

## Differential Equations. Week 1

1. (Filipov 79) A large chamber contains 200 cubic meters of gas, 0.15% of which is carbon dioxide (CO<sub>2</sub>). A ventilator exchanges 20 cubic meters per minute of this gas with new gas containing only 0.04% CO<sub>2</sub>. Write down a differential equation that describes the change of the concentration of CO<sub>2</sub>.

Answer:  $y' = 0.008 - 0.1y$ .

2. (Filipov 83) A raft is being slowed down by resistance of the water, the resistance being proportional to the speed of the raft. Write down a differential equation that describes how the speed of the boat changes in time.

Answer:  $v' = -kv$ .

3. (Filipov 85) It is experimentally determined that every gram of radium loses 0.44 milligrams in 1 year. Write down a differential equation that describes how the mass of the radium changes in time.

Answer:  $m' = -km$ .

4. (Filipov 98) A boat is held by a cable that is wound around a post, the end being held by a laborer. Write down a differential equation that defines how braking force changes with the number of wounds around the post. The coefficient of friction is  $\frac{1}{3}$ .

Answer:  $\frac{dT}{d\theta} = \frac{T}{3}$ .

5. (Filipov 100) The mass of a rocket, including a full chamber of fuel, is  $M$ ; its net mass (without fuel) is  $m$ . The products of combustion are ejected with velocity  $c$ . If the rocket starts from rest, find Ciolkovskii's formula, which gives the speed imparted to the rocket by the burning of the fuel, neglecting the resistance of the atmosphere.

Answer:  $-m \frac{dV}{dt} = c \frac{dm}{dt}$ .

Use the method of isoclines to solve the differential equation given.

6. (Filipov 6)  $xy' = 2y$
7. (Filipov 7)  $xy' + y = 0$
8. (Filipov 16) Give a method for finding the locus of the inflection points of the solutions of the equation.
- (a)  $y' = y - x^2$
- (b)  $y' = x - e^y$

Find the differential equations which correspond to the following families of curves.

9. (Filipov 20)  $y = \sin(x + C)$
10. (Filipov 25)  $y = ax^2 + be^x$
11. (Filipov 30) Write the differential equation of circles of radius 1 which have centers on the line  $y = 2x$ .
12. (Filipov 33) Write the differential equation of all parabolas which have axis parallel to the y axis and pass through the origin.

Find the differential equations of the trajectories which intersect the given family isogonally at the angle  $\phi$ :

13. (Filipov 38)  $y^2 = x + C$ ,  $\phi = 90^\circ$ .

14. (Filipov 40)  $x^2 + y^2 = a^2$ ,  $\phi = 45^\circ$ .

15. (Filipov 42)  $3x^2 + y^2 = C$ ,  $\phi = 30^\circ$ .

**Homework:** Filipov 12, 34, 36, 42.