Multiperiod Production Smoothing Problem

Problem Description

A company is planning the manufacture of a product for March, April, May and June of next year. The company has a permanent workforce but can meet fluctuating production needs by hiring and firing temporary workers. There are extra cost incurred for hiring and firing a temp in any month. The production of permanent workers differs from the temporary worker due to their level of experience. If the company can produce more than needed in any month, the surplus can be carried over to a succeeding month at a holding cost. Develop an optimal hiring/firing policy over the 4-month planning horizon.

Mathematical Model

Parameters/Inputs that need to be specified by user:

d_i = demand for month i, $i = 1(\text{March}), 2(\text{April}), 3(\text{May}), 4(\text{June})$

N_p = no of permanent workers

P_p = production of a permanent worker

P_t = production of a temporary worker

h = holding cost per unit per month

I_0 = Initial inventory level

I_f = Final inventory level

C_h = Hiring cost of a temporary worker

C_f = Firing cost of a temporary worker

Variables for month i that need to be obtained by solving the model:

x_i = net number of temporary workers at the start of month i after any hiring or firing

S_i = Number of temporary workers hired or fired at the start of month i

I_i = Units of ending inventory for month i

The permanent workers can be accounted for by subtracting the units they produce from the respective monthly demand. The remaining demand is then satisfied through hiring and firing of temps. Thus,

Remaining demand for March = $d_1 - (P_p \cdot N_p)$

Remaining demand for April = $d_2 - (P_p \cdot N_p)$

Remaining demand for May = $d_3 - (P_p \cdot N_p)$

Remaining demand for June = $d_4 - (P_p \cdot N_p)$

Constraints

Inventory constraints

March : $I_0 + P_t \cdot x_1 = \ d_1 - \left(P_p \cdot N_p\right) + I_1$

April : $I_1 + P_t \cdot x_2 = \ d_2 - \left(P_p \cdot N_p\right) + I_2$

May : $I_2 + P_t \cdot x_3 = \ d_3 - \left(P_p \cdot N_p\right) + I_3$

June : $I_3 + P_t \cdot x_4 = \ d_4 - \left(P_p \cdot N_p\right) + I_f$

$x_1, x_2, x_3, x_4 \geq 0, I_1, I_2, I_3 \geq 0$

Number of temporary workers

Beginning of March : $x_1 = S_1$

Beginning of April : $x_2 = x_1 + S_2$

Beginning of May: $x_3 = x_2 + S_3$

Beginning of June : $x_4 = x_3 + S_4$

$x_1, x_2, x_3, x_4 \geq 0, S_1, S_2, S_3, S_4$ unrestricted in sign

If the variable S_i is positive, hiring take place in month i. If it is negative, then firing occurs.

Therefore, this can be translated as

$S_i = S_i^+ - S_i^-$, where $S_i^+, S_i^- \geq 0$

S_i^+ is the number of temporary workers hired and S_i^- is the number fired.

$$x_1 = S_1^+ - S_1^-$$

$$x_2 = x_1 + S_2^+ - S_2^-$$

$$x_3 = x_2 + S_3^+ - S_3^-$$

$$x_4 = x_3 + S_4^+ - S_4^-$$

Objective Function

Inventory Holding Cost

Inventory Holding Cost = $h(I_1 + I_2 + I_3)$

Hiring and Firing Cost

Hiring Cost = $C_h(S_1^+ + S_2^+ + S_3^+ + S_4^+)$

Firing Cost = $C_f(S_1^- + S_2^- + S_3^- + S_4^-)$

Complete Model

Minimize $z = h(I_1 + I_2 + I_3) + C_h(S_1^+ + S_2^+ + S_3^+ + S_4^+) + C_f(S_1^- + S_2^- + S_3^- + S_4^-)$

subject to

$$I_0 + P_t \cdot x_1 = d_1 - (P_p \cdot N_p) + I_1$$

$$I_1 + P_t \cdot x_2 = d_2 - (P_p \cdot N_p) + I_2$$

$$I_2 + P_t \cdot x_3 = d_3 - (P_p \cdot N_p) + I_3$$

$$I_3 + P_t \cdot x_4 = d_4 - (P_p \cdot N_p) + I_f$$

$$x_1 = S_1^+ - S_1^-$$

$$x_2 = x_1 + S_2^+ - S_2^-$$

$$x_3 = x_2 + S_3^+ - S_3^-$$

$$x_4 = x_3 + S_4^+ - S_4^-$$

$$x_1, x_2, x_3, x_4 \geq 0$$

$$I_1, I_2, I_3 \geq 0$$

$$S_1^+, S_1^-, S_2^+, S_2^-, S_3^+, S_3^-, S_4^+, S_4^- \geq 0$$

Solution Method

The optimal solution can be obtained by using Simplex Method / Branch and Bound Method. The optimization module can be found in optimization software such as LINGO, GAMS, etc. Need to check whether this optimization module can be integrated in our system or need to code the optimization algorithm/procedure too.