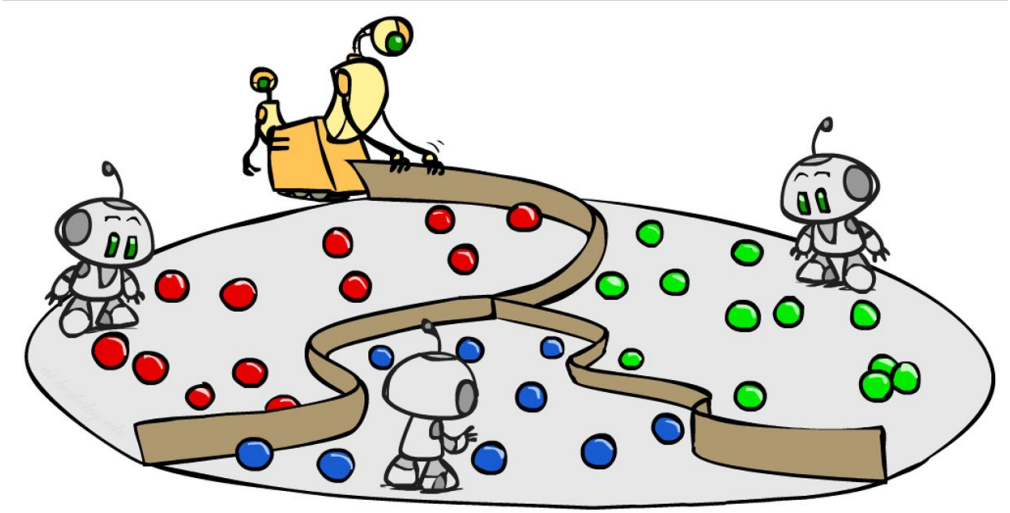
CS-ELEC2C: Machine Learning

Loss Functions, Evaluation, and Linear Regression



Legend		Machine Learning A.Y. 2024 - 2025, 2nd Term
Onsite	Major Exam	Instructor: Red M Castilla
Online	No Class	College of Information and Computing Sciences
Project		Schedule: Saturday 8AM - 10AM | 10:30AM - 1:30PM

Date	Day	Week	Session	Activity / Lesson
18-Jan	Saturday	Week #1	Lecture	Overview of Advanced Intelligent Systems
			Laboratory	Pre-Test Examination and Introductions
25-Jan	Saturday	Week #2	Lecture	Loss Functions and Linear Regression
			Laboratory	Summative Lab Exercise #1 Discussion
1-Feb	Saturday	Week #3	Lecture	Linear Classification and Logistic Regression
			Laboratory	Summative Lab Exercise #1 Deadline + Formative Lab Exercise #1
8-Feb	Saturday	Week #4	Lecture	Naïve Bayes and Decision Trees
			Laboratory	Long Examination #1
15-Feb	Saturday	Week #5	Lecture	k-Nearest Neighbors and Support Vector Machines
			Laboratory	Formative Lab Exercise #2

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What goes in the Machine Learning Workflow?

***Preprocessing** the data. **Creating models** specific for the task. **Evaluating** if the model has performed with respect to expectations*

Recap on the Machine Learning Setup

What are the elements required for machine learning?

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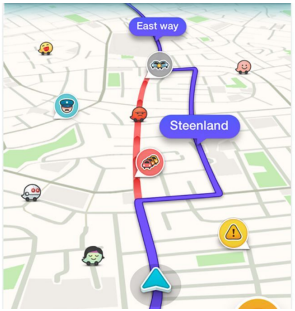

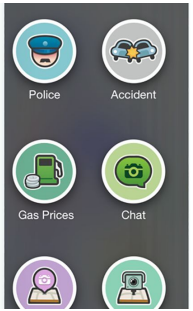

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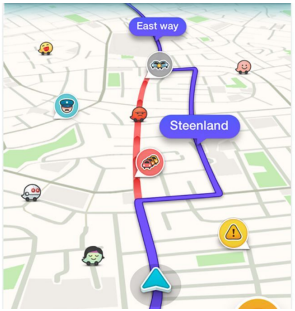

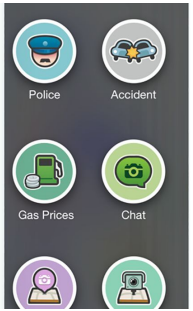

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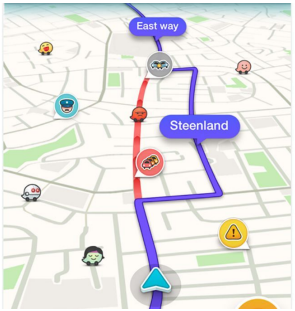

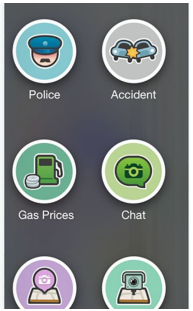

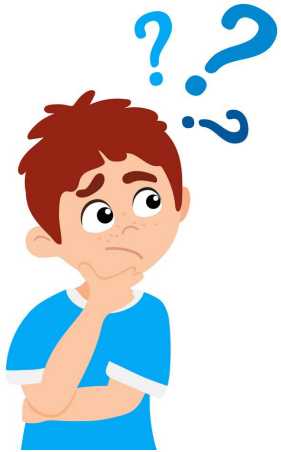

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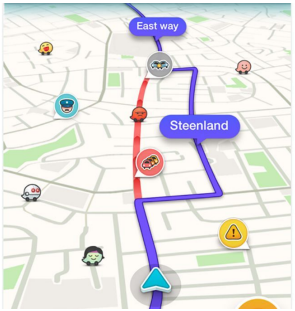

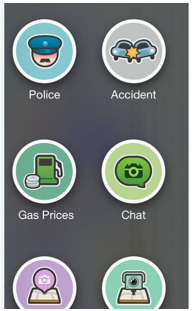

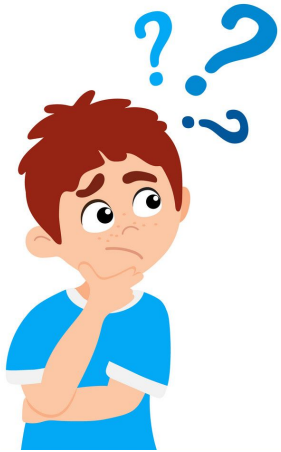

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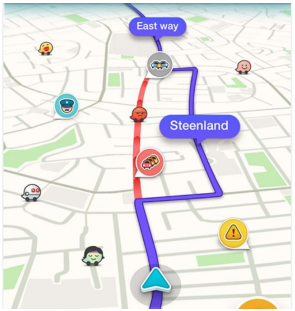

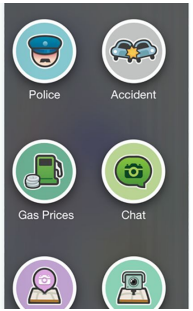

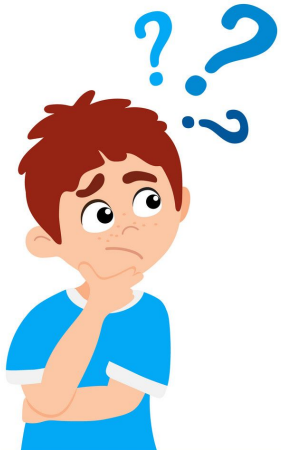

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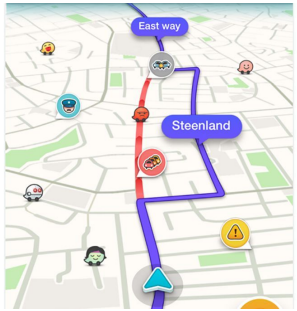

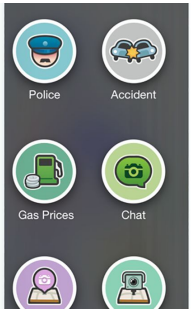

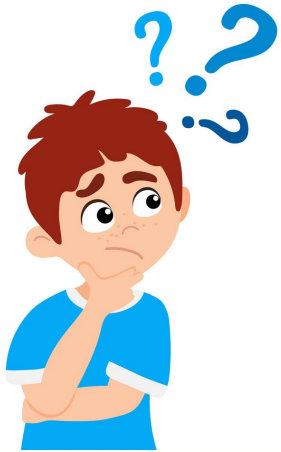

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What did Waze estimate?

How far was Waze's estimate from the actual time of arrival?

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In mathematical optimization and decision theory, a **loss function** or **cost function** (sometimes also called an **error function**) is a function that **maps an event or values of one or more variables onto a real number** intuitively representing some "cost" associated with the event. An **optimization problem seeks to minimize a loss function.**

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$$Y = f(x)$$

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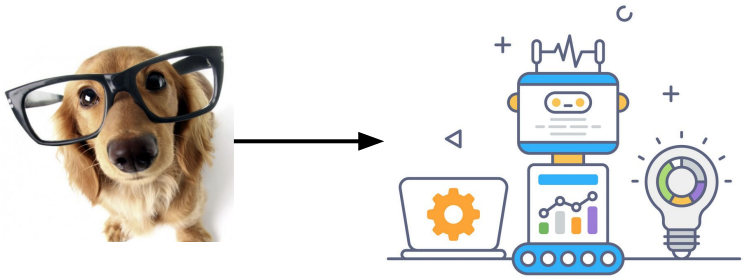

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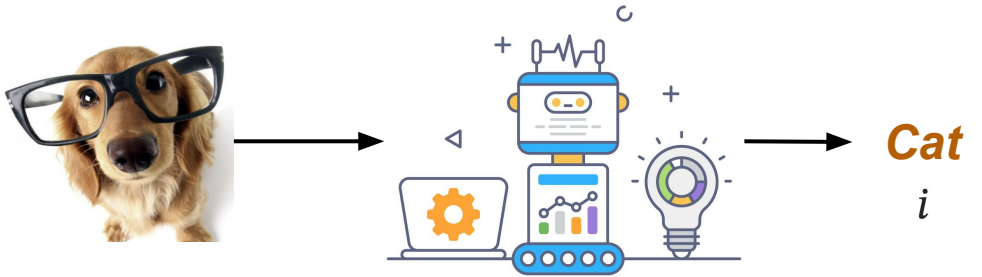

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i

Output

Dog

j

True Value

$$L(i,j) = \begin{cases} 0 & i = j \\ 1 & i \neq j \end{cases}$$

Simple Loss Function

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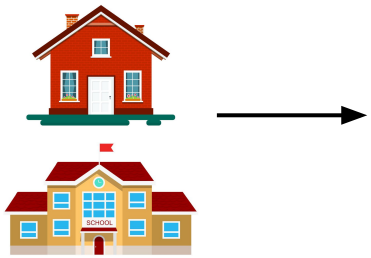

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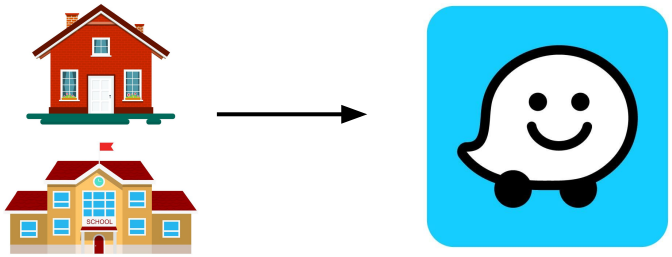

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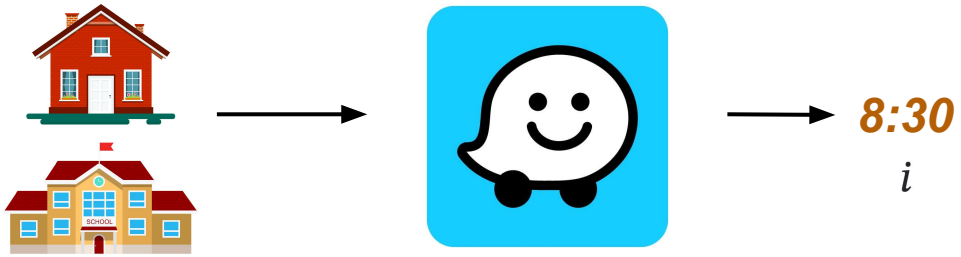

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8:30

i

Output

8:50

j

True Value

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i

Output

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j

True Value

$$L(i,j) = j - i$$

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Sum of Squared Error

$$RSS = \sum_{i=1}^{n}(y_i - f(x_i))^2$$

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$$\text{RMSD} = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \hat{x}_i)^2}{N}}$$

Sum of Squared Error

$$RSS = \sum_{i=1}^{n}(y_i - f(x_i))^2$$

Classification Problems

Binary Cross Entropy Loss

$$H_p(q) = -\frac{1}{N}\sum_{i=1}^{N} y_i \cdot log(p(y_i)) + (1 - y_i) \cdot log(1 - p(y_i))$$

Loss Functions

Why do we need this?

*It **quantifies the difference** between **predicted** and **actual values** in a machine learning model.*

What are mostly used loss functions?

Regression Problems

Mean Squared Error

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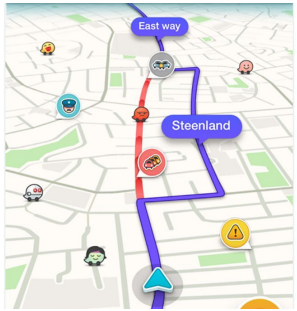
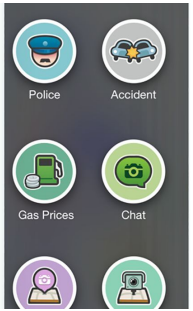
$$L(\hat{y}, y) = -\sum_{k}^{K} y^{(k)} \log \hat{y}^{(k)}$$

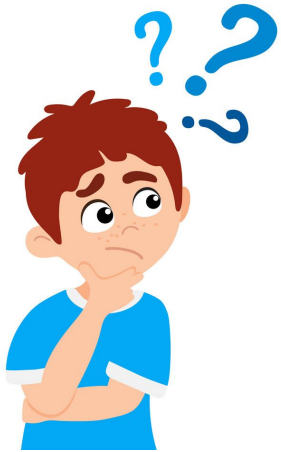
Waze Scenario

Scenario:

Suppose you are **travelling** from **your home** to **UST** to attend your morning classes. Your **class is at 9:00 AM** and Waze has **estimated that you will arrive at 8:30AM.** However, due to heavy traffic, **you arrived at 8:50AM**







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What are were the inputs to Waze?

What did Waze estimate?

How far was Waze's estimate from the actual time of arrival?

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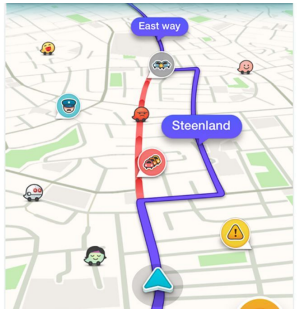

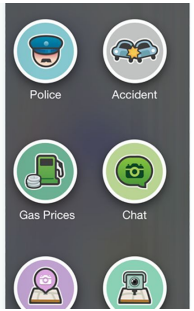

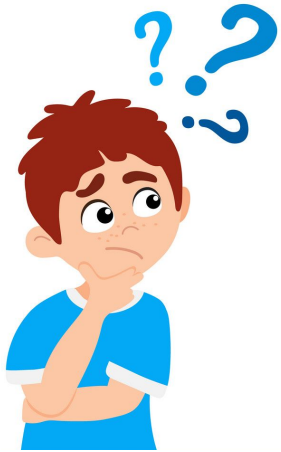

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What factors do you think Waze considered for estimating?

Recap on the Machine Learning Setup

What are the elements required for machine learning?

Output Space

Target variable that is desired to be **estimated**. These are **all possible outputs** that can be generated by the model based on the inputs.

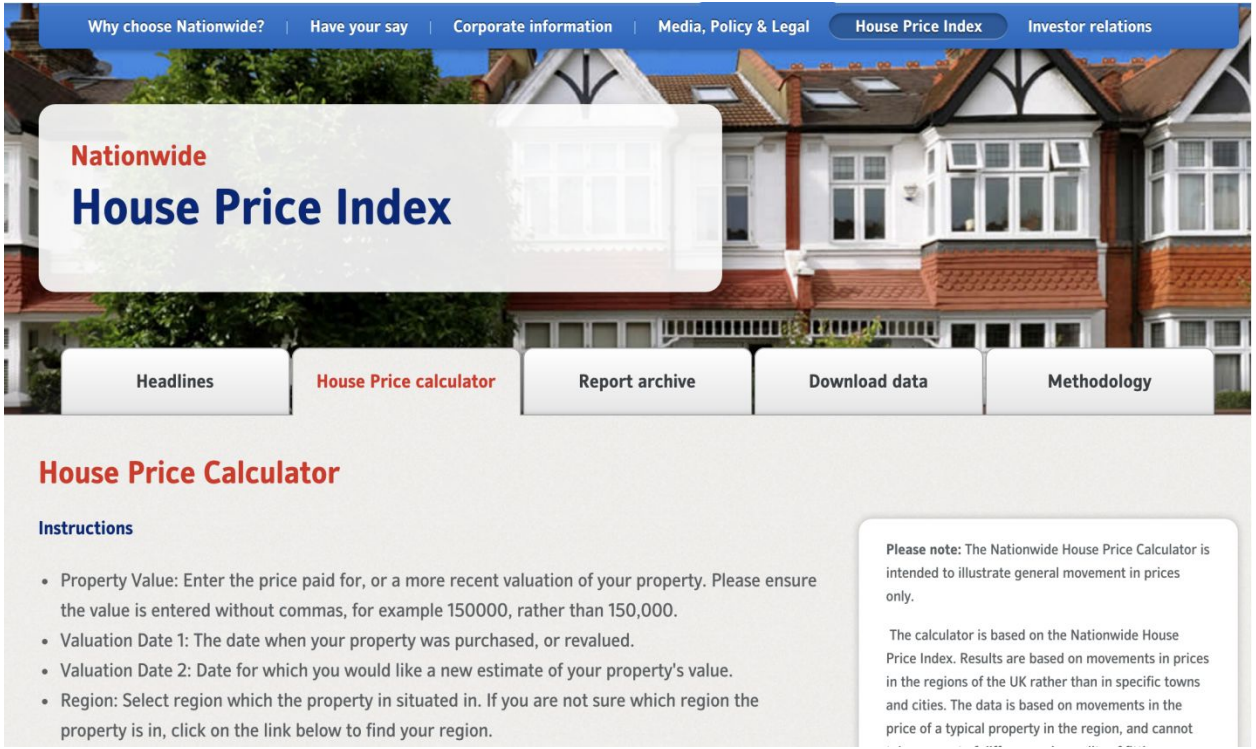
Hypothesis

Some **speculative relationship** between the input space and the output space. It is **expressed as a collection of parameters characterizing the behavior of the model**.

Input Space

This is the **input data**. These can be either called as variables, features, and attributes. The input space comprises **all potential sets of values for input**.

How can we…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Methodology
House Price Calculator
Instructions
Property Value: Enter the price paid for, or a more recent valuation of your property. Please ensure the value is entered without commas, for example 150000, rather than 150,000.
Valuation Date 1: The date when your property was purchased, or revalued.
Valuation Date 2: Date for which you would like a new estimate of your property's value.
Region: Select region which the property in situated in. If you are not sure which region the property is in, click on the link below to find your region.
Please note: The Nationwide House Price Calculator is intended to illustrate general movement in prices only.
The calculator is based on the Nationwide House Price Index. Results are based on movements in prices in the regions of the UK rather than in specific towns and cities. The data is based on movements in the price of a typical property in the region, and cannot

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Optimization: A way of finding the parameters of our model that minimizes the loss function

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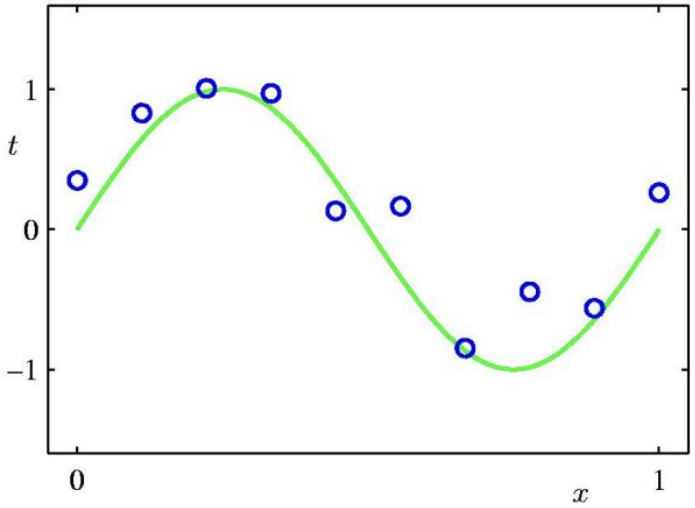

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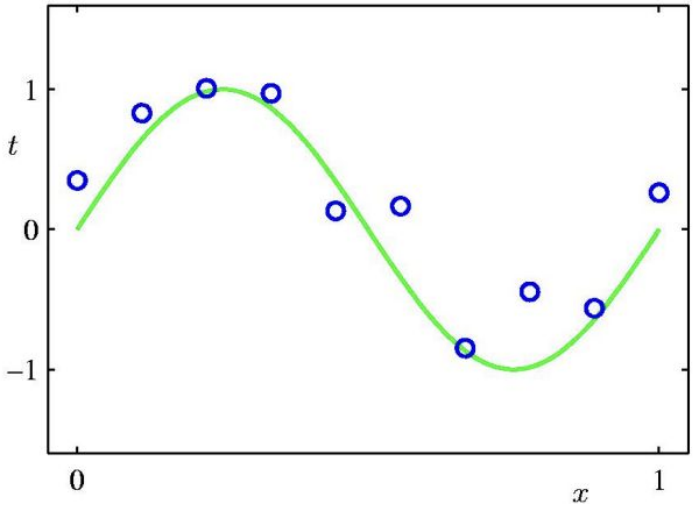

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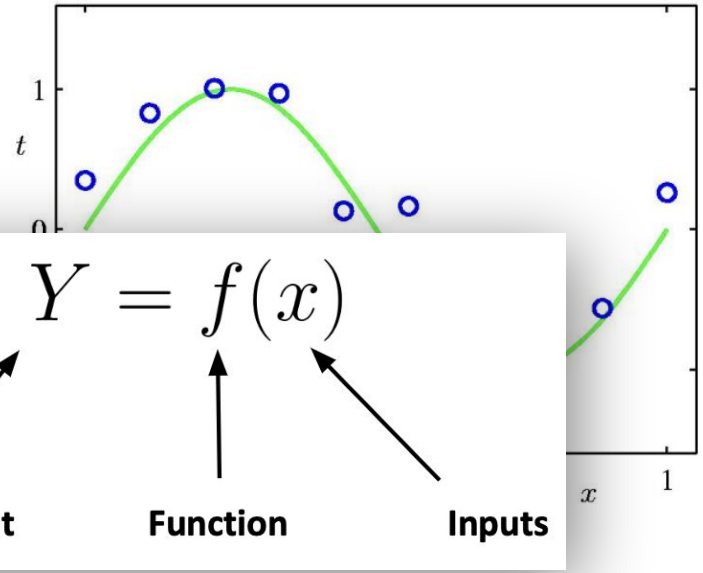

Recall $Y = f(x)$

Target Output **Function** **Inputs**

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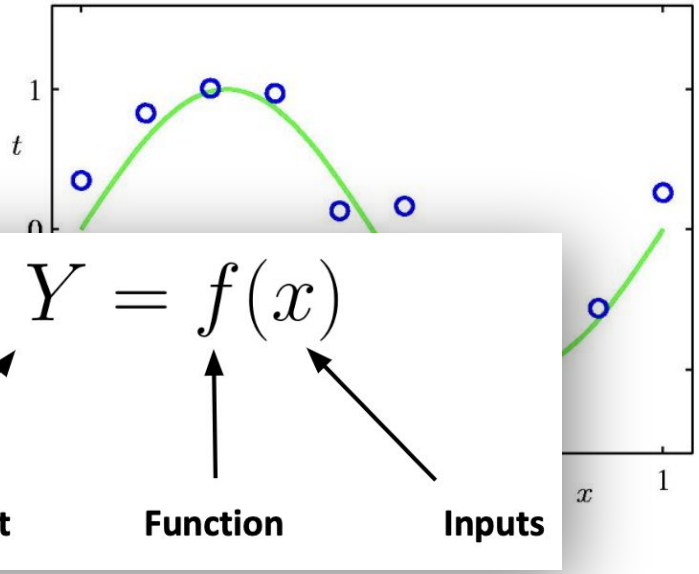
The function *f* is the model that the algorithm wants to estimate.



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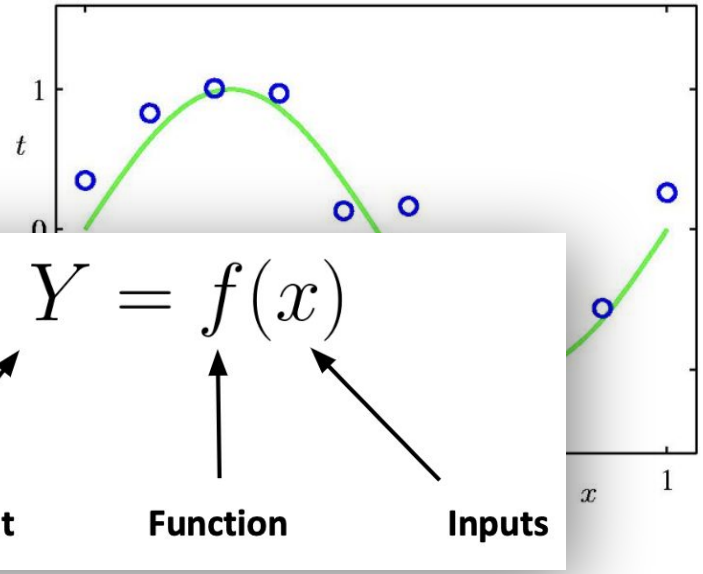

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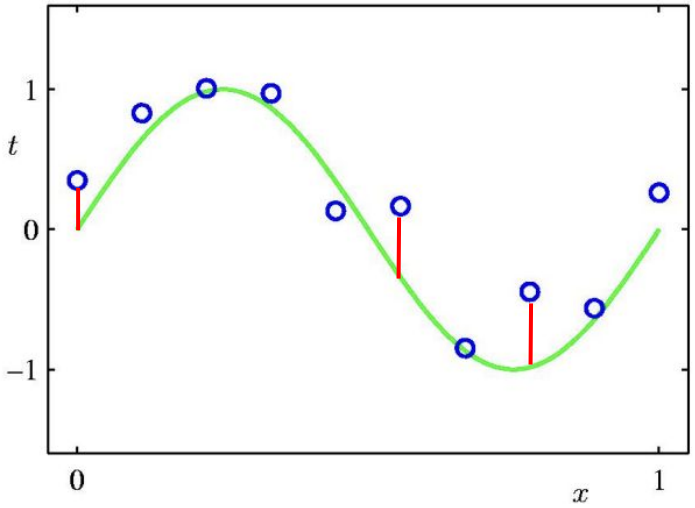

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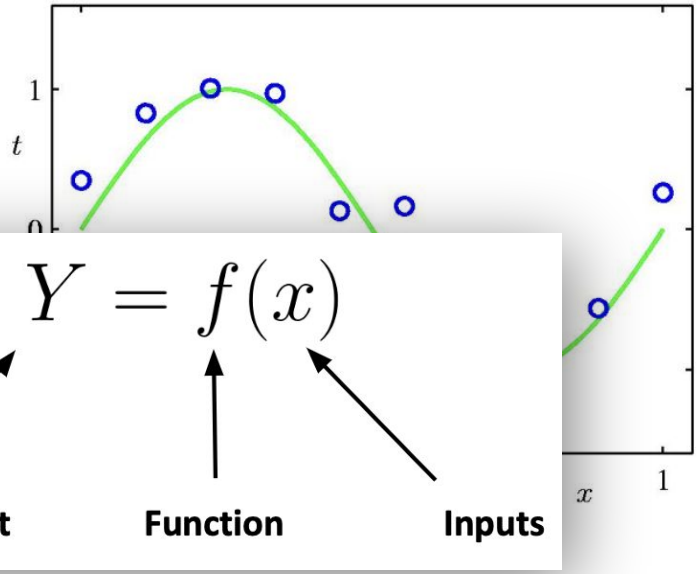
The target given that there is also error involved since the algorithm isn't perfect



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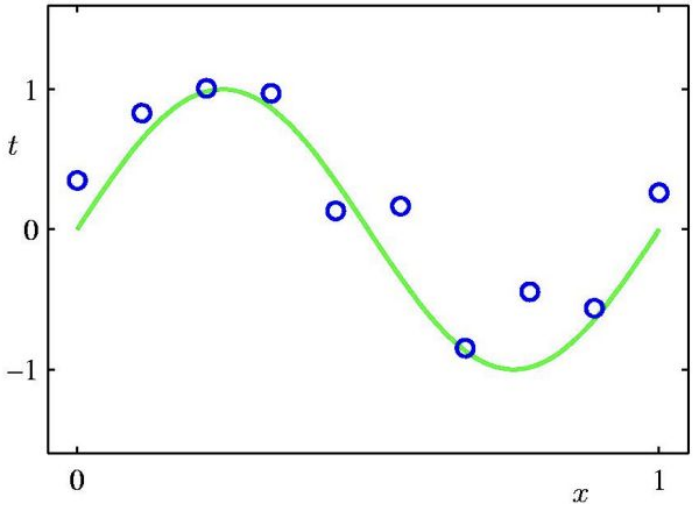

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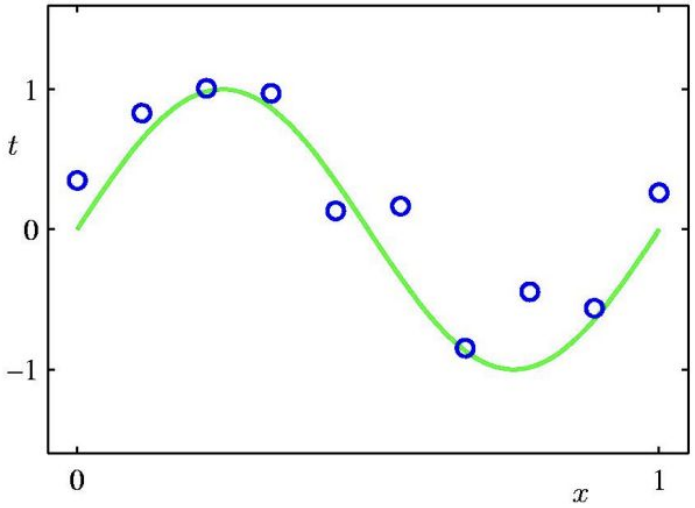
Goal: We want to **fit a curve** to the **data points**



Working Example: Boston Housing Data

Objective: **Estimate median house price in a neighborhood** based on neighborhood statistics

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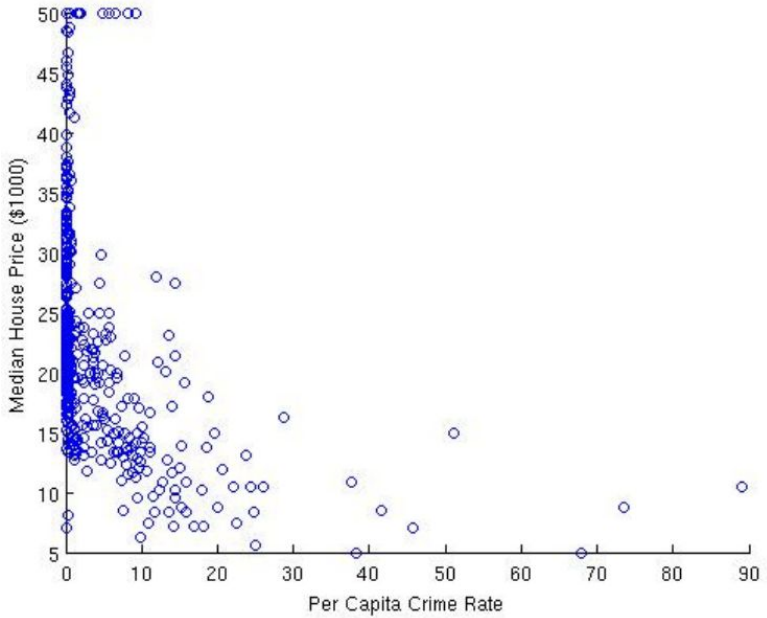

First possible feature: Per Capita Crime Rate

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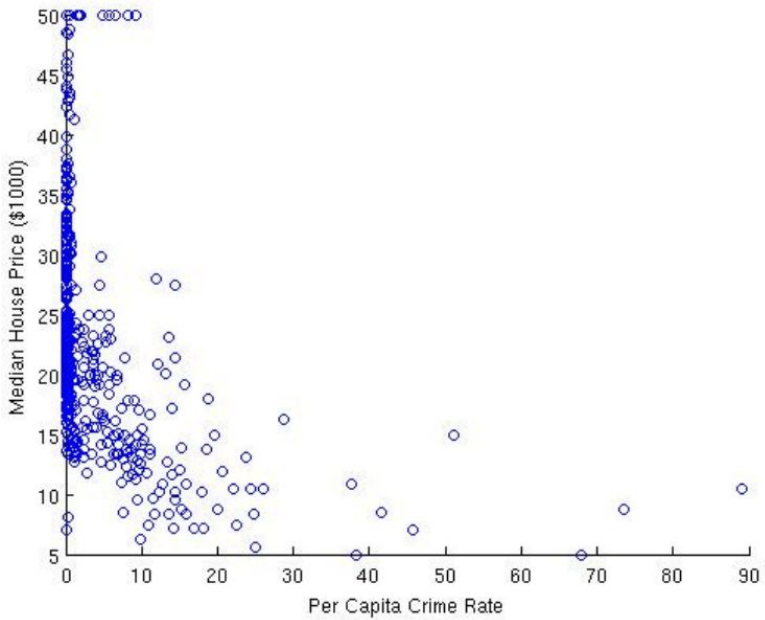

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Do you think this is a **good feature?**

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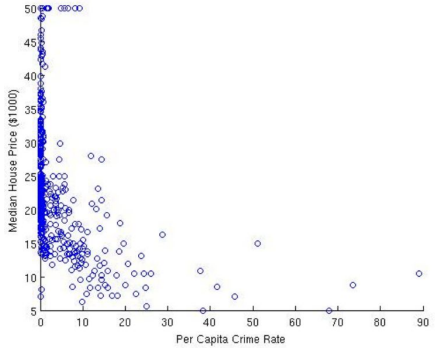

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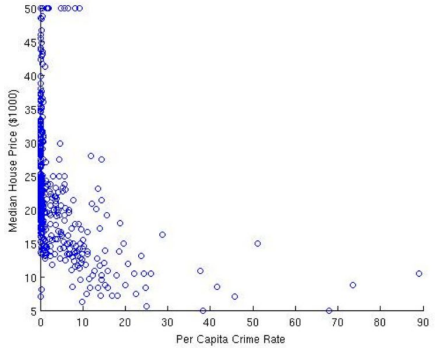

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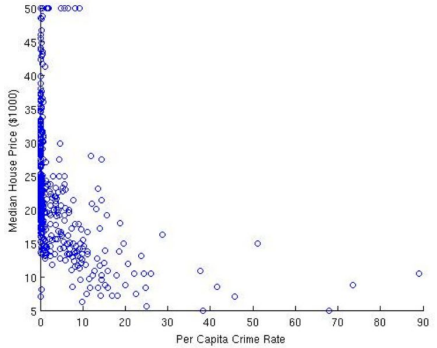

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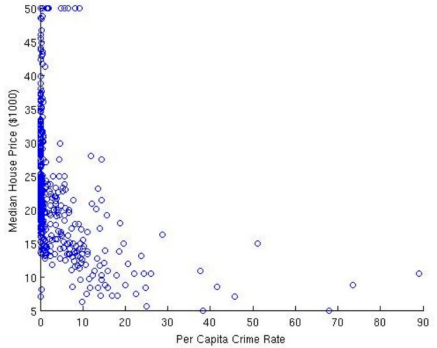

How does it look like for our example?

The **Median House Price** is t

The **Per Capita Crime Rate** is x

Each **dot in the plot is one data point** $(x^{(1)}, t^{(1)}), \cdots$

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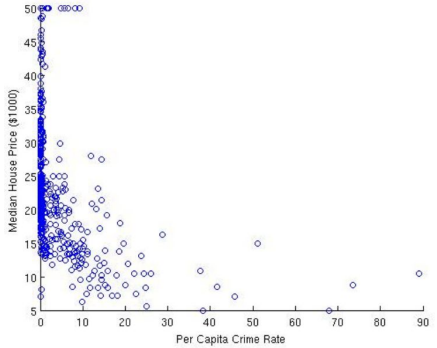

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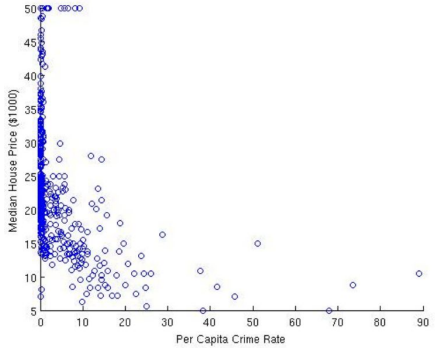

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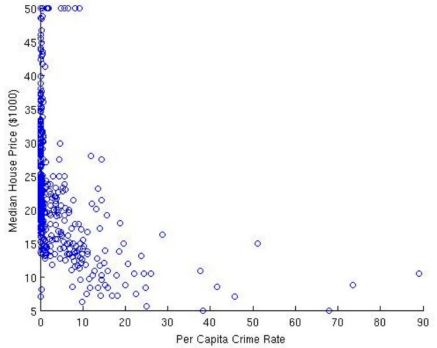

How does it look like for our example?

Working Example: Boston Housing Data

Objective: **Estimate median house price in a neighborhood** based on neighborhood statistics

Features

Training Data

Model

Loss

Optimization

How to represent the data?

Data is described as pairs $D = \{(x^{(1)}, t^{(1)}), \cdots, (x^{(N)}, t^{(N)})\}$

$x \in \mathbb{R}$ is the **input feature** (per capita crime rate)

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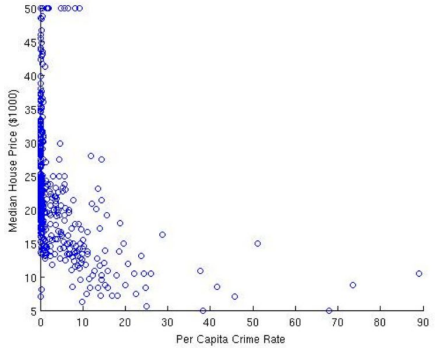

How does it look like for our example?

The **Median House Price** is t

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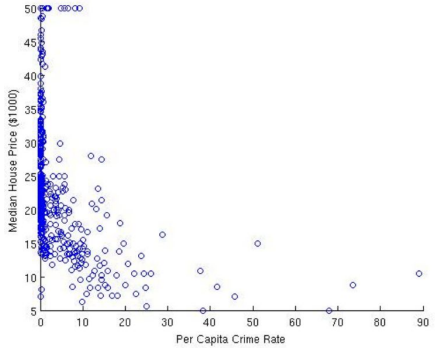

How does it look like for our example?

The **Median House Price** is t

The **Per Capita Crime Rate** is x

Working Example: Boston Housing Data

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- *Features*
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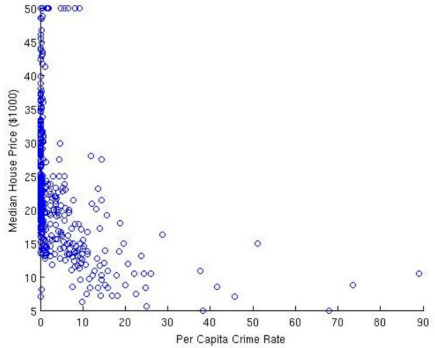

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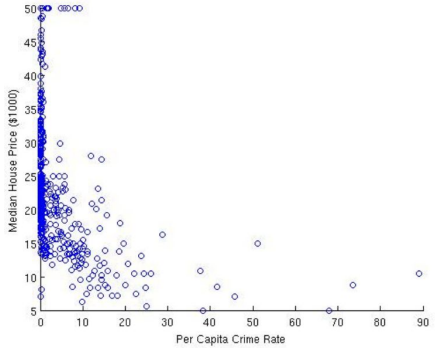
We have 1 feature / variable. Is there any model that you can think of?



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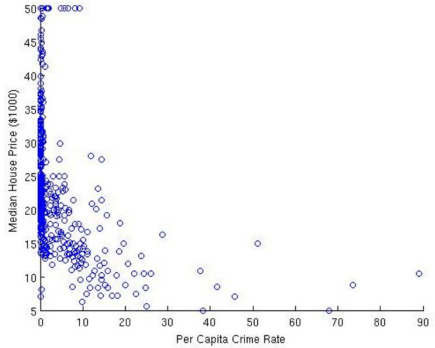

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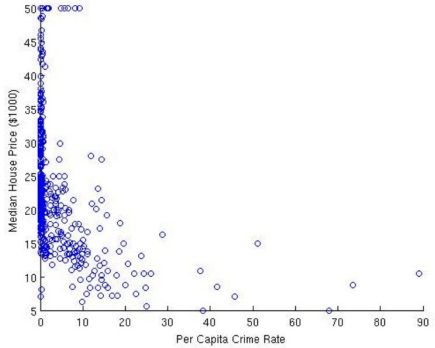

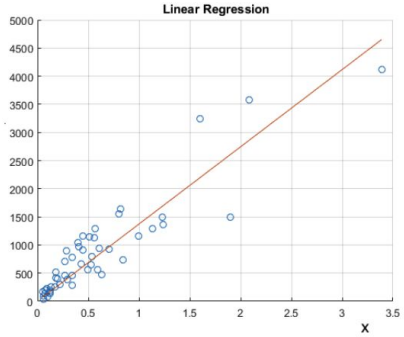

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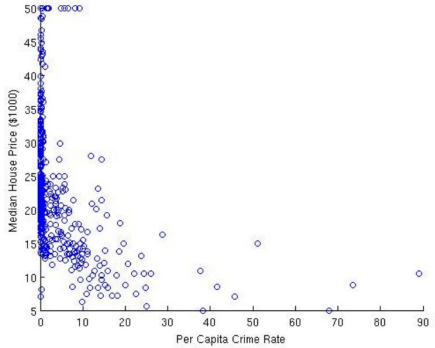

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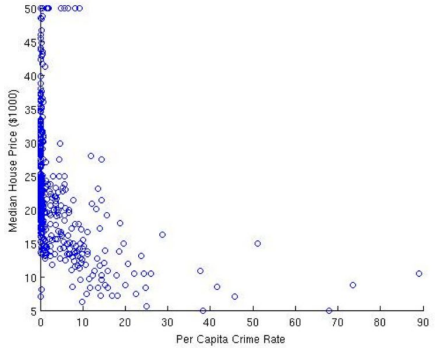
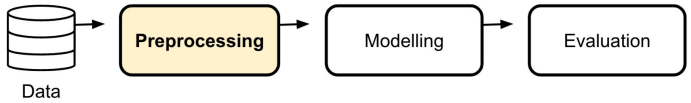


The Usual Machine Learning Workflow



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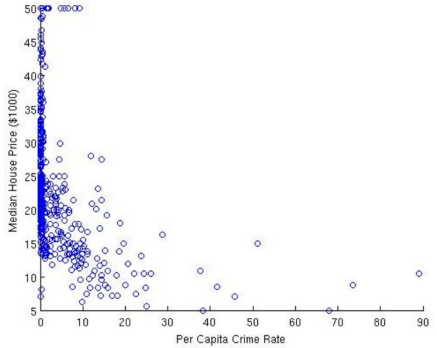

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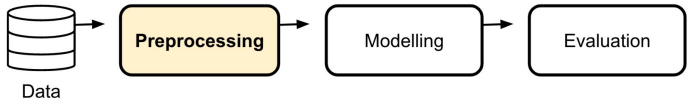

In practice, at this point, the data should have been split already to **training and testing sets**. The model should map the input x to y

Working Example: Boston Housing Data

Objective: **Estimate median house price in a neighborhood** based on neighborhood statistics

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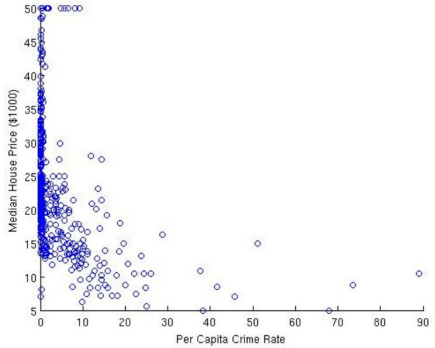
What about noise?



Working Example: Boston Housing Data

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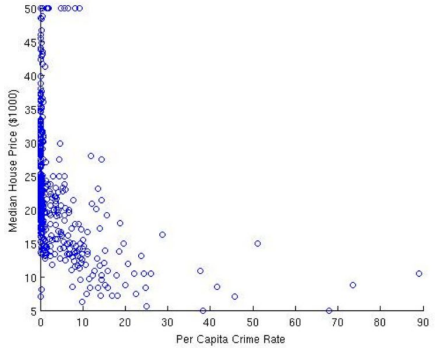
What about noise?

A simple model typically **does not exactly fit the data** – lack of fit can be considered **noise**



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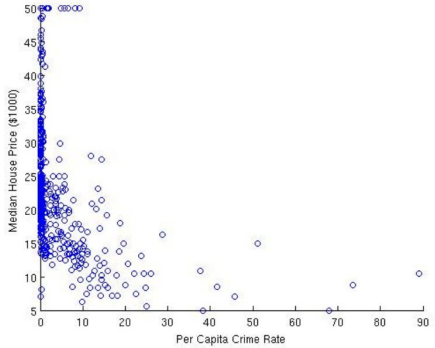

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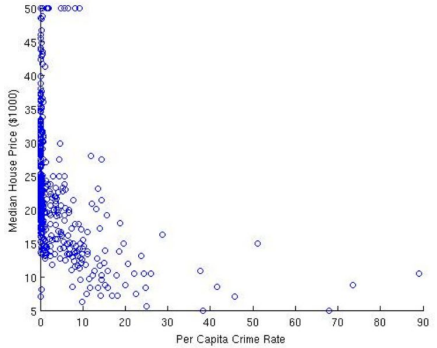

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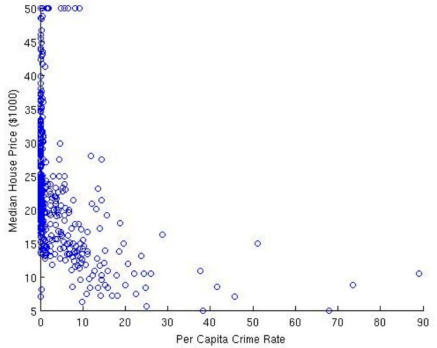
Errors in data targets (mislabelling)



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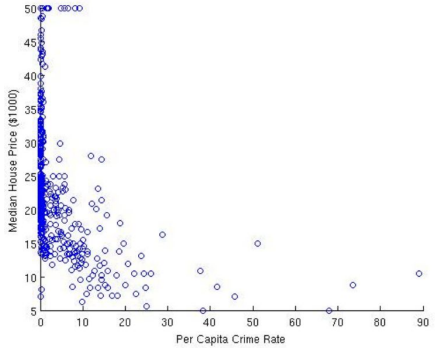

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Additional attributes **not taken into account** by data attributes, affect target values (latent variables)

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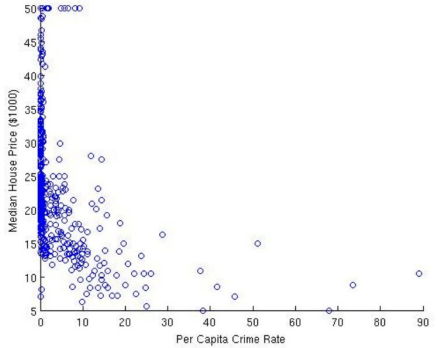

What are the sources of noise?

Imprecision in data Attributes (input noise)

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Additional attributes **not taken into account** by data attributes, affect target values (latent variables)

Model may be too simple to account for data targets

Identifying the Loss Function

Objective: **Estimate median house price in a neighborhood** based on neighborhood statistics

What loss do we use?

Features

Training Data

Model

Loss

Optimization

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The loss function measures the **squared error between true labels**

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$$\ell(\mathbf{w}) = \sum_{n=1}^{N} [t^{(n)} - (w_0 + w_1 x^{(n)})]^2$$

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This is the model that predicts the output given feature *x*

Identifying the Loss Function

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This is the true labels of the example

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Why?

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Why do we need to square the difference?

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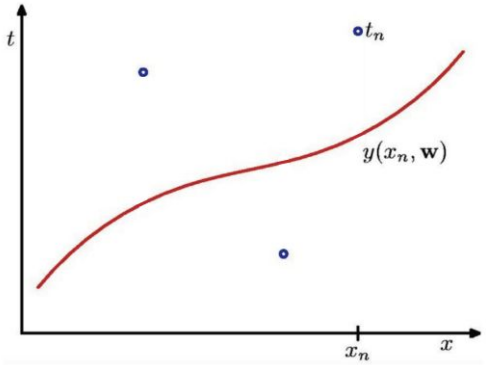

Using Sum of Squared Error Loss

The loss function measures the **squared error between true labels**

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For a particular hypothesis (y(x) defined by a choice of ***w***, drawn in red), **what does the loss represent geometrically**?

Identifying the Loss Function

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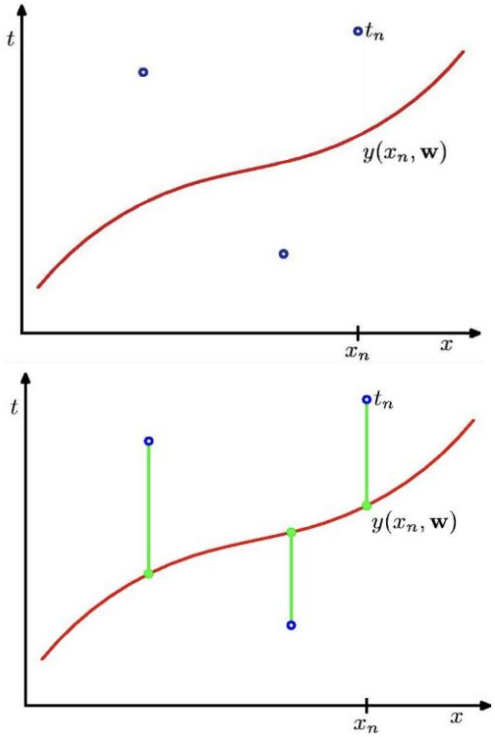
The **loss for the red hypothesis is the sum of the squared vertical errors**



Optimizing the Objective

Objective: **Estimate median house price in a neighborhood** based on neighborhood statistics

How to figure out the weights?

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Training Data

Model

Loss

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Initialize $\mathbf{w}$ (e.g. random initialization)

Sample: $y(x) = 0.5 + 0.25x$

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Initialize $\mathbf{w}$ (e.g. random initialization)

Repeatedly update $\mathbf{w}$ based on the gradient

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$$\mathbf{w} \leftarrow \mathbf{w} - \lambda \frac{\partial \ell}{\partial \mathbf{w}}$$

λ is the **learning rate**

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Partial derivative of the loss

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Partial derivative of the loss ***with respect to weights***

Optimizing the Objective

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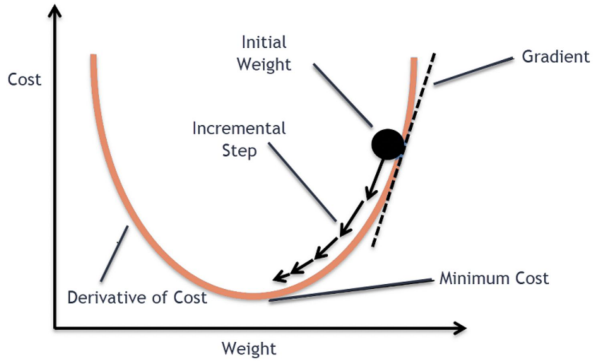


Optimizing the Objective

Objective: **Estimate median house price in a neighborhood** based on neighborhood statistics

Features

Training Data

Model

Loss

Optimization

How to figure out the weights?

$$y(x) = w_0 + w_1 x$$

We **need to find weights** $\mathbf{w}$ such that it minimizes the loss $\ell(\mathbf{w})$

Using Gradient Descent for one example

Initialize $\mathbf{w}$ (e.g. random initialization)

Repeatedly update $\mathbf{w}$ based on the gradient

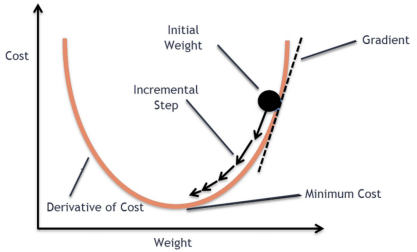
$$\mathbf{w} \leftarrow \mathbf{w} - \lambda \frac{\partial \ell}{\partial \mathbf{w}}$$

λ is the **learning rate**

For a single case, this gives the **least mean squares** update rule:

Sample: $y(x) = 0.5 + 0.25x$

Sample Update: $y(x) = 0.60 + 0.75x$



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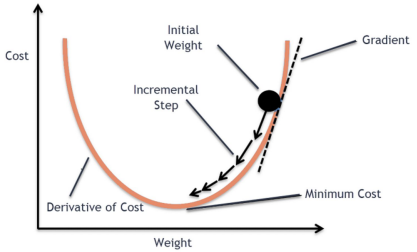

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Objective: **Estimate median house price in a neighborhood** based on neighborhood statistics

- *Features*
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After solving the equation, it equates to the following

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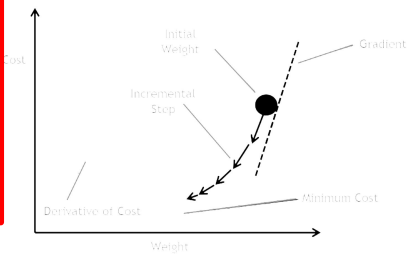

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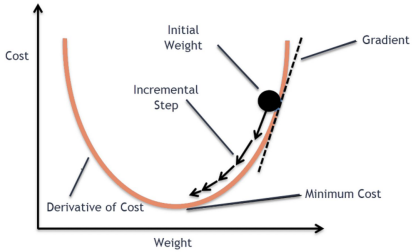

Note: *As the error approaches 0, so does the update (* $\mathbf{w}$ *stops changing)*

Optimizing Across the Dataset

Objective: **Estimate median house price in a neighborhood** based on neighborhood statistics

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How to optimize across the entire dataset?

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Two ways to generalize this for **all examples in the training set**

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Two ways to generalize this for **all examples in the training set**

1. **Batch updates:** Sum of average updates across every example n, then try to change parameter values

$$\mathbf{w} \leftarrow \mathbf{w} + 2\lambda \sum_{n=1}^{N} (t^{(n)} - y(x^{(n)}))x^{(n)}$$

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Algorithm 1 Stochastic gradient descent

1: Randomly shuffle examples in the training set
2: **for** $i = 1$ to N **do**
3: Update:

$$\mathbf{w} \leftarrow \mathbf{w} + 2\lambda(t^{(i)} - y(x^{(i)}))x^{(i)} \qquad \text{(update for a linear model)}$$

4: **end for**

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The underlying assumption is that each sample is independent and identically distributed

Visualized Optimization Process

Objective: **Estimate median house price in a neighborhood** based on neighborhood statistics

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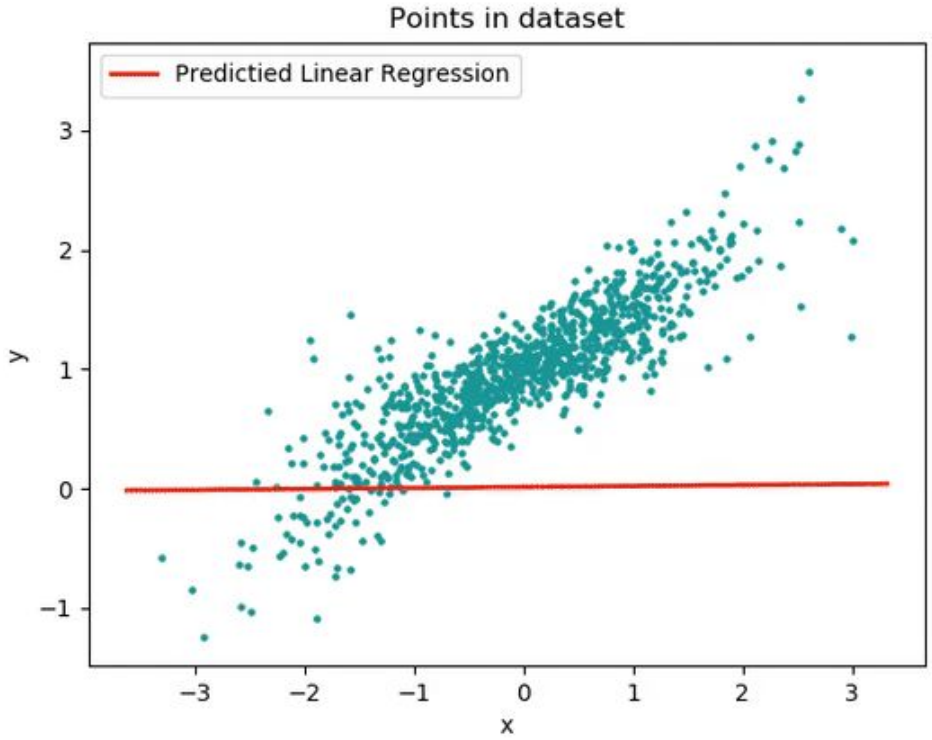

Working Example: Boston Housing Data

Objective: **Estimate median house price in a neighborhood** based on neighborhood statistics

Is there a way to improve the model?

Features

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Working Example: Boston Housing Data

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- *Features*
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Is there a way to improve the model?

One method of extending the model is to **consider other input dimensions**

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$$y(\mathbf{x}) = w_0 + w_1 x_1 + w_2 x_2$$

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x here is a vector now

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In the Boston house pricing example, **number of rooms can also be explored as a feature**

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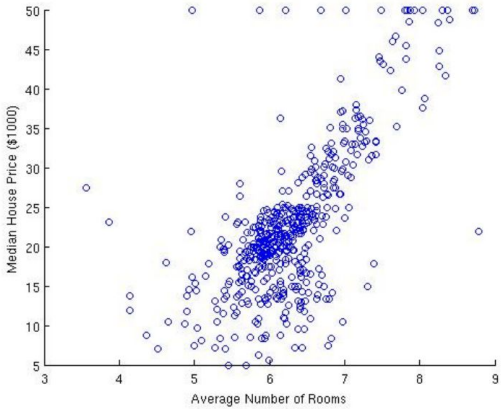

Working with Multidimensional Inputs

Objective: **Estimate median house price in a neighborhood** based on neighborhood statistics

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How do we represent multiple features / multidimensional inputs?

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How do we represent multiple features / multidimensional inputs?

Each house is a data point n, with observations indexed by j:

$$\mathbf{x}^{(n)} = \left(x_1^{(n)}, \ldots, x_j^{(n)}, \ldots, x_d^{(n)}\right)$$

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$$y(x) = w_0 + w_1 x$$

Basically the same but accounting for more features in the input data

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We can **use gradient descent to solve for each coefficient**, or compute $\mathbf{w}$ analytically.

Increasing Complexity

Objective: **Estimate median house price in a neighborhood** based on neighborhood statistics

What if the linear model is not good? Can we create a more complicated model?

Features

Training Data

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Increasing Complexity

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We can create a more complicated model by **defining input variables that are combinations of components** of $\mathbf{x}$

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An M-th order polynomial function one dimensional feature x:

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$$y(x, \mathbf{w}) = w_0 + \sum_{j=1}^{M} w_j x^j$$

where x^j is the j-th power of x

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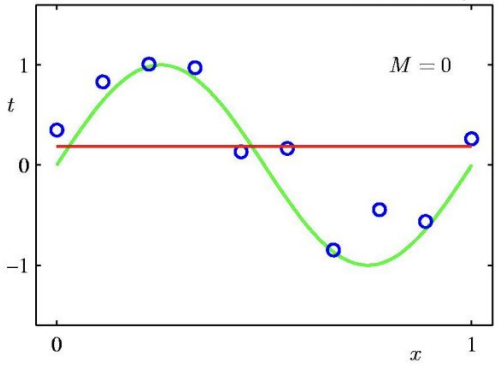
We can use the **same approach to optimize** for the weights $\mathbf{w}$

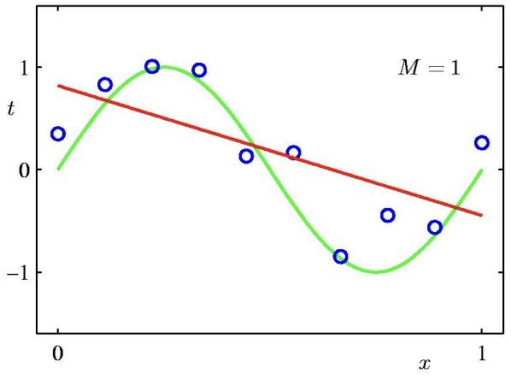
Increasing Complexity

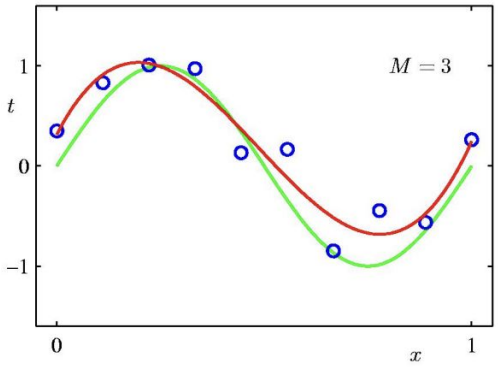
Which fit is the best?

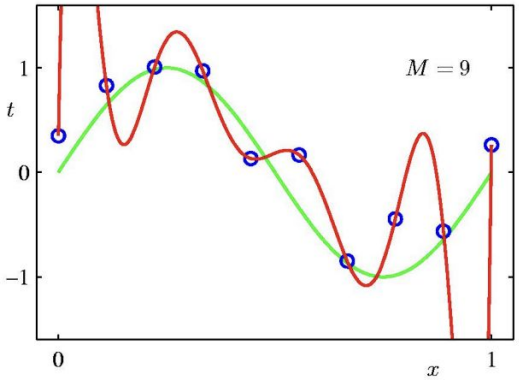
- *Features*
- *Training Data*
- ***Model***
- *Loss*
- *Optimization*

from Bishop









Generalization

What is generalization?

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Model

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Optimization

Generalization is the **model's ability to predict the held out data**

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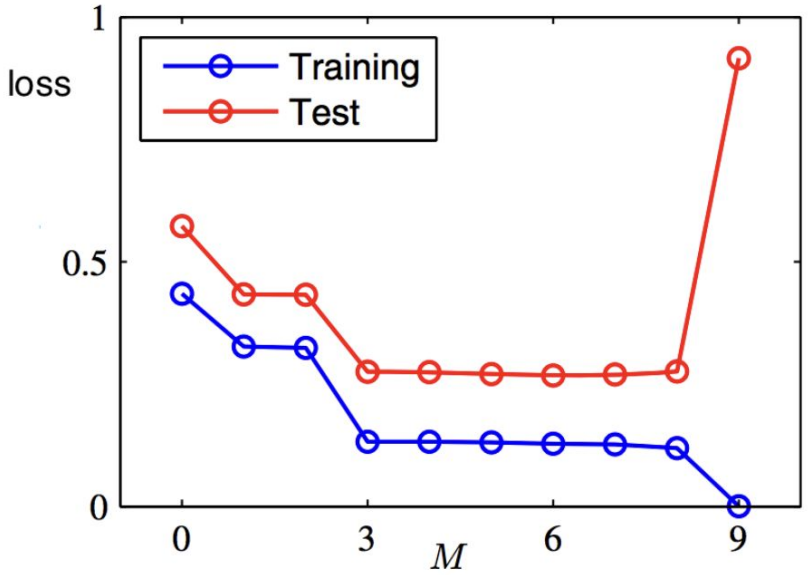
Generalization is the **model's ability to predict the held out data**

What is happening?



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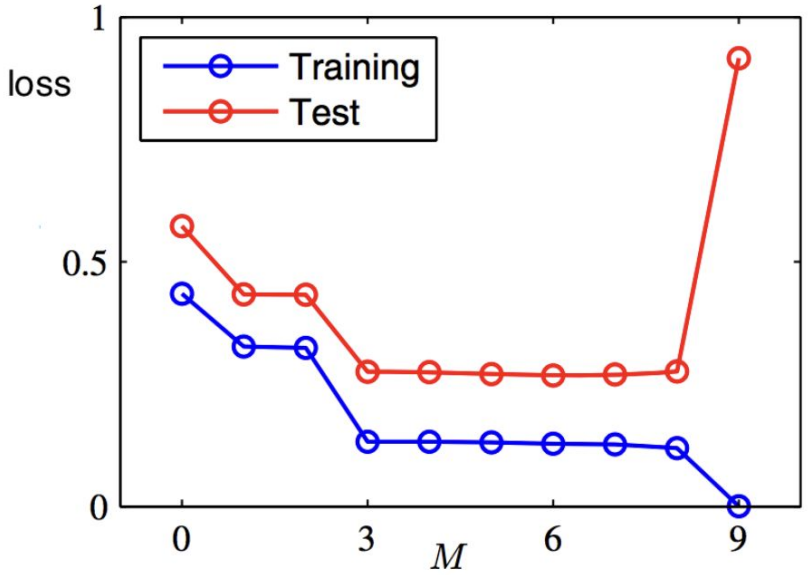
Generalization is the **model's ability to predict the held out data**

What is happening? Our model with M = 9 **overfits the data** (it models also noise)



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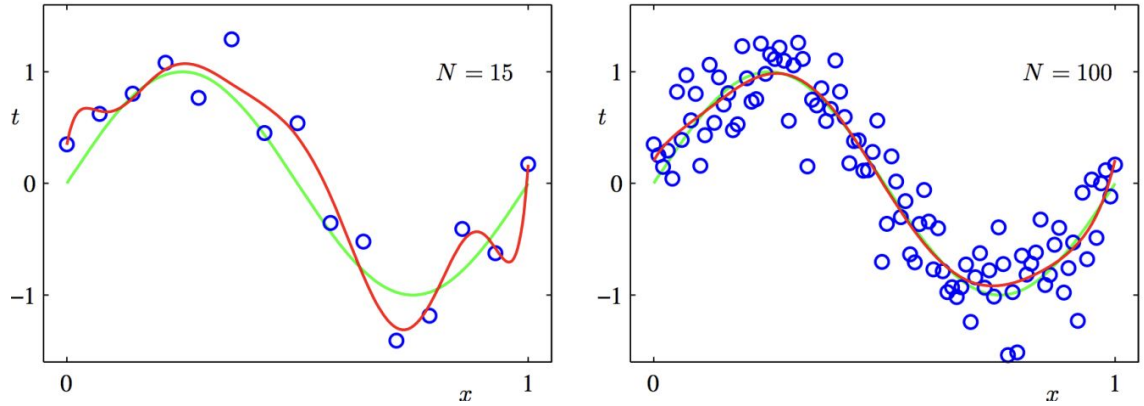

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Let's look at the **estimated weights** for various M in the case of fewer examples

	$M=0$	$M=1$	$M=6$	$M=9$
$w_0^\star$	0.19	0.82	0.31	0.35
$w_1^\star$		-1.27	7.99	232.37
$w_2^\star$			-25.43	-5321.83
$w_3^\star$			17.37	48568.31
$w_4^\star$				-231639.30
$w_5^\star$				640042.26
$w_6^\star$				-1061800.52
$w_7^\star$				1042400.18
$w_8^\star$				-557682.99
$w_9^\star$				125201.43

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The **weights are becoming huge to compensate** for the noise

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$w_4^\star$				-231639.30
$w_5^\star$				640042.26
$w_6^\star$				-1061800.52
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The **weights are becoming huge to compensate** for the noise

One way of dealing with this is to **encourage the weights to be small** (this way no input dimension will have too much influence on prediction). This is called **regularization**.

Regularization

How to regularize?

- *Features*
- *Training Data*
- *Model*
- ***Loss***
- *Optimization*

Regularization

How to regularize?

Features

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Model

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Goal: Select the appropriate model complexity automatically

Regularization

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Goal: Select the appropriate model complexity automatically

Standard approach: **Regularization**

$$\tilde{\ell} = \sum_{n=1}^{N} \left[t^{(n)} - (w_0 + w_1 x^{(n)}) \right]^2 + \alpha \mathbf{w}^T \mathbf{w}$$

Regularization

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Penalty term

Regularization

How to regularize?

- *Features*
- *Training Data*
- *Model*
- ***Loss***
- *Optimization*

Goal: Select the appropriate model complexity automatically

Standard approach: **Regularization**

$$\tilde{\ell} = \sum_{n=1}^{N} \left[t^{(n)} - (w_0 + w_1 x^{(n)}) \right]^2 + \alpha \mathbf{w}^T \mathbf{w}$$

Intuition: Since we are minimizing the loss, the second term will encourage smaller values in

Regularization

How to regularize?

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- *Training Data*
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Intuition: Since we are minimizing the loss, the second term will encourage smaller values in

The penalty on the squared weights is known as **ridge regression in statistics**

Regularization

How to regularize?

Features

Training Data

Model

Loss

Optimization

Goal: Select the appropriate model complexity automatically

Standard approach: **Regularization**

$$\tilde{\ell} = \sum_{n=1}^{N} \left[t^{(n)} - (w_0 + w_1 x^{(n)}) \right]^2 + \alpha \mathbf{w}^T \mathbf{w}$$

Intuition: Since we are minimizing the loss, the second term will encourage smaller values in

The penalty on the squared weights is known as **ridge regression in statistics**

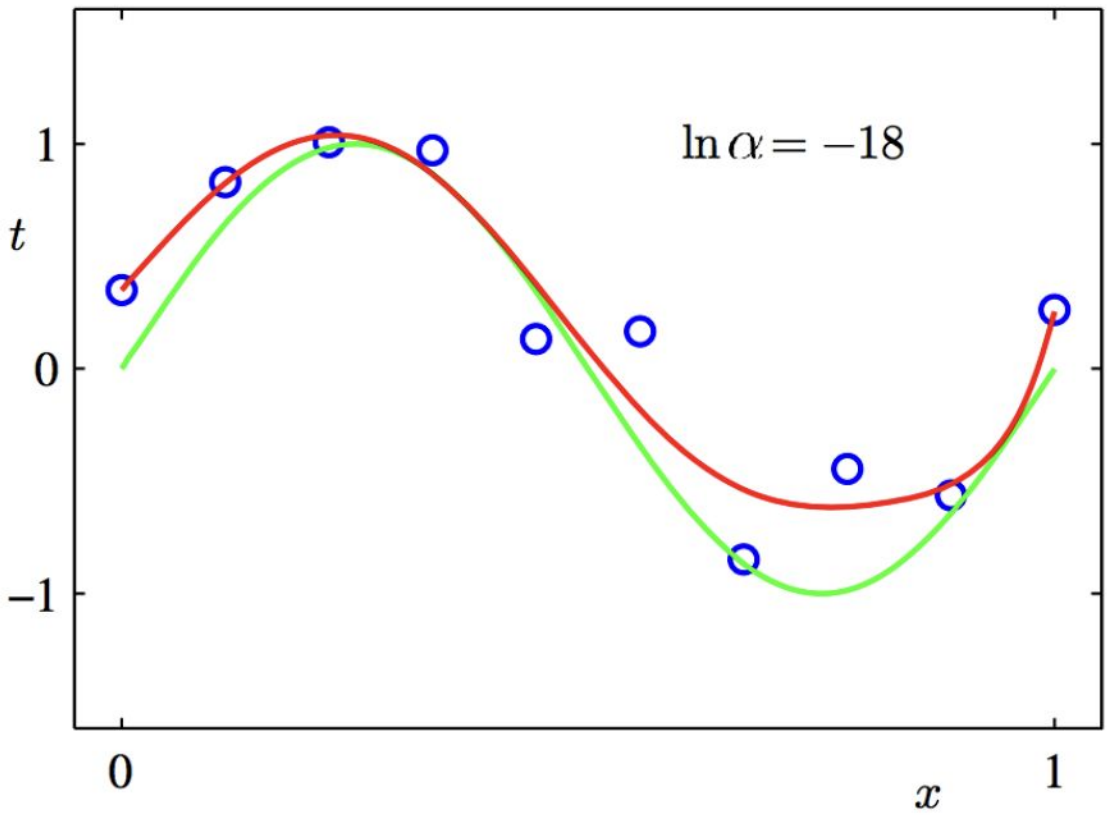
Leads to a **modified update rule** for gradient descent:

$$\mathbf{w} \leftarrow \mathbf{w} + 2\lambda \left[\sum_{n=1}^{N} (t^{(n)} - y(x^{(n)})) x^{(n)} - \alpha \mathbf{w} \right]$$

Regularization

Choose α carefully

- *Features*
- *Training Data*
- *Model*
- ***Loss***
- *Optimization*



In Summary

What have we learned for today?

- ***Features***
- *Training Data*
- *Model*
- *Loss*
- *Optimization*

Features: **Per Capita Crime Rate** and **Average Number of Rooms**

In Summary

What have we learned for today?

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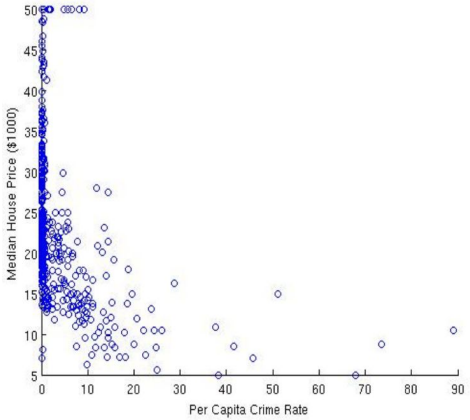
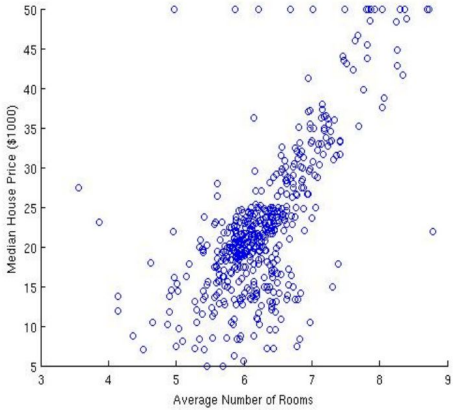
Features: **Per Capita Crime Rate** and **Average Number of Rooms**





In Summary

What have we learned for today?

- *Features*
- ***Training Data***
- *Model*
- *Loss*
- *Optimization*

Features: **Per Capita Crime Rate** and **Average Number of Rooms**

***Training* Data Structure**

$$D = \{(\mathbf{x}^{(1)}, t^{(1)}), \ldots, (\mathbf{x}^{(n)}, t^{(n)})\}$$

In Summary

What have we learned for today?

- *Features*
- ***Training Data***
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Features: **Per Capita Crime Rate** and **Average Number of Rooms**

***Training* Data Structure**

$$D = \{(\mathbf{x}^{(1)}, t^{(1)}), \ldots, (\mathbf{x}^{(n)}, t^{(n)})\}$$

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In Summary

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$$D = \{(\mathbf{x}^{(1)}, t^{(1)}), \ldots, (\mathbf{x}^{(n)}, t^{(n)})\}$$

such that

$$\mathbf{x} = (x_1^{(n)}, x_2^{(n)})$$

In Summary

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$$x_1 = \text{per capita crime rate}$$

In Summary

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- ***Training Data***
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Features: **Per Capita Crime Rate** and **Average Number of Rooms**

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$$D = \{(\mathbf{x}^{(1)}, t^{(1)}), \ldots, (\mathbf{x}^{(n)}, t^{(n)})\}$$

such that

$$\mathbf{x} = (x_1^{(n)}, x_2^{(n)})$$

$$x_1 = \text{per capita crime rate}$$

$$x_2 = \text{number of rooms}$$

In Summary

What have we learned for today?

- *Features*
- ***Training Data***
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Features: **Per Capita Crime Rate** and **Average Number of Rooms**

***Training* Data Structure**

$$D = \{(\mathbf{x}^{(1)}, t^{(1)}), \ldots, (\mathbf{x}^{(n)}, t^{(n)})\}$$

such that

$$\mathbf{x} = (x_1^{(n)}, x_2^{(n)})$$

$$x_1 = \text{per capita crime rate}$$
$$x_2 = \text{number of rooms}$$
$$t = \text{median house price}$$

In Summary

What have we learned for today?

- *Features*
- ***Training Data***
- *Model*
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- *Optimization*

Features: **Per Capita Crime Rate** and **Average Number of Rooms**

***Training* Data Structure**

$$D = \{(\mathbf{x}^{(1)}, t^{(1)}), \ldots, (\mathbf{x}^{(n)}, t^{(n)})\}$$

such that

$$\mathbf{x} = (x_1^{(n)}, x_2^{(n)})$$

$x_1 = \text{per capita crime rate}$
$x_2 = \text{number of rooms}$
$t = \text{median house price}$
$n = \text{number of data points}$

In Summary

What have we learned for today?

Features

Training Data

Model

Loss

Optimization

Features: **Per Capita Crime Rate** and **Average Number of Rooms**

***Training* Data Structure**

$$D = \{(\mathbf{x}^{(1)}, t^{(1)}), \ldots, (\mathbf{x}^{(n)}, t^{(n)})\}$$

such that

$$\mathbf{x} = (x_1^{(n)}, x_2^{(n)})$$

$x_1 =$ per capita crime rate
$x_2 =$ number of rooms
$t =$ median house price
$n =$ number of data points

Sample Data:

Neighborhood #1

$x_1 = 40 \quad x_2 = 3 \quad t = 3,400$

Neighborhood #2

$x_1 = 20 \quad x_2 = 5 \quad t = 4,500$

Neighborhood #3

$x_1 = 30 \quad x_2 = 2 \quad t = 2,800$

In Summary

What have we learned for today?

- *Features*
- *Training Data*
- ***Model***
- *Loss*
- *Optimization*

Features: **Per Capita Crime Rate** and **Average Number of Rooms**

***Training* Data Structure**

$$D = \{(\mathbf{x}^{(1)}, t^{(1)}), \ldots, (\mathbf{x}^{(n)}, t^{(n)})\}$$

Multivariate Linear Regression

$$y(\mathbf{x}) = w_0 + w_1 x_1 + w_2 x_2$$

In Summary

What have we learned for today?

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Features: **Per Capita Crime Rate** and **Average Number of Rooms**

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Multivariate Linear Regression

$$y(\mathbf{x}) = w_0 + w_1 x_1 + w_2 x_2$$

***Sum of Squares* Error**

$$\ell(\mathbf{w}) = \sum_{n=1}^{N} [t^{(n)} - (w_0 + w_1 x^{(n)})]^2$$

In Summary

What have we learned for today?

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- *Training Data*
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Features: **Per Capita Crime Rate** and **Average Number of Rooms**

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Multivariate Linear Regression

$$y(\mathbf{x}) = w_0 + w_1 x_1 + w_2 x_2$$

***Sum of Squares* Error**

$$\ell(\mathbf{w}) = \sum_{n=1}^{N} [t^{(n)} - (w_0 + w_1 x^{(n)})]^2$$

Optimization using Gradient Descent

$$\mathbf{w} \leftarrow \mathbf{w} + 2\lambda \sum_{n=1}^{N} (t^{(n)} - y(x^{(n)}))x^{(n)}$$

In Summary

What have we learned for today?

Multivariate Linear Regression $y(\mathbf{x}) = w_0 + w_1 x_1 + w_2 x_2$

- *Features*
- *Training Data*
- *Model*
- *Loss*
- ***Optimization***

Sample Data:

Neighborhood #1

$x_1 = 40 \quad x_2 = 3 \quad t = 3,400$

Neighborhood #2

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In Summary

What have we learned for today?

- Features
- Training Data
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- **Optimization**

Multivariate Linear Regression $y(\mathbf{x}) = w_0 + w_1 x_1 + w_2 x_2$

Gradient Descent

Sample Data:

Neighborhood #1

$x_1 = 40 \quad x_2 = 3 \quad t = 3,400$

Neighborhood #2

$x_1 = 20 \quad x_2 = 5 \quad t = 4,500$

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In Summary

What have we learned for today?

Features

Training Data

Model

Loss

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Multivariate Linear Regression $y(\mathbf{x}) = w_0 + w_1 x_1 + w_2 x_2$

Gradient Descent

Step #1: Initialize Weights

Sample Data:

Neighborhood #1

$x_1 = 40 \quad x_2 = 3 \quad t = 3,400$

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What have we learned for today?

- Features
- Training Data
- Model
- Loss
- **Optimization**

Multivariate Linear Regression $y(\mathbf{x}) = w_0 + w_1 x_1 + w_2 x_2$

Gradient Descent

Step #1: Initialize Weights

$$y(\mathbf{x}) = 200 + 5(x_1) + 200\,(x_2)$$

Sample Data:

Neighborhood #1

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Multivariate Linear Regression $y(\mathbf{x}) = w_0 + w_1 x_1 + w_2 x_2$

Gradient Descent

Step #1: Initialize Weights

$$y(\mathbf{x}) = 200 + 5(x_1) + 200\,(x_2)$$

Step #2: Update Weights

$$\mathbf{w} \leftarrow \mathbf{w} - \lambda \frac{\partial \ell}{\partial \mathbf{w}} \qquad \mathbf{w} \leftarrow \mathbf{w} + 2\lambda(t^{(n)} - y(x^{(n)}))x^{(n)}$$

Sample Data:

Neighborhood #1

$x_1 = 40 \quad x_2 = 3 \quad t = 3,400$

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Gradient Descent

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$$y(\mathbf{x}) = 200 + 5(x_1) + 200\,(x_2)$$

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Neighborhood #1

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Gradient Descent

Step #1: Initialize Weights

$$y(\mathbf{x}) = 200 + 5(x_1) + 200\,(x_2)$$

Step #2: Update Weights

$$\mathbf{w} \leftarrow \mathbf{w} - \lambda \frac{\partial \ell}{\partial \mathbf{w}} \qquad \mathbf{w} \leftarrow \mathbf{w} + 2\lambda(t^{(n)} - y(x^{(n)}))x^{(n)}$$

Neighborhood #1

$$w_0 = 200 + 2(1)(3400 - (200 + 5(40) + 200(3)))1$$

Sample Data:

Neighborhood #1

$x_1 = 40 \quad x_2 = 3 \quad t = 3,400$

Neighborhood #2

$x_1 = 20 \quad x_2 = 5 \quad t = 4,500$

Neighborhood #3

$x_1 = 30 \quad x_2 = 2 \quad t = 2,800$

In Summary

What have we learned for today?

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Multivariate Linear Regression $y(\mathbf{x}) = w_0 + w_1 x_1 + w_2 x_2$

Gradient Descent

Step #1: Initialize Weights

$$y(\mathbf{x}) = 200 + 5(x_1) + 200\,(x_2)$$

Step #2: Update Weights

$$\mathbf{w} \leftarrow \mathbf{w} - \lambda \frac{\partial \ell}{\partial \mathbf{w}} \qquad \mathbf{w} \leftarrow \mathbf{w} + 2\lambda(t^{(n)} - y(x^{(n)}))x^{(n)}$$

Neighborhood #1

$$w_0 = 200 + 2(1)(3400 - (200 + 5(40) + 200(3)))1$$

$$w_1 = 5 + 2(1)(3400 - (200 + 5(40) + 200(3)))40$$

Sample Data:

Neighborhood #1

$x_1 = 40 \quad x_2 = 3 \quad t = 3,400$

Neighborhood #2

$x_1 = 20 \quad x_2 = 5 \quad t = 4,500$

Neighborhood #3

$x_1 = 30 \quad x_2 = 2 \quad t = 2,800$

In Summary

What have we learned for today?

Features

Training Data

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Optimization

Multivariate Linear Regression $y(\mathbf{x}) = w_0 + w_1 x_1 + w_2 x_2$

Gradient Descent

Step #1: Initialize Weights

$$y(\mathbf{x}) = 200 + 5(x_1) + 200\,(x_2)$$

Step #2: Update Weights

$$\mathbf{w} \leftarrow \mathbf{w} - \lambda \frac{\partial \ell}{\partial \mathbf{w}} \qquad \mathbf{w} \leftarrow \mathbf{w} + 2\lambda(t^{(n)} - y(x^{(n)}))x^{(n)}$$

Neighborhood #1

$$w_0 = 200 + 2(1)(3400 - (200 + 5(40) + 200(3)))1$$

$$w_1 = 5 + 2(1)(3400 - (200 + 5(40) + 200(3)))40$$

$$w_2 = 200 + 2(1)(3400 - (200 + 5(40) + 200(3)))3$$

Sample Data:

Neighborhood #1

$x_1 = 40 \quad x_2 = 3 \quad t = 3,400$

Neighborhood #2

$x_1 = 20 \quad x_2 = 5 \quad t = 4,500$

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In Summary

What have we learned for today?

- *Features*
- *Training Data*
- *Model*
- *Loss*
- ***Optimization***

Multivariate Linear Regression $y(\mathbf{x}) = w_0 + w_1 x_1 + w_2 x_2$

Gradient Descent

Step #1: Initialize Weights

$$y(\mathbf{x}) = 200 + 5(x_1) + 200\,(x_2)$$

Step #2: Update Weights

$$\mathbf{w} \leftarrow \mathbf{w} - \lambda \frac{\partial \ell}{\partial \mathbf{w}} \qquad \mathbf{w} \leftarrow \mathbf{w} + 2\lambda(t^{(n)} - y(x^{(n)}))x^{(n)}$$

Neighborhood #1

$$w_0 = 200 + 2(1)(3400 - (200 + 5(40) + 200(3)))1 \qquad w_0 = 5,000$$

$$w_1 = 5 + 2(1)(3400 - (200 + 5(40) + 200(3)))40 \qquad w_1 = 192,005$$

$$w_2 = 200 + 2(1)(3400 - (200 + 5(40) + 200(3)))3 \qquad w_2 = 15,400$$

Sample Data:

Neighborhood #1

$x_1 = 40 \quad x_2 = 3 \quad t = 3,400$

Neighborhood #2

$x_1 = 20 \quad x_2 = 5 \quad t = 4,500$

Neighborhood #3

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In Summary

What have we learned for today?

- *Features*
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- ***Optimization***

Multivariate Linear Regression $y(\mathbf{x}) = w_0 + w_1 x_1 + w_2 x_2$

Gradient Descent

Step #1: Initialize Weights

$$y(\mathbf{x}) = 200 + 5(x_1) + 200\,(x_2)$$

Step #2: Update Weights

$$\mathbf{w} \leftarrow \mathbf{w} - \lambda \frac{\partial \ell}{\partial \mathbf{w}} \qquad \mathbf{w} \leftarrow \mathbf{w} + 2\lambda (t^{(n)} - y(x^{(n)}))x^{(n)}$$

Neighborhood #2

$$w_0 = 5000 + 2(1)(4500 - (5000 + 192005(20) + 15400(5)))1$$

Sample Data:

Neighborhood #1

$x_1 = 40 \quad x_2 = 3 \quad t = 3,400$

Neighborhood #2

$x_1 = 20 \quad x_2 = 5 \quad t = 4,500$

Neighborhood #3

$x_1 = 30 \quad x_2 = 2 \quad t = 2,800$

In Summary

What have we learned for today?

- *Features*
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- *Loss*
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Multivariate Linear Regression $y(\mathbf{x}) = w_0 + w_1 x_1 + w_2 x_2$

Gradient Descent

Step #1: Initialize Weights

$$y(\mathbf{x}) = 200 + 5(x_1) + 200\,(x_2)$$

Step #2: Update Weights

$$\mathbf{w} \leftarrow \mathbf{w} - \lambda \frac{\partial \ell}{\partial \mathbf{w}} \qquad \mathbf{w} \leftarrow \mathbf{w} + 2\lambda(t^{(n)} - y(x^{(n)}))x^{(n)}$$

Neighborhood #2

$$w_0 = 5000 + 2(1)(4500 - (5000 + 192005(20) + 15400(5)))1$$

$$w_1 = 192005 + 2(1)(4500 - (5000 + 192005(20) + 15400(5)))20$$

Sample Data:

Neighborhood #1

$x_1 = 40 \quad x_2 = 3 \quad t = 3,400$

Neighborhood #2

$x_1 = 20 \quad x_2 = 5 \quad t = 4,500$

Neighborhood #3

$x_1 = 30 \quad x_2 = 2 \quad t = 2,800$

In Summary

What have we learned for today?

Features

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Optimization

Multivariate Linear Regression $y(\mathbf{x}) = w_0 + w_1 x_1 + w_2 x_2$

Gradient Descent

Step #1: Initialize Weights

$$y(\mathbf{x}) = 200 + 5(x_1) + 200\,(x_2)$$

Step #2: Update Weights

$$\mathbf{w} \leftarrow \mathbf{w} - \lambda \frac{\partial \ell}{\partial \mathbf{w}} \qquad \mathbf{w} \leftarrow \mathbf{w} + 2\lambda(t^{(n)} - y(x^{(n)}))x^{(n)}$$

Neighborhood #2

$$w_0 = 5000 + 2(1)(4500 - (5000 + 192005(20) + 15400(5)))1$$
$$w_1 = 192005 + 2(1)(4500 - (5000 + 192005(20) + 15400(5)))20$$
$$w_2 = 15400 + 2(1)(4500 - (5000 + 192005(20) + 15400(5)))5$$

Sample Data:

Neighborhood #1

$x_1 = 40 \quad x_2 = 3 \quad t = 3,400$

Neighborhood #2

$x_1 = 20 \quad x_2 = 5 \quad t = 4,500$

Neighborhood #3

$x_1 = 30 \quad x_2 = 2 \quad t = 2,800$

In Summary

What have we learned for today?

Features

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Multivariate Linear Regression $y(\mathbf{x}) = w_0 + w_1 x_1 + w_2 x_2$

Gradient Descent

Step #1: Initialize Weights

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$$\mathbf{w} \leftarrow \mathbf{w} - \lambda \frac{\partial \ell}{\partial \mathbf{w}} \qquad \mathbf{w} \leftarrow \mathbf{w} + 2\lambda(t^{(n)} - y(x^{(n)}))x^{(n)}$$

Neighborhood #2

$$w_0 = 5000 + 2(1)(4500 - (5000 + 192005(20) + 15400(5)))1 \qquad w_0 = -7,830,200$$

$$w_1 = 192005 + 2(1)(4500 - (5000 + 192005(20) + 15400(5)))20 \qquad w_1 = -156,551,995$$

$$w_2 = 15400 + 2(1)(4500 - (5000 + 192005(20) + 15400(5)))5 \qquad w_2 = -36,160,600$$

Sample Data:

Neighborhood #1

$x_1 = 40 \quad x_2 = 3 \quad t = 3,400$

Neighborhood #2

$x_1 = 20 \quad x_2 = 5 \quad t = 4,500$

Neighborhood #3

$x_1 = 30 \quad x_2 = 2 \quad t = 2,800$

In Summary

What have we learned for today?

- ***Features***
- ***Training Data***
- ***Model***
- ***Loss***
- ***Optimization***

We have learned…

What are Loss Functions and how Linear Regression is done

We have identified the necessary components for linear regression

Features, Training Data, Model, Loss, and Optimization

We discussed concepts about…

Noie Generalization, Regularization, etc.