S11 - IS-LM-BP problem

Problem 1

Consider the case of an open economy described by the IS-LM-BP model, for which we know the following data: the marginal propensity to consume c=0.7; the tax rate t= 30%; b=250; k=0,2; h=2500; autonomous consumption C_0=300 bil.€; autonomous investments I_0=420 bil.€; G= 900 bil.€; autonomous taxes T_0=100 bil.€; real money supply= 500 bil.€ and transfers to households are zero. Also, the net export equation is known: $NX = NX_0 - m \cdot Y$, where $m = 0.1$ and $NX_0 = 530$ bil.€.

Determine:

a) The IS, LM and BP equations both analytically and numerically.
b) The equilibrium point (Y^*, r^*), as well as the budgetary deficit (BD), C, I, T and NX.
c) The budgetary policy multiplier, Γ_{BP}, and the monetary policy multiplier, Γ_{PM}.
d) The effects of the following policies on the equilibrium point (both numerically and graphically):
   - an increase of 100 bil. € in government expenditures;
   - a decrease of 100 bil. € in money supply;
   - a reduction of 5 p.p. in the tax rate.

Solution

a) First write the IS-LM-BP model:

$$\begin{cases} Y = C + I + G + NX \\ C = C_0 + c \cdot Y_{disp}, \quad c \in (0,1) \\ Y_{disp} = Y - T + TR \\ T = T_0 + t \cdot Y, \quad t \in (0,1) \\ I = I_0 - b \cdot r, \quad b > 0 \\ NX = NX_0 - mY, \quad m > 0 \\ \frac{M}{p} = kY - hr, \quad k, h > 0 \end{cases}$$

We rewrite the IS equation which describes the equilibrium in the goods and services market in analytical form:

$$\text{IS: } Y = \frac{1}{1-c(1-t)+m}[C_0 + cTR - cT_0 + G + I_0 + NX_0 - b \cdot r],$$

where $\alpha_G = \frac{1}{1-c(1-t)+m}$ is the government spending multiplier on the market for goods and services.

Substituting the numerical values, we obtain the numerical IS equation:

$$\text{IS: } Y = \frac{1}{1-0.7(1-0.30)+0.1}[300 - 0.7 \cdot 100 + 900 + 420 + 530 - 250 \cdot r]$$

$\Rightarrow Y = 1.639 \cdot [2080 - 250\, r]$.

So the numerical IS equation is:

$Y = 3409.1 - 409.8 \cdot r$

Then we write the equation describing the equilibrium on the monetary market, when real money demand equals real money supply:

$$\frac{M}{p} = k \cdot Y - h \cdot r.$$

And obtain the analytical LM equation:

$$Y = \frac{1}{k}\frac{M}{p} + \frac{h}{k}r$$

Substituting the numerical values:

$$Y = \frac{1}{0.2}500 + \frac{2500}{0.2}r\ .$$

We obtain the numerical LM equation:

$$Y = 2500 + 12500 \cdot r$$

The analytical form of the balance of payment equation is: (BP): $NX = NX_0 - m \cdot Y$

Substituting the numerical values:

(BP): $NX = 530 - 0.1 \cdot Y$.

When BP is set to zero, we have an external equilibrium. For NX= 0, Y=5300 bil. €.

b) Solving the system:

$$\begin{cases} Y = 3409.1 - 409.8 \cdot r \\ Y = 2500 + 12500 \cdot r \end{cases}$$

we get $r^* = 0.0704$ (7%) and $Y^* = 3380$ bil. €.
The equilibrium point (Y^*, r^*) is thus: (3380; 0.07)
To determine if the economy is experiencing surplus or budget deficit, we write the budget deficit equation:

$$BD = G - T + TR \Rightarrow BD = G - T_0 - tY + TR$$

Substituting the numerical values:

$BD = 900 - 100 - 0.30 * 3380 + 0 = -214$ bil. €.

Therefore, the economy is facing a budget surplus of 214 bil. €.
The level of private investments is:

$I = I_0 - b \cdot r^* = 420 - 250 * 0.07 = 402.5$ bil. €.

The net export is:

$NX = NX_0 - m \cdot Y^* = 530 - 0.1 \cdot 3380 = 192$ bil. €.

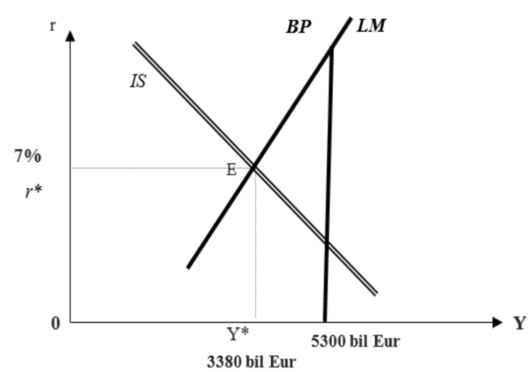
The level of taxes is:

$T = T_0 + t \cdot Y^* = 100 + 0.3 \cdot 3380 = 1114$ bil. €.

The level of private consumption is:

$C = C_0 + c(1-t) \cdot Y - c \cdot T_0 + c \cdot TR \Rightarrow$

$C = 300 + 0.7 \cdot (1 - 0.3) \cdot 3380 - 0.7 \cdot 100 = 1885.5$ bil. €.



c) In order to determine the multipliers of the budgetary and monetary policy in the IS-LM-BP model, we need to solve the system analytically.

From the LM equation, we extract the interest rate:

$$r = \frac{k}{h}Y - \frac{1}{h}\frac{M}{p}$$

After replacing r in the analytical IS equation:

$$Y = \frac{1}{1-c(1-t)+m}\left[C_0 + cTR - cT_0 + G + I_0 + NX_0 - b \cdot \left(\frac{k}{h}Y - \frac{1}{h}\frac{M}{p}\right)\right]$$

It results:

$$\begin{cases} Y^* = \dfrac{1}{1-c(1-t)+m+\dfrac{b\cdot k}{h}}\left[C_0 + cTR - cT_0 + G + I_0 + NX_0 + \dfrac{b}{h}\dfrac{M}{p}\right] \\ r^* = \dfrac{k}{h}Y^* - \dfrac{1}{h}\dfrac{M}{p} \end{cases}$$

from where we can identify the budgetary policy multiplier Γ_{BP}, as well as the monetary policy multiplier Γ_{MP}:

$$\Gamma_{BP} = \frac{1}{1-c(1-t)+m+\frac{bk}{h}} = \frac{1}{1-0.7(1-0.30)+0.1+\frac{250\cdot 0.2}{2500}} = 1.587$$

while $\Gamma_{MP} = \Gamma_{BP}\cdot\frac{b}{h} = 1.587\cdot\frac{250}{2500} = 0.1587$

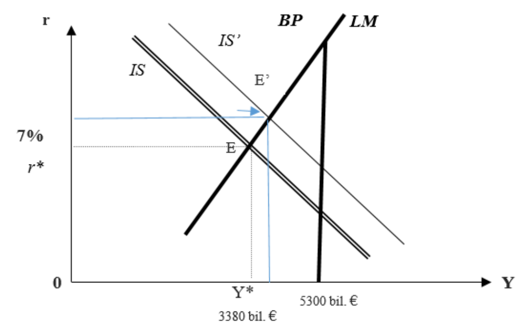
Comparing the two multipliers, we can conclude that the budgetary policy is ten times more efficient than the monetary policy, since $\Gamma_{BP} > \Gamma_{MP}$.

d1) In case of an increase in the government expenditures by 100 bil. € we have:

$\Delta Y = \Gamma_{BP}\cdot\Delta G = 1.587\cdot 100 = 158.7$ bil. €.

$\Delta r = \frac{k}{h}\Delta Y = \frac{0.2}{2500}(158.7) = 0.0127$ or 1.27 p.p.

Therefore, the level of GDP increases by 158.7 bil. €, while the interest rate increases by 1.27 percentage points.

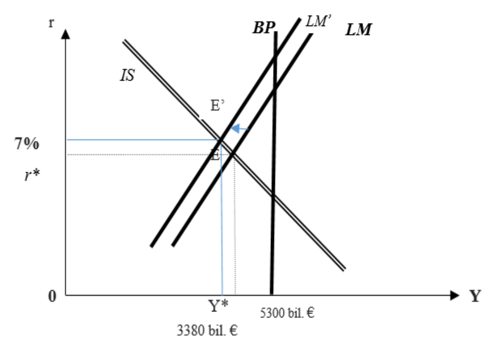


d2) In case of a drop in the real money supply by 100 bil. €, we have:

$\Delta Y = \Gamma_{MP}\cdot\Delta\frac{M}{p} = 0.1587\cdot(-100) = -15.87$ bil. €.

$\Delta r = \frac{k}{h}\Delta Y - \frac{1}{h}\frac{\Delta M}{p} = \frac{0.2}{2500}(-15.87) - \frac{1}{2500}(-100) = 0.0387$ or 3.87 p.p.

The LM line will shift to the left and the GDP level will decrease by 15.87 bil. €, while the interest rate will increase by 3.87 percentage points.



d3) To determine the impact of a fiscal policy of decreasing the tax rate by 5 percentage points on the initial equilibrium point (Y^*, r^*), we use the following relation:

$$\Delta Y = \frac{-c\Delta t\, Y_0}{1-c(1-t_1)+m+\frac{bk}{h}} = \frac{-0.7\cdot(-0.05)\cdot 3380}{1-0.7(1-0.25)+0.1+\frac{250\cdot 0.2}{2500}} = \frac{118.3}{0.595} = 198.8 \text{ bil.€.}$$

$\Delta r = \frac{k}{h}\Delta Y = \frac{0.2}{2500}(198.8) = 0.0159$ or 1.59 p.p.

In this case, the IS line will rotate to the right, which will lead to a 198.8 bil. € increase of the GDP level, as well as a rise of 1.59 percentage points of the interest rate. This will result in a reduction of 19.88 bil. € of the trade balance surplus.