Differential Equations. Week 12

Using the definition of Lyapunov stability determine whether the solution of initial value problem is stable.

1. (Filippov 881a) $3(t-1)\dot{x} = x$, $x(2) = 0$.
2. (Filippov 881b) $\dot{x} = 4x - t^2 x$, $x(0) = 0$.
3. (Filippov 881c) $\dot{x} = t - x$, $x(0) = 1$.

Determine if the trivial solution of the system is stable.

4. (Filippov 890) $x = C_1 \cos^2 t - C_2 e^{-t}$, $y = C_1 t^4 e^{-t} + 2C_2$.
5. (Filippov 891) $x = \frac{C_1 - C_2 t}{1+t^2}$, $y = (C_1 t^3 + C_2)e^{-t}$.

Using the first Lyapunov theorem determine the stability of the trivial solution ($x = y = z = 0$).

6. (Filippov 901) $\begin{cases} \dot{x} = e^{x+2y} - \cos 3x \\ \dot{y} = \sqrt{4+8x} - 2e^y \end{cases}$
7. (Filippov 903) $\begin{cases} \dot{x} = \ln(3e^y - 2\cos x) \\ \dot{y} = 2e^x - \sqrt[3]{8+12y} \end{cases}$
8. (Filippov 905) $\begin{cases} \dot{x} = \tan(z-y) - 2x \\ \dot{y} = \sqrt{9+12x} - 3e^y \\ \dot{z} = -3y \end{cases}$

Determine the stability of the system given in a polar coordinates:

9. (Filippov 1040) $\begin{cases} \dot{r} = r(1-r^2) \\ \dot{\phi} = 1 \end{cases}$
10. (Filippov 1042) $\begin{cases} \dot{r} = r(1-r)^2 \\ \dot{\phi} = 1 \end{cases}$
11. (Filippov 1043) $\begin{cases} \dot{r} = \sin r \\ \dot{\phi} = 1 \end{cases}$

Homework: Filippov 1045, 906.