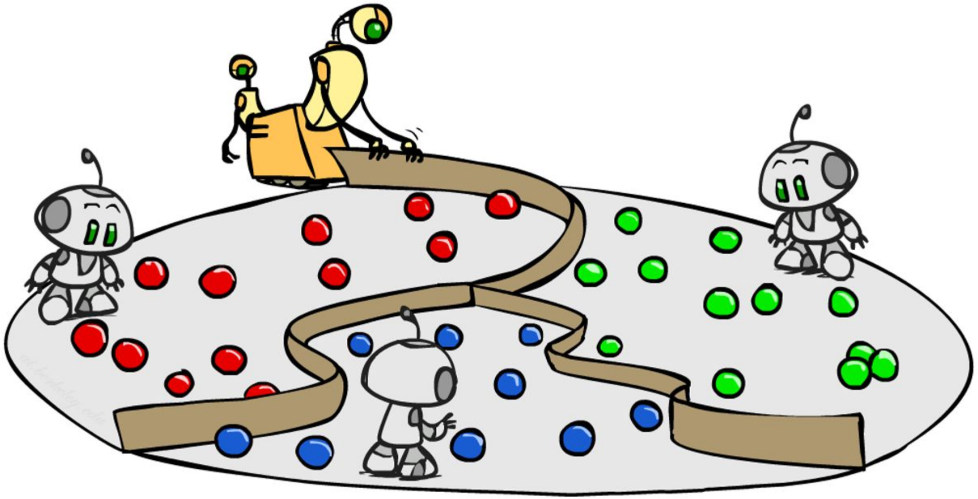
CS-ELEC2C: Machine Learning

Linear Classification and Logistic Regression



What have we learned in the previous lecture?

What have we learned in the previous lecture?

Step-by-step Example

Features

Training Data

Model

Loss

Optimization

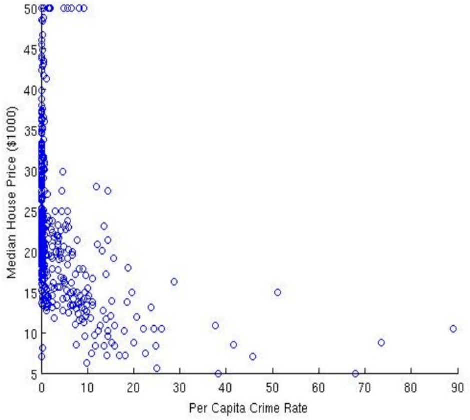
Features: Per Capita Crime Rate and Average Number of Rooms

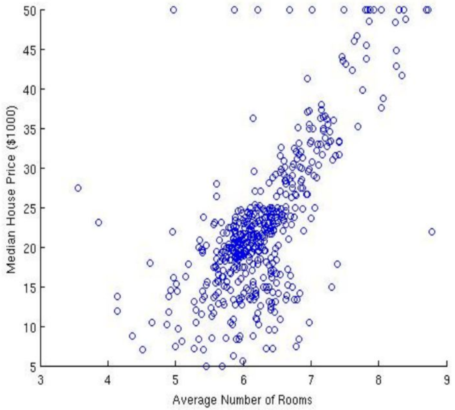
What have we learned in the previous lecture?

Step-by-step Example

- ***Features***
- *Training Data*
- *Model*
- *Loss*
- *Optimization*

Features: **Per Capita Crime Rate** and **Average Number of Rooms**





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Features: **Per Capita Crime Rate** and **Average Number of Rooms**

Training Data Structure

$$D = \{(\mathbf{x}^{(1)}, t^{(1)}), \ldots, (\mathbf{x}^{(n)}, t^{(n)})\}$$

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$$D = \{(\mathbf{x}^{(1)}, t^{(1)}), \ldots, (\mathbf{x}^{(n)}, t^{(n)})\}$$

such that

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$$D = \{(\mathbf{x}^{(1)}, t^{(1)}), \ldots, (\mathbf{x}^{(n)}, t^{(n)})\}$$

such that

$$\mathbf{x} = (x_1^{(n)}, x_2^{(n)})$$

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Step-by-step Example

- *Features*
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Features: **Per Capita Crime Rate** and **Average Number of Rooms**

Training Data Structure

$$D = \{(\mathbf{x}^{(1)}, t^{(1)}), \ldots, (\mathbf{x}^{(n)}, t^{(n)})\}$$

such that

$$\mathbf{x} = (x_1^{(n)}, x_2^{(n)})$$

$$x_1 = \text{per capita crime rate}$$

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Features: **Per Capita Crime Rate** and **Average Number of Rooms**

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$$D = \{(\mathbf{x}^{(1)}, t^{(1)}), \ldots, (\mathbf{x}^{(n)}, t^{(n)})\}$$

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$$\mathbf{x} = (x_1^{(n)}, x_2^{(n)})$$

$$x_1 = \text{per capita crime rate}$$
$$x_2 = \text{number of rooms}$$

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$$\mathbf{x} = (x_1^{(n)}, x_2^{(n)})$$

$$x_1 = \text{per capita crime rate}$$
$$x_2 = \text{number of rooms}$$
$$t = \text{median house price}$$

What have we learned in the previous lecture?

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$$x_2 = \text{number of rooms}$$
$$t = \text{median house price}$$
$$n = \text{number of data points}$$

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$$x_2 = \text{number of rooms}$$
$$t = \text{median house price}$$
$$n = \text{number of data points}$$

Sample Data:

Neighborhood #1

$x_1 = 40 \quad x_2 = 3 \quad t = 3,400$

Neighborhood #2

$x_1 = 20 \quad x_2 = 5 \quad t = 4,500$

Neighborhood #3

$x_1 = 30 \quad x_2 = 2 \quad t = 2,800$

What have we learned in the previous lecture?

Step-by-step Example

- *Features*
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Features: **Per Capita Crime Rate** and **Average Number of Rooms**

***Training* Data Structure**

$$D = \{(\mathbf{x}^{(1)}, t^{(1)}), \ldots, (\mathbf{x}^{(n)}, t^{(n)})\}$$

Multivariate Linear Regression

$$y(\mathbf{x}) = w_0 + w_1 x_1 + w_2 x_2$$

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$$y(\mathbf{x}) = w_0 + w_1 x_1 + w_2 x_2$$

Sum of Squares Error

$$\ell(\mathbf{w}) = \sum_{n=1}^{N} [t^{(n)} - (w_0 + w_1 x^{(n)})]^2$$

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Features: **Per Capita Crime Rate** and **Average Number of Rooms**

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Sum of Squares Error

$$\ell(\mathbf{w}) = \sum_{n=1}^{N} [t^{(n)} - (w_0 + w_1 x^{(n)})]^2$$

Optimization using Gradient Descent

$$\mathbf{w} \leftarrow \mathbf{w} + 2\lambda \sum_{n=1}^{N} (t^{(n)} - y(x^{(n)}))x^{(n)}$$

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Step #2: Update Weights

$$\mathbf{w} \leftarrow \mathbf{w} - \lambda \frac{\partial \ell}{\partial \mathbf{w}} \qquad \mathbf{w} \leftarrow \mathbf{w} + 2\lambda(t^{(n)} - y(x^{(n)}))x^{(n)}$$

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$$\mathbf{w} \leftarrow \mathbf{w} - \lambda \frac{\partial \ell}{\partial \mathbf{w}} \qquad \mathbf{w} \leftarrow \mathbf{w} + 2\lambda(t^{(n)} - y(x^{(n)}))x^{(n)}$$

Neighborhood #1

$$w_0 = 200 + 2(1)(3400 - (200 + 5(40) + 200(3)))1$$

Sample Data:

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$x_1 = 40 \quad x_2 = 3 \quad t = 3,400$

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Neighborhood #1

$$w_0 = 200 + 2(1)(3400 - (200 + 5(40) + 200(3)))1$$

$$w_1 = 5 + 2(1)(3400 - (200 + 5(40) + 200(3)))40$$

Sample Data:

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Neighborhood #1

$$w_0 = 200 + 2(1)(3400 - (200 + 5(40) + 200(3)))1$$

$$w_1 = 5 + 2(1)(3400 - (200 + 5(40) + 200(3)))40$$

$$w_2 = 200 + 2(1)(3400 - (200 + 5(40) + 200(3)))3$$

Sample Data:

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Neighborhood #1

$$w_0 = 200 + 2(1)(3400 - (200 + 5(40) + 200(3)))1 \qquad w_0 = 5,000$$

$$w_1 = 5 + 2(1)(3400 - (200 + 5(40) + 200(3)))40 \qquad w_1 = 192,005$$

$$w_2 = 200 + 2(1)(3400 - (200 + 5(40) + 200(3)))3 \qquad w_2 = 15,400$$

Sample Data:

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Neighborhood #2

$$w_0 = 5000 + 2(1)(4500 - (5000 + 192005(20) + 15400(5)))1$$

Sample Data:

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Neighborhood #2

$x_1 = 20 \quad x_2 = 5 \quad t = 4,500$

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Neighborhood #2

$$w_0 = 5000 + 2(1)(4500 - (5000 + 192005(20) + 15400(5)))1$$

$$w_1 = 192005 + 2(1)(4500 - (5000 + 192005(20) + 15400(5)))20$$

Sample Data:

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Neighborhood #2

$$w_0 = 5000 + 2(1)(4500 - (5000 + 192005(20) + 15400(5)))1$$

$$w_1 = 192005 + 2(1)(4500 - (5000 + 192005(20) + 15400(5)))20$$

$$w_2 = 15400 + 2(1)(4500 - (5000 + 192005(20) + 15400(5)))5$$

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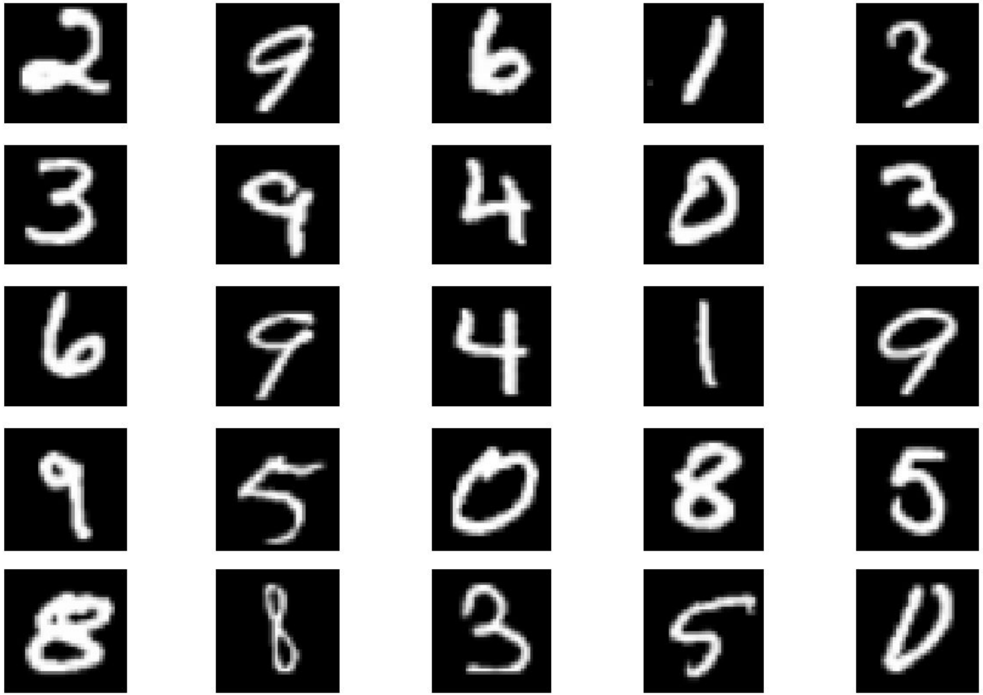
Neighborhood #2

$$w_0 = 5000 + 2(1)(4500 - (5000 + 192005(20) + 15400(5)))1 \qquad w_0 = -7,830,200$$

$$w_1 = 192005 + 2(1)(4500 - (5000 + 192005(20) + 15400(5)))20 \qquad w_1 = -156,551,995$$

$$w_2 = 15400 + 2(1)(4500 - (5000 + 192005(20) + 15400(5)))5 \qquad w_2 = -36,160,600$$

What Problem is This?



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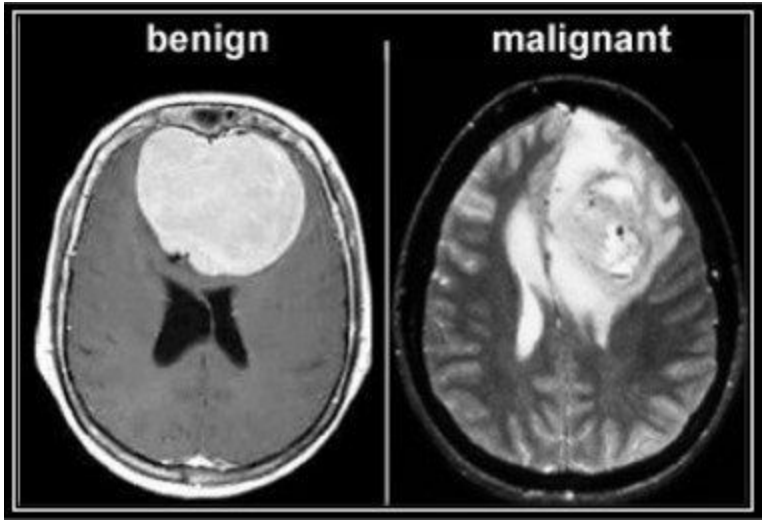


What Problem is This?

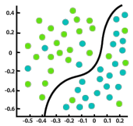
Introduction: Linear Classification

Classification

Introduction: Linear Classification

Classification

In information science, a classification scheme is ***arranging things according to its kind*** *or into* ***groups or classes***



Introduction: Linear Classification

Classification

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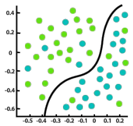

How is it different from Regression?

Introduction: Linear Classification

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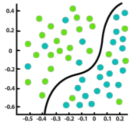
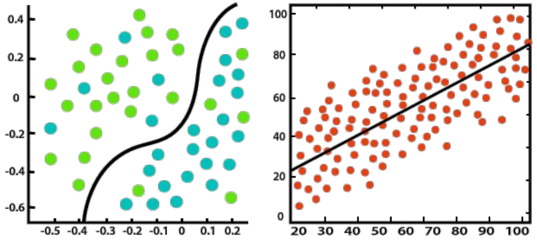


How is it different from Regression?

In **regression problems**, the targets or outputs are **continuous variables**. However, in **classification problems**, they have **categorical outputs**. Intuitively, classification can be thought of as **assigning each input to one of a finite number of labels**



Classification Regression

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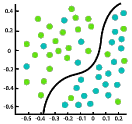
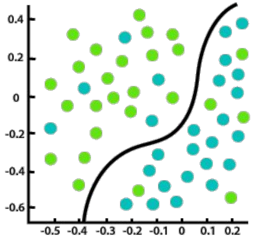


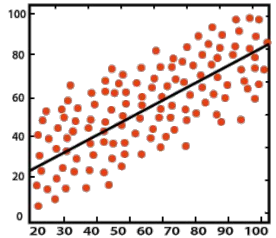
How is it different from Regression?

In **regression problems**, the targets or outputs are **continuous variables**. However, in **classification problems**, they have **categorical outputs**. Intuitively, classification can be thought of as **assigning each input to one of a finite number of labels**

There are two types of classification problems: **binary classification** and **multiclass**



Classification



Regression

Linear Classification

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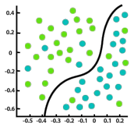

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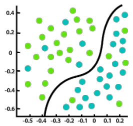

Can we frame classification as a regression problem?

Linear Classification

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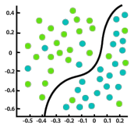

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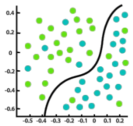

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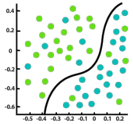

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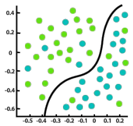

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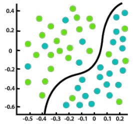

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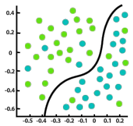

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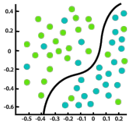

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$$\ell_{\text{square}}(\mathbf{w}, t) = \frac{1}{N}\sum_{n=1}^{N}(t^{(n)} - \mathbf{w}^T\mathbf{x})^2$$

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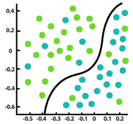

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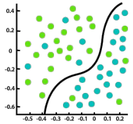

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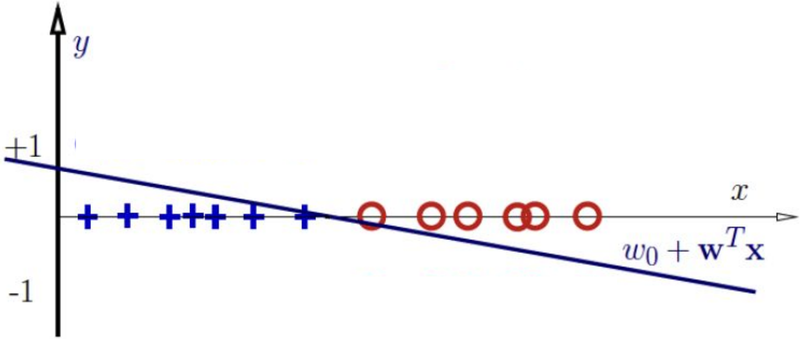

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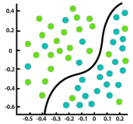

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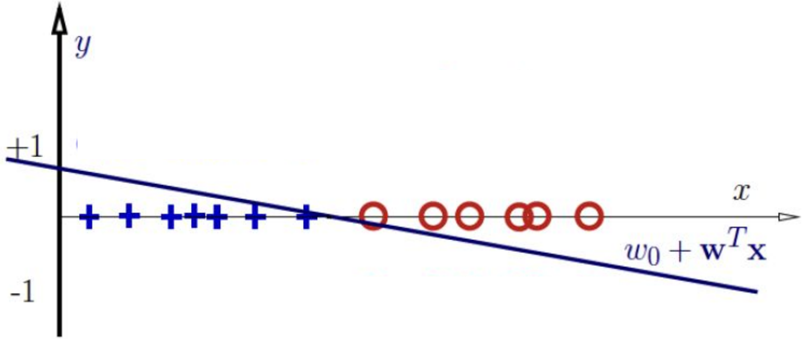

Assume that the classifier has the following form

$$f(\mathbf{x}, \mathbf{w}) = w_0 + \mathbf{w}^T\mathbf{x}$$

A reasonable **decision rule** would be

$$y = \begin{cases} 1, & \text{if } f(\mathbf{x}, \mathbf{w}) \geq 0 \\ -1, & \text{otherwise} \end{cases}$$

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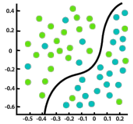

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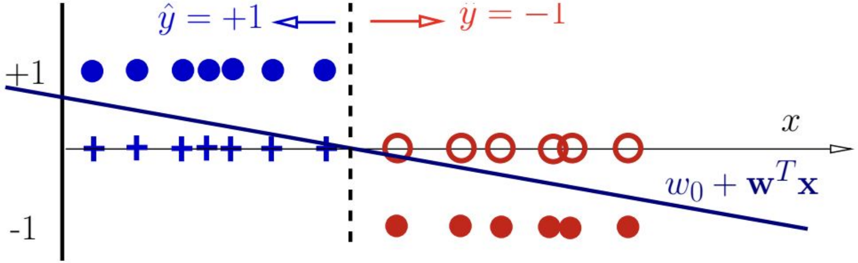

How do we mathematically represent that rule?

$$y = \mathrm{sign}(w_0 + \mathbf{w}^T\mathbf{x})$$

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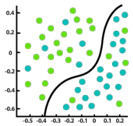

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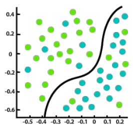

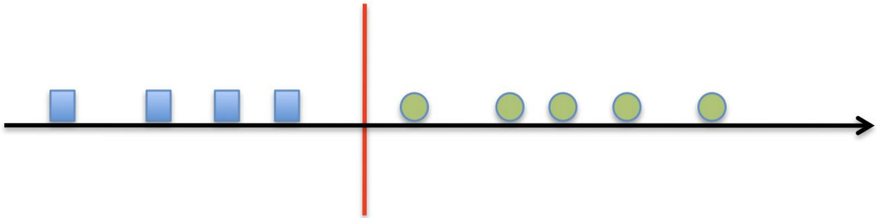

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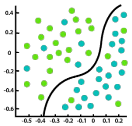

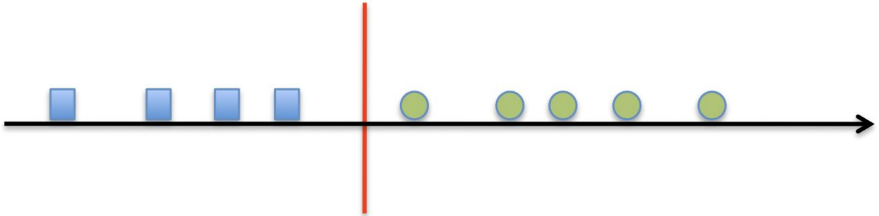

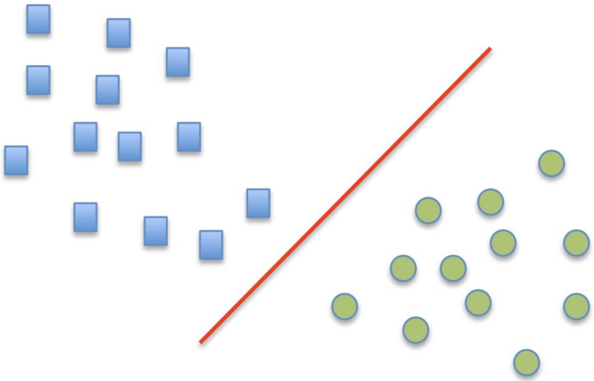

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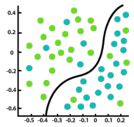

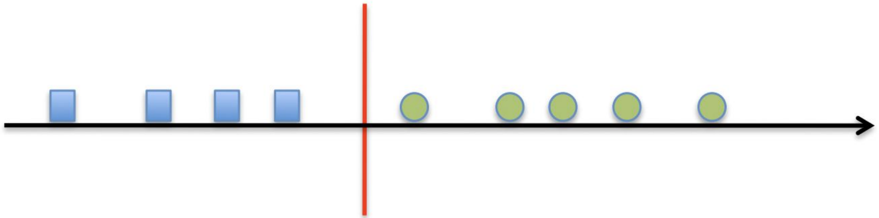

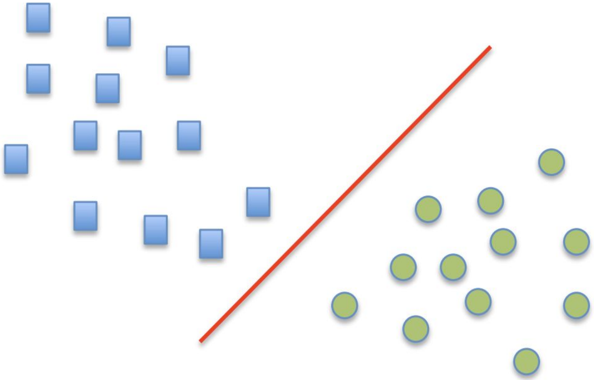

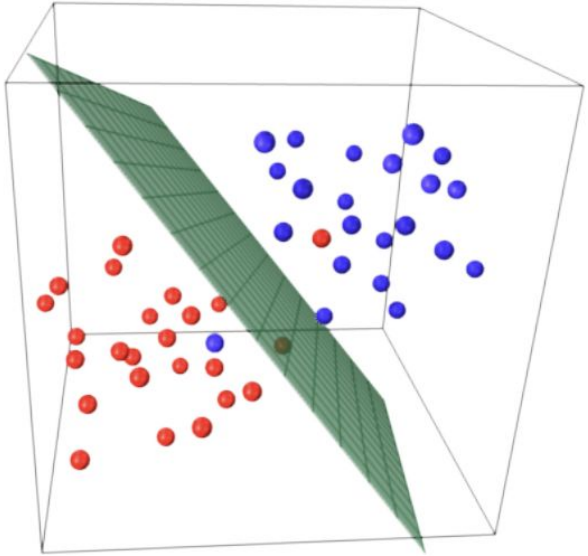

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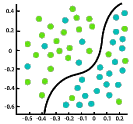
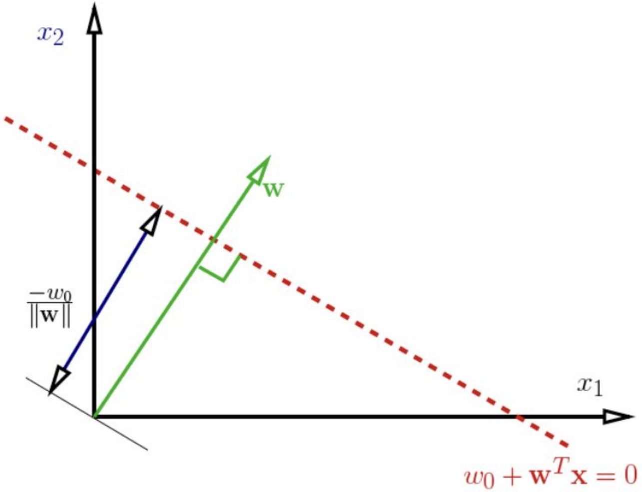


What does this represent geometrically?

$\mathbf{w}^T\mathbf{x} = 0$ is a line passing through the origin and is orthogonal to $\mathbf{w}$

$\mathbf{w}^T\mathbf{x} + w_0 = 0$ shifts the line by w_0



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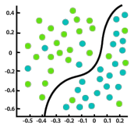

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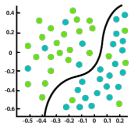

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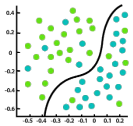

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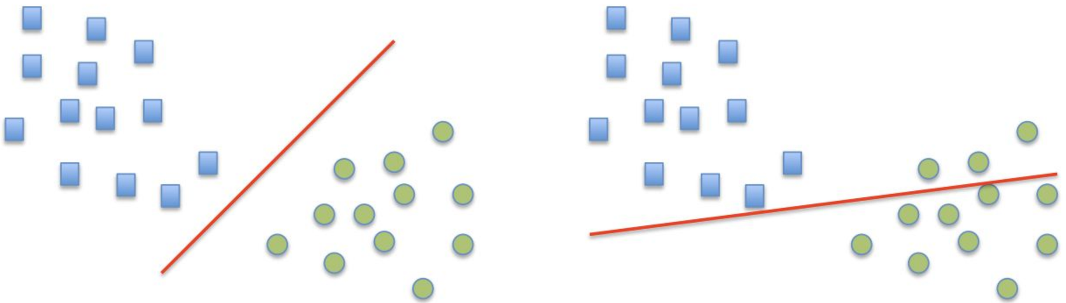

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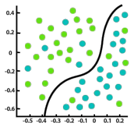

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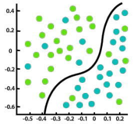

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Zero/one loss for a classifier

$$L_{0-1}(y(\mathbf{x}), t) = \begin{cases} 0 & \text{if } y(\mathbf{x}) = t \\ 1 & \text{if } y(\mathbf{x}) \neq t \end{cases}$$

Asymmetric Binary Loss

$$L_{ABL}(y(\mathbf{x}), t) = \begin{cases} \alpha & \text{if } y(\mathbf{x}) = 1 \wedge t = 0 \\ \beta & \text{if } y(\mathbf{x}) = 0 \wedge t = 1 \\ 0 & \text{if } y(\mathbf{x}) = t \end{cases}$$

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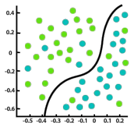

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Squared (quadratic) loss

$$L_{squared}(y(\mathbf{x}), t) = (t - y(\mathbf{x}))^2$$

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Absolute Error

$$L_{absolute}(y(\mathbf{x}), t) = |t - y(\mathbf{x})|$$

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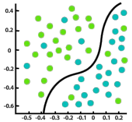

Can classes always be separated?

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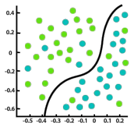

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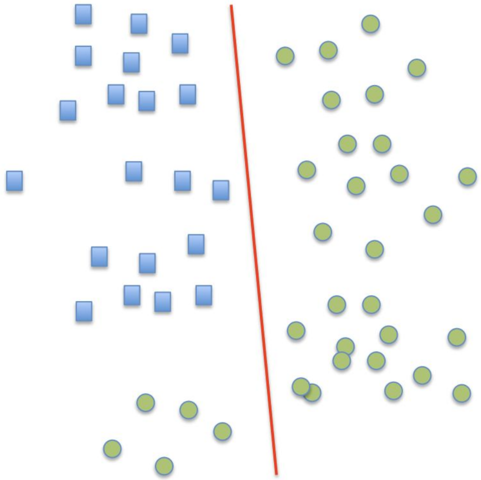

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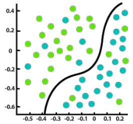

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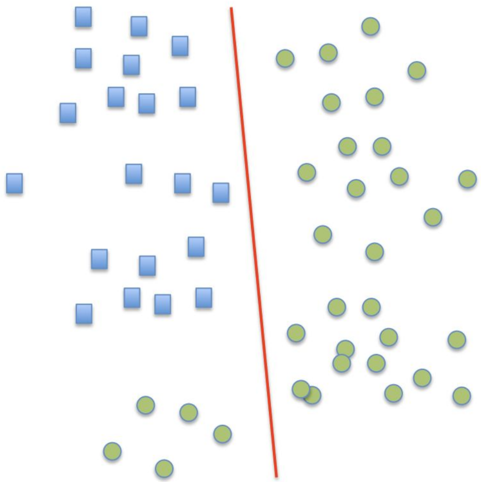

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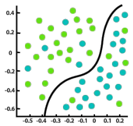

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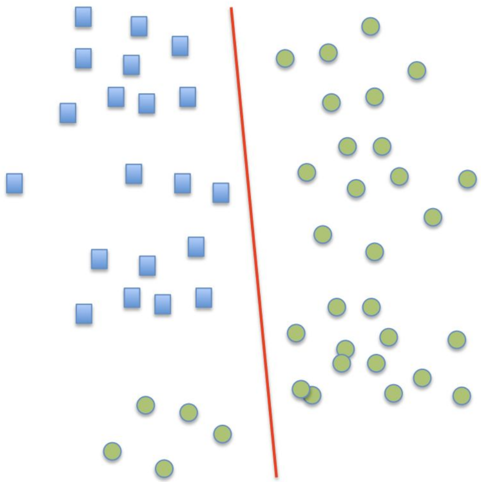

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Causes of Non-Perfect Separation:

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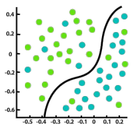

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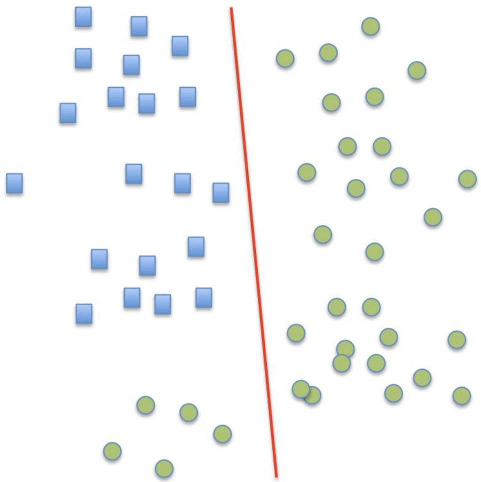

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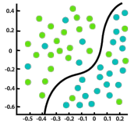

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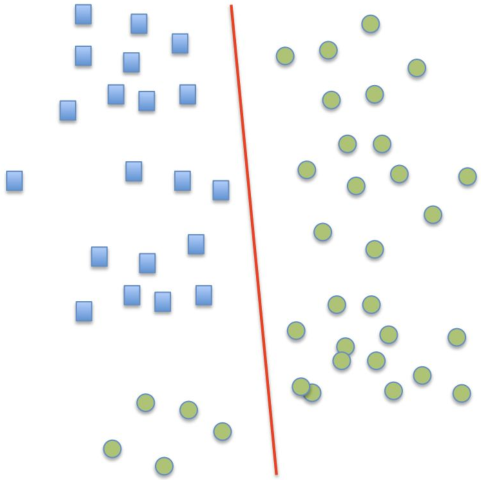

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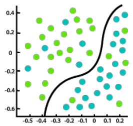

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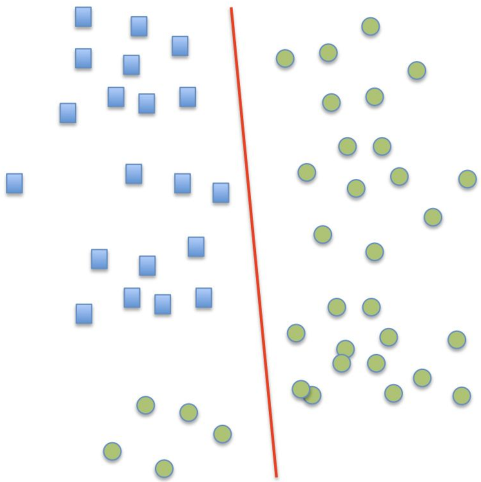

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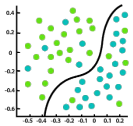

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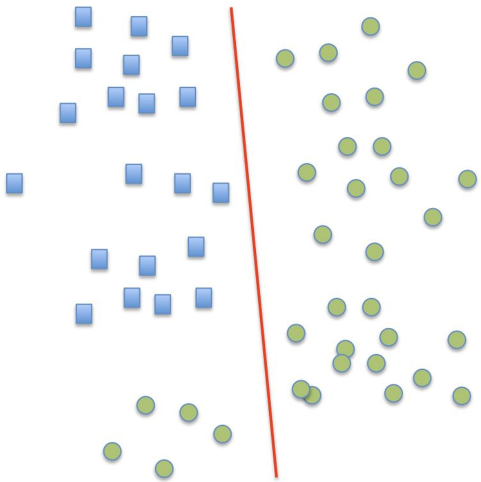

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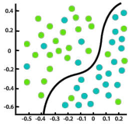

What is the cost of getting things wrong?

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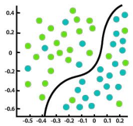

What is the cost of getting things wrong?

For medical diagnosis: For a diabetes screening test is it better to have **false positives** or **false negatives**?

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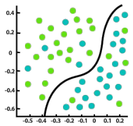

What is the cost of getting things wrong?

For medical diagnosis: For a diabetes screening test is it better to have **false positives** or **false negatives**?

For movie ratings: The ”truth” is that Alice thinks E.T. is worthy of a 4. How bad is it to predict a 5? How about a 2?

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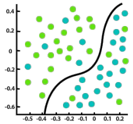

How to Assess Correctness?

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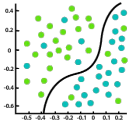

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		Actual Values	
		Positive	Negative
Predicted Values	Positive	True Positive	False Positive
	Negative	False Negative	True Negative

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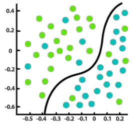
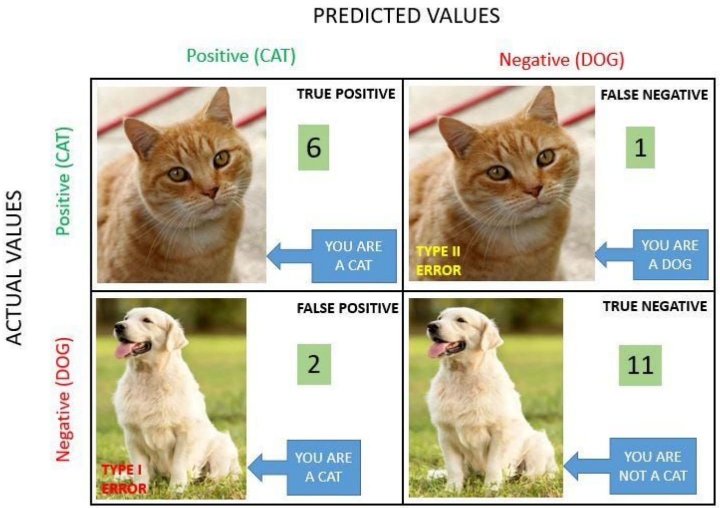


How to Assess Correctness?

		Actual Values: Positive	Actual Values: Negative
Predicted Values	Positive	True Positive	False Positive
Predicted Values	Negative	False Negative	True Negative



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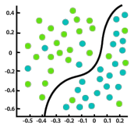

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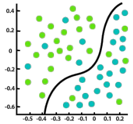

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Recall: is the fraction of relevant instances that are retrieved

$$R = \frac{TP}{TP + FN} = \frac{TP}{\text{all groundtruth instances}}$$

Precision: is the fraction of retrieved instances that are relevant

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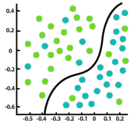

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Accuracy: fraction of correct instances

$$A = \frac{TP+TN}{TP+TN+FP+FN}$$

F1 score: harmonic mean of precision and recall

$$F1 = 2\frac{P \cdot R}{P + R}$$

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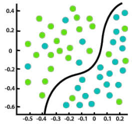

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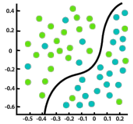


What is the difference between these metrics and loss?

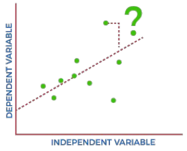
Metrics on the dataset is what we usually care about. This is what is usually referred to as **performance**.

Typically, it is **not possible to directly optimize for these metrics**. The **loss function should reflect the problem we are solving**. We then hope it will yield models that will do well on our dataset

Introduction: Logistic Regression

What is Regression?

***Regression** is a **statistical technique** that **relates a dependent variable** to **one or more independent** variables*

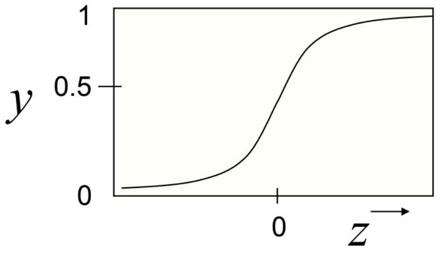


Is there a way to improve Linear Classification?

As an alternative, the *sign()* function can be replaced by the **sigmoid** or **logistic function.** Sigmoid applied to linear function of the data

$$\sigma(z) = \frac{1}{1 + \exp(-z)} \qquad y(\mathbf{x}) = \sigma\left(\mathbf{w}^T\mathbf{x} + w_0\right)$$

The output is a smooth function of the inputs and the weights. It can be seen as a **smoothed** and **differentiable** alternative to sign()



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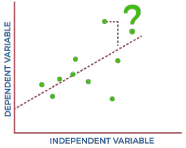

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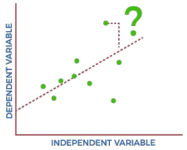

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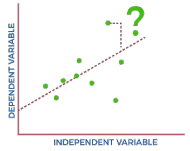

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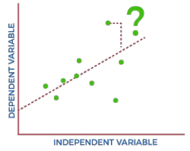

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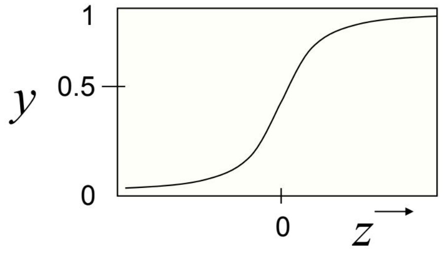

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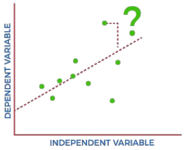
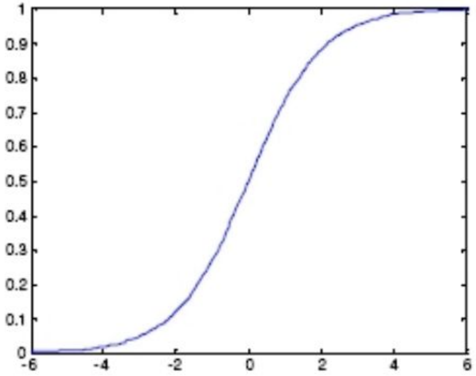
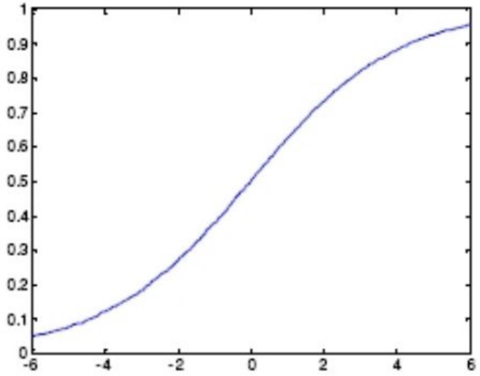


$$y = \sigma\left(w_1 x + w_0\right)$$

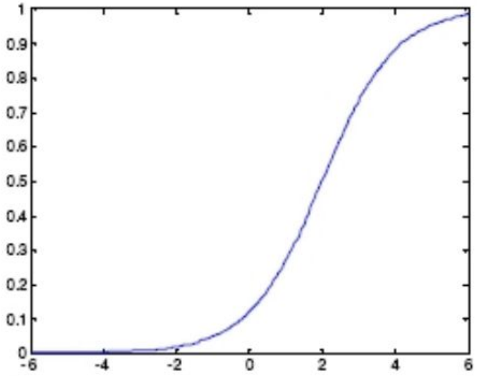
$w_0 = 0, w_1 = 1$



$w_0 = 0, w_1 = 0.5$



$w_0 = -2, w_1 = 1$



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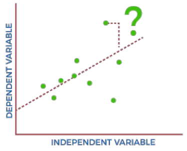
Models the probability of an event taking place *by having the* ***log-odds for the event be a linear combination*** *of one or more independent variables.*



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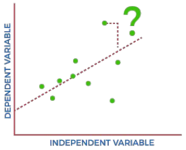

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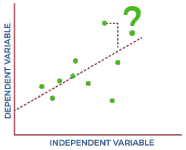

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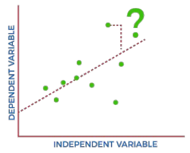

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By substitution…

$$p(C=0|\mathbf{x}) = \frac{1}{1+\exp\left(-\mathbf{w}^T\mathbf{x} - w_0\right)}$$

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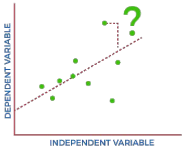

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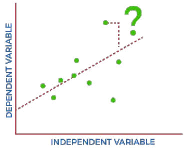

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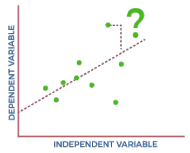

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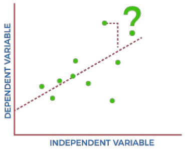

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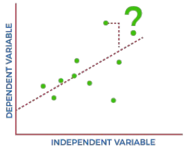

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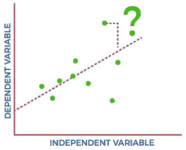

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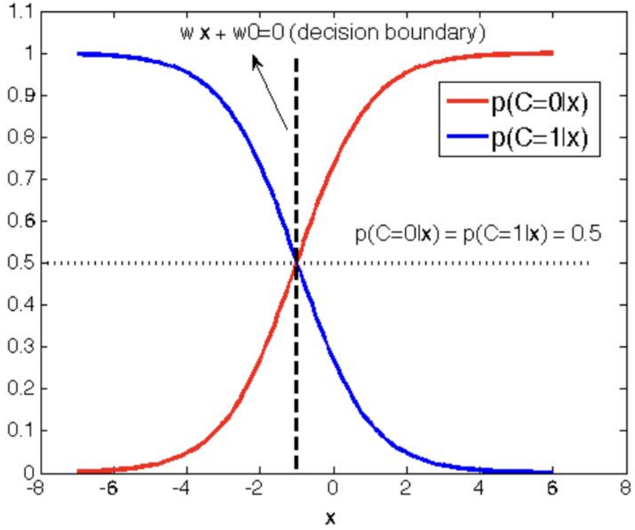

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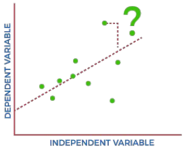
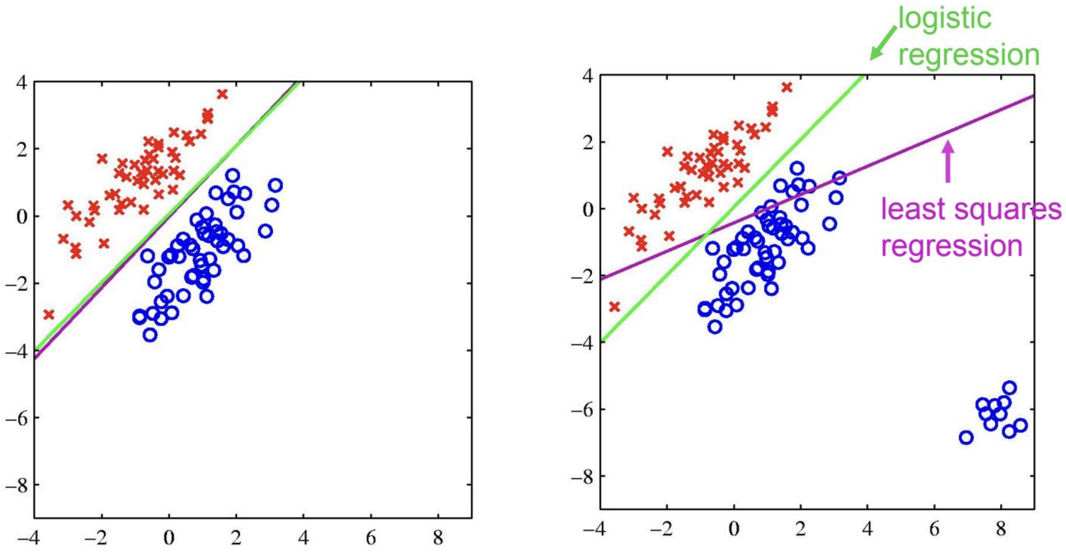


Logistic Regression vs. Least Squares



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Hours	0.50	0.75	1.00	1.25	1.50	1.75	1.75	2.00	2.25	2.50	2.75	3.00	3.25	3.50	4.00	4.25	4.50	4.75	5.00	5.50
Pass	0	0	0	0	0	0	1	0	1	0	1	0	1	0	1	1	1	1	1	1

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Learn the weights for our model (How to do so will come shortly after this)

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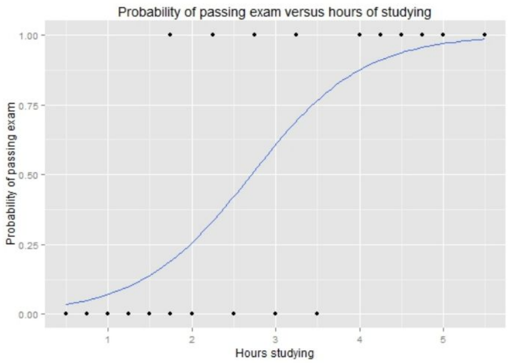
Pass	0	0	0	0	0	0	1	0	1	0	1	0	1	0	1	1	1	1	1	1

Learn the weights for our model (How to do so will come shortly after this)

Make predictions:



Hours of study	Probability of passing exam
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5	0.97

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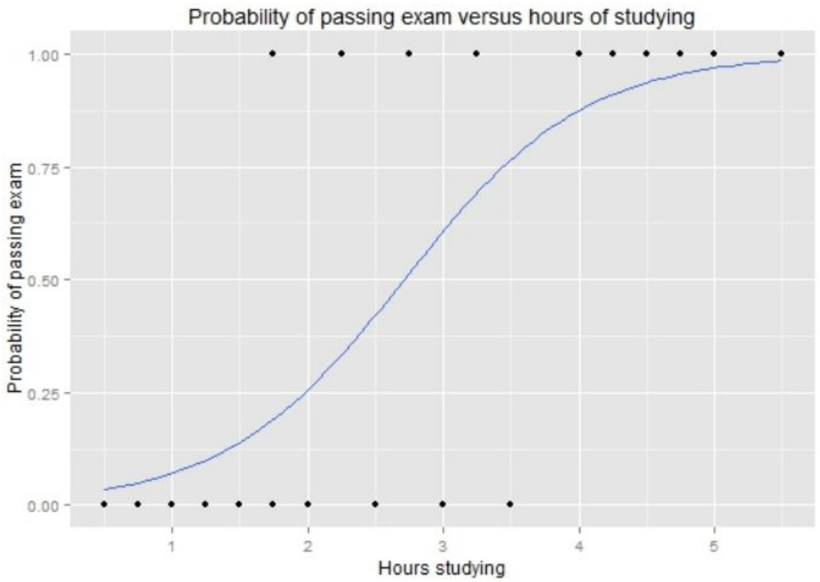

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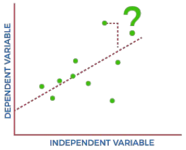

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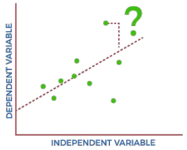

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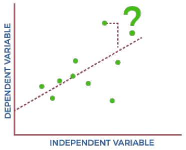

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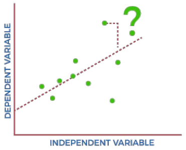

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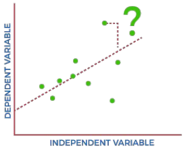

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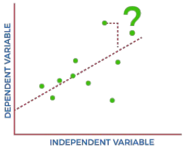

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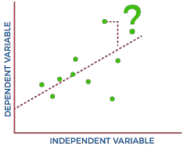

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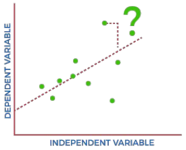

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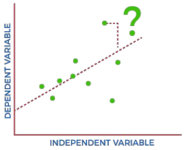

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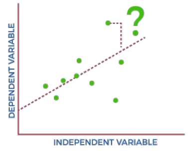

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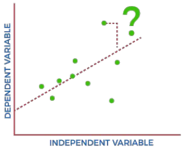

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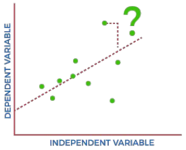

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This part is using the property of marginal probability

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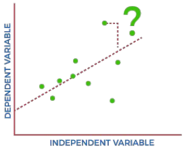

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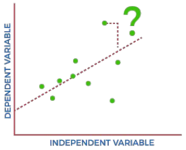

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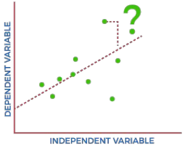

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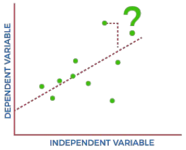

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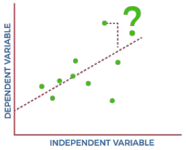

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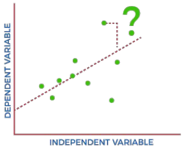

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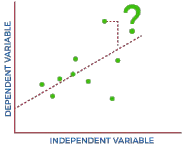

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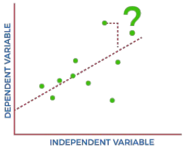

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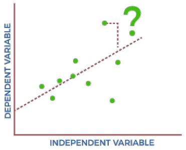

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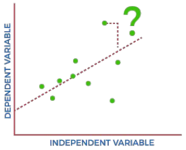

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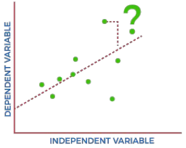

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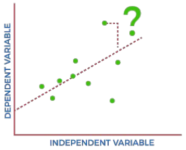

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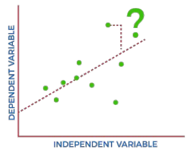

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$$p(C=0|\mathbf{x}) = \frac{1}{1 + \exp(-\mathbf{w}^T \mathbf{x} - w_0)}, \quad p(C=1|\mathbf{x}) = \frac{\exp(-\mathbf{w}^T \mathbf{x} - w_0)}{1 + \exp(-\mathbf{w}^T \mathbf{x} - w_0)}$$

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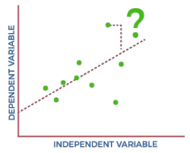

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The **loss** is $$\ell_{log-loss}(\mathbf{w}) = -\sum_{i=1}^{N} t^{(i)} \log p(C=1|\mathbf{x}^{(i)}, \mathbf{w}) - \sum_{i=1}^{N} (1-t^{(i)}) \log p(C=0|\mathbf{x}^{(i)}, \mathbf{w})$$

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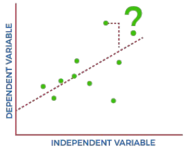

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Such that:

$$p(C=0|\mathbf{x},\mathbf{w}) = \frac{1}{1+\exp(-z)} \qquad p(C=1|\mathbf{x},\mathbf{w}) = \frac{\exp(-z)}{1+\exp(-z)} \qquad z = \mathbf{w}^T\mathbf{x} + w_0$$

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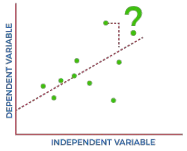

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$$\ell_{log-loss}(\mathbf{w}) = -\sum_{i=1}^{N} t^{(i)} \log p(C=1|\mathbf{x}^{(i)}, \mathbf{w}) - \sum_{i=1}^{N} (1-t^{(i)}) \log p(C=0|\mathbf{x}^{(i)}, \mathbf{w})$$

Such that:

$$p(C=0|\mathbf{x}, \mathbf{w}) = \frac{1}{1+\exp(-z)} \qquad p(C=1|\mathbf{x}, \mathbf{w}) = \frac{\exp(-z)}{1+\exp(-z)} \qquad z = \mathbf{w}^T\mathbf{x} + w_0$$

Can **simplify** by using properties of logarithms:

Logistic Regression

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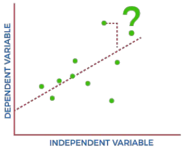
Models the probability of an event taking place *by having the* ***log-odds for the event be a linear combination*** *of one or more independent variables.*



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$$\begin{aligned}
\ell(\mathbf{w})_{log-loss} &= \sum_i t^{(i)} \log(1+\exp(-z^{(i)})) + \sum_i t^{(i)} z^{(i)} + \sum_i (1-t^{(i)}) \log(1+\exp(-z^{(i)})) \\
&= \sum_i \log(1+\exp(-z^{(i)})) + \sum_i t^{(i)} z^{(i)}
\end{aligned}$$

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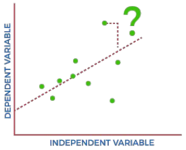

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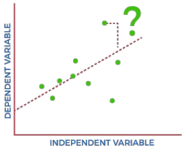

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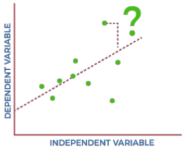

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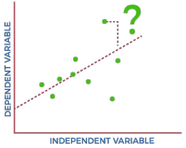

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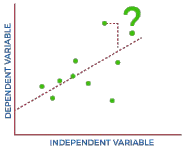

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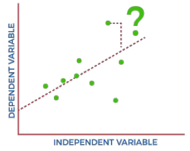

How to correctly learn the weights?

Gradient descent for logistic regression:

$$w_j^{(t+1)} \leftarrow w_j^{(t)} - \lambda \sum_i x_j^{(i)} \left(t^{(i)} - p(C = 1 | \mathbf{x}^{(i)}; \mathbf{w}) \right)$$

where:

$$p(C = 1 | \mathbf{x}^{(i)}; \mathbf{w}) = \frac{\exp(-\mathbf{w}^T \mathbf{x} - w_0)}{1 + \exp\left(-\mathbf{w}^T \mathbf{x} - w_0\right)} = \frac{1}{1 + \exp\left(\mathbf{w}^T \mathbf{x} + w_0\right)}$$