S12 - IS-LM-BP problem

Problem 1

Consider the case of an open economy described by the IS-LM-BP model, for which we know the following data: the marginal propensity to consume c=0.8; the tax rate t= 30%; b=410; k=0.5; h=90; autonomous consumption C_0=60 mil.€; autonomous investments I_0=1500 mil.€; G= 1000 mil.€; autonomous taxes T_0=10 mil.€; real money supply= 900 mil.€ and transfers to households are zero. Also, the net export equation is known: NX = $NX_0 - m \cdot Y$, the capital has imperfect capital mobility, CF = $n \cdot r$, where m = 0.3, n = -50 (CF shows net capital outflows) and NX_0 = 500 mil.€.

Determine:

a) The IS, LM and BP equations both analytically and numerically.
b) The equilibrium point (Y^*, r^*), as well as the budgetary deficit (BD), C, I, T and NX.
c) The budgetary policy multiplier, Γ_{BP}, and the monetary policy multiplier, Γ_{PM}.
d) The effects of the following policies on the equilibrium point (both numerically and graphically):
   - a 10 mil. € decrease in government expenditures;
   - a 10 mil. € increase in the money supply;
   - a 5 p.p. increase in the tax rate.

Solution

a) First write the IS-LM-BP model:

$$\begin{cases} Y = C + I + G + NX \\ C = C_0 + c \cdot Y_{disp}, \quad c \in (0,1) \\ Y_{disp} = Y - T + TR \\ T = T_0 + t \cdot Y, \quad t \in (0,1) \\ I = I_0 - b \cdot r, \quad b > 0 \\ NX = NX_0 - mY, \quad m > 0 \\ \frac{M}{p} = kY - hr, \quad k, h > 0 \end{cases}$$

We rewrite the IS equation in analytical form, which describes the equilibrium in the goods and services market:

IS: $Y = \frac{1}{1-c(1-t)+m}[C_0 + cTR - cT_0 + G + I_0 + NX_0 - b \cdot r]$,

where $\alpha_G = \frac{1}{1-c(1-t)+m}$ is the government spending multiplier on the goods and services market.

Substituting the numerical values, we obtain the numerical IS equation:

IS: Y= 1.351· [3052 – 410· r].

So the numerical IS equation is:

Y= 4124.3 – 554 ·r

Then we write the equilibrium between the real money demand and the real money supply:

$\frac{M}{p} = k \cdot Y - h \cdot r.$

and obtain the analytical LM equation:

$$Y = \frac{1}{k}\frac{M}{p} + \frac{h}{k}r$$

Substituting the numerical values:

$$Y = \frac{1}{0.5}900 + \frac{90}{0.5}r .$$

We obtain the numerical LM equation:

$Y = 1800 + 180 \cdot r$

The analytical form of the balance of payment equation is:

(BP): NX= CF, therefore $NX_0 - m \cdot Y = n \cdot r$

Substituting the numerical values:

(BP): $500 - 0.3 \cdot Y = -50 \cdot r$, therefore:

$Y = 5000/3 - 500/3 \cdot r$,

b) Solving the system:

$$\begin{cases} Y = 4124.3 - 554 \cdot r \\ Y = 1800 + 180 \cdot r \end{cases}$$

we get $r^* = 3.17\%$ and $Y^* = 2370$ mil. €.

The equilibrium point **(Y^*, r^*)** is thus: **(2370; 3.17%)**

To determine if the economy is experiencing surplus or budget deficit, we write the budget deficit equation:

$BD^* = G - T^* + TR \Rightarrow BD = G - T_0 - tY^* + TR$

Substituting the numerical values:

$BD^* = 1000 - 10 - 0.30 * 2370 + 0 = 279$ mil. €.

Therefore, the economy is facing a budget deficit of 279 mil €.

Private investments are:

$I^* = I_0 - b \cdot r^* = 1500 - 410 * 3.17 = 201.69$ mil. €.

Net exports are: $NX = NX_0 - m \cdot Y^* - n \cdot r^*$.

$NX^* = 500 - 0.3 \cdot 2370 = -211$ mil. € (trade balance deficit).

Total taxes are:

$T^* = T_0 + t \cdot Y^* = 10 + 0.3 \cdot 2370 = 721$ mil. €.

The private consumption is: $C = C_0 + c(1 - t) \cdot Y - c \cdot T_0 + c \cdot TR \Rightarrow$

$C^* = 60 + 0.8 \cdot (2370 - 721) = 1379.2$ mil. €.

Net capital outflows:

$CF^* = n \cdot r^* = -50 \cdot 3.17 = -158.33 < 0$ (net capital outflows of -158.33 mil. €. or net capital inflows of 158.33 mil. €.)

The balance of paymments:

$BP^* = NX^* - CF^*$ or $BP^* = -211 + 158.33 = -52.67$ mil. € (BP deficit of 52.67 mil. €.)

c) In order to determine the multipliers of the budgetary and monetary policy in the IS-LM-BP model, we need to solve the system analytically.

From the LM equation, we extract the interest rate:

$r = \frac{k}{h}Y - \frac{1}{h}\frac{M}{p}$

After replacing *r* in the analytical IS equation:

$$Y = \frac{1}{1-c(1-t)+m}\left[C_0 + cTR - cT_0 + G + I_0 + NX_0 - b \cdot \left(\frac{k}{h}Y - \frac{1}{h}\frac{M}{p}\right)\right]$$

So:

$$\begin{cases} Y^* = \dfrac{1}{1-c(1-t)+m+\dfrac{b \cdot k}{h}}\left[C_0 + cTR - cT_0 + G + I_0 + NX_0 + \dfrac{b}{h}\dfrac{M}{p}\right] \\ r^* = \dfrac{k}{h}Y^* - \dfrac{1}{h}\dfrac{M}{p} \end{cases}$$

from where we can identify the budgetary policy multiplier Γ_{BP}, as well as the monetary policy multiplier Γ_{MP}:

$$\Gamma_{BP} = \frac{1}{1-c(1-t)+m+\frac{bk}{h}} = \frac{1}{1-0.8(1-0.30)+0.3+\frac{410\cdot 0.5}{90}} = 0.331$$

while $\Gamma_{MP} = \Gamma_{BP} \cdot \frac{b}{h} = 0.331 \cdot \frac{410}{90} = 1.509$

Comparing the two multipliers, we can conclude that the monetary policy is much more efficient than the budgetary one, since $\Gamma_{MP} > \Gamma_{BP}$.

d1) In case of a decrease in the government expenditures by 10 mil. € we have:

$\Delta Y = \Gamma_{BP} \cdot \Delta G = 0.331 \cdot (-10) = -3.31$ mil. €.

$\Delta r = \frac{k}{h}\Delta Y = \frac{0.5}{90}(-3.31) = -0.018$

Therefore, the level of GDP decreases by 3.31 mil. €, while the interest rate decreases by 0.018 p.p.

d2) In case of an increase in the real money supply by 10 bil. €, we have

$\Delta Y = \Gamma_{MP} \cdot \Delta\frac{M}{p} = 1.509 \cdot (10) = 15.09$ mil. €.

$\Delta r = \frac{k}{h}\Delta Y - \frac{1}{h}\frac{\Delta M}{p} = \frac{0.5}{90}(15.09) - \frac{1}{90}(10) = -0.0273$

Therefore, the level of GDP increases by € 15.09 mil. €, while the interest rate decreases by 0.0273 p.p.

d3) To determine the impact of a fiscal policy of increasing the tax rate by 5 percentage points on the initial equilibrium point (Y^*, r^*), we use the following relation:

$$\Delta Y = \frac{-c\Delta t\, Y_0}{1-c(1-t_1)+m+\frac{bk}{h}} = \frac{-0.8\cdot(0.05)\cdot 2370}{1-0.8(1-0.35)+0.3+\frac{410\cdot 0.5}{90}} = \frac{-94.8}{3.018} = -31.41 \text{ mil. €.}$$

$\Delta r = \frac{k}{h}\Delta Y = \frac{0.5}{90}(-31.41) = -0.1745$

Therefore, the level of GDP decreases by € 31.41 mil. €. while the interest rate drops by 0.1745 p.p.

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