Differential Equations. Week 2

1. (Filippov 221) Construct the approximations y_0, y_1, y_2 for the initial value problem.
   (a) $y' = x - y^2$, $y(0) = 0$;
   (b) $y' = y^2 + 3x^2 - 1$, $y(1) = 1$;
   (c) $y' = y + e^{y-1}$, $y(0) = 1$;
   (d) $y' = 1 + x \sin y$, $y(\pi) = 2\pi$
2. (Filippov 223) Indicate any interval of an existence of unique solutions of the initial value problem.
   (a) $y' = x + y^3$, $y(0) = 0$;
   (b) $y' = 2y^2 - x$, $y(1) = 1$;
   (c) $\frac{dx}{dt} = t + e^x$, $x(1) = 0$.
3. (Filippov 225) Show the region of x-y plane where specified differential equation has a unique solution.
   (a) $y' = 2xy + y^2$;
   (b) $(x-2)y' = \sqrt{y} - x$;
   (c) $(y-x)y' = y \ln x$;
   (d) $y' = 2 + \sqrt[3]{y - 2x}$;
   (e) $y' = 1 + \tan y$.

   Find general solutions or a solution of an initial value problem.
4. (Filippov 51) $xydx + (x+1)dy = 0$
5. (Filippov 53) $(x^2 - 1)y' + 2xy^2 = 0$, $y(0) = 1$.
6. (Filippov 67) $3y^2y' + 16x = 2xy^3$, $y(x)$ is bounded as $x \to +\infty$
7. (Filippov 69) Show that any integral curve of the equation $y' = \sqrt[3]{\frac{y^2+1}{x^4+1}}$ has two horizontal asymptotes

   Homework: Filippov 52, 56, 66, 236(2), 228(1).