Differential Equations. Week 11

Draw the trajectories and deduce properties of the solutions as $t \to \infty$:

1. (Filippov 1001) $\ddot{x} + 4x = 0$
2. (Filippov 1003) $\ddot{x} - x + x^2 = 0$
3. (Filippov 1009) $\ddot{x} - \sin x = 0$
4. (Filippov 1015) $\ddot{x} + \dot{x} + 2x - x^2 = 0$

Draw trajectories and investigate singular points:

5. (Filippov 1021) $\begin{cases} \dot{x} = 2x + y^2 - 1 \\ \dot{y} = 6x - y^2 + 1 \end{cases}$
6. (Filippov 1025) $\begin{cases} \dot{x} = 2 + y - x^2 \\ \dot{y} = 2x(x - y) \end{cases}$
7. (Filippov 1031) $\begin{cases} \dot{x} = (2x - y)^2 - 9 \\ \dot{y} = (x - 2y)^2 - 9 \end{cases}$
8. (FitzHugh-Nagumo neuron model)
   $\begin{cases} \dot{V} = V - \frac{V^3}{3} - w + I \\ \tau\dot{w} = V - a - bw \end{cases}$, $a = -0.7$, $b = 0.8$, $\tau = \frac{1}{0.08}$, I - external current.
9. Derive an equation for movement of pendulum under external force, which equals to one half of the pendulum weight and directed clockwise on a tangent to the circus of the pendulum motion. Draw trajectories of the motion and explain their type.
10. Draw on a phase plane small oscillations of a pendulum, considering the pendulum length during upward movement is l and the pendulum length during downward movement is L, where $L > l$. Define a change of amplitude after one oscillation.

Homework: Filippov 1014, 1026.
Draw trajectories and investigate singular points for Hindmarsh-Rose model of neuron activity:
$\begin{cases} \dot{x} = y + 3x^2 - x^3 - z + I \\ \dot{y} = 1 - 5x^2 - y \\ \dot{z} = r(4(x + \frac{8}{5}) - z) \end{cases}$, $r = \sqrt{x^2 + y^2 + z^2} \approx 0.01$, I - external current.