

Differential Equations. Week 4

Find all solutions of given equations. Indicate singular solutions if there are any. Draw the integral curves.

1. (Filippov 241) $y'^2 - y^2 = 0$
2. (Filippov 245) $y'^2 - 4y^3 = 0$
3. (Filippov 249) $y'^3 + y^2 = yy'(y' + 1)$

Solve given equation using an additional parameter.

4. (Filippov 267) $x = y'^3 + y'$
5. (Filippov 271) $y = y'^2 + 2y'^3$
6. (Filippov 283) $y' = e^{\frac{xy'}{y}}$

Solve the following Clairaut's and Lagrange's equations.

7. (Filippov 291) $y'^3 = 3(xy' - y)$
8. (Filippov 293) $xy' - y = \ln y'$
9. (Filippov 297.3) Find singular solution of a differential equation for a family of curves $y = C(x - C)^2$
10. (Filippov 298) Find a curve such that its tangent cuts off a triangle of area $2a^2$ on the coordinate axes.
11. (Filippov 103) Find a general solution of homogeneous equation $(y^2 - 2xy)dx + x^2dy = 0$ using a change of variable $y = xu(x)$.
12. (Filippov 118) Find a general solution of the equation $y' = 2\left(\frac{y+2}{x+y-1}\right)^2$ using changes of variables $y+2 = t$ and $x-3 = u(t)$.

Homework: Filippov 297.1, 242.