S10 - IS-LM problem

Consider the case of an economy described by the IS-LM model, for which we know the following data: the marginal propensity to consume c=0.7; the tax rate t= 0.16; b=1000; k=0.6; h= 2000; autonomous consumption C_0=30 bil.€; autonomous investments I_0=25 bil.€; G= 95 bil.€; autonomous taxes T_0=5 bil.€; real money supply= 80 bil.€; transfers to households are zero and the trade balance is balanced (NX = 0).
Determine:

a) The IS and LM equations both analytically and numerically.
b) The equilibrium point (Y^*, r^*), as well as the budgetary deficit (BD).
c) The government spending multiplier on the goods and services market α_G, as well as the budgetary policy Γ_{BP} and the monetary policy Γ_{PM} multipliers.
d) Determine the effects of a mixed policy to reduce government spending by 10 bil. € and also to increase transfers by 10 bil. € on the initial equilibrium point (Y^*, r^*) and on the budget deficit. Comment.
e) Propose a monetary policy with the aim of increasing the real interest rate by 1p.p. (Δr= 0.01) and estimate its effect on the level of GDP.
f) The impact of an increase in the tax rate by 4 p.p. (Δt= 0.04) upon the initial equilibrium point (Y^*, r^*).

Solution

a) The equations that describe the IS-LM model (open economy) are the following:

$$\begin{cases} Y = C + I + G + NX \\ C = C_0 + c \cdot Y^D, \qquad c \in (0,1) \\ Y^D = Y - T + TR \\ T = T_0 + t \cdot Y, \quad t \in (0,1) \\ I = I_0 - b \cdot r, \qquad b > 0 \\ \frac{M}{p} = kY - hr, \quad k,h > 0 \end{cases}$$

We rewrite the IS equation in analytical form, which describes the equilibrium in the goods and services market:

$$Y = \frac{1}{1-c(1-t)} \cdot [C_0 - cT_0 + cTR + G + I_0 - b \cdot r]$$

where $\alpha_G = \frac{1}{1-c(1-t)}$ is the government spending multiplier on the goods and services market.

Substituting the numerical values, we obtain the numerical IS equation:

$$\text{IS: } Y = \frac{1}{1-0.7(1-0.16)}[30 - 0.7 \cdot 5 + 95 + 25 - 1000 \cdot r]$$

=> Y= 2.427· [146.5 – 1000 r].

So the numerical IS equation is:

Y= 355.6 – 2427 · r

Then we write the equilibrium between the real money demand and the real money supply:

$$\frac{M}{p} = k \cdot Y - h \cdot r.$$

And obtain the analytical LM equation:

$$Y = \frac{1}{k}\frac{M}{p} + \frac{h}{k}r$$

Substituting the numerical values:

$$Y = \frac{1}{0.6}80 + \frac{2000}{0.6}r\ .$$

We obtain the numerical LM equation:

Y= 133.3 +3333.3 · r

b) Solving the system:

$$\begin{cases} Y = 355.6 - 2427 \cdot r \\ Y = 133.3 + 3333.3 \cdot r \end{cases}$$

we get r^*= 0.039 (3.9%) and Y^*= 261.94 bil. €

The equilibrium point (Y^*, r^*) is thus: (261.94; 0.039)

To determine if the economy is experiencing surplus or budget deficit, we write the budget deficit equation:
BD= G – T +TR => BD= G – T_0 – tY + TR

BD= 95 – 5 – 0,16* 261,94 + 0= 48.09 bil. €

Therefore, the economy is facing a budget deficit of 48.09 bil. €.

c) The government spending multiplier on the goods and services market α_G was determined at point a):

$$\alpha_G = \frac{1}{1-c(1-t)} = 2.427$$

In order to determine the multipliers of the budgetary and monetary policy in the IS-LM model, we need to solve the IS-LM system analytically.

From the LM equation, we extract the interest rate:

$$r = \frac{k}{h}Y - \frac{1}{h}\frac{M}{p}$$

After replacing *r* in the analytical IS equation:

$$Y = \frac{1}{1-c(1-t)}\left[C_0 + cTR - cT_0 + G + I_0 - b\cdot\left(\frac{k}{h}Y - \frac{1}{h}\frac{M}{p}\right)\right]$$

It results:

$$\begin{cases} Y^* = \dfrac{1}{1-c(1-t)+\frac{b\cdot k}{h}}\left[C_0 + cTR - cT_0 + G + I_0 + \dfrac{b}{h}\dfrac{M}{p}\right] \\ r^* = \dfrac{k}{h}Y^* - \dfrac{1}{h}\dfrac{M}{p} \end{cases},$$

From where we can identify the budgetary policy multiplier Γ_{BP}, as well as the monetary policy multiplier Γ_{MP}:

$$\Gamma_{BP} = \frac{1}{1-c(1-t)+\frac{bk}{h}} = \frac{1}{1-0,7(1-0,16)+\frac{1000\cdot 0,6}{2000}} = 1.405$$

while $\Gamma_{MP} = \Gamma_{BP}\cdot\frac{b}{h} = 1{,}405\cdot\frac{1000}{2000} = 0.7025$

Comparing the two multipliers, we can conclude that the budgetary policy is much more efficient than the monetary one, since $\Gamma_{BP} > \Gamma_{MP}$.

d) If a mixed policy of reducing government spending by 10 bil. € takes place simultaneously with an increase of the transfers by 10 bil. €, we call on the budget policy multiplier determined in point c) to determine the effects on the equilibrium point:

$$\Delta Y = \Gamma_{BP}\,[\Delta G + c\Delta TR] = 1.405\,[-10 + 0.7\cdot 10] = -4.215 \text{ bil. €}$$

$$\Delta r = \frac{k}{h}\Delta Y - \frac{1}{h}\frac{\Delta M}{p} = \frac{0.6}{2000}(-4.215) = -0.00126$$

Therefore, the level of GDP decreases by 4.215 bil. €, while the interest rate drops by 1.26%.

Regarding the effect of the mixed policy on the budget deficit, we obtain:

ΔBD= ΔG + ΔTR – t·ΔY = -10 +10 -0.16· (-4,215) = 0.6744 bil. €

As a result, the budget deficit deepens by 0.6744 bil. €.

e) To propose a monetary policy with the aim of increasing the interest rate by 1 percentage point we use the relation:

$$\Delta r = \frac{k}{h}\Delta Y - \frac{1}{h}\Delta\frac{M}{p} = 0.01$$

where $\Delta Y = \Gamma_{MP} \cdot \Delta\frac{M}{p}$.

So:

$$\Delta r = \frac{k}{h}\Gamma_{MP} \cdot \Delta\frac{M}{p} - \frac{1}{h}\Delta\frac{M}{p} = \Delta\frac{M}{p}\left(\frac{k}{h}\Gamma_{MP} - \frac{1}{h}\right)$$

So: $$\Delta\frac{M}{p} = \frac{\Delta r}{\left(\frac{k}{h}\Gamma_{MP} - \frac{1}{h}\right)} = \frac{0{,}01}{\left(\frac{0{,}6}{2000}0{,}7025 - \frac{1}{2000}\right)} = -34.6 \text{ bil. €}$$

$$\Delta Y = \Gamma_{PM} \cdot \Delta\frac{M}{p} = 0.7025 \cdot (-34.6) = -24.3 \text{ bil. €}$$

f) To determine the impact of a fiscal policy of increasing the tax rate by 4 percentage points on the initial equilibrium point (Y^*, r^*) determined at point b), we use the following relation:

$$\Delta Y = Y_1 - Y_0 = \frac{-c\Delta t\, Y_0}{1 - c(1-t_1) + \frac{bk}{h}} = \frac{-0{,}7 \cdot 0{,}04 \cdot 261{,}94}{1 - 0{,}7(1-0{,}2) + \frac{1000 \cdot 0{,}6}{2000}} = \frac{-7{,}3343}{0{,}74} = -9.91 \text{ bil. €}$$

$$\Delta r = \frac{k}{h}\Delta Y = \frac{0.6}{2000}(-9{,}91) = -0.003$$

Both GDP and the real interest rate are dropping because of the increase of the tax rate.