

Quantum Computing Cheat Sheet

Valentin Taillandier

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State vectors

$$|\psi\rangle = \begin{pmatrix} \cos \frac{\theta}{2} \\ e^{i\varphi} \sin \frac{\theta}{2} \end{pmatrix}$$

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

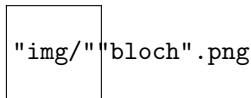
$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

$$|-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

$$|+\prime\rangle = \frac{|0\rangle + i|1\rangle}{\sqrt{2}}$$

$$|-\prime\rangle = \frac{|0\rangle - i|1\rangle}{\sqrt{2}}$$

Bloch sphere



$$\begin{pmatrix} r_x \\ r_y \\ r_z \end{pmatrix} = \begin{pmatrix} \sin \theta \cos \varphi \\ \sin \theta \sin \varphi \\ \cos \theta \end{pmatrix}$$

Pauli matrices

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$\vec{r} \times$	X	Y	Z
X	I	iZ	$-iY$
Y	$-iZ$	I	iX
Z	iY	$-iX$	I

$$\begin{aligned} X &= |+\rangle \langle +| - |-\rangle \langle -| \\ Y &= |+\prime\rangle \langle +\prime| - |-\prime\rangle \langle -\prime| \\ Z &= |0\rangle \langle 0| - |1\rangle \langle 1| \end{aligned}$$

Rotations

$$R_x(\theta) = e^{-i\theta X/2} = \cos \frac{\theta}{2} I - i \sin \frac{\theta}{2} X$$

$$R_y(\theta) = e^{-i\theta Y/2} = \cos \frac{\theta}{2} I - i \sin \frac{\theta}{2} Y$$

$$R_z(\theta) = e^{-i\theta Z/2} = \cos \frac{\theta}{2} I - i \sin \frac{\theta}{2} Z$$

Density matrix

$$\rho = |\psi\rangle \langle\psi|$$

$$\rho = \frac{1}{2}(I + \sin\theta \cos\varphi X + \sin\theta \sin\varphi Y + \cos\theta Z)$$

$$\rho = \frac{1}{2}(I + r_x X + r_y Y + r_z Z)$$

$$\psi \text{ is a pure-state} \Leftrightarrow \text{Tr}(\rho^2) = 1 \Leftrightarrow r_x^2 + r_y^2 + r_z^2 = 1$$

Tomography

$$r_x = \text{Tr}(X\rho) = \langle + | \rho | + \rangle - \langle - | \rho | - \rangle = \mathbb{P} | + \rangle - \mathbb{P} | - \rangle$$

$$r_y = \text{Tr}(Y\rho) = \langle + | \rho | +' \rangle - \langle - | \rho | -' \rangle = \mathbb{P} | +' \rangle - \mathbb{P} | -' \rangle$$

$$r_z = \text{Tr}(Z\rho) = \langle 0 | \rho | 0 \rangle - \langle 1 | \rho | 1 \rangle = \mathbb{P} | 0 \rangle - \mathbb{P} | 1 \rangle$$

Gates

Hadamard Gate

The Hadamard Gate can be decomposed in two rotations:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = |+\rangle \langle 0| + |-\rangle \langle 1| = R_x(\pi)R_y\left(\frac{\pi}{2}\right) = -iX \cdot R_y\left(\frac{\pi}{2}\right)$$

Phase Gate

$$S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} = |0\rangle \langle 0| + i |1\rangle \langle 1|$$

Controlled Not (CNOT, CX)

$$CX = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & X \end{pmatrix}$$

"img/" "CNOT_gate".pdf

EPR pairs

The EPR pairs are the Bell states denoted by $|\Phi^+\rangle$, $|\Phi^-\rangle$, $|\Psi^+\rangle$ and $|\Psi^-\rangle$.

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle_B + |11\rangle_B)$$

"img/" "h_cnot".png