Quantum Computing Cheat Sheet

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State vectors

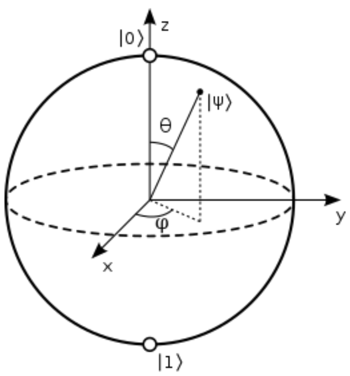
$$|\psi\rangle = \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{i\varphi}\sin\frac{\theta}{2} \end{pmatrix}$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}} \qquad |\text{-}\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$|+'\rangle = \frac{|0\rangle + i\,|1\rangle}{\sqrt{2}} \qquad |\text{-}'\rangle = \frac{|0\rangle - i\,|1\rangle}{\sqrt{2}}$$

Bloch sphere



$$\begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{pmatrix}$$

Pauli matrices

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$\Rsh\ \times$	X	Y	Z
X	I	iZ	$-iY$
Y	$-iZ$	I	iX
Z	iY	$-iX$	I

$$X = |+\rangle\langle+| - |\text{-}\rangle\langle\text{-}|$$
$$Y = |+'\rangle\langle+'| - |\text{-}'\rangle\langle\text{-}'|$$
$$Z = |0\rangle\langle0| - |1\rangle\langle1|$$

Rotations

$$
\begin{aligned}
R_x(\theta) &= e^{-i\theta X/2} = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}X \\
R_y(\theta) &= e^{-i\theta Y/2} = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}Y \\
R_z(\theta) &= e^{-i\theta Z/2} = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}Z
\end{aligned}
$$

Density matrix

$$\rho = |\psi\rangle\langle\psi|$$

$$\rho = \frac{1}{2}(I + \sin\theta\cos\varphi X + \sin\theta\sin\varphi Y + \cos\theta Z)$$

$$\rho = \frac{1}{2}(I + r_x X + r_y Y + r_z Z)$$

ψ is a pure-state $\Leftrightarrow Tr(\rho^2) = 1 \Leftrightarrow r_x^2 + r_y^2 + r_z^2 = 1$

Tomography

$$
\begin{aligned}
r_x &= Tr(X\rho) = \langle +|\rho|+\rangle - \langle -|\rho|-\rangle = \mathbb{P}\,|+\rangle - \mathbb{P}\,|-\rangle \\
r_y &= Tr(Y\rho) = \langle +'|\rho|+'\rangle - \langle -'|\rho|-'\rangle = \mathbb{P}\,|+'\rangle - \mathbb{P}\,|-'\rangle \\
r_z &= Tr(Z\rho) = \langle 0|\rho|0\rangle - \langle 1|\rho|1\rangle = \mathbb{P}\,|0\rangle - \mathbb{P}\,|1\rangle
\end{aligned}
$$

Gates

Hadamard Gate

The Hadamard Gate can be decomposed in two rotations:

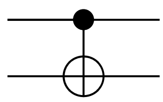
$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = |+\rangle\langle 0| + |-\rangle\langle 1| = R_x(\pi)R_y(\frac{\pi}{2}) = -iX \cdot R_y(\frac{\pi}{2})$$

Phase Gate

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} = |0\rangle\langle 0| + i\,|1\rangle\langle 1|$$

Controlled Not (CNOT, CX)

$$CX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & X \end{pmatrix}$$



EPR pairs

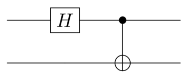
The EPR pairs are the Bell states denoted by $|\Phi^+\rangle$, $|\Phi^-\rangle$, $|\Psi^+\rangle$ and $|\Psi^-\rangle$. The circuit is $(H \otimes I_2) \cdot CX$ and can output every Bell states:

$$|\Phi^+\rangle = [(H \otimes I_2) \cdot CX] \, |00\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$|\Psi^+\rangle = [(H \otimes I_2) \cdot CX] \, |01\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

$$|\Phi^-\rangle = [(H \otimes I_2) \cdot CX] \, |10\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$

$$|\Psi^-\rangle = [(H \otimes I_2) \cdot CX] \, |11\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$



Which sums up to:

$$[(H \otimes I_2) \cdot CX] \, |xy\rangle = \frac{1}{\sqrt{2}}(|0y\rangle + (-1)^x |1\bar{y}\rangle)$$