vOptSolver:
an ecosystem for multi-objective linear optimization

JuliaCon 2021 — JuMP-dev

Xavier GANDIBLEUX
with
Gauthier SOLEILHAC, Anthony PRZYBYLSKI et al.

http://github.com/vOptSolver

Optimization problems targeted

Computing Y_N for Multiobjective linear Optimization Problems (MOP):

$$\begin{array}{rcl} \min\ F(x) & = & Cx \\ \text{s/t} \quad Tx & \leqq & d \end{array}$$

$x \in \mathbb{R}^{n_1}$	$x \in \mathbb{R}^{n_1} \times \mathbb{Z}^{n_2}$	$x \in \mathbb{Z}^{n_2}$
Multi Objective Linear Problem (MOLP)	Multi Objective Mixed-Integer Linear Problem (MOMILP)	Multi Objective Integer Problem (MOIP)

MO(M)ILP + structure
MultiObjective Combinatorial Optimization (MOCO)

where:

$$\begin{array}{rcl} T \in \mathbb{Z}^{m\times n} & \longrightarrow & m \text{ constraints, } i = 1, \dots, m \\ C \in \mathbb{Z}^{n\times p} & \longrightarrow & \text{the objective matrix} \\ X = \{x \in \mathbb{R}^{n_1} \times \mathbb{Z}^{n_2} \mid Tx \leqslant d\} \subseteq \mathbb{R}^n & \longrightarrow & \text{the set of feasible solutions} \\ Y = F(X) \subseteq \mathbb{R}^p & \longrightarrow & \text{the set of images} \end{array}$$

Context

Computing Y_N for Multiobjective linear Optimization Problems (MOP):

$$\begin{array}{lrcl} \min & F(x) & = & Cx \\ \text{s/t} & Tx & \leqq & d \end{array}$$

$x \in \mathbb{R}^{n_1}$ $\qquad$ $x \in \mathbb{R}^{n_1} \times \mathbb{Z}^{n_2}$ $\qquad$ $x \in \mathbb{Z}^{n_2}$

$$y = F(x)$$

$y^* \in Y$ is nondominated, if $\nexists\, y \in Y$ such that $y_i \leqslant y_i^*,\ \forall i$ and $y \neq y^*$.

Y_N is the set of nondominated points.

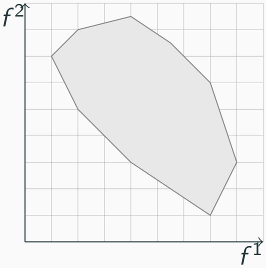
$x^* \in X$ is efficient if y^* is nondominated.

X_E is a complete set of efficient solutions.

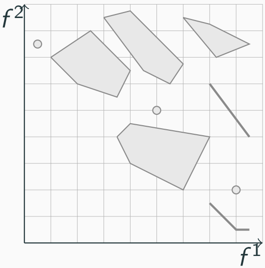
Examples of Y and Y_N when $p = 2$

Computing Y_N for Multiobjective (linear) Optimization Problems (MOP):

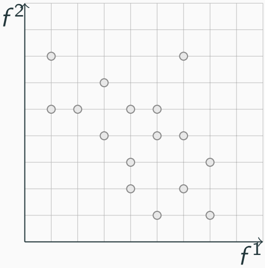
$x \in \mathbb{R}^{n_1}$



$x \in \mathbb{R}^{n_1} \times \mathbb{Z}^{n_2}$



$x \in \mathbb{Z}^{n_2}$



Nondominated set composed of edges that are either closed, half-open, open or reduced to a point (Vincent et al., 2013)

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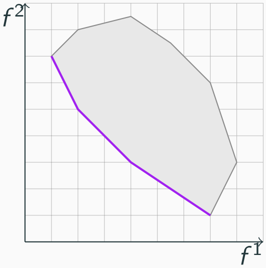
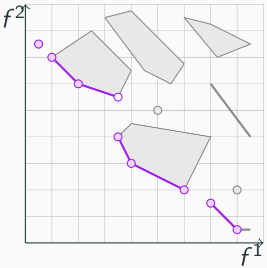


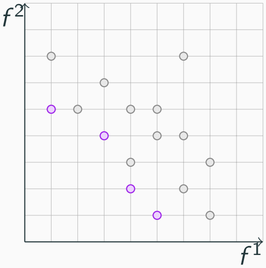
Continuous nondominated set (edges)

$x \in \mathbb{R}^{n_1} \times \mathbb{Z}^{n_2}$



Nondominated set composed of edges that are either closed, half-open, open or reduced to a point (Vincent et al., 2013)

$x \in \mathbb{Z}^{n_2}$



Discrete set of nondominated points

vOptSolver

vOptSpecific.jl and vOptGeneric.jl
First release in July 2017

June 2021:

tested on
macOS 11.4 and Ubuntu 18.04.5 LTS

compliant with
Julia 1.6.1 and JuMP 0.21.8
GLPK 5, GUROBI 9.1.2

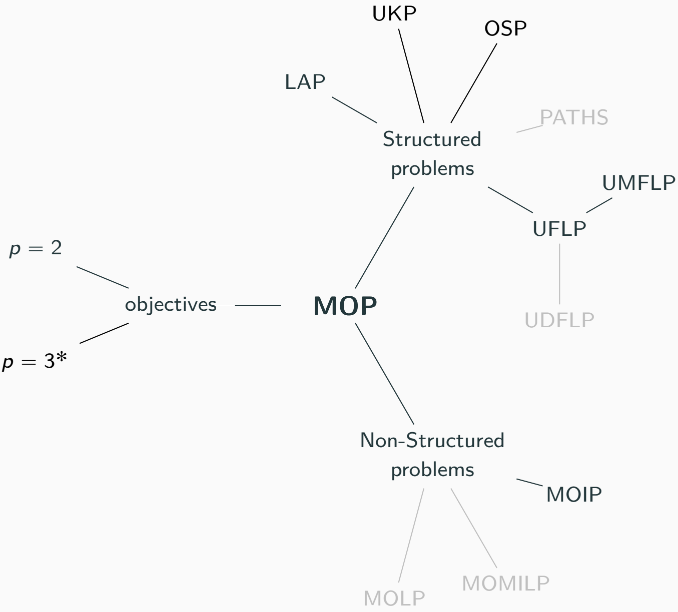
Purposes

Natural and **intuitive** use for mathematicians, informaticians, engineers

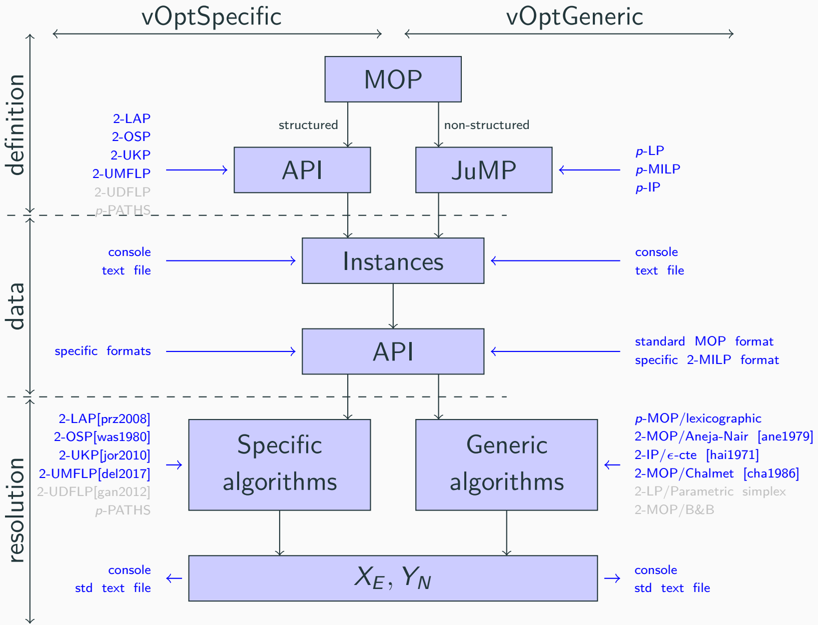
- **Research needs:**
  support and primitives for the development of new algorithms
- **Solving needs:**
  methods and algorithms for performing numerical experiments
- **Pedagogic needs:**
  environment for practicing of theories and algorithms

Easy to formulate a problem,
provide the data,
solve a problem,
collect the outputs,
analyze the solutions.

Optimization problems currently covered (and in project)



Design of vOptSolver



Example: 2-LAP

$$
\left[\begin{array}{lcll}
\min z^k & = & \displaystyle\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}^k x_{ij} & k = 1,\ldots,2 \\
s/c & & \displaystyle\sum_{i=1}^{n} x_{ij} = 1 & j = 1,\ldots,n \\
 & & \displaystyle\sum_{j=1}^{n} x_{ij} = 1 & i = 1,\ldots,n \\
 & & x_{ij} = (0,1) & i = 1,\ldots,n \\
 & & & j = 1,\ldots,n
\end{array}\right]
$$

```
C1 = [ 3 9 0 0 6 ;
      16 0 6 12 19 ;
       2 7 11 15 8 ;
       4 11 7 16 3 ;
       2 5 1 9 0 ]

C2 = [16 5 6 19 12 ;
      15 7 13 7 7 ;
       1 2 13 2 3 ;
      14 7 8 1 7 ;
      10 10 1 0 0 ]

n = size(C2,1)
```

Example: 2-LAP and vOptGeneric

Algorithm: ϵ-constraint

Yacov V. Haimes, Leon S. Lasdon, David A. Wismer: On a bicriterion formation of the problems of integrated system identification and system optimization. *IEEE Transactions on Systems, Man and Cybernetics*. Volume SMC-1, Issue 3, 296-297, 1971.

MILP: GLPK

Output: $X_E \subseteq \mathbb{N}^n$, $Y_N \subseteq \mathbb{Z}^2$

Program:

```
using JuMP, GLPK, vOptGeneric
m = vModel( GLPK.Optimizer )
@variable( m, x[1:n,1:n] , Bin )
@addobjective( m, Min, sum( C1[i,j]*x[i,j] for i=1:n,j=1:n ))
@addobjective( m, Min, sum( C2[i,j]*x[i,j] for i=1:n,j=1:n ))
@constraint( m, cols[i=1:n], sum(x[i,j] for j=1:n) == 1 )
@constraint( m, rows[j=1:n], sum(x[i,j] for i=1:n) == 1 )
vSolve( m, method = :epsilon , step = 1.0 )
printX_E( m )
getY_N( m )
```

Example: 2-LAP and vOptSpecific

Algorithm: Two phases

A. Przybylski, X. Gandibleux, and M. Ehrgott. Two phase algorithms for the bi-objective assignment problem. *European Journal of Operational Research*, 185(2):509–533, 2008.

Routine: algorithm provided in language C

Output: $X_E \subseteq \mathbb{N}^n$, $Y_N \subseteq \mathbb{Z}^2$

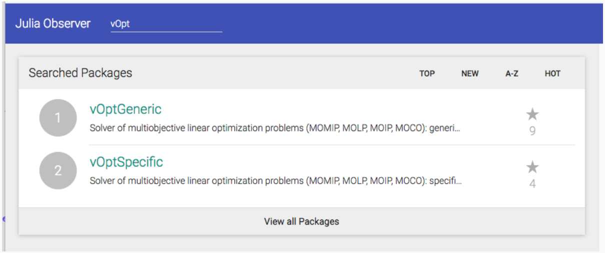
Program:

```
using vOptSpecific
m = set2LAP( n , C1 , C2 )
solver = LAP_Przybylski2008( )
z1, z2, S = vSolve( m , solver )
```

or simply (using default options)

```
using vOptSpecific
m = set2LAP( n , C1 , C2 )
z1, z2, S = vSolve( m )
```

vOptSolver: `http://github.com/vOptSolver`



Xavier Gandibleux, Gauthier Soleilhac, Anthony Przybylski, Flavien Lucas, Stefan Ruzika, Pascal Halffmann. vOptSolver, a "get and run" solver of multiobjective linear optimization problems built on Julia and JuMP. MCDM2017: 24th International Conference on Multiple Criteria Decision Making. July 10-14, 2017. Ottawa (Canada).

Xavier Gandibleux, Gauthier Soleilhac, Anthony Przybylski, Stefan Ruzika. vOptSolver: an open source software environment for multiobjective mathematical optimization. IFORS2017: 21st Conference of the International Federation of Operational Research Societies. July 17-21, 2017. Quebec City (Canada).