S8 - IS-LM problem

The equations that describe the IS-LM model for the case of a closed economy are the following:

$$\begin{cases} Y = C + I + G \\ C = C_0 + c \cdot Y_{disp}, \quad c \in (0,1) \\ Y_{disp} = Y - T + TR \\ T = T_0 + t \cdot Y, \quad t \in (0,1) \\ I = I_0 - b \cdot r, \quad b > 0 \\ \frac{M}{p} = kY - hr, \quad k, h > 0 \end{cases}$$

We rewrite the IS equation in analytical form, which describes the equilibrium in the goods and services market:

$$Y = \frac{1}{1-c(1-t)} \cdot [C_0 - cT_0 + cTR + G + I_0 - b \cdot r]$$

In the money market, the real money demand $\frac{M^D}{p}$ equals the real money supply $\frac{M^S}{p}$. It results the LM equation:

$$Y = \frac{1}{k}\frac{M}{p} + \frac{h}{k}r$$

From the LM equation we extract the interest rate: $r = \frac{k}{h}Y - \frac{1}{h}\frac{M}{p}$

Solving the IS-LM system by substituting the interest rate in the IS equation we can determine the level of the total income Y and respectively, the level of the real interest rate, depending on the exogenous variables:

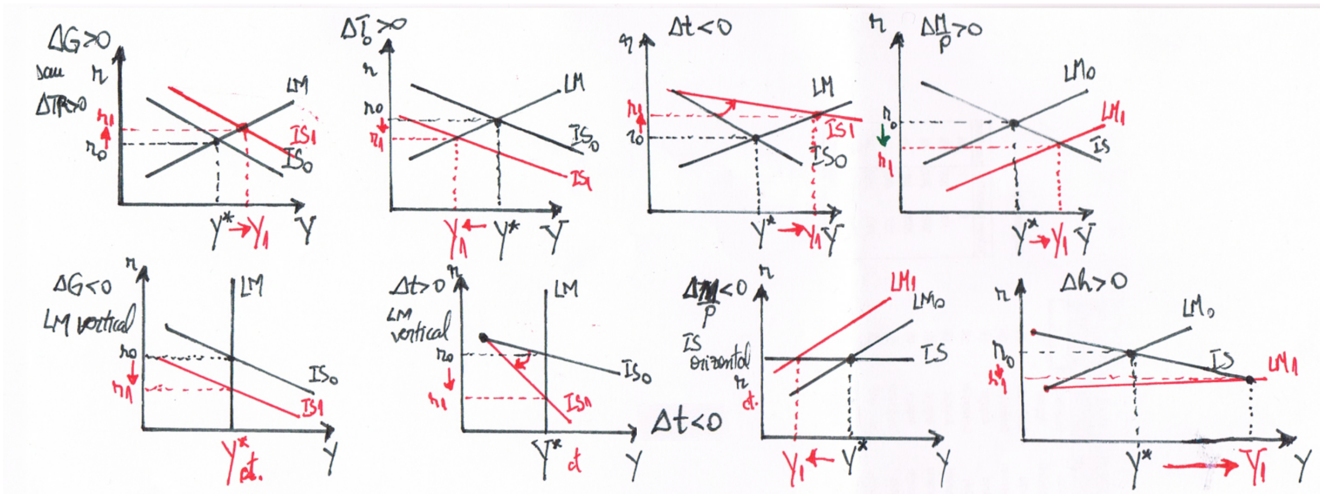
$$\begin{cases} Y^* = \frac{1}{1-c(1-t)+\frac{b \cdot k}{h}}\left[C_0 + cTR - cT_0 + G + I_0 + \frac{b}{h}\frac{M}{p}\right] \\ r^* = \frac{k}{h}Y^* - \frac{1}{h}\frac{M}{p} \end{cases}$$

Based on these equations, we can now determine:

- the budgetary/fiscal policy multiplier $\Gamma_{BP} = \frac{\Delta Y}{\Delta G} = \frac{1}{1-c(1-t)+\frac{bk}{h}}$,
- the monetary policy multiplier : $\Gamma_{MP} = \frac{\Delta Y}{\frac{\Delta M}{p}} = \Gamma_{BP} \cdot \frac{b}{h}$

	CHANGES OF THE IS CURVE	CHANGES OF THE LM CURVE
Slope modification (rotations)	▪ *b - investment sensitivity in relation to the real interest rate*. The higher the sensitivity to interest rate is, the closer IS tends to horizontal; ▪ *t – tax rate.* Increasing the tax rate rotates the IS curve to the left (tends towards vertical), while lowering *t* rotates the IS curve to the right (tends to horizontal).	▪ *(h) sensitivity towards speculative money demand with respect to interest rates*. The higher it is, the closer LM tends to horizontal (a flat line) ▪ *(k) sensitivity towards current transactions demand with respect to changes in aggregate income*. The higher it is, the closer LM tends to vertical (a higher slope).
Translations (shifts)	▪ **ΔG** : An increase in G will shift IS to the right, while a reduction will shift it to the left; ▪ **ΔTR**: An increase in TR will shift IS to the right, while a reduction will shift it to the left; ▪ **ΔT_0**: An increase in taxes will shift IS to the left, while a reduction will shift it to the right	- $\Delta\frac{M}{p}$ – changes in the real money supply (if $\Delta\frac{M}{p} > 0$ = > LM shifts to the right)

Graphical representations:



Problem 1

Let's consider the case of an economy described by the IS-LM model, for which the following data are known: the propensity of consumption c= 0.8; tax rate t= 0; b= 1000; k= 1; h = 10000; autonomous consumption C_0=100 mil.€; autonomous investments I_0=200 mil.€; G= 550 mil.€; autonomous taxes T_0=500 mil.€; real money supply= 900 mil.€; transfers to households are zero and the trade balance is balanced (NX = 0).

Determine:

a) The IS and LM equations both analytically and numerically.
b) The equilibrium values for GDP, interest rate, private consumption and investments
c) The budgetary policy Γ_{BP} and the monetary policy Γ_{PM}
d) Due to the decrease in the investors' confidence, the autonomous component of the investment decreased by 90 mil. €. How will the equilibrium point (Y, r) be affected?
e) How much should the money supply be modified to bring the GDP back to its original level? What will be the new interest rate?
f) Build the graphs to illustrate the equilibrium solutions from points d and e.

Solution

a) The analytical IS equation is:

$$\text{IS: } Y = \frac{1}{1-c(1-t)}[C_0 + cTR - cT_0 + G + I_0 - b \cdot r],$$

where $\alpha_G = \frac{1}{1-c(1-t)}$ represents the multiplier of budgetary expenditures on the market for goods and services.

Substituting the numerical values, we obtain:

$$\text{IS: } Y = \frac{1}{1-0{,}8(1-0)}[100 - 0{,}8 \cdot 500 + 550 + 200 - 1000 \cdot r]$$

=> Y= 5· [450 – 1000 r].

Thus, we obtain the numerical IS equation:

Y= 2250 – 5000 · r

Then we write the equilibrium between the demand and the real money supply: $\frac{M}{p} = k \cdot Y - h \cdot r.$

Hence the analytical LM equation is:

$$Y = \frac{1}{k}\frac{M}{p} + \frac{h}{k}r$$

Substituting the numerical values, we obtain:

$$Y = \frac{1}{1}900 + \frac{10000}{1}r\ .$$

Thus we obtain the numerical LM equation:

Y= 900 +10000 · r

b) In order to determine the equilibrium point, the system of equations is solved:

$$\begin{cases} Y = 2250 - 5000 \cdot r \\ Y = 900 + 10000 \cdot r \end{cases}$$

It results that r^*= 0,09 (9%) and Y^*= 1800 mil. €;

The equilibrium point (Y^*, r^*) is thus: (1800; 0,09)

Then we compute the level of private consumption and investments:

C = 100 + 0,8 (1800 – 500) = 1140 mil. €.

I = 200 – 1000 · 0.09 = 110 mil. €.

Indeed: Y = C + I + G = 1140 + 110 + 550 = 1800 mil. €.

c) In order to determine the multipliers we need to reach the final form of the model.

So, from the LM equation we extract the interest rate: $r = \frac{k}{h}Y - \frac{1}{h}\frac{M}{p}$

After solving the IS-LM system by substituting the interest rate in the IS equation we can determine the level of the total income Y and respectively, the level of the real interest rate, depending on the exogenous variables:

$$\begin{cases} Y^* = \frac{1}{1-c(1-t)+\frac{b \cdot k}{h}}\left[C_0 + cTR - cT_0 + G + I_0 + \frac{b}{h}\frac{M}{p}\right] \\ r^* = \frac{k}{h}Y^* - \frac{1}{h}\frac{M}{p} \end{cases}$$

Based on these equations, we can now determine:

- The budgetary/fiscal policy multiplier $\Gamma_{BP} = \frac{1}{1-c(1-t)+\frac{bk}{h}} = \frac{1}{1-0{,}8+\frac{1000}{10000}} = \frac{10}{3} = 3{,}33$
- The monetary policy multiplier: $\Gamma_{MP} = \frac{\Delta Y}{\frac{\Delta M}{p}} = \Gamma_{BP} \cdot \frac{b}{h} = \frac{1000}{10000} 3{,}33 = 0{,}33$

Comparing the two multipliers, we can conclude that the budgetary policy is much more efficient than the monetary one, since$\Gamma_{BP} > \Gamma_{MP}$.

d) As the autonomous component of the investment decreases by 90 mil.€ (ΔI_0= – 90), the GDP drops by 300 mil. €.

$\Delta Y = \Gamma_{BP} \cdot \Delta I_0$ = 10/3 (– 90) = – 300 mil. €. => Y_1 = 1800-300 = 1500 mil. €.

Next the new real interest rate is determined:

$\Delta r = \frac{k}{h}\ \Delta Y = \frac{-300}{10000}$ = – 0,0 3 (-3%) => r_1 =9% - 3% = 6%

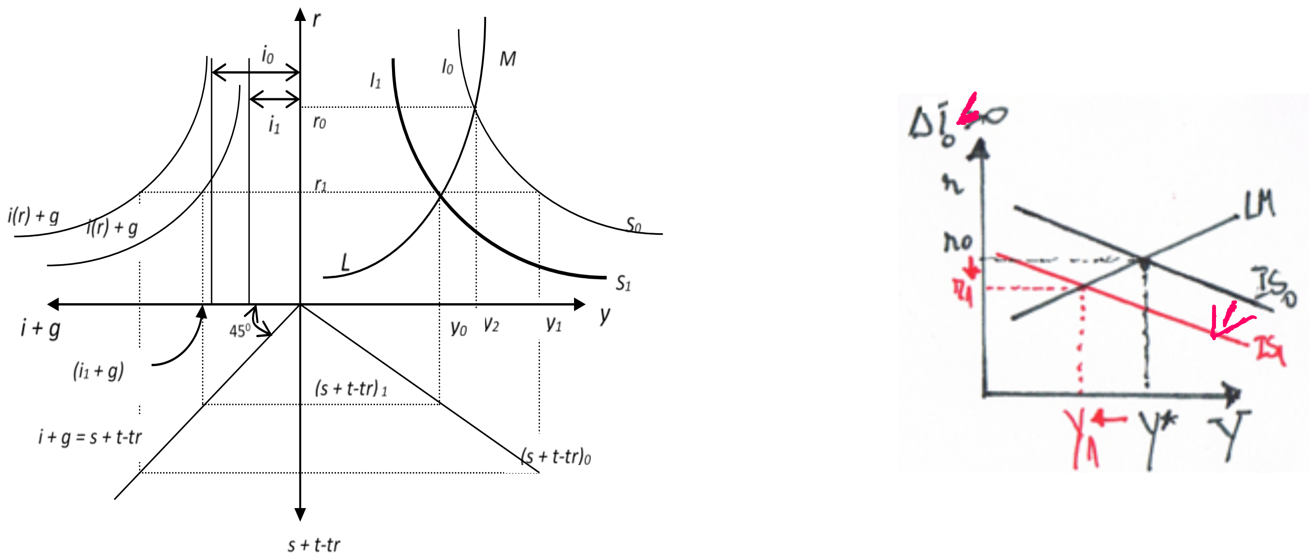
e) In order to restore the GDP to its initial level, it mean that ΔY = 300 mil. €. As the monetary policy multiplier is 1/3, it is necessary to increase the money supply by 900 mil. €.

$$\Delta\ Y = \Gamma_{MP} \cdot \frac{\Delta M}{p} = 300 \text{ mil. €} \Rightarrow \frac{\Delta M}{p} = \frac{300}{0{,}33} = 900 \text{ mil. €}.$$

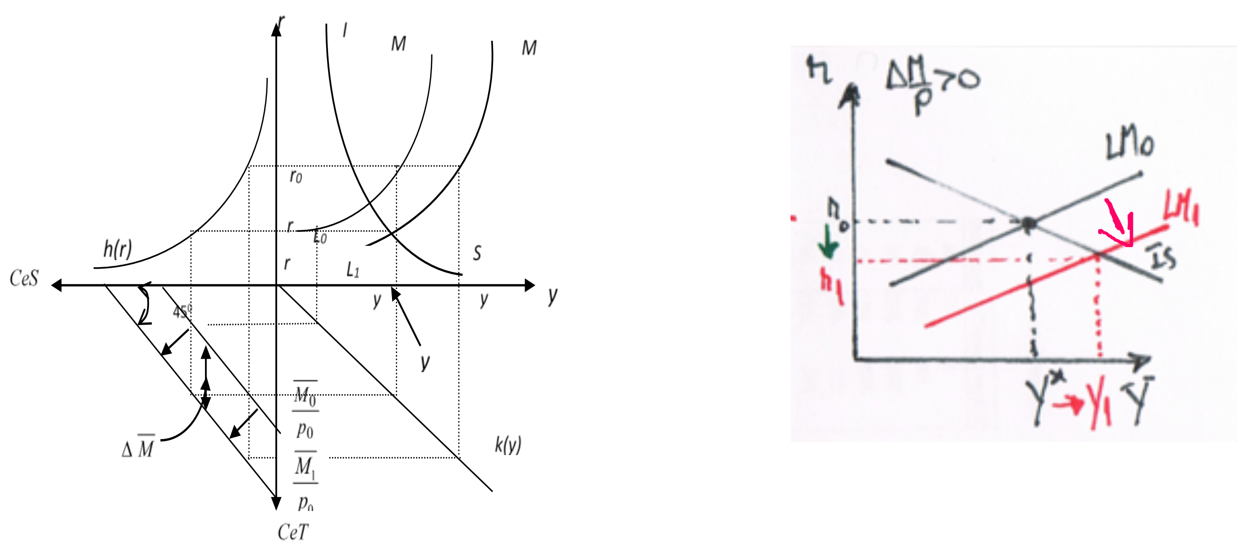
Using the interest rate equation, we find that the level of real interest rate will further decrease by 6%:

$$\Delta r = \frac{k}{h}\ \Delta Y - \frac{1}{h}\ \frac{\Delta M}{p}\ = \frac{300-900}{10000}\ = -\ 0{,}06\ \ (-6\%)$$

The representation of the two graphs was made in a system of 4 quadrants, following that the changes to the initial equilibrium point (regarding the level of GDP and the interest rate) can be observed in the upper right quadrant.



Changing the equilibrium point in case of reduced investments



Modification of the equilibrium point in case of a money supply increase