

Peter Naur's contribution to formal notations and beyond

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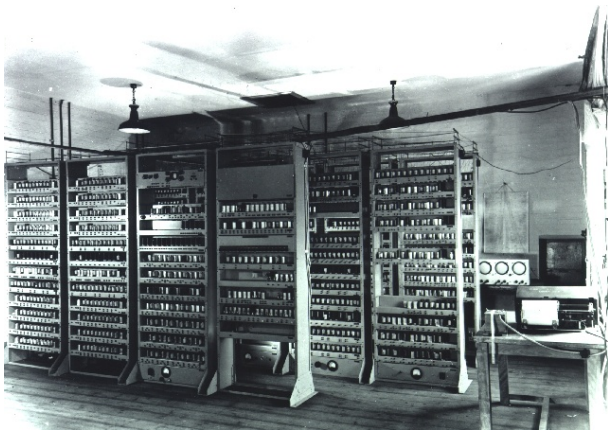
Seminar Turing Award Winners and Their Contributions
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Formal notations in history



New need for formal notations



Peter Naur



Peter Naur's contribution

Citation

For fundamental contributions to programming language design and the definition of Algol 60, to compiler design, and to the art and practice of computer programming.

Phrase structure grammar

Definition

Quadruple $G = (V, \Sigma, P, S)$, where

- V is vocabulary
- $\Sigma \subseteq V$ *terminal symbols*
- $S \in (V - \Sigma)$ *start symbol*
- $P \subseteq V^*NV^* \times V^*$ *productions*

Phrase structure

Example

$$G = (\{S, NP, VP, V, \text{the man}, \text{the book}, \text{took}\}, \\ \{\text{the man}, \text{the book}, \text{took}\}, P, S)$$

$P :$

$S \rightarrow NPVP$

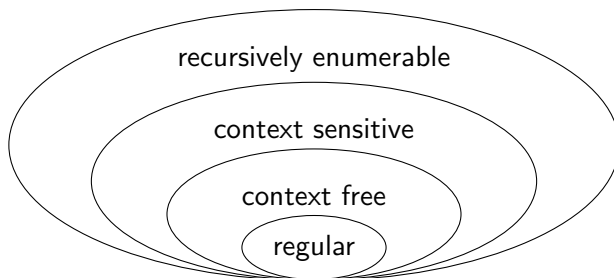
$VP \rightarrow VNP$

$NP \rightarrow \text{the man}$

$NP \rightarrow \text{the book}$

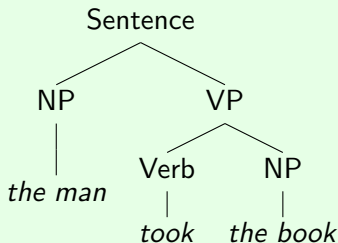
$V \rightarrow \text{took}$

Chomsky Hierarchy



Phrase structure in tree form

Example



Backus Normal form

Example

$$\begin{aligned} \langle S \rangle &: \equiv \langle NP \rangle \ \langle VP \rangle \\ \langle VP \rangle &: \equiv \langle V \rangle \ \langle NP \rangle \\ \langle NP \rangle &: \equiv \text{the man } \textit{or} \ \text{the book} \\ \langle V \rangle &: \equiv \text{took} \end{aligned}$$

Backus Naur form

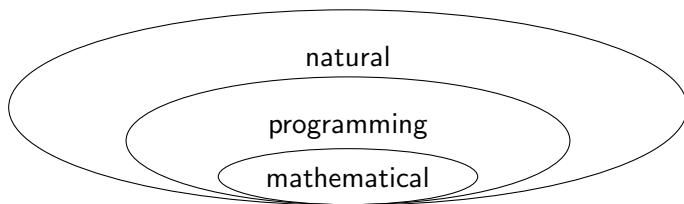
Example

$\langle \textit{sentence} \rangle ::= \langle \textit{noun_phrase} \rangle \ \langle \textit{verb_phrase} \rangle$
 $\langle \textit{verb_phrase} \rangle ::= \langle \textit{verb} \rangle \ \langle \textit{noun_phrase} \rangle$
 $\langle \textit{noun_phrase} \rangle ::= \text{the man} \mid \text{the book}$
 $\langle \textit{verb} \rangle ::= \text{took}$

Comparison of levels of formalization

- Natural languages
- Mathematical languages
- Programming languages

Language hierarchy



Historical context



Fortran

GENERAL FORM	EXAMPLES
1 to 5 decimal digits. A preceding + or — sign is optional. The magnitude of the constant must be less than 32768.	3 +1 –28987

Algol 58

3.31 Integers and numbers

$\langle \text{digit} \rangle : \equiv 0 \text{ or } 1 \text{ or } 2 \text{ or } 3 \text{ or } 4 \text{ or } 5 \text{ or } 6 \text{ or } 7 \text{ or } 8 \text{ or } 9$

$\langle \text{integer} \rangle : \equiv \langle \text{digit} \rangle \text{ or } \langle \text{integer} \rangle \langle \text{digit} \rangle$

$\langle \text{dn} \rangle : \equiv \langle \text{integer} \rangle \text{ or } \langle \text{integer} \rangle \text{ or } \langle \text{integer} \rangle \text{ or } \langle \text{dn} \rangle \langle \text{integer} \rangle$

$\langle \text{si} \rangle : \equiv + \langle \text{integer} \rangle \text{ or } - \langle \text{integer} \rangle \text{ or } \langle \text{integer} \rangle$

Algol 60

```
<digit> ::= 0|1|2|3|4|5|6|7|8|9
```

Digits are used for forming numbers, identifiers, and strings.

```
<unsigned integer> ::= <digit>|<unsigned integer><digit>
```

```
<integer> ::= <unsigned integer>|+<unsigned integer>|-<unsigned integer>
```

+ Additional examples, semantics and types

Algol 68

b) An arithmetic value has a "length number", i.e., a positive integer characterizing the degree of discrimination with which the value is kept in the computer. The number of integers (real numbers) of given length number that can be distinguished increases with the length number up to a certain length number, the number of different lengths of integers (real numbers) {10.1.a,c}, after which it is constant.

c) For each pair of integers (real numbers) of the same length number, the relationship "to be smaller than" is defined {10.2.3.a, 10.2.4.a}. For each pair of integers of the same length number, a third integer of that length number may exist, the first integer "minus" the other one {10.2.3.g}. Finally, for each pair of real numbers of the same length number, three real numbers of that length number may exist, the first real number "minus" ("times", "divided by") the other one {10.2.4.g,l,m}; these real numbers are obtained "in the sense of numerical analysis", i.e., by performing the operations known in mathematics by these terms on real numbers which may deviate slightly from the given ones {; this deviation is left undefined in this Report}.

Properties of Algol 60

- Block scope
- Call-by-value, call-by-name and call-by-reference parameter passing
- Formal specification
- No I/O facilities

Block scope

Example

```
begin
  integer X;
  X := 5;
  begin
    integer X, Y;
    X := 4;
    Y := 8;
  end
  print(X);
  Y := 12;
end;
```

Call-by-value

- Callee parameter Value of arguments given
- No modification of caller argument

Call-by-name

- Pass thunk
- Similar to call-by-reference
- Parameter is passed without evaluation

Call-by-name

Example

```
procedure swap (a, b);  
integer a, b, temp;  
begin  
    temp := a;  
    a := b;  
    b := temp;  
end;
```

Call-by-name

Example

swap(x, y):

temp := x;

x := y;

y := temp

sqap(i, x[i]):

temp := i;

i := x[i];

x[i] := temp

I/O facilities

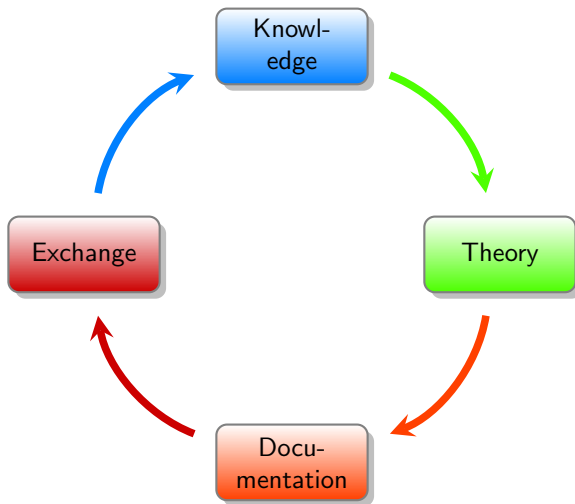
Example

```
begin
    print('Hello world!');
    comment error!
end;
```

Programming as theory building

- Source code contains theory
- Transfer theory to next programmer
- Theory should be observable

Role of formal descriptions



Problems of over-formalization

- Hard to exchange
- Prone to errors
- Goal in itself
- Disconnected from environment

Computing Versus Human Thinking

- Description of computers
- Description of human thinking

Discussion

Do you think that human thinking will be formally describable?

Importance of formal notation

For achieving clarity any formal mode of expression should be used, not as a goal in itself, but wherever it appears to be helpful to authors and readers alike.

(Peter Naur)

The End

References



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