Differential Equations. Week 13

Using Lyapunov's function and Lyapunov's or Chetaev's theorems check the stability of a system at the origin.

1. (Filippov 923) $\begin{cases} \dot{x} = x^3 - y \\ \dot{y} = x + y^3 \end{cases}$

2. (Filippov 924) $\begin{cases} \dot{x} = y - x + xy \\ \dot{y} = x - y - x^2 - y^3 \end{cases}$

3. (Filippov 925) $\begin{cases} \dot{x} = 2y^3 - x^5 \\ \dot{y} = -x - y^3 + y^5 \end{cases}$

4. (Filippov 927) $\begin{cases} \dot{x} = y - 3x - x^3 \\ \dot{y} = 6x - 2y \end{cases}$

Check the stability of the system $\ddot{x} + p(t)x = 0$ for a given a and b at the origin, where $p(t) = \begin{cases} a^2,\ 2\pi k < t < 2\pi k + \pi \\ b^2,\ 2\pi k + \pi \le t \le 2\pi k + 2\pi \end{cases}$, $k \in \mathrm{Z}$.

5. (Filippov 959 (1)) $a = 0.5$, $b = 0$.

6. (Filippov 959 (3)) $a = 0.5$, $b = 1.5$.

7. (Filippov 959 (5)) $a = 1$, $b = 0$.

Homework: Filippov 929, 959(2).