Probability and Statistics. Week 1

1. A cube with all of its faces painted is sawn into 343 smaller cubes of equal sizes. One of the small cubes is chosen at random. Find the probability that this cube:

   (a) has exactly one painted face;

   (b) has exactly two painted faces;

   (c) has at least two painted faces.

   Answer: (a) $\frac{150}{343}$, (b) $\frac{60}{343}$, (c) $\frac{68}{343}$.

2. List the elements of each of the following sample spaces:

   (a) the set $S = \{x | 9 - 3x^2 \geq 0 \text{ and } x > 0\}$

   (b) the set $S = \{x | 2x - 4 \geq 0 \text{ and } x < 1\}$.

   Answer: (a) $S = (0, \sqrt{3}]$, (b) $\emptyset$

3. m white balls and n black balls are placed at random in a row. Find the probability that k^{th} place is occupied with a black ball.

   Answer: $\frac{n}{m+n}$.

4. Six volumes are placed on a bookshelf in a random order. Find the probability that at least one of the volumes is not on its correct position.

   Answer: $\frac{719}{720}$.

5. There were 26 white balls and 14 black balls in the urn. One of the balls has disappeared. After that one ball is taken at random from the urn. Determine the probability that the ball is black.

   Answer: 0.35.

6. Four people enter the lift at the ground floor of a nine-storey building. Find the probability that at least two of them get off at the same floor.

   Note: We use the following conventions for problems with lifts if it is not stated otherwise. 1) The floors are counted this way: ground floor, first floor, second floor, etc. 2) We assume that all people who have entered the lift at the ground floor can go out of it on each floor starting with the first one with equal probabilities and independently from the others. 3) No other people enter the lift as it goes up.

   Answer: $\frac{151}{256}$.

7. Seventy numbers are chosen at random from integers $1, 2, 3, \ldots, 100$. What is the probability that the largest number chosen is 98?

   Answer: $\frac{29}{462}$.

8. m white balls, n black balls and k yellow balls are placed at random in a row. Determine the probability that if looking at the balls from left to right, we will see a white ball before we see a black one?

   Answer: $\frac{m}{m+n}$.

9. Six letters are chosen at random from the ones of the word PROPORTION. Determine the probability that it is possible to form the following words from these letters:

(a) OPTION;

(b) PORT;

(c) RIOT.

Answer: (a) $\frac{1}{35}$, (b) $\frac{79}{210}$, (c) $\frac{5}{21}$.

10. Twenty participants of a chess tournament are casting lots in order to get into one of two groups (each of the groups has 10 players in it). Find the probability that two of the strongest players are in the same group.

    Answer: $\frac{9}{19}$.

11. Two different numbers are chosen at random from the set $\{1, 2, \ldots, 100\}$. What is the probability that the first one is greater than the second?

    Answer: 0.5.

12. What is the probability to get two "threes" when rolling two dice given that the sum of the two numbers is a multiple of 3?

    Answer: $\frac{1}{12}$.

13. An urn contains 6 white balls, 4 black balls and 2 orange balls. Three balls are taken at random out of the urn. What is the probability that the balls are of the same colour?

    Answer: $\frac{6}{55}$.

14. A card is drawn at random from a deck of 36 cards. Let A = the card is a jack and B = the card has a black suit. Find the probability of $A \cup B$.

    Answer: $\frac{5}{9}$.

15. A coin is flipped until one does not get a sequence "tails-tails-tails". What is the probability that the coin has been flipped

    (a) exactly 6 times;

    (b) exactly 7 times.

    Answer: (a) $\frac{1}{16}$, (b) $\frac{7}{128}$.

16. A coin is flipped until one does not get a sequence "heads-tails-heads". What is the probability that the coin has been flipped

    (a) exactly 6 times;

    (b) exactly 7 times.

    Answer: (a) $\frac{5}{64}$, (b) $\frac{9}{128}$.

17. Three dice are rolled. What is the probability that the product of all numbers rolled is 24?

    Answer: $\frac{5}{72}$.

18. A woman arrives at the airport between 11 and 12 o'clock, waits for her husband for 10 minutes, and if the does not arrive, she hails a taxi and leaves the airport alone (and the husband has a lot of trouble!). Her husband shows up at the airport at some time between 11 and 11:40. He waits for 20 minues, and if he does not meet his wife, he goes home (and he is in a lot of trouble!) What is the probability that the husband ends up in having lots of trouble?

    Answer: $\frac{25}{48}$.