Modern Physics II - HW 7 Solution - Winter 2016

1. Enumerate all states of the hydrogen atom corresponding to the principal quantum number n = 3, giving the set of three spatial quantum numbers, the spin quantum numbers, and the corresponding spectroscopic notation. What is the energy level of all of these states in Nm, Ws, and eV? 5 points

So we have $n = 3$, which implies $\ell = 0,1,2$ and $m_\ell = -\ell \ldots \ell$. Putting this into a table with spectroscopic notation and spin:

$\ell \backslash m_\ell$	-2	1	0	1	2
0	-	-	$3s, m_s = \pm\frac{1}{2}$	-	-
1	-	$3p, m_s = \pm\frac{1}{2}$	$3p, m_s = \pm\frac{1}{2}$	$3p, m_s = \pm\frac{1}{2}$	-
2	$3d, m_s = \pm\frac{1}{2}$	$3d, m_s = \pm\frac{1}{2}$	$3d, m_s = \pm\frac{1}{2}$	$3d, m_s = \pm\frac{1}{2}$	$3d, m_s = \pm\frac{1}{2}$

Table 1: Hydrogen n=3 states

So the enumerated states comes out to 18 since we have spin up/down possibility for each orbital and magnetic quantum number(9 shells in total). Assuming no external magnetic fields (Zeeman effect) we can determine the energy from the principal quantum number alone, which is 3 for this problem:

$$E_3 \approx -\frac{13.6_{eV}}{3^2} \approx -1.511_{eV} \approx -2.421\text{E-19}_J \approx -2.421\text{E-19}_{N\cdot m} \approx -2.421\text{E-19}_{W\cdot s}$$

Note that Joules, Newton meters, and Watt seconds are all equivalent units.

2. What is the atom that has all 1s, 2s, 2p, 3s, and 3p states filled with two electrons of opposite spin? Write out its electronic structure, find approximate values for its ionization energy, radius, and volume (from appropriate graphs or tabulations) 4 points in total if all the units are correct. Given its electronic structure, will this atom be highly reactive chemically so that it cannot be found in nature in its element form? 1 point (5 in total)

Assuming this thing isn't already ionized we can just count up the electrons required to fill up everything and map that to the number of protons for the elemental number. We know that there are two electrons per filled *s* shell and 6 per filled *p* shell. Adding up we have:

$$2_e + 2_e + 6_e + 2_e + 6_e = 18_e$$

Looking this up on a periodic table indicates that this is argon, which should make sense give the *s* and *p* states are all filled, so it meets the requirement of being a noble gas. The electronic structure/electron configuration for argon is:

$$\text{Ar} = 1s^2 2s^2 2p^6 3s^2 3p^6$$

The ionization energy for argon is about 15.755_{eV}, which is fairly high though is expected for a noble gas.

The radius depends on how you are talking about it. For covalent(single) bonds it is listed as 97_{pm} , but there are some error bars on this value (disagreement abound). The van der Waals radius is listed at 188_{pm} , and the calculated atomic radius is 71_{pm} . Conditions obviously apply when trying to talk about how big these things are since the energy changes depending on how it is packed.

Volume is a bit vague(what condition?) so this will depend on how you calculate. The van der Waal radius is usually employed when speaking of atoms as small little hard balls, so let us say the radius is 188_{pm} . So we can calculate the volume of a sphere in the normal way, $4/3\pi r^3$, which gives about $2.78E\text{-}29_{m}^{3}$ or 0.0278_{nm}^{3} depending on your preference.

Finally of course we need to address its reactivity, of which it has nearly none. Anyone who has taken a chemistry course (also known as applied physics) will know that noble gasses are not reactive. While argon isn't impossible to form compounds with (some tricks with fluorine and cold temperatures) it generally will be found all by itself in its elemental form unless it is under some rather specific conditions(btw you can get Nature papers if you figure out how to make compounds out of noble gasses).