

# Quantum Computing Cheat Sheet

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## State vectors

$$|\psi\rangle = \begin{pmatrix} \cos \frac{\theta}{2} \\ e^{i\varphi} \sin \frac{\theta}{2} \end{pmatrix}$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

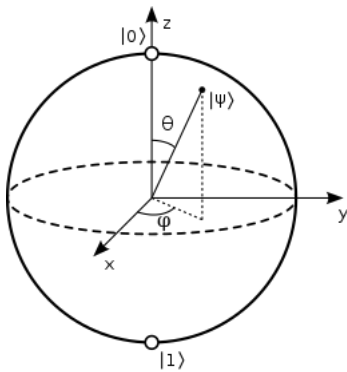
$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$|-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$|+\rangle = \frac{|0\rangle + i|1\rangle}{\sqrt{2}}$$

$$|-\rangle = \frac{|0\rangle - i|1\rangle}{\sqrt{2}}$$

## Bloch sphere



$$\begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix} = \begin{pmatrix} \sin \theta \cos \varphi \\ \sin \theta \sin \varphi \\ \cos \theta \end{pmatrix}$$

## Pauli matrices

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$\vec{r} \times$	X	Y	Z
X	I	iZ	-iY
Y	-iZ	I	iX
Z	iY	-iX	I

$$X = |+\rangle \langle +| - |-\rangle \langle -|$$

$$Y = |+\rangle \langle +'| - |-\rangle \langle -'|$$

$$Z = |0\rangle \langle 0| - |1\rangle \langle 1|$$

## Rotations

$$R_x(\theta) = e^{-i\theta X/2} = \cos \frac{\theta}{2} I - i \sin \frac{\theta}{2} X$$

$$R_y(\theta) = e^{-i\theta Y/2} = \cos \frac{\theta}{2} I - i \sin \frac{\theta}{2} Y$$

$$R_z(\theta) = e^{-i\theta Z/2} = \cos \frac{\theta}{2} I - i \sin \frac{\theta}{2} Z$$

## Density matrix

$$\rho = |\psi\rangle \langle\psi|$$

$$\rho = \frac{1}{2}(I + \sin \theta \cos \varphi X + \sin \theta \sin \varphi Y + \cos \theta Z)$$

$$\rho = \frac{1}{2}(I + r_x X + r_y Y + r_z Z)$$

$$\psi \text{ is a pure-state} \Leftrightarrow \text{Tr}(\rho^2) = 1 \Leftrightarrow r_x^2 + r_y^2 + r_z^2 = 1$$

## Tomography

$$r_x = \text{Tr}(X\rho) = \langle +|\rho|+ \rangle - \langle -|\rho|- \rangle = \mathbb{P}|+ \rangle - \mathbb{P}|- \rangle$$

$$r_y = \text{Tr}(Y\rho) = \langle +'\rho|+'\rangle - \langle -'\rho|-'\rangle = \mathbb{P}|+'\rangle - \mathbb{P}|-' \rangle$$

$$r_z = \text{Tr}(Z\rho) = \langle 0|\rho|0 \rangle - \langle 1|\rho|1 \rangle = \mathbb{P}|0 \rangle - \mathbb{P}|1 \rangle$$

## Gates

### Hadamard Gate

The Hadamard Gate can be decomposed in two rotations:

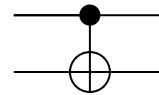
$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = |+\rangle \langle 0| + |-\rangle \langle 1| = R_x(\pi)R_y(\frac{\pi}{2}) = -iX \cdot R_y(\frac{\pi}{2})$$

### Phase Gate

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} = |0\rangle \langle 0| + i|1\rangle \langle 1|$$

### Controlled Not (CNOT, CX)

$$CX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & X \end{pmatrix}$$



### EPR pairs

The EPR pairs are the Bell states denoted by  $|\Phi^+\rangle$ ,  $|\Phi^-\rangle$ ,  $|\Psi^+\rangle$  and  $|\Psi^-\rangle$ .

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle_B + |11\rangle_B)$$

