CS15210: More Waves and Signals

Helen Miles (hem23@aber.ac.uk)

29/01/16

(based on slides by Mike Clarke)

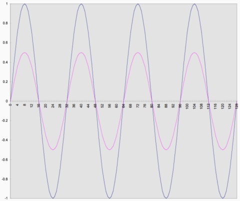
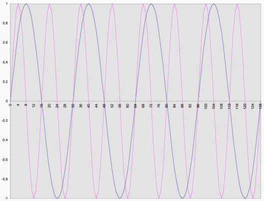
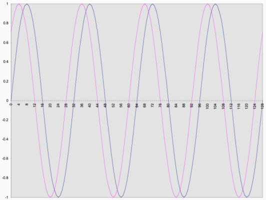
Previously, in CS15210...

- Signals are sent as varying voltage over time
- How signals become damaged:
  - attenuation, dispertion, distortion

- Signals and data can be analogue or digital
  - Can send either kind of data as either kind of signal using converters (ADC, DAC)

Previously, in CS15210...

- Anatomy of a wave:
  - cycle, amplitude, wavelength, frequency
- How two waves can be compared:
  - amplitude, frequency, phase

Contents

Measuring frequency

Frequency is measured in Hertz (cycles per second)

kilohertz (kHz)	1×10^3 Hz	1,000 cycles/second
megahertz (MHz)	1×10^6 Hz	1,000,000 cycles/second
gigahertz (GHz)	1×10^9 Hz	1,000,000,000 cycles/second

Some calculations...

Example Question: A wave has a frequency of 10 MHz.
How long does it take to complete one cycle?

- There are various tricks for helping us to do calculations with frequencies
- These are not particularly difficult; its just a question of remembering the techniques

Indices

$$\boxed{a^n}$$

a^n means n a's multiplied together:

$$2^3 = 2 \times 2 \times 2 = 8$$

$$10^2 = 10 \times 10 = 100$$

Indices

$$\boxed{a^1 = a}$$

a^1 is just a on it's own!

$$2^1 = 2$$

$$10^1 = 10$$

Indices

$$\boxed{a^m \times a^n = a^{(m+n)}}$$

When two exponents are multiplied, if the base is the same, you can add powers together:

$$2^3 \times 2^2 = (2 \times 2 \times 2) \times (2 \times 2) = 2^5$$

$$10^2 \times 10^1 = (10 \times 10) \times (10) = 10^3$$

Indices

$$a^m \times a^n = a^{(m+n)}$$

What happens if $m = 0$?
a^0 becomes 1:

$$a^0 \times a^n = a^{(0+n)} = 1 \times a^n$$

$$10^0 \times 10^2 = 10^{(0+2)} = 1 \times 10^2$$

Indices

$$\boxed{a^0 = 1}$$

a^0 is equal to 1

$$2^0 = 1$$

$$10^0 = 1$$

Special case... $0^0 = 0$

Indices in Large Units

1 kHz	1000 Hz	10^0 Hz $\times$ 10^3 Hz	10^3 Hz
1 MHz	1000 kHz	10^3 Hz $\times$ 10^3 Hz	10^6 Hz
1 GHz	1000 MHz	10^6 Hz $\times$ 10^3 Hz	10^9 Hz

Indices

$$a^m$$

What happens if m is negative?
Division is the opposite of multiplication:

$$\begin{aligned} 2^{-3} &= 1 \div 2 \div 2 \div 2 \\ &= 1 \div (2 \times 2 \times 2) \\ &= \frac{1}{2^3} \end{aligned}$$

Indices

$$\boxed{a^{-n} = \frac{1}{a^n}}$$

a^{-n} is 1 divided by a^{+n}

$$2^{-3} = \frac{1}{2^3} = 0.125$$

$$10^{-5} = \frac{1}{10^5} = 0.00001$$

Indices

$$a^m \times a^n = a^{(m+n)}$$

What happens if n is negative?
Division is the opposite of multiplication:

$$a^m \times a^{-n} = a^{(m-n)}$$

$$= a^m \times \frac{1}{a^n}$$

$$= \frac{a^m}{a^n}$$

Indices

$$\boxed{a^{(m-n)} = \frac{a^m}{a^n}}$$

$a^{(m-n)}$ is a^m divided by a^n

$$2^{(5-3)} = \frac{2^5}{2^3} = \frac{32}{8} = 4$$

$$10^{(8-5)} = \frac{10^8}{10^5} = \frac{100,000,000}{100,000} = 1000$$

Indices

"Start with '1' and then multiply or divide as many times as the exponent says"

a^n	$\dots$
a^2	$1 \times a \times a$
a^1	$1 \times a$
a^0	1
a^{-1}	$1 \div a$
a^{-2}	$1 \div a \div a$
a^{-n}	$\dots$

Rod Pierce, 2014, 'Negative Exponents', Math Is Fun,

http://www.mathsisfun.com/algebra/negative-exponents.html

Small Units of Time

We will be dealing with small units of time (measured in seconds)

millisecond (ms)	$1 \times 10^{-3}\,\mathrm{s}$	$^1/_{1,000}$ seconds
microsecond (μs)	$1 \times 10^{-6}\,\mathrm{s}$	$^1/_{1,000,000}$ seconds
nanosecond (ns)	$1 \times 10^{-9}\,\mathrm{s}$	$^1/_{1,000,000,000}$ seconds
picosecond (ps)	$1 \times 10^{-12}\,\mathrm{s}$	$^1/_{1,000,000,000,000}$ seconds

Example

A wave has a frequency of 10 MHz.
How long does it take to complete one cycle?

Example

A wave has a frequency of 10 MHz.
How long does it take to complete one cycle?

$$10\,\text{MHz} = 10 \times 10^6\,\text{Hz}$$

The wave is performing 10,000,000 (1×10^7) cycles per second

To complete one cycle, it therefore takes $^1/_{10,000,000}$ seconds

$$= 1 \times 10^{-7}\,\text{s}$$

$$= 10^{-1} \times 10^{-6}\,\text{s} = 0.1\,\mu\text{s}$$
$$= 10^{2} \times 10^{-9}\,\text{s} = 100\,\text{ns}$$

Example

An electrical signal travels at a speed of
2×10^8 metres per second in copper wire*.

How long does it take to travel the length of a 1 km cable?

$$\boxed{\mathit{speed} = \frac{\mathit{distance}}{\mathit{time}}}$$

*This is around $^2/_3$ the speed of light in a vacuum ($3 \times 10^8\,\mathrm{m\,s^{-1}}$)

Example

An electrical signal travels at a speed of
2×10^8 metres per second in copper wire.
How long does it take to travel the length of a 1 km cable?

$$2 \times 10^8 = \frac{1000}{time} \longrightarrow time = \frac{10^3}{2 \times 10^8}$$

$$= 10^3 \times 2^{-1} \times 10^{-8}$$

$$= \tfrac{1}{2^1} \times 10^{(3-8)}$$

$$= 0.5 \times 10^{-5}\,\mathrm{s}$$

$$= 5 \times 10^{-6}\,\mathrm{s} = 5\,\mu\mathrm{s}$$

Example

An FM radio station broadcasts on 100 MHz.
Radio waves travel at the speed of light ($3 \times 10^8\,\mathrm{m\,s^{-1}}$).

Given that the ideal[†] length for a normal radio aerial is $1/4$ of the wavelength[‡], how long should my aerial be?

$$\boxed{\textit{distance} = \textit{speed} \times \textit{time}}$$

(distance = wavelength)

[†] there are variants, but in this module we use $1/4\lambda$

[‡] if you're interested why, see: `https://en.wikipedia.org/wiki/Dipole_antenna#Quarter-wave_monopole`

Example

An FM radio station broadcasts on 100 MHz.
Radio waves travel at the speed of light ($3 \times 10^8\,\mathrm{m\,s^{-1}}$).
Given that the ideal length for a normal radio aerial is $1/4$ of the wavelength, how long should my aerial be?

If frequency is 100 MHz (10^8 Hz), one cycle takes 10^{-8} s

$$\textit{distance} = (3 \times 10^8) \times 10^{-8}$$

$$= 3 \times 10^{(8-8)} = 3 \times 1$$

Station wavelength $= 3\,\mathrm{m}$

Aerial length $= 3/4\mathrm{m} = 75\,\mathrm{cm}$

Alternative Formula

$$\boxed{v = f\lambda}$$

$$velocity = frequency \times wavelength$$

$$wavelength = \frac{velocity}{frequency} = \frac{3 \times 10^8}{10^8} = 3\,\mathrm{m}$$

Why Waves are Important

- Any analogue signal can be represented as a combination of waves of different frequencies and amplitudes
  - Fourier Analysis - break down the wave into a number of simpler waves
  - see: `http://en.wikipedia.org/wiki/Fourier_analysis`
  - You don't need to be able to do the maths for this module, but it's worth reading up on

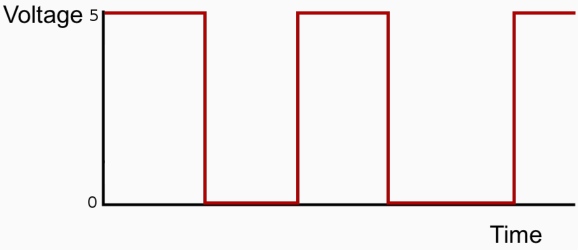
- Attenuation, dispersion and distortion affect different frequencies and mediums (cables, etc.) to a different extent and in different ways

Why Waves are Important

- Each transmission medium has a range of frequencies that it can transmit
    - A range of frequencies that it can transmit with least damage
    - e.g. Wi-Fi at 2.4 GHz and 5 GHz

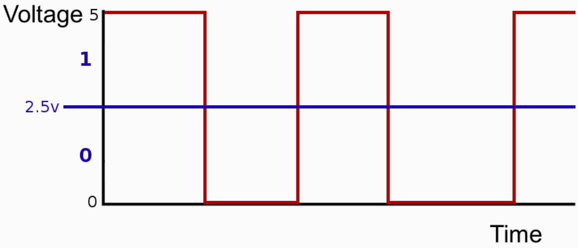
- The most widely available network is the telephone network (PSTN - Public Switched Telephone Network)
    - Designed to transmit analogue signals for voice
    - Uses lots of different technologies, more in a later lecture...

Binary signals



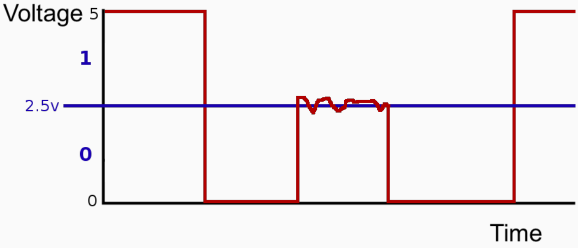
- Binary signals are digital signals with only two possible values
  - Conventionally written 0 and 1
  - But may be any pair of contrasting values,
    e.g. {long, short}, {yes, no}, {hot, cold}

Binary signals



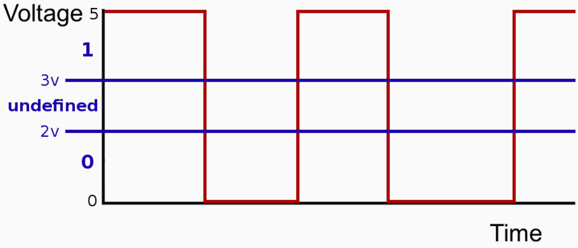
- Could use a basic threshold:
  - Anything below 2.5 V is a 0
  - Anything above 2.5 V is a 1

Binary signals



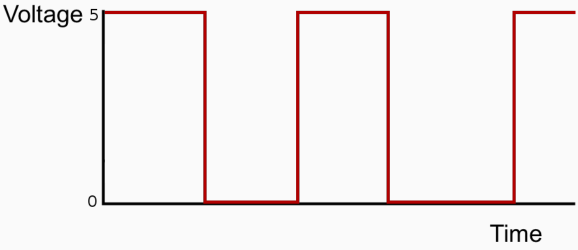
But... signals aren't perfect

Binary signals



- So instead we can use ranges:
  - 0 V to 2 V is a 0
  - 3 V to 5 V is a 1
  - Anything in the middle (2 V to 3 V) is undefined
  - This will at least give us a hint when something might be going wrong...

Binary signals



- Binary signals are very widely used because they are technically easy to generate and to recognise
- Any digital data can be represented as a sequence of binary signals

The important things to remember:

- How to do basic calculations for waves
  - use the indices tricks
- Different SI units:
  - giga (G), mega (M), kilo (k), milli (m), micro (μ), nano (n), ...
- Binary signals are a specific digital signal with only two values
  - don't have to be $\{1, 0\}$

Indices crib sheet

$$\boxed{a^n}$$

a^n means n a's multiplied together

Special cases:

$$a^1 = a$$

$$a^0 = 1$$

$$0^0 = 0$$

Useful tricks:

$$a^m \times a^n = a^{(m+n)}$$

$$a^m \div a^n = a^{(m-n)}$$

$$a^{-n} = {}^{1}\!/\!_{a^n}$$

Next time...

Modes and Media
(Monday at 13:10, HO-A12)