/300
1/1 [==============================] - 1s 682ms/sample - loss: 0.2520 -
binary_accuracy: 0.5003
Epoch 22/300
1/1 [==============================] - 1s 664ms/sample - loss: 0.2539 -
binary_accuracy: 0.5003
Epoch 23/300
1/1 [==============================] - 1s 757ms/sample - loss: 0.2555 -
binary_accuracy: 0.5003
Epoch 24/300
1/1 [==============================] - 1s 702ms/sample - loss: 0.2564 -
binary_accuracy: 0.5003
Epoch 25/300
1/1 [==============================] - 1s 681ms/sample - loss: 0.2564 -
binary_accuracy: 0.5003
Epoch 26/300
```

```
1/1 [==============================] - 1s 697ms/sample - loss: 0.2557 -
binary_accuracy: 0.5003
Epoch 27/300
1/1 [==============================] - 1s 733ms/sample - loss: 0.2545 -
binary_accuracy: 0.5003
Epoch 28/300
1/1 [==============================] - 1s 735ms/sample - loss: 0.2530 -
binary_accuracy: 0.5003
Epoch 29/300
1/1 [==============================] - 1s 669ms/sample - loss: 0.2518 -
binary_accuracy: 0.5003
Epoch 30/300
1/1 [==============================] - 1s 697ms/sample - loss: 0.2509 -
binary_accuracy: 0.4400
Epoch 31/300
1/1 [==============================] - 1s 687ms/sample - loss: 0.2505 -
binary_accuracy: 0.4897
Epoch 32/300
1/1 [==============================] - 1s 674ms/sample - loss: 0.2505 -
binary_accuracy: 0.4897
Epoch 33/300
1/1 [==============================] - 1s 769ms/sample - loss: 0.2509 -
binary_accuracy: 0.4897
Epoch 34/300
1/1 [==============================] - 1s 681ms/sample - loss: 0.2514 -
binary_accuracy: 0.5773
Epoch 35/300
1/1 [==============================] - 1s 687ms/sample - loss: 0.2518 -
binary_accuracy: 0.5773
Epoch 36/300
1/1 [==============================] - 1s 698ms/sample - loss: 0.2521 -
binary_accuracy: 0.5773
Epoch 37/300
1/1 [==============================] - 1s 670ms/sample - loss: 0.2520 -
binary_accuracy: 0.5773
Epoch 38/300
1/1 [==============================] - 1s 710ms/sample - loss: 0.2517 -
binary_accuracy: 0.5773
Epoch 39/300
1/1 [==============================] - 1s 697ms/sample - loss: 0.2513 -
binary_accuracy: 0.5773
Epoch 40/300
1/1 [==============================] - 1s 699ms/sample - loss: 0.2507 -
binary_accuracy: 0.5773
Epoch 41/300
1/1 [==============================] - 1s 700ms/sample - loss: 0.2503 -
binary_accuracy: 0.5533
Epoch 42/300
```

```
1/1 [==============================] - 1s 697ms/sample - loss: 0.2499 -
binary_accuracy: 0.4897
Epoch 43/300
1/1 [==============================] - 1s 860ms/sample - loss: 0.2498 -
binary_accuracy: 0.4897
Epoch 44/300
1/1 [==============================] - 1s 709ms/sample - loss: 0.2497 -
binary_accuracy: 0.4897
Epoch 45/300
1/1 [==============================] - 1s 718ms/sample - loss: 0.2498 -
binary_accuracy: 0.4213
Epoch 46/300
1/1 [==============================] - 1s 729ms/sample - loss: 0.2499 -
binary_accuracy: 0.5003
Epoch 47/300
1/1 [==============================] - 1s 738ms/sample - loss: 0.2500 -
binary_accuracy: 0.5003
Epoch 48/300
1/1 [==============================] - 1s 720ms/sample - loss: 0.2500 -
binary_accuracy: 0.5003
Epoch 49/300
1/1 [==============================] - 1s 681ms/sample - loss: 0.2499 -
binary_accuracy: 0.5003
Epoch 50/300
1/1 [==============================] - 1s 703ms/sample - loss: 0.2498 -
binary_accuracy: 0.5003
Epoch 51/300
1/1 [==============================] - 1s 697ms/sample - loss: 0.2496 -
binary_accuracy: 0.5003
Epoch 52/300
1/1 [==============================] - 1s 714ms/sample - loss: 0.2494 -
binary_accuracy: 0.5003
Epoch 53/300
1/1 [==============================] - 1s 708ms/sample - loss: 0.2493 -
binary_accuracy: 0.4213
Epoch 54/300
1/1 [==============================] - 1s 717ms/sample - loss: 0.2491 -
binary_accuracy: 0.4897
Epoch 55/300
1/1 [==============================] - 1s 702ms/sample - loss: 0.2491 -
binary_accuracy: 0.5773
Epoch 56/300
1/1 [==============================] - 1s 721ms/sample - loss: 0.2490 -
binary_accuracy: 0.5773
Epoch 57/300
1/1 [==============================] - 1s 689ms/sample - loss: 0.2490 -
binary_accuracy: 0.5773
Epoch 58/300
```

```
1/1 [==============================] - 1s 743ms/sample - loss: 0.2490 -
binary_accuracy: 0.5773
Epoch 59/300
1/1 [==============================] - 1s 704ms/sample - loss: 0.2490 -
binary_accuracy: 0.5600
Epoch 60/300
1/1 [==============================] - 1s 745ms/sample - loss: 0.2489 -
binary_accuracy: 0.5357
Epoch 61/300
1/1 [==============================] - 1s 731ms/sample - loss: 0.2488 -
binary_accuracy: 0.5477
Epoch 62/300
1/1 [==============================] - 1s 694ms/sample - loss: 0.2487 -
binary_accuracy: 0.5773
Epoch 63/300
1/1 [==============================] - 1s 696ms/sample - loss: 0.2486 -
binary_accuracy: 0.5773
Epoch 64/300
1/1 [==============================] - 1s 708ms/sample - loss: 0.2485 -
binary_accuracy: 0.5773
Epoch 65/300
1/1 [==============================] - 1s 688ms/sample - loss: 0.2483 -
binary_accuracy: 0.5773
Epoch 66/300
1/1 [==============================] - 1s 707ms/sample - loss: 0.2482 -
binary_accuracy: 0.5773
Epoch 67/300
1/1 [==============================] - 1s 704ms/sample - loss: 0.2481 -
binary_accuracy: 0.5623
Epoch 68/300
1/1 [==============================] - 1s 707ms/sample - loss: 0.2480 -
binary_accuracy: 0.5413
Epoch 69/300
1/1 [==============================] - 1s 701ms/sample - loss: 0.2479 -
binary_accuracy: 0.5090
Epoch 70/300
1/1 [==============================] - 1s 659ms/sample - loss: 0.2478 -
binary_accuracy: 0.5090
Epoch 71/300
1/1 [==============================] - 1s 708ms/sample - loss: 0.2477 -
binary_accuracy: 0.5090
Epoch 72/300
1/1 [==============================] - 1s 688ms/sample - loss: 0.2476 -
binary_accuracy: 0.5090
Epoch 73/300
1/1 [==============================] - 1s 674ms/sample - loss: 0.2474 -
binary_accuracy: 0.5090
Epoch 74/300
```

```
1/1 [==============================] - 1s 689ms/sample - loss: 0.2473 -
binary_accuracy: 0.5090
Epoch 75/300
1/1 [==============================] - 1s 735ms/sample - loss: 0.2471 -
binary_accuracy: 0.5413
Epoch 76/300
1/1 [==============================] - 1s 679ms/sample - loss: 0.2470 -
binary_accuracy: 0.5413
Epoch 77/300
1/1 [==============================] - 1s 650ms/sample - loss: 0.2469 -
binary_accuracy: 0.5413
Epoch 78/300
1/1 [==============================] - 1s 710ms/sample - loss: 0.2467 -
binary_accuracy: 0.5413
Epoch 79/300
1/1 [==============================] - 1s 685ms/sample - loss: 0.2466 -
binary_accuracy: 0.5623
Epoch 80/300
1/1 [==============================] - 1s 700ms/sample - loss: 0.2464 -
binary_accuracy: 0.5623
Epoch 81/300
1/1 [==============================] - 1s 703ms/sample - loss: 0.2463 -
binary_accuracy: 0.5623
Epoch 82/300
1/1 [==============================] - 1s 748ms/sample - loss: 0.2461 -
binary_accuracy: 0.5413
Epoch 83/300
1/1 [==============================] - 1s 712ms/sample - loss: 0.2460 -
binary_accuracy: 0.5413
Epoch 84/300
1/1 [==============================] - 1s 720ms/sample - loss: 0.2458 -
binary_accuracy: 0.5413
Epoch 85/300
1/1 [==============================] - 1s 678ms/sample - loss: 0.2457 -
binary_accuracy: 0.5413
Epoch 86/300
1/1 [==============================] - 1s 673ms/sample - loss: 0.2455 -
binary_accuracy: 0.5413
Epoch 87/300
1/1 [==============================] - 1s 676ms/sample - loss: 0.2454 -
binary_accuracy: 0.5183
Epoch 88/300
1/1 [==============================] - 1s 694ms/sample - loss: 0.2452 -
binary_accuracy: 0.5090
Epoch 89/300
1/1 [==============================] - 1s 675ms/sample - loss: 0.2451 -
binary_accuracy: 0.5090
Epoch 90/300
```

```
1/1 [==============================] - 1s 667ms/sample - loss: 0.2450 -
binary_accuracy: 0.5090
Epoch 91/300
1/1 [==============================] - 1s 684ms/sample - loss: 0.2448 -
binary_accuracy: 0.5090
Epoch 92/300
1/1 [==============================] - 1s 716ms/sample - loss: 0.2447 -
binary_accuracy: 0.5090
Epoch 93/300
1/1 [==============================] - 1s 701ms/sample - loss: 0.2445 -
binary_accuracy: 0.5090
Epoch 94/300
1/1 [==============================] - 1s 670ms/sample - loss: 0.2444 -
binary_accuracy: 0.5090
Epoch 95/300
1/1 [==============================] - 1s 688ms/sample - loss: 0.2442 -
binary_accuracy: 0.5090
Epoch 96/300
1/1 [==============================] - 1s 678ms/sample - loss: 0.2441 -
binary_accuracy: 0.5090
Epoch 97/300
1/1 [==============================] - 1s 694ms/sample - loss: 0.2439 -
binary_accuracy: 0.5090
Epoch 98/300
1/1 [==============================] - 1s 694ms/sample - loss: 0.2438 -
binary_accuracy: 0.5090
Epoch 99/300
1/1 [==============================] - 1s 671ms/sample - loss: 0.2436 -
binary_accuracy: 0.5090
Epoch 100/300
1/1 [==============================] - 1s 730ms/sample - loss: 0.2435 -
binary_accuracy: 0.5090
Epoch 101/300
1/1 [==============================] - 1s 714ms/sample - loss: 0.2433 -
binary_accuracy: 0.5090
Epoch 102/300
1/1 [==============================] - 1s 669ms/sample - loss: 0.2432 -
binary_accuracy: 0.5090
Epoch 103/300
1/1 [==============================] - 1s 722ms/sample - loss: 0.2430 -
binary_accuracy: 0.5090
Epoch 104/300
1/1 [==============================] - 1s 708ms/sample - loss: 0.2429 -
binary_accuracy: 0.5090
Epoch 105/300
1/1 [==============================] - 1s 705ms/sample - loss: 0.2427 -
binary_accuracy: 0.5090
Epoch 106/300
```

```
1/1 [==============================] - 1s 691ms/sample - loss: 0.2425 -
binary_accuracy: 0.5090
Epoch 107/300
1/1 [==============================] - 1s 811ms/sample - loss: 0.2424 -
binary_accuracy: 0.5090
Epoch 108/300
1/1 [==============================] - 1s 674ms/sample - loss: 0.2422 -
binary_accuracy: 0.5090
Epoch 109/300
1/1 [==============================] - 1s 690ms/sample - loss: 0.2421 -
binary_accuracy: 0.5090
Epoch 110/300
1/1 [==============================] - 1s 666ms/sample - loss: 0.2419 -
binary_accuracy: 0.5090
Epoch 111/300
1/1 [==============================] - 1s 705ms/sample - loss: 0.2417 -
binary_accuracy: 0.5090
Epoch 112/300
1/1 [==============================] - 1s 698ms/sample - loss: 0.2416 -
binary_accuracy: 0.5090
Epoch 113/300
1/1 [==============================] - 1s 664ms/sample - loss: 0.2414 -
binary_accuracy: 0.5090
Epoch 114/300
1/1 [==============================] - 1s 686ms/sample - loss: 0.2413 -
binary_accuracy: 0.5090
Epoch 115/300
1/1 [==============================] - 1s 670ms/sample - loss: 0.2411 -
binary_accuracy: 0.5090
Epoch 116/300
1/1 [==============================] - 1s 732ms/sample - loss: 0.2410 -
binary_accuracy: 0.5090
Epoch 117/300
1/1 [==============================] - 1s 687ms/sample - loss: 0.2408 -
binary_accuracy: 0.5090
Epoch 118/300
1/1 [==============================] - 1s 713ms/sample - loss: 0.2406 -
binary_accuracy: 0.5090
Epoch 119/300
1/1 [==============================] - 1s 686ms/sample - loss: 0.2405 -
binary_accuracy: 0.5090
Epoch 120/300
1/1 [==============================] - 1s 724ms/sample - loss: 0.2403 -
binary_accuracy: 0.5090
Epoch 121/300
1/1 [==============================] - 1s 691ms/sample - loss: 0.2402 -
binary_accuracy: 0.5090
Epoch 122/300
```

```
1/1 [==============================] - 1s 721ms/sample - loss: 0.2401 -
binary_accuracy: 0.5090
Epoch 123/300
1/1 [==============================] - 1s 724ms/sample - loss: 0.2399 -
binary_accuracy: 0.5090
Epoch 124/300
1/1 [==============================] - 1s 719ms/sample - loss: 0.2398 -
binary_accuracy: 0.5090
Epoch 125/300
1/1 [==============================] - 1s 671ms/sample - loss: 0.2397 -
binary_accuracy: 0.5090
Epoch 126/300
1/1 [==============================] - 1s 720ms/sample - loss: 0.2395 -
binary_accuracy: 0.5090
Epoch 127/300
1/1 [==============================] - 1s 728ms/sample - loss: 0.2394 -
binary_accuracy: 0.5090
Epoch 128/300
1/1 [==============================] - 1s 699ms/sample - loss: 0.2393 -
binary_accuracy: 0.5090
Epoch 129/300
1/1 [==============================] - 1s 694ms/sample - loss: 0.2392 -
binary_accuracy: 0.5090
Epoch 130/300
1/1 [==============================] - 1s 705ms/sample - loss: 0.2391 -
binary_accuracy: 0.5090
Epoch 131/300
1/1 [==============================] - 1s 707ms/sample - loss: 0.2390 -
binary_accuracy: 0.5090
Epoch 132/300
1/1 [==============================] - 1s 693ms/sample - loss: 0.2389 -
binary_accuracy: 0.5090
Epoch 133/300
1/1 [==============================] - 1s 736ms/sample - loss: 0.2388 -
binary_accuracy: 0.5090
Epoch 134/300
1/1 [==============================] - 1s 675ms/sample - loss: 0.2387 -
binary_accuracy: 0.5090
Epoch 135/300
1/1 [==============================] - 1s 720ms/sample - loss: 0.2386 -
binary_accuracy: 0.5090
Epoch 136/300
1/1 [==============================] - 1s 668ms/sample - loss: 0.2386 -
binary_accuracy: 0.5090
Epoch 137/300
1/1 [==============================] - 1s 700ms/sample - loss: 0.2385 -
binary_accuracy: 0.5090
Epoch 138/300
```

```
1/1 [==============================] - 1s 692ms/sample - loss: 0.2384 -
binary_accuracy: 0.5090
Epoch 139/300
1/1 [==============================] - 1s 654ms/sample - loss: 0.2384 -
binary_accuracy: 0.5090
Epoch 140/300
1/1 [==============================] - 1s 697ms/sample - loss: 0.2383 -
binary_accuracy: 0.5090
Epoch 141/300
1/1 [==============================] - 1s 701ms/sample - loss: 0.2382 -
binary_accuracy: 0.5090
Epoch 142/300
1/1 [==============================] - 1s 697ms/sample - loss: 0.2382 -
binary_accuracy: 0.5090
Epoch 143/300
1/1 [==============================] - 1s 675ms/sample - loss: 0.2381 -
binary_accuracy: 0.5090
Epoch 144/300
1/1 [==============================] - 1s 687ms/sample - loss: 0.2381 -
binary_accuracy: 0.5090
Epoch 145/300
1/1 [==============================] - 1s 707ms/sample - loss: 0.2380 -
binary_accuracy: 0.5090
Epoch 146/300
1/1 [==============================] - 1s 734ms/sample - loss: 0.2380 -
binary_accuracy: 0.5090
Epoch 147/300
1/1 [==============================] - 1s 695ms/sample - loss: 0.2379 -
binary_accuracy: 0.5090
Epoch 148/300
1/1 [==============================] - 1s 666ms/sample - loss: 0.2379 -
binary_accuracy: 0.5090
Epoch 149/300
1/1 [==============================] - 1s 688ms/sample - loss: 0.2378 -
binary_accuracy: 0.5090
Epoch 150/300
1/1 [==============================] - 1s 721ms/sample - loss: 0.2378 -
binary_accuracy: 0.5090
Epoch 151/300
1/1 [==============================] - 1s 711ms/sample - loss: 0.2377 -
binary_accuracy: 0.5090
Epoch 152/300
1/1 [==============================] - 1s 741ms/sample - loss: 0.2377 -
binary_accuracy: 0.5090
Epoch 153/300
1/1 [==============================] - 1s 685ms/sample - loss: 0.2376 -
binary_accuracy: 0.5090
Epoch 154/300
```

```
1/1 [==============================] - 1s 706ms/sample - loss: 0.2376 -
binary_accuracy: 0.5090
Epoch 155/300
1/1 [==============================] - 1s 736ms/sample - loss: 0.2375 -
binary_accuracy: 0.5090
Epoch 156/300
1/1 [==============================] - 1s 744ms/sample - loss: 0.2375 -
binary_accuracy: 0.5090
Epoch 157/300
1/1 [==============================] - 1s 722ms/sample - loss: 0.2374 -
binary_accuracy: 0.5090
Epoch 158/300
1/1 [==============================] - 1s 696ms/sample - loss: 0.2374 -
binary_accuracy: 0.5090
Epoch 159/300
1/1 [==============================] - 1s 676ms/sample - loss: 0.2374 -
binary_accuracy: 0.5090
Epoch 160/300
1/1 [==============================] - 1s 687ms/sample - loss: 0.2373 -
binary_accuracy: 0.5090
Epoch 161/300
1/1 [==============================] - 1s 689ms/sample - loss: 0.2373 -
binary_accuracy: 0.5090
Epoch 162/300
1/1 [==============================] - 1s 691ms/sample - loss: 0.2372 -
binary_accuracy: 0.5090
Epoch 163/300
1/1 [==============================] - 1s 693ms/sample - loss: 0.2372 -
binary_accuracy: 0.5090
Epoch 164/300
1/1 [==============================] - 1s 697ms/sample - loss: 0.2372 -
binary_accuracy: 0.5090
Epoch 165/300
1/1 [==============================] - 1s 682ms/sample - loss: 0.2371 -
binary_accuracy: 0.5090
Epoch 166/300
1/1 [==============================] - 1s 681ms/sample - loss: 0.2371 -
binary_accuracy: 0.5090
Epoch 167/300
1/1 [==============================] - 1s 714ms/sample - loss: 0.2371 -
binary_accuracy: 0.5090
Epoch 168/300
1/1 [==============================] - 1s 692ms/sample - loss: 0.2370 -
binary_accuracy: 0.5090
Epoch 169/300
1/1 [==============================] - 1s 728ms/sample - loss: 0.2370 -
binary_accuracy: 0.5090
Epoch 170/300
```

```
1/1 [==============================] - 1s 758ms/sample - loss: 0.2370 -
binary_accuracy: 0.5090
Epoch 171/300
1/1 [==============================] - 1s 721ms/sample - loss: 0.2369 -
binary_accuracy: 0.5090
Epoch 172/300
1/1 [==============================] - 1s 671ms/sample - loss: 0.2369 -
binary_accuracy: 0.5090
Epoch 173/300
1/1 [==============================] - 1s 696ms/sample - loss: 0.2369 -
binary_accuracy: 0.5090
Epoch 174/300
1/1 [==============================] - 1s 685ms/sample - loss: 0.2368 -
binary_accuracy: 0.5090
Epoch 175/300
1/1 [==============================] - 1s 659ms/sample - loss: 0.2368 -
binary_accuracy: 0.5090
Epoch 176/300
1/1 [==============================] - 1s 674ms/sample - loss: 0.2368 -
binary_accuracy: 0.5090
Epoch 177/300
1/1 [==============================] - 1s 691ms/sample - loss: 0.2368 -
binary_accuracy: 0.5090
Epoch 178/300
1/1 [==============================] - 1s 698ms/sample - loss: 0.2367 -
binary_accuracy: 0.5090
Epoch 179/300
1/1 [==============================] - 1s 668ms/sample - loss: 0.2367 -
binary_accuracy: 0.5090
Epoch 180/300
1/1 [==============================] - 1s 643ms/sample - loss: 0.2367 -
binary_accuracy: 0.5090
Epoch 181/300
1/1 [==============================] - 1s 692ms/sample - loss: 0.2366 -
binary_accuracy: 0.5090
Epoch 182/300
1/1 [==============================] - 1s 660ms/sample - loss: 0.2366 -
binary_accuracy: 0.5523
Epoch 183/300
1/1 [==============================] - 1s 661ms/sample - loss: 0.2366 -
binary_accuracy: 0.5523
Epoch 184/300
1/1 [==============================] - 1s 663ms/sample - loss: 0.2366 -
binary_accuracy: 0.5523
Epoch 185/300
1/1 [==============================] - 1s 683ms/sample - loss: 0.2366 -
binary_accuracy: 0.5523
Epoch 186/300
```

```
1/1 [==============================] - 1s 685ms/sample - loss: 0.2365 -
binary_accuracy: 0.5523
Epoch 187/300
1/1 [==============================] - 1s 696ms/sample - loss: 0.2365 -
binary_accuracy: 0.5523
Epoch 188/300
1/1 [==============================] - 1s 665ms/sample - loss: 0.2365 -
binary_accuracy: 0.5877
Epoch 189/300
1/1 [==============================] - 1s 677ms/sample - loss: 0.2365 -
binary_accuracy: 0.5880
Epoch 190/300
1/1 [==============================] - 1s 689ms/sample - loss: 0.2364 -
binary_accuracy: 0.5880
Epoch 191/300
1/1 [==============================] - 1s 675ms/sample - loss: 0.2364 -
binary_accuracy: 0.5880
Epoch 192/300
1/1 [==============================] - 1s 681ms/sample - loss: 0.2364 -
binary_accuracy: 0.5880
Epoch 193/300
1/1 [==============================] - 1s 682ms/sample - loss: 0.2364 -
binary_accuracy: 0.5880
Epoch 194/300
1/1 [==============================] - 1s 684ms/sample - loss: 0.2364 -
binary_accuracy: 0.5880
Epoch 195/300
1/1 [==============================] - 1s 677ms/sample - loss: 0.2363 -
binary_accuracy: 0.5880
Epoch 196/300
1/1 [==============================] - 1s 690ms/sample - loss: 0.2363 -
binary_accuracy: 0.5880
Epoch 197/300
1/1 [==============================] - 1s 673ms/sample - loss: 0.2363 -
binary_accuracy: 0.5880
Epoch 198/300
1/1 [==============================] - 1s 707ms/sample - loss: 0.2363 -
binary_accuracy: 0.5880
Epoch 199/300
1/1 [==============================] - 1s 701ms/sample - loss: 0.2363 -
binary_accuracy: 0.5880
Epoch 200/300
1/1 [==============================] - 1s 679ms/sample - loss: 0.2363 -
binary_accuracy: 0.5880
Epoch 201/300
1/1 [==============================] - 1s 670ms/sample - loss: 0.2362 -
binary_accuracy: 0.5880
Epoch 202/300
```

```
1/1 [==============================] - 1s 704ms/sample - loss: 0.2362 -
binary_accuracy: 0.5880
Epoch 203/300
1/1 [==============================] - 1s 687ms/sample - loss: 0.2362 -
binary_accuracy: 0.5880
Epoch 204/300
1/1 [==============================] - 1s 675ms/sample - loss: 0.2362 -
binary_accuracy: 0.5880
Epoch 205/300
1/1 [==============================] - 1s 673ms/sample - loss: 0.2362 -
binary_accuracy: 0.5880
Epoch 206/300
1/1 [==============================] - 1s 657ms/sample - loss: 0.2362 -
binary_accuracy: 0.5880
Epoch 207/300
1/1 [==============================] - 1s 700ms/sample - loss: 0.2362 -
binary_accuracy: 0.5880
Epoch 208/300
1/1 [==============================] - 1s 670ms/sample - loss: 0.2361 -
binary_accuracy: 0.5880
Epoch 209/300
1/1 [==============================] - 1s 697ms/sample - loss: 0.2361 -
binary_accuracy: 0.5880
Epoch 210/300
1/1 [==============================] - 1s 658ms/sample - loss: 0.2361 -
binary_accuracy: 0.5880
Epoch 211/300
1/1 [==============================] - 1s 680ms/sample - loss: 0.2361 -
binary_accuracy: 0.5880
Epoch 212/300
1/1 [==============================] - 1s 675ms/sample - loss: 0.2361 -
binary_accuracy: 0.5880
Epoch 213/300
1/1 [==============================] - 1s 707ms/sample - loss: 0.2361 -
binary_accuracy: 0.5880
Epoch 214/300
1/1 [==============================] - 1s 672ms/sample - loss: 0.2361 -
binary_accuracy: 0.5880
Epoch 215/300
1/1 [==============================] - 1s 689ms/sample - loss: 0.2360 -
binary_accuracy: 0.5880
Epoch 216/300
1/1 [==============================] - 1s 695ms/sample - loss: 0.2360 -
binary_accuracy: 0.5880
Epoch 217/300
1/1 [==============================] - 1s 698ms/sample - loss: 0.2360 -
binary_accuracy: 0.5880
Epoch 218/300
```

```
1/1 [==============================] - 1s 739ms/sample - loss: 0.2360 -
binary_accuracy: 0.5880
Epoch 219/300
1/1 [==============================] - 1s 710ms/sample - loss: 0.2360 -
binary_accuracy: 0.5880
Epoch 220/300
1/1 [==============================] - 1s 716ms/sample - loss: 0.2360 -
binary_accuracy: 0.5880
Epoch 221/300
1/1 [==============================] - 1s 698ms/sample - loss: 0.2360 -
binary_accuracy: 0.5880
Epoch 222/300
1/1 [==============================] - 1s 697ms/sample - loss: 0.2360 -
binary_accuracy: 0.5880
Epoch 223/300
1/1 [==============================] - 1s 669ms/sample - loss: 0.2360 -
binary_accuracy: 0.5880
Epoch 224/300
1/1 [==============================] - 1s 693ms/sample - loss: 0.2360 -
binary_accuracy: 0.5880
Epoch 225/300
1/1 [==============================] - 1s 712ms/sample - loss: 0.2359 -
binary_accuracy: 0.5880
Epoch 226/300
1/1 [==============================] - 1s 706ms/sample - loss: 0.2359 -
binary_accuracy: 0.5880
Epoch 227/300
1/1 [==============================] - 1s 685ms/sample - loss: 0.2359 -
binary_accuracy: 0.5880
Epoch 228/300
1/1 [==============================] - 1s 698ms/sample - loss: 0.2359 -
binary_accuracy: 0.5880
Epoch 229/300
1/1 [==============================] - 1s 697ms/sample - loss: 0.2359 -
binary_accuracy: 0.5880
Epoch 230/300
1/1 [==============================] - 1s 720ms/sample - loss: 0.2359 -
binary_accuracy: 0.5880
Epoch 231/300
1/1 [==============================] - 1s 657ms/sample - loss: 0.2359 -
binary_accuracy: 0.5880
Epoch 232/300
1/1 [==============================] - 1s 693ms/sample - loss: 0.2359 -
binary_accuracy: 0.5880
Epoch 233/300
1/1 [==============================] - 1s 660ms/sample - loss: 0.2359 -
binary_accuracy: 0.5880
Epoch 234/300
```

```
1/1 [==============================] - 1s 697ms/sample - loss: 0.2359 -
binary_accuracy: 0.5880
Epoch 235/300
1/1 [==============================] - 1s 701ms/sample - loss: 0.2359 -
binary_accuracy: 0.5880
Epoch 236/300
1/1 [==============================] - 1s 679ms/sample - loss: 0.2358 -
binary_accuracy: 0.5880
Epoch 237/300
1/1 [==============================] - 1s 676ms/sample - loss: 0.2358 -
binary_accuracy: 0.5880
Epoch 238/300
1/1 [==============================] - 1s 678ms/sample - loss: 0.2358 -
binary_accuracy: 0.5880
Epoch 239/300
1/1 [==============================] - 1s 712ms/sample - loss: 0.2358 -
binary_accuracy: 0.5880
Epoch 240/300
1/1 [==============================] - 1s 688ms/sample - loss: 0.2358 -
binary_accuracy: 0.5880
Epoch 241/300
1/1 [==============================] - 1s 668ms/sample - loss: 0.2358 -
binary_accuracy: 0.5880
Epoch 242/300
1/1 [==============================] - 1s 714ms/sample - loss: 0.2358 -
binary_accuracy: 0.5880
Epoch 243/300
1/1 [==============================] - 1s 672ms/sample - loss: 0.2358 -
binary_accuracy: 0.5880
Epoch 244/300
1/1 [==============================] - 1s 671ms/sample - loss: 0.2358 -
binary_accuracy: 0.5880
Epoch 245/300
1/1 [==============================] - 1s 701ms/sample - loss: 0.2358 -
binary_accuracy: 0.5880
Epoch 246/300
1/1 [==============================] - 1s 689ms/sample - loss: 0.2358 -
binary_accuracy: 0.5880
Epoch 247/300
1/1 [==============================] - 1s 681ms/sample - loss: 0.2358 -
binary_accuracy: 0.5880
Epoch 248/300
1/1 [==============================] - 1s 695ms/sample - loss: 0.2358 -
binary_accuracy: 0.5880
Epoch 249/300
1/1 [==============================] - 1s 698ms/sample - loss: 0.2358 -
binary_accuracy: 0.5880
Epoch 250/300
```

```
1/1 [==============================] - 1s 684ms/sample - loss: 0.2357 -
binary_accuracy: 0.5880
Epoch 251/300
1/1 [==============================] - 1s 687ms/sample - loss: 0.2357 -
binary_accuracy: 0.5880
Epoch 252/300
1/1 [==============================] - 1s 678ms/sample - loss: 0.2357 -
binary_accuracy: 0.5880
Epoch 253/300
1/1 [==============================] - 1s 668ms/sample - loss: 0.2357 -
binary_accuracy: 0.5880
Epoch 254/300
1/1 [==============================] - 1s 693ms/sample - loss: 0.2357 -
binary_accuracy: 0.5880
Epoch 255/300
1/1 [==============================] - 1s 687ms/sample - loss: 0.2357 -
binary_accuracy: 0.5880
Epoch 256/300
1/1 [==============================] - 1s 703ms/sample - loss: 0.2357 -
binary_accuracy: 0.5880
Epoch 257/300
1/1 [==============================] - 1s 685ms/sample - loss: 0.2357 -
binary_accuracy: 0.5880
Epoch 258/300
1/1 [==============================] - 1s 672ms/sample - loss: 0.2357 -
binary_accuracy: 0.5880
Epoch 259/300
1/1 [==============================] - 1s 667ms/sample - loss: 0.2357 -
binary_accuracy: 0.5880
Epoch 260/300
1/1 [==============================] - 1s 685ms/sample - loss: 0.2357 -
binary_accuracy: 0.5880
Epoch 261/300
1/1 [==============================] - 1s 685ms/sample - loss: 0.2357 -
binary_accuracy: 0.5880
Epoch 262/300
1/1 [==============================] - 1s 674ms/sample - loss: 0.2357 -
binary_accuracy: 0.5880
Epoch 263/300
1/1 [==============================] - 1s 675ms/sample - loss: 0.2357 -
binary_accuracy: 0.5880
Epoch 264/300
1/1 [==============================] - 1s 681ms/sample - loss: 0.2357 -
binary_accuracy: 0.5880
Epoch 265/300
1/1 [==============================] - 1s 714ms/sample - loss: 0.2357 -
binary_accuracy: 0.5880
Epoch 266/300
```

```
1/1 [==============================] - 1s 687ms/sample - loss: 0.2357 -
binary_accuracy: 0.5880
Epoch 267/300
1/1 [==============================] - 1s 704ms/sample - loss: 0.2357 -
binary_accuracy: 0.5880
Epoch 268/300
1/1 [==============================] - 1s 703ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 269/300
1/1 [==============================] - 1s 701ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 270/300
1/1 [==============================] - 1s 678ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 271/300
1/1 [==============================] - 1s 717ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 272/300
1/1 [==============================] - 1s 708ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 273/300
1/1 [==============================] - 1s 663ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 274/300
1/1 [==============================] - 1s 703ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 275/300
1/1 [==============================] - 1s 681ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 276/300
1/1 [==============================] - 1s 706ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 277/300
1/1 [==============================] - 1s 682ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 278/300
1/1 [==============================] - 1s 699ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 279/300
1/1 [==============================] - 1s 682ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 280/300
1/1 [==============================] - 1s 682ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 281/300
1/1 [==============================] - 1s 683ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 282/300
```

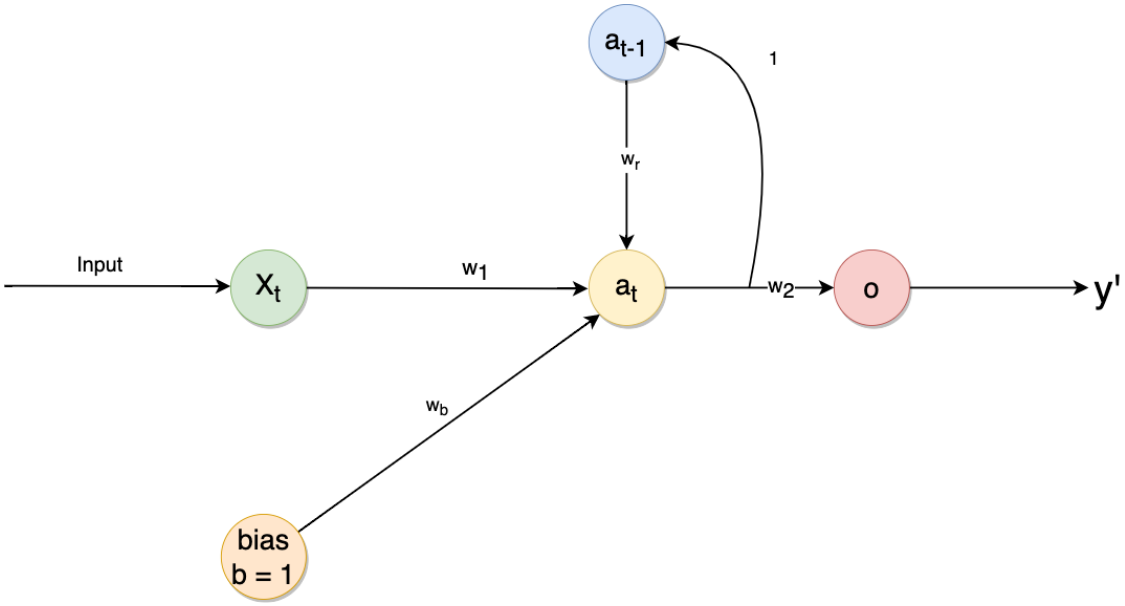
```
1/1 [==============================] - 1s 695ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 283/300
1/1 [==============================] - 1s 653ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 284/300
1/1 [==============================] - 1s 703ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 285/300
1/1 [==============================] - 1s 684ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 286/300
1/1 [==============================] - 1s 659ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 287/300
1/1 [==============================] - 1s 688ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 288/300
1/1 [==============================] - 1s 698ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 289/300
1/1 [==============================] - 1s 685ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 290/300
1/1 [==============================] - 1s 685ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 291/300
1/1 [==============================] - 1s 687ms/sample - loss: 0.2356 -
binary_accuracy: 0.5880
Epoch 292/300
1/1 [==============================] - 1s 655ms/sample - loss: 0.2355 -
binary_accuracy: 0.5880
Epoch 293/300
1/1 [==============================] - 1s 683ms/sample - loss: 0.2355 -
binary_accuracy: 0.5880
Epoch 294/300
1/1 [==============================] - 1s 676ms/sample - loss: 0.2355 -
binary_accuracy: 0.5880
Epoch 295/300
1/1 [==============================] - 1s 672ms/sample - loss: 0.2355 -
binary_accuracy: 0.5880
Epoch 296/300
1/1 [==============================] - 1s 653ms/sample - loss: 0.2355 -
binary_accuracy: 0.5880
Epoch 297/300
1/1 [==============================] - 1s 662ms/sample - loss: 0.2355 -
binary_accuracy: 0.5880
Epoch 298/300
```

```
1/1 [==============================] - 1s 673ms/sample - loss: 0.2355 -
binary_accuracy: 0.5880
Epoch 299/300
1/1 [==============================] - 1s 686ms/sample - loss: 0.2355 -
binary_accuracy: 0.5880
Epoch 300/300
1/1 [==============================] - 1s 686ms/sample - loss: 0.2355 -
binary_accuracy: 0.5880
```

Plot the loss and accuracy progression of the model's training by using the `loss` and `binary_accuracy` keys of dictionary `history.history` like before. What is the final accuracy of the model?

3.3 Exercise 3

A) Draw the architecture diagram of the SRN that solves the XOR problem. You can represent connections from one layer to another by a single arrow.



B) It turns out that the accuracy will not get higher than 0.66. Why specifically this number (HINT: What is theoretically possible? You can see for yourself by writing out some data and trying to guess yourself what the next input will be.) Explain your answer.

Let us consider 3 separate sets of inputs in a single sequence. WHen the model is fed input 1, it returns a blind prediction with a probability of the answer being correct = 0.5. The probability for the second trial with the second sequence of inputs is also 0.5. With the third trial, the chances are improved based on the knowledge of the previous trials taht the model has learned.

Thus all third predictions have a larger probability of being accurate on teh XOR data. The following equation depicts the probability calculation for the third trial being correct.

$$((_1 =) + (_2 =) + (=))/3 = (0.5 + 0.5 + 1)/3 = 2/3 = 0.66$$

C) This is a weird way to model XOR. But it was very important for Elman to do it this way. Why? Explain your answer.

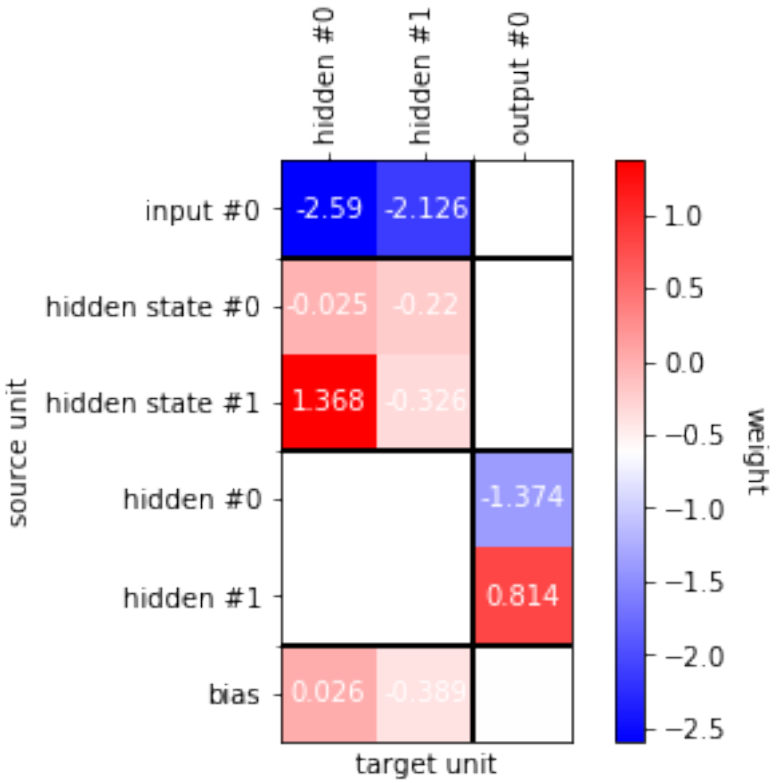
Elman was trying to represent time as an intrinsic property and understand if the neural network with context could be utilized to model linguistic structures. The implicit representation of time is necessary to study the effect of time on the congnition of language and the understanding of linguistic patterns.

3.4 Network analysis

[62]:
```
from plot_functions import plot_weight_matrix, plot_activation_matrix
```

This network is still quite simple, so let's look at the fitted weights.

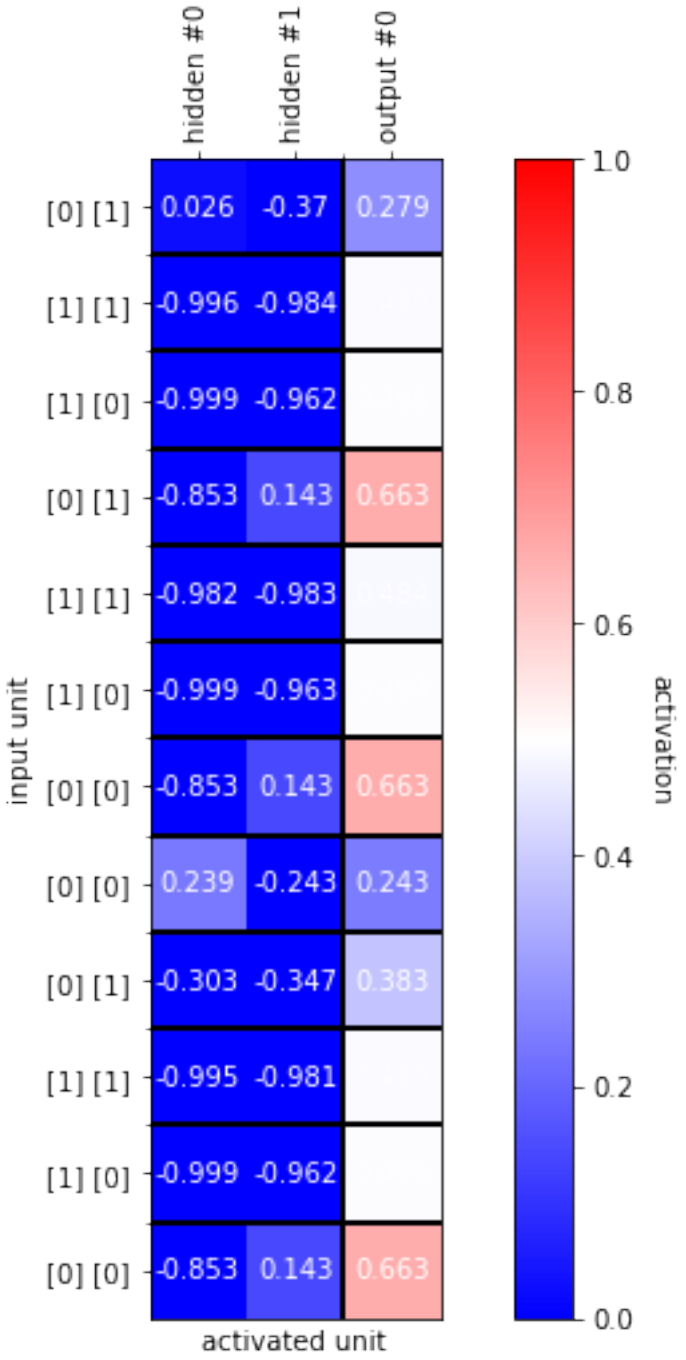
[63]:
```
plot_weight_matrix(model_rnn1)
```



Can you make sense of the weights and what it does? We can make analysis a bit easier by looking the activations of a subset of our data. In the following heatmap, you can see the input values and corresponding target outputs on the y-axis. The actual outputs are in the final column.

[64]:
```
plt.figure(figsize=(10,8))
plot_activation_matrix(model_rnn1, X, y,
                       subset=(0, slice(0, 12), slice(None)),
```

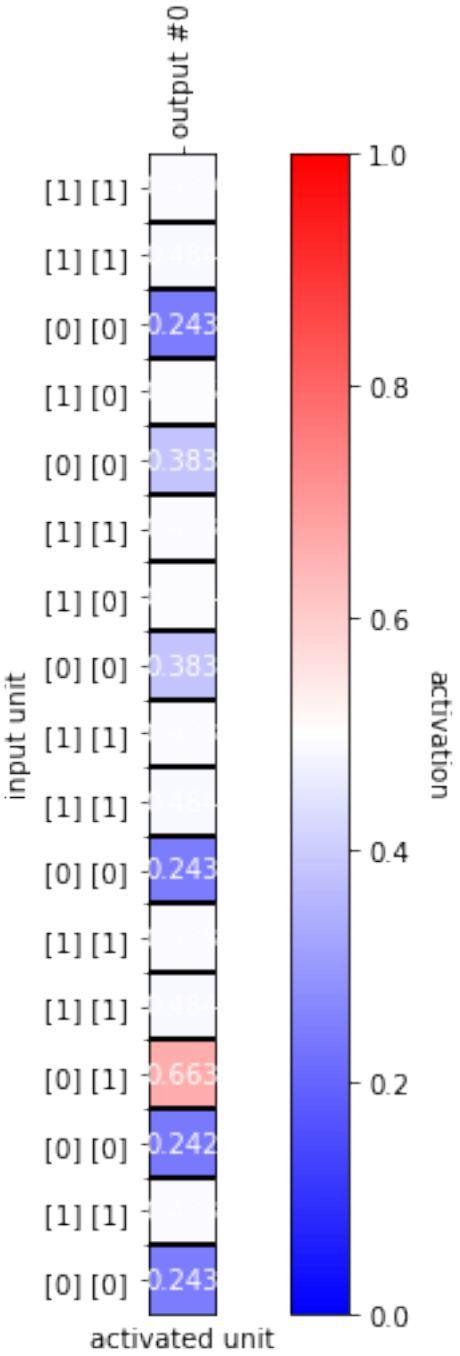
```
                    vmin=0, vmax=1)
```



The `subset` argument allows us to take a subset of our data using the Python `slice` function. We take only the first item of the inputs (`0`), the first 12 time steps (`slice(0, 12)`) and all input/output dimensions (`slice(None)`).

Of every three consecutive time steps, every second output should be correct since that output is the XOR of the current and previous input. We can make seeing this easier by just looking at the output layer activations and only the predictable time steps:

[65]:
```
plt.figure(figsize=(10,8))
plot_activation_matrix(model_rnn1, X, y, layers=[model_rnn1.layers[1]],
                       subset=(0, slice(1, 50, 3), slice(None)),
                       vmin=0, vmax=1)
```



Note that the output is always correct with respect to the target output. This indicates that the network applies XOR on these items. What does the network do with the other two items, which are not predictable?

This network shows that neural networks are able to find structure in data, even though two thirds of the data is unpredictable.

3.5 Exercise 4

A) Report the connection weights for the SRN after the training. Are they negative or positive?

The connection weights are largely in the range -2.5 to $+0.8$. The only weight that is larger than 1 is the weight between the hidden nodes.

B) How can you see if the network has learned to predict XOR? Explain where you see how well the network is doing. (Hint: read the bottom of page 186-187 in Elman 1990)

Based on Elman's observations, we can know if a model learned to model the XOR problem by checking the activations at the output nodes. The inputs $[0, 0]$ and $[1, 1]$ would activate one of the outputs whereas the alternated inputs $[0, 1]$ and $[1, 0]$ would produce activations at the other output node.

Based on the outputs and the activations observed, the network works at separating $[0, 1]$ and $[0, 0]$ but fails at separating $[1, 0]$ from $[1, 1]$.

C) How does the network eventually solve the problem? Explain your answer.

The network, upon training over multiple epochs, would be able to create a representation utilizing the weights learned over training. In this representation, the points $[0, 0]$ and $[1, 1]$ would be closer to each other than to the points $[0, 1]$ and $[1, 0]$.

Such a representation can be likened to the effect of folding the vector space. This is achieved by using a deeper layer to extract different features from the data.

4 Part 3: Modelling Simple Sentences with Simple Recurrent Neural Networks

The previous tutorial showed that simple RNNs are able to find structure in sequential data, even though the structure is not deterministic. Elman (1990) took this a step further in his final experiment in which he trained a simple RNN on a sequence of generated English sentences. In this tutorial, we make a reproduction of his original network.

Elman, J. L. (1990). Finding structure in time. Cognitive science, 14(2), 179-211.

4.1 Data generation

Use the small python program `sentence_generator.py`to generate the training set. This program follows the details that are described by Elman (1990) as closely as possible. (Tip: read the paper!) Just like in the original experiment, we will be using a concatenation of 10,000 random 2-3 word

sentences without any sentence boundaries. The 29 unique words are one-hot-encoded (like Elman did!) with two extra bits that are always zero.

[27]:
```
from sentence_generator import generate_sentences
from sklearn.preprocessing import LabelBinarizer
```

[28]:
```
sents = generate_sentences(10000)
words = [word for words in sents for word in words]
```

What do these sentences look like?

[29]:
```
sents[:3]
```

[29]:
```
[['boy', 'think'], ['woman', 'move', 'rock'], ['monster', 'eat', 'cookie']]
```

These seem like decent simple sentences. But remember that the actual data is a concatenation of these sentences:

[30]:
```
words[:8]
```

[30]:
```
['boy', 'think', 'woman', 'move', 'rock', 'monster', 'eat', 'cookie']
```

Now we have to encode the data so that our simple RNN can process it.

[31]:
```
all_words = list(set(words)) + ['zog', 'zog2']

# Use one-hot-encoding to assign a unique input unit to each word
encoder = LabelBinarizer()
encoder.fit(all_words)
word_data = encoder.transform(words)

# The first axis will have size 1, just like the previous tutorial
X = word_data[np.newaxis, :]
y = np.append(word_data[1:], word_data[:1], axis=0)[np.newaxis, :]

X.shape, y.shape
```

[31]:
```
((1, 27500, 31), (1, 27500, 31))
```

`X` and `y` both have the correct shape, so we can now create the network.

4.2 Model definition

Now you will create a network that is very similar to the sequential XOR SRN, but follows the architecture choices of Elman.

[32]:
```
# complete code here
model = Sequential([
    SimpleRNN(20,
              input_shape=(None, 31),
```

```
                return_sequences=True,
                activation='tanh',
                name='hidden'),
    TimeDistributed(Dense(31, activation='softmax', name='output'))
])
```

The important differences between the sequential XOR SRN and this network are: * Greater number of hidden units (how many?), which enables the network to learn more complex information. * The input shape can be defined as `(None, 31)`. The `None` value indicates that the network works with any number of time steps. Just like the output, the input must be 31-dimensional. * We use the `softmax` activation function in the final layer. This is the recommended choice for the output layer if the output is categorical (which it is). * The loss function is `categorical_crossentropy` for the same reason as above. In simple terms, categorical crossentropy is better for one-hot-encoded data because it gives more importance to that the target unit activation should be highest. MSE for instance weighs all deviations from zero for the other units equally important, so optimization gets easily stuck at the minimum of only zero outputs. * Our network also differs in a number of ways from Elman's because we have some different functions available. However, our results should still be comparable (and informative).

[33]:
```
model.compile(loss='categorical_crossentropy',
              optimizer=SGD(lr=0.1, momentum=0.9),
              metrics=['categorical_accuracy', 'mean_squared_error'])

history = model.fit(X, y, epochs=100, verbose=2)
```

```
Train on 1 samples
Epoch 1/100
1/1 - 41s - loss: 3.4569 - categorical_accuracy: 0.0359 - mean_squared_error:
0.0313
Epoch 2/100
1/1 - 47s - loss: 3.4434 - categorical_accuracy: 0.0380 - mean_squared_error:
0.0312
Epoch 3/100
1/1 - 46s - loss: 3.4203 - categorical_accuracy: 0.0483 - mean_squared_error:
0.0312
Epoch 4/100
1/1 - 48s - loss: 3.3920 - categorical_accuracy: 0.0674 - mean_squared_error:
0.0311
Epoch 5/100
1/1 - 49s - loss: 3.3614 - categorical_accuracy: 0.0949 - mean_squared_error:
0.0311
Epoch 6/100
1/1 - 48s - loss: 3.3294 - categorical_accuracy: 0.1127 - mean_squared_error:
0.0310
Epoch 7/100
1/1 - 50s - loss: 3.2959 - categorical_accuracy: 0.1131 - mean_squared_error:
0.0309
Epoch 8/100
```

```
1/1 - 52s - loss: 3.2607 - categorical_accuracy: 0.1130 - mean_squared_error:
0.0308
Epoch 9/100
1/1 - 49s - loss: 3.2233 - categorical_accuracy: 0.1130 - mean_squared_error:
0.0307
Epoch 10/100
1/1 - 49s - loss: 3.1850 - categorical_accuracy: 0.1130 - mean_squared_error:
0.0306
Epoch 11/100
1/1 - 53s - loss: 3.1484 - categorical_accuracy: 0.1130 - mean_squared_error:
0.0305
Epoch 12/100
1/1 - 62s - loss: 3.1160 - categorical_accuracy: 0.1130 - mean_squared_error:
0.0304
Epoch 13/100
1/1 - 54s - loss: 3.0892 - categorical_accuracy: 0.1136 - mean_squared_error:
0.0304
Epoch 14/100
1/1 - 50s - loss: 3.0688 - categorical_accuracy: 0.1109 - mean_squared_error:
0.0303
Epoch 15/100
1/1 - 47s - loss: 3.0537 - categorical_accuracy: 0.1074 - mean_squared_error:
0.0302
Epoch 16/100
1/1 - 55s - loss: 3.0408 - categorical_accuracy: 0.1112 - mean_squared_error:
0.0302
Epoch 17/100
1/1 - 52s - loss: 3.0270 - categorical_accuracy: 0.1200 - mean_squared_error:
0.0301
Epoch 18/100
1/1 - 51s - loss: 3.0109 - categorical_accuracy: 0.1274 - mean_squared_error:
0.0301
Epoch 19/100
1/1 - 51s - loss: 2.9933 - categorical_accuracy: 0.1359 - mean_squared_error:
0.0300
Epoch 20/100
1/1 - 57s - loss: 2.9750 - categorical_accuracy: 0.1469 - mean_squared_error:
0.0299
Epoch 21/100
1/1 - 50s - loss: 2.9569 - categorical_accuracy: 0.1472 - mean_squared_error:
0.0298
Epoch 22/100
1/1 - 54s - loss: 2.9391 - categorical_accuracy: 0.1448 - mean_squared_error:
0.0297
Epoch 23/100
1/1 - 60s - loss: 2.9216 - categorical_accuracy: 0.1423 - mean_squared_error:
0.0297
Epoch 24/100
```

```
1/1 - 49s - loss: 2.9046 - categorical_accuracy: 0.1426 - mean_squared_error:
0.0296
Epoch 25/100
1/1 - 49s - loss: 2.8880 - categorical_accuracy: 0.1424 - mean_squared_error:
0.0295
Epoch 26/100
1/1 - 54s - loss: 2.8711 - categorical_accuracy: 0.1444 - mean_squared_error:
0.0295
Epoch 27/100
1/1 - 55s - loss: 2.8534 - categorical_accuracy: 0.1472 - mean_squared_error:
0.0294
Epoch 28/100
1/1 - 52s - loss: 2.8348 - categorical_accuracy: 0.1510 - mean_squared_error:
0.0293
Epoch 29/100
1/1 - 54s - loss: 2.8163 - categorical_accuracy: 0.1546 - mean_squared_error:
0.0292
Epoch 30/100
1/1 - 52s - loss: 2.7988 - categorical_accuracy: 0.1588 - mean_squared_error:
0.0292
Epoch 31/100
1/1 - 51s - loss: 2.7823 - categorical_accuracy: 0.1635 - mean_squared_error:
0.0291
Epoch 32/100
1/1 - 51s - loss: 2.7663 - categorical_accuracy: 0.1705 - mean_squared_error:
0.0290
Epoch 33/100
1/1 - 52s - loss: 2.7504 - categorical_accuracy: 0.1773 - mean_squared_error:
0.0290
Epoch 34/100
1/1 - 54s - loss: 2.7348 - categorical_accuracy: 0.1822 - mean_squared_error:
0.0289
Epoch 35/100
1/1 - 48s - loss: 2.7195 - categorical_accuracy: 0.1861 - mean_squared_error:
0.0289
Epoch 36/100
1/1 - 51s - loss: 2.7047 - categorical_accuracy: 0.1907 - mean_squared_error:
0.0288
Epoch 37/100
1/1 - 51s - loss: 2.6902 - categorical_accuracy: 0.1927 - mean_squared_error:
0.0287
Epoch 38/100
1/1 - 52s - loss: 2.6759 - categorical_accuracy: 0.1946 - mean_squared_error:
0.0287
Epoch 39/100
1/1 - 55s - loss: 2.6619 - categorical_accuracy: 0.1961 - mean_squared_error:
0.0286
Epoch 40/100
```

```
1/1 - 52s - loss: 2.6483 - categorical_accuracy: 0.1972 - mean_squared_error:
0.0286
Epoch 41/100
1/1 - 53s - loss: 2.6350 - categorical_accuracy: 0.1974 - mean_squared_error:
0.0285
Epoch 42/100
1/1 - 47s - loss: 2.6218 - categorical_accuracy: 0.1974 - mean_squared_error:
0.0285
Epoch 43/100
1/1 - 48s - loss: 2.6088 - categorical_accuracy: 0.2001 - mean_squared_error:
0.0284
Epoch 44/100
1/1 - 47s - loss: 2.5963 - categorical_accuracy: 0.2032 - mean_squared_error:
0.0284
Epoch 45/100
1/1 - 47s - loss: 2.5843 - categorical_accuracy: 0.2061 - mean_squared_error:
0.0283
Epoch 46/100
1/1 - 48s - loss: 2.5724 - categorical_accuracy: 0.2092 - mean_squared_error:
0.0283
Epoch 47/100
1/1 - 48s - loss: 2.5608 - categorical_accuracy: 0.2106 - mean_squared_error:
0.0282
Epoch 48/100
1/1 - 48s - loss: 2.5496 - categorical_accuracy: 0.2150 - mean_squared_error:
0.0282
Epoch 49/100
1/1 - 47s - loss: 2.5390 - categorical_accuracy: 0.2171 - mean_squared_error:
0.0282
Epoch 50/100
1/1 - 47s - loss: 2.5290 - categorical_accuracy: 0.2193 - mean_squared_error:
0.0281
Epoch 51/100
1/1 - 49s - loss: 2.5196 - categorical_accuracy: 0.2216 - mean_squared_error:
0.0281
Epoch 52/100
1/1 - 47s - loss: 2.5107 - categorical_accuracy: 0.2224 - mean_squared_error:
0.0281
Epoch 53/100
1/1 - 47s - loss: 2.5022 - categorical_accuracy: 0.2231 - mean_squared_error:
0.0280
Epoch 54/100
1/1 - 47s - loss: 2.4939 - categorical_accuracy: 0.2249 - mean_squared_error:
0.0280
Epoch 55/100
1/1 - 48s - loss: 2.4859 - categorical_accuracy: 0.2256 - mean_squared_error:
0.0280
Epoch 56/100
```

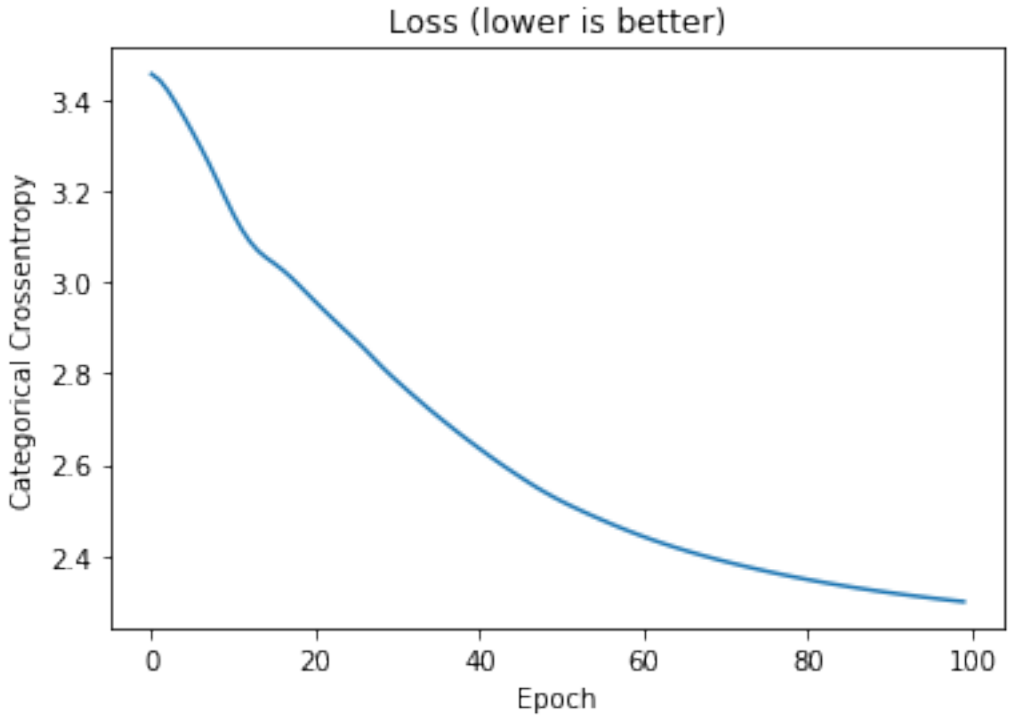
```
1/1 - 48s - loss: 2.4781 - categorical_accuracy: 0.2260 - mean_squared_error:
0.0279
Epoch 57/100
1/1 - 46s - loss: 2.4704 - categorical_accuracy: 0.2265 - mean_squared_error:
0.0279
Epoch 58/100
1/1 - 48s - loss: 2.4629 - categorical_accuracy: 0.2274 - mean_squared_error:
0.0279
Epoch 59/100
1/1 - 47s - loss: 2.4557 - categorical_accuracy: 0.2282 - mean_squared_error:
0.0278
Epoch 60/100
1/1 - 47s - loss: 2.4487 - categorical_accuracy: 0.2287 - mean_squared_error:
0.0278
Epoch 61/100
1/1 - 50s - loss: 2.4420 - categorical_accuracy: 0.2294 - mean_squared_error:
0.0278
Epoch 62/100
1/1 - 52s - loss: 2.4355 - categorical_accuracy: 0.2306 - mean_squared_error:
0.0277
Epoch 63/100
1/1 - 52s - loss: 2.4294 - categorical_accuracy: 0.2306 - mean_squared_error:
0.0277
Epoch 64/100
1/1 - 53s - loss: 2.4234 - categorical_accuracy: 0.2308 - mean_squared_error:
0.0277
Epoch 65/100
1/1 - 52s - loss: 2.4176 - categorical_accuracy: 0.2320 - mean_squared_error:
0.0277
Epoch 66/100
1/1 - 54s - loss: 2.4121 - categorical_accuracy: 0.2332 - mean_squared_error:
0.0276
Epoch 67/100
1/1 - 50s - loss: 2.4068 - categorical_accuracy: 0.2333 - mean_squared_error:
0.0276
Epoch 68/100
1/1 - 51s - loss: 2.4017 - categorical_accuracy: 0.2344 - mean_squared_error:
0.0276
Epoch 69/100
1/1 - 47s - loss: 2.3968 - categorical_accuracy: 0.2357 - mean_squared_error:
0.0276
Epoch 70/100
1/1 - 54s - loss: 2.3920 - categorical_accuracy: 0.2359 - mean_squared_error:
0.0276
Epoch 71/100
1/1 - 52s - loss: 2.3874 - categorical_accuracy: 0.2360 - mean_squared_error:
0.0275
Epoch 72/100
```

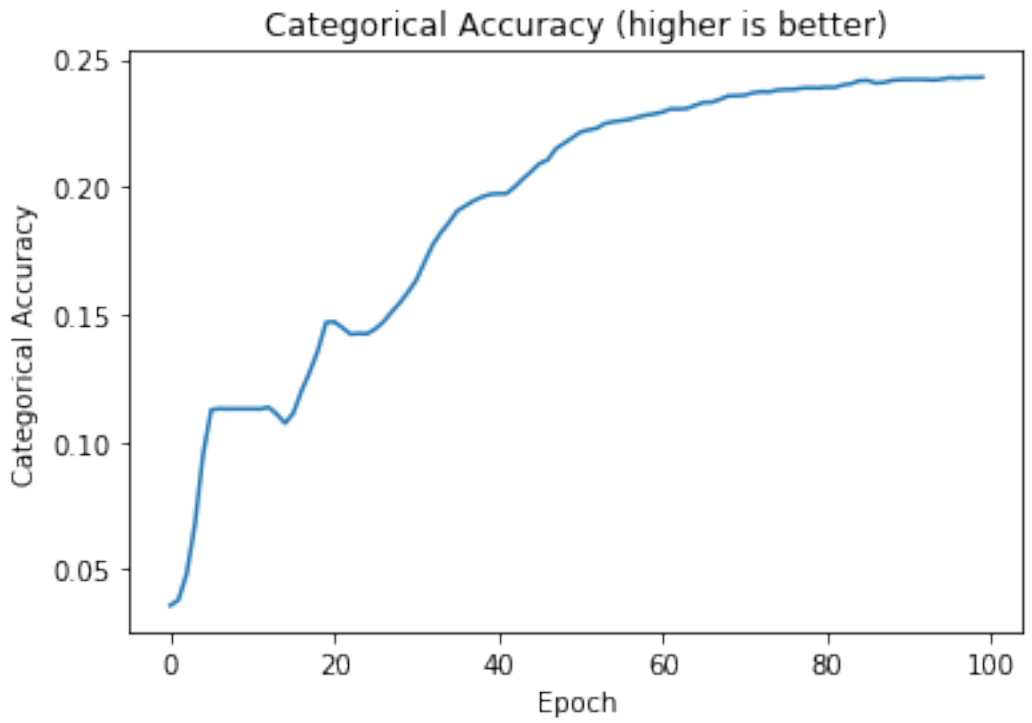
```
1/1 - 51s - loss: 2.3829 - categorical_accuracy: 0.2369 - mean_squared_error:
0.0275
Epoch 73/100
1/1 - 50s - loss: 2.3785 - categorical_accuracy: 0.2374 - mean_squared_error:
0.0275
Epoch 74/100
1/1 - 48s - loss: 2.3742 - categorical_accuracy: 0.2373 - mean_squared_error:
0.0275
Epoch 75/100
1/1 - 52s - loss: 2.3701 - categorical_accuracy: 0.2380 - mean_squared_error:
0.0275
Epoch 76/100
1/1 - 54s - loss: 2.3661 - categorical_accuracy: 0.2383 - mean_squared_error:
0.0274
Epoch 77/100
1/1 - 53s - loss: 2.3622 - categorical_accuracy: 0.2383 - mean_squared_error:
0.0274
Epoch 78/100
1/1 - 50s - loss: 2.3585 - categorical_accuracy: 0.2388 - mean_squared_error:
0.0274
Epoch 79/100
1/1 - 47s - loss: 2.3549 - categorical_accuracy: 0.2389 - mean_squared_error:
0.0274
Epoch 80/100
1/1 - 52s - loss: 2.3514 - categorical_accuracy: 0.2388 - mean_squared_error:
0.0274
Epoch 81/100
1/1 - 50s - loss: 2.3479 - categorical_accuracy: 0.2391 - mean_squared_error:
0.0274
Epoch 82/100
1/1 - 51s - loss: 2.3447 - categorical_accuracy: 0.2391 - mean_squared_error:
0.0273
Epoch 83/100
1/1 - 52s - loss: 2.3414 - categorical_accuracy: 0.2400 - mean_squared_error:
0.0273
Epoch 84/100
1/1 - 50s - loss: 2.3383 - categorical_accuracy: 0.2405 - mean_squared_error:
0.0273
Epoch 85/100
1/1 - 47s - loss: 2.3353 - categorical_accuracy: 0.2417 - mean_squared_error:
0.0273
Epoch 86/100
1/1 - 49s - loss: 2.3324 - categorical_accuracy: 0.2418 - mean_squared_error:
0.0273
Epoch 87/100
1/1 - 52s - loss: 2.3296 - categorical_accuracy: 0.2408 - mean_squared_error:
0.0273
Epoch 88/100
```

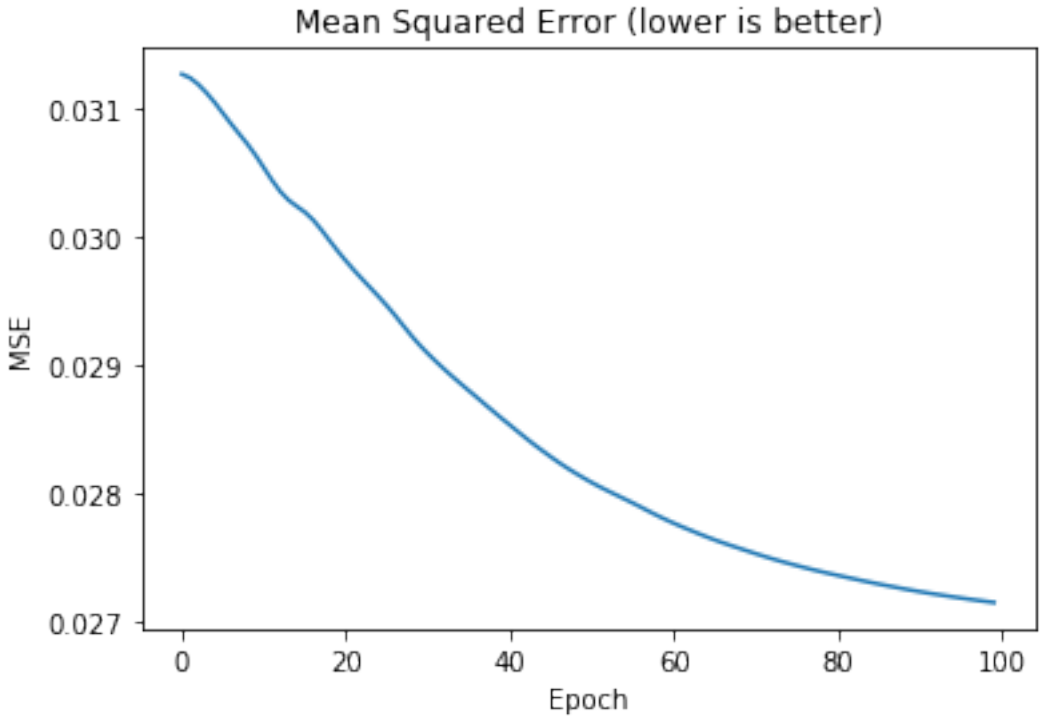
```
1/1 - 53s - loss: 2.3269 - categorical_accuracy: 0.2410 - mean_squared_error:
0.0273
Epoch 89/100
1/1 - 52s - loss: 2.3242 - categorical_accuracy: 0.2419 - mean_squared_error:
0.0273
Epoch 90/100
1/1 - 52s - loss: 2.3216 - categorical_accuracy: 0.2421 - mean_squared_error:
0.0272
Epoch 91/100
1/1 - 47s - loss: 2.3191 - categorical_accuracy: 0.2422 - mean_squared_error:
0.0272
Epoch 92/100
1/1 - 48s - loss: 2.3167 - categorical_accuracy: 0.2422 - mean_squared_error:
0.0272
Epoch 93/100
1/1 - 48s - loss: 2.3143 - categorical_accuracy: 0.2422 - mean_squared_error:
0.0272
Epoch 94/100
1/1 - 49s - loss: 2.3120 - categorical_accuracy: 0.2420 - mean_squared_error:
0.0272
Epoch 95/100
1/1 - 47s - loss: 2.3098 - categorical_accuracy: 0.2423 - mean_squared_error:
0.0272
Epoch 96/100
1/1 - 52s - loss: 2.3076 - categorical_accuracy: 0.2429 - mean_squared_error:
0.0272
Epoch 97/100
1/1 - 52s - loss: 2.3055 - categorical_accuracy: 0.2427 - mean_squared_error:
0.0272
Epoch 98/100
1/1 - 52s - loss: 2.3034 - categorical_accuracy: 0.2430 - mean_squared_error:
0.0272
Epoch 99/100
1/1 - 58s - loss: 2.3015 - categorical_accuracy: 0.2429 - mean_squared_error:
0.0272
Epoch 100/100
1/1 - 52s - loss: 2.2995 - categorical_accuracy: 0.2431 - mean_squared_error:
0.0271
```

[34]:
```
plt.plot(history.history['loss'])
plt.title('Loss (lower is better)')
plt.ylabel('Categorical Crossentropy')
plt.xlabel('Epoch')
plt.show()

plt.plot(history.history['categorical_accuracy'])
plt.title('Categorical Accuracy (higher is better)')
```

```
plt.ylabel('Categorical Accuracy')
plt.xlabel('Epoch')
plt.show()

plt.plot(history.history['mean_squared_error'])
plt.title('Mean Squared Error (lower is better)')
plt.ylabel('MSE')
plt.xlabel('Epoch')
plt.show()
```




Categorical Accuracy (higher is better)
Categorical Accuracy
0.25
0.20
0.15
0.10
0.05
0
20
40
60
80
100
Epoch


Mean Squared Error (lower is better)
MSE
0.031
0.030
0.029
0.028
0.027
0
20
40
60
80
100
Epoch

After 100 epochs, the loss gets nearly stable. The MSE metric for the final epoch is:

[35]:
```
history.history['mean_squared_error'][-1] * 31
```

[35]:
```
0.8416352607309818
```

It appears that our MSE is a bit lower than the score of Elman. Elman did however train only 6 epochs while we trained 100 epochs. (Poor guy! It probably took him days, if not weeks!)

4.3 Exercise 5

A) What parameters did you use to define the Elman SRN?

The Elman SRN defined had the following structure: * Simple Recurrent Unit: * Number of hidden units: 20 * Input shape: None, 31 * Activation: tanh * Time Distributed Dense Unit: * Number of outputs: 31 * Activation: softmax * Optimizer: * Stochastic Gradient Descent with Momentum * Learning Rate: 0.1 * Momentum: 0.9 * Loss function: Categorical Cross Entropy

B) Elman states: "Recall that the prediction task is non-deterministic". Explain why this is so, and why using this network to solve a non-deterministic problem is more interesting than solving a deterministic one.

Deterministic problems would always produce the same outputs for a given particular input. This problem is non-deterministic in the sense the machine could produce different predictions for the third word in a sequence given the first words, due to the many possible underlying grammatical relationships between words. In the context of learning word representations and relationships between words, considering the word sequences to be temporal sequences, thus providing a contextual memory to the network, enhances the learning of word sequences and the underlying relationships.

Learning word sequences as a deterministic task would lead to generating explicit maps (if ABC is a sequence and AB is an input, machine only learns to output C as the next in sequence). Considering it to be a non-deterministic task allows the model to understand deeper relationships between words and form a simple representation of the grammatical and lexical structure of the language

For Example: If the network is able to relate the action of eating with food related nouns, it also implies the network is able to group such words and form an abstract relationship between the words in a sentence by considering the sentence to be a time dependant series.

C) Commonly when training and evaluating neural networks, we are concerned about how quickly the loss decreases and how low we can get it to be. But consider: are these useful evaluation metrics for this particular problem? Why or why not? Explain your answer.

The accuracy of our predictions have to be measured by testing the correctness of the prdicted sequence. This could mean syntactic correctness or semantic correctness. An alternative metric would be to compare the predictions against the classes defined in the dataset for correctness.

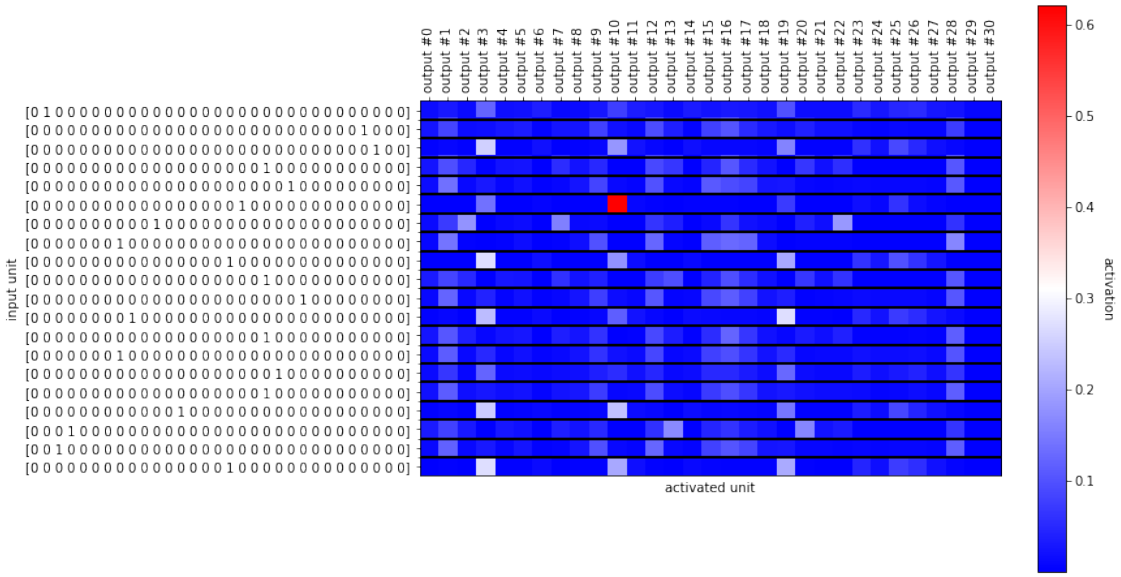
Loss is not always the best metric to test the performance or the learning that has been achieved by a model.

4.4 Network Analysis

Let's take a look at our network again. With the amount of units in our network, our weight matrix is however not very useful anymore. We can still take a look at some input output combinations in our activation matrix.

[36]:
```
from plot_functions import plot_activation_matrix

plt.figure(figsize=(10,8))
plot_activation_matrix(model, X, layers=[model.layers[1]],
                       subset=(0, slice(0, 20), slice(None)),
                       show_values=False)
```

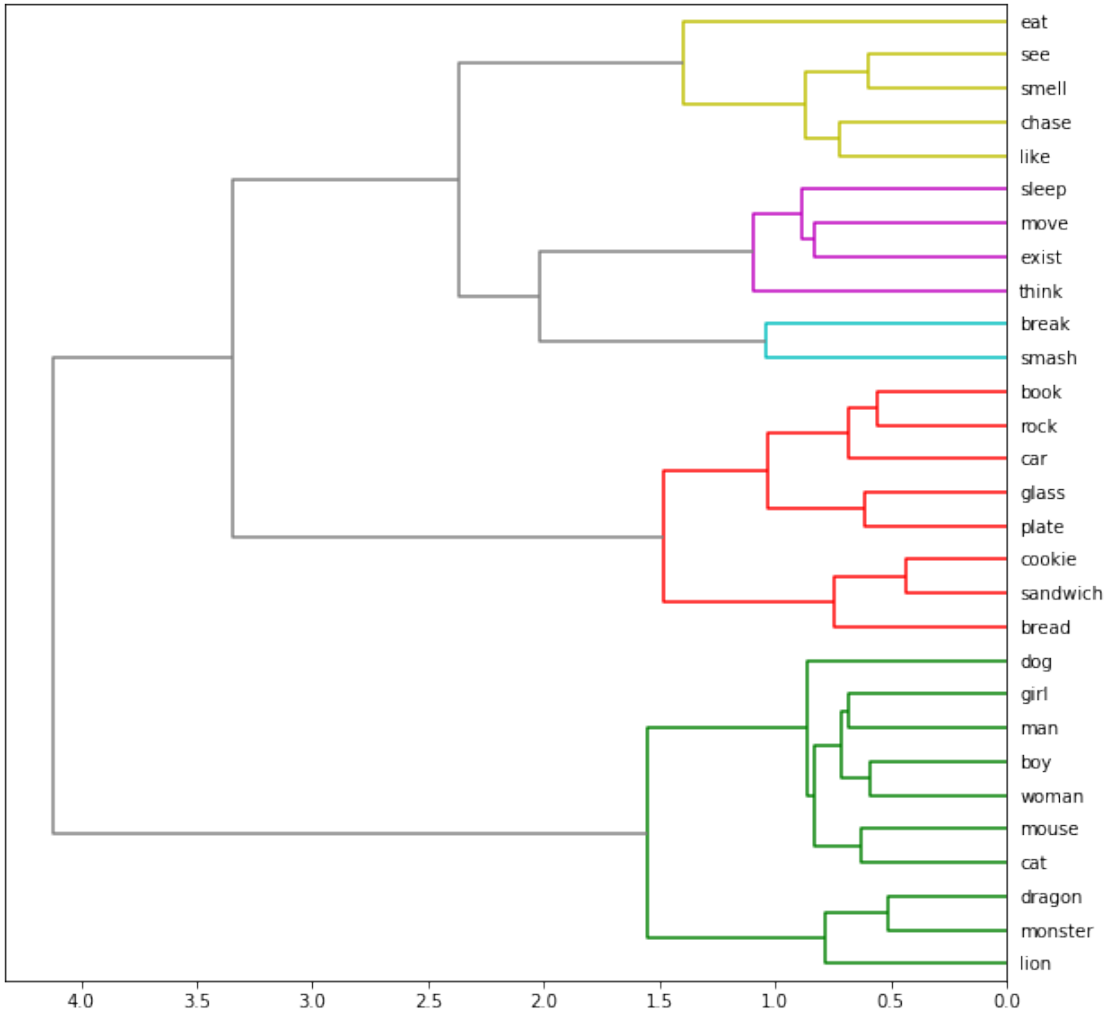
```
/Users/varunravivarma/py3_virtualenv/jupe/lib/python3.7/site-
packages/matplotlib/text.py:1150: FutureWarning: elementwise comparison failed;
returning scalar instead, but in the future will perform elementwise comparison
  if s != self._text:
```



We can at least see that the network makes predictions and some words are more easily predictable than others. Elman (1990) created a hierarchical cluster of the mean hidden layer activation values per input word. Elman found that the hidden layer activations of similar words are also similar. Let's reproduce his dendrogram:

[37]:
```
from utils import get_activation_matrix

import pandas as pd
from scipy.cluster.hierarchy import dendrogram, linkage
```

[38]:
```
# Use pandas to easily calculate group means for the hidden layer
activations = pd.DataFrame(get_activation_matrix(model, X, layers=[model.
 ↪layers[0]])[0])
mean_activations = activations.groupby(words).mean()

# Use scipy to create a hierarchical cluster
Z = linkage(mean_activations, method='complete', metric='euclidean',␣
 ↪optimal_ordering=True)

plt.figure(figsize=(10,10))
dendrogram(Z,
           labels=mean_activations.index,
           leaf_rotation=0,
           orientation='left',
           color_threshold=2.0,
           above_threshold_color='grey')
plt.show()
```



Clearly, the network did learn lexical categories! If you take a look at the lexicon in `sentence_generator.py`, you can see that the clusters match the lexical categories.

4.5 Exercise 6 (To be answered individually)

A) Ideally we would want to include several homonyms; use the same representations for both nouns and verbs. How would you modify the `sentence_generator.py` to include several homonyms? Explain what you would do.

The `sentence_generator.py` already contains certain homonyms (such as the verb break, which can be used as a transitive verb in multiple contexts, with multiple meanings). Upon adding new homonyms, the structures for the `lexicons` and the `grammar` would have to be updated to match the possible combinations of lexicons resulting from the addition of homonyms. (Updates can be in the form of adding the homonyms to the correct tags in the lexicon.)

Using the same representations could lead to a drop in the accuracy of the model, due to the homonyms having the same encoded value. This could cause the homonyms to be classified erroneously.

B) In the first lecture 5 advantages for computational modelling were presented (1. Explicitness,2. Study complex predictions or interactions, 3. Inspiration, 4. Practicality and 5. Control and Explanation). Explain how the SRN model of nouns and verbs relates to each one of the advantages. Your answers should not be longer than 400 Words.

1. *Study complex interactions* The SRN model is able to utilize context in encoded sequences and genderate a complex grammatical structure from being fed a simple input series. Th emodel enables a study of how linguistic learning has a temporal component and is affected by the contextual component.

2. *Practicality* The effect of treating a task as a temporal task could not be tested easily in humans. Since the learning of a language is not a very conscious task which allows us to analyze the processes involved, it would be easier to simulate an experiment that treated the learning process as temporal and verify hypothesis that are difficult to observe otherwise.

3. *Inspiration* The SRN model inspired more complex temporal neural network (such as LSTM's (*Hochreiter, S., & Schmidhuber, J. (1997). Long short-term memory. Neural computation, 9(8), 1735-1780.*). It also furthered research into representations of tasks in the temporal context. Elman also suggests some possible extensions of his work in the paper.

4. *Control and Explanation* The definition of the model and the language the model trains on are all constrained by the programmer. This allows for a controlled environment for the simulations. In a similar experiment for adults, people could not function like on the simulation due to varying attention spans, low memory quotient, etc. which are extrinsic to the scope of this study.

5. *Explicitness* The experiment design for the SRN model allows us to control all possible variables in the world. The following constraints are applied to the experiment:

    - Limited lexicon language

- Restricted grammar This constrains the possible outcomes from the experiment to a favorable pool. The model architecture and optimization are also constrained.

C) Elman (1990) at the end of the paper states " Some problems change their nature when expressed as temporal events", and in the paper the main example of this is the modelling of XOR in a simple feed forward network compared to an SRN. In the lab we also modelled word sequences and saw that parts-of-speech like nouns and verbs could emerge from modelling sequences. But this problem was presented as a serial one. Can you speculate on how noun and verb detection could be learned with using a simple feedforward network? What would the input look like and do you think it could solve the problem? What would that then mean?

Part-of-speech tagging can be learnt using a simple neural network by using PoS tagged words as inputs and the word tags as output labels (while training the model). This allows us to only understand the PoS associated with a word, with no clarity on the relationship between sequences of words or contextual input (homonyms that belong to multiple PoS classes).

Using a simple network with specifically tagged words could generate similar outputs to the RNN, but we would lose a lot of contextual information using a simple network. Context assists in understanding the correct PoS tag for a word more easily compared to explicit labelled datasets.

Hence the SRN is a better choice for the task at hand.
