# S9 - IS-LM problem

**Problem 1**

Consider the case of an economy described by the IS-LM model, for which we know the following data: the propensity of consumption c=0,9; the tax rate t= 1/3 (33.3%); b=1000; k=1; h= 10000; autonomous consumption $C_0$=90 mil.€; autonomous investments $I_0$=200 mil.€; G= 710 mil.€; autonomous taxes $T_0$=0 mil.€; real money supply= 500 mil.€; transfers to households are zero and the trade balance is balanced (NX = 0).

Determine:

a) The IS and LM equations both analytically and numerically.
b) The equilibrium point ($Y^*$, $r^*$), as well as the budgetary deficit ($BD_0$).
c) The budgetary policy $\Gamma_{BP}$ and the monetary policy $\Gamma_{MP}$
d) The change in the money supply ($\frac{\Delta M}{p}$ =? ) that could lead to a balanced budgetary deficit ($BD_1$=0).
e) The effects of a budgetary policy ($\Delta$G = –50 mil. €) upon the initial equilibrium point ($Y^*$, $r^*$).
f) The new level of the tax rate $t_1$ that could lead to an increase of the GDP by 100 mil.€

*Solution*

a) The equations that describe the IS-LM model are the following:

$$\begin{cases} Y = C + I + G + NX \\ \mathrm{C} = \mathrm{C}_0 + \mathrm{c} \cdot Y^D, \qquad \mathrm{c} \in (0,1) \\ Y^D = Y - T + TR \\ T = T_0 + t \cdot Y, \quad \mathrm{t} \in (0,1) \\ I = I_0 - b \cdot r, \qquad b > 0 \\ \frac{M}{p} = kY - hr, \quad k, h > 0 \end{cases}$$

We rewrite the IS equation in analytical form, which describes the equilibrium in the goods and services market:

$$Y = \frac{1}{1-c(1-t)} \cdot [C_0 - cT_0 + cTR + G + I_0 - b \cdot r]$$

Substituting the numerical values, we obtain the numerical IS equation:

Y = 2500 – 2500 r

Then we write the equilibrium between the real money demand and the real money supply:

$$Y = \frac{1}{k}\frac{M}{p} + \frac{h}{k} r$$

Substituting the numerical values, we obtain the numerical LM equation:

Y = 500 + 10000 r

b) Solving the system:

Y = 2500 – 2500 r

Y = 500 + 10000 r → r = 0.16 (16%), while Y=2100 mil. €

The equilibrium point **($Y^*$, $r^*$)** is thus: **(2100; 0.16)**

Substituting in the functions of investment and budget deficit we obtain:

I = 200 – 1000 × 0.16 = 40 mil. €.

BD = G -T = 710- 1/3 x 2100 = 10 mil. €. (the economy is facing a budget deficit of 10 mil. €)

c) From the LM equation we extract the interest rate: $r = \frac{k}{h}Y - \frac{1}{h}\frac{M}{p}$

After solving the IS-LM system by substituting the interest rate in the IS equation we can determine the level of the total income Y and respectively, the level of the real interest rate, depending on the exogenous variables:

$$\begin{cases} Y^* = \frac{1}{1-c(1-t)+\frac{b \cdot k}{h}}\left[C_0 + cTR - cT_0 + G + I_0 + \frac{b}{h}\frac{M}{p}\right] \\ r^* = \frac{k}{h}Y^* - \frac{1}{h}\frac{M}{p} \end{cases}$$

Based on these equations, we can now determine:

- The budgetary/fiscal policy multiplier $\Gamma_{BP} = \frac{1}{1-c(1-t)+\frac{bk}{h}} = \frac{1}{1-0{,}9\frac{2}{3}+\frac{1000}{10000}} = 2$
- The monetary policy multiplier: $\Gamma_{MP} = \frac{\Delta Y}{\frac{\Delta M}{p}} = \Gamma_{BP} \cdot \frac{b}{h} = \frac{1000}{10000} 2 = 0.2$

Comparing the two multipliers, we can conclude that the budgetary policy is much more efficient than the monetary one, since$\Gamma_{BP} > \Gamma_{MP}$.

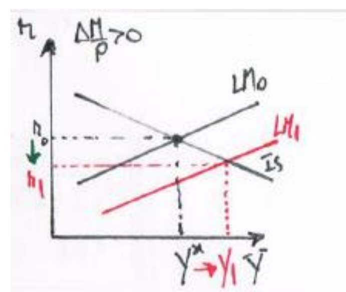
d) In order to have a balanced budgetary deficit ($BD_1$ =0), the budgetary deficit has to drop by 10 mil.€ (ΔBD= -10 mil. €)

$\Delta BD = -t\, \Delta Y$.

It results that $\Delta Y$= 30 mil. €

$\Delta Y = \Gamma_{MP} \cdot \Delta(M/p)$ = 30 mil. €.

So $\Delta(M/P)$ = 150 mil. €.

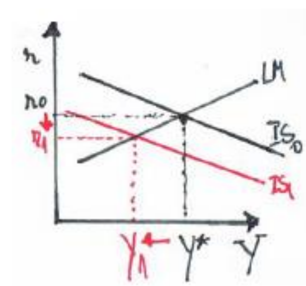



In order to balance the budget deficit it is necessary to increase the volume of taxes by € 10 million. Therefore GDP should increase by € 30 million. As the monetary policy multiplier is 0.2, the money supply must increase by € 150 million.

e) In case the government spendings decrease by 50 mil. € then:

$\Delta Y = \Gamma_{BP} \times \Delta G = 2\times(-50) = -100$ mil. €

$\Delta r = \frac{k}{h}\Delta Y = (-100)/10000 = -0.01$




Under the conditions of the decrease of the governmental expenses by 50 million €, the level of the GDP diminishes by 100 million €, while the interest rate decreases by 1%.

f) To determine the new tax rate $t_1$, which will lead to an increase of the GDP level by 100 million €, the following relation is used:

$$\Delta Y = \frac{-c\Delta t\, Y_0}{1-(c(1-t_1)+\frac{bk}{h}} = 100 \text{ mil.€}$$

Then:

$100\times(1-0.9+ 0.9\, t_1 +0.1) = (-0.9) \times (t_1- 1/3)\, 2100$ =>

=> $0.2 +0.9\, t_1 =-0.9 \times 21\times t_1 +0.3\times 21$

Thus we get the new tax rate $t_1 = 0.305$ (ie 30.5%, which is lower than $t_0$ of 33.3%). This aspect is also confirmed graphically.