2. GIVEN: $[B[\sigma]] = (1-\sigma^2)[I_{3\times3}] + 2[\tilde{\sigma}] + 2\sigma\sigma^T$

AND IDENTITY: $[\tilde{\sigma}]^2 = \sigma\sigma^T - \sigma^2[I_{3\times3}]$

FIND: $[B]^{-1}$

A: $\tilde{\sigma}$ IS SKEW-SYMMETRIC, PROPERTY: $A^T = -A$

$\therefore [B]^T = (1-\sigma^2)[I_{3\times3}] - 2[\tilde{\sigma}] + 2\sigma\sigma^T$

$$[B][B]^T = ((\underbrace{(1-\sigma^2)[I_{3\times3}]}_{(a} + \underbrace{2[\tilde{\sigma}]}_{+\ b} + \underbrace{2\sigma\sigma^T}_{+\ c)})\cdot(\underbrace{(1-\sigma^2)[I_{3\times3}]}_{(a} - \underbrace{2[\tilde{\sigma}]}_{+\ b} + \underbrace{2\sigma\sigma^T}_{+\ c)})$$

$$= a^2 - \cancel{ba} + ca + \cancel{ba} - b^2 + \cancel{bc} + ca - \cancel{cb} + c^2$$

$$= a^2 - b^2 + 2ac + c^2$$

$$= (1-\sigma^2)^2[I_{3\times3}] - (2[\tilde{\sigma}])^2 + 2((1-\sigma^2)[I_{3\times3}]\cdot 2\sigma\sigma^T) + (2\sigma\sigma^T)^2$$

$$= (1-\sigma^2)^2[I_{3\times3}] - 4\underbrace{\boxed{[\tilde{\sigma}]}}_{\text{IDENTITY}}{}^2 + 4((1-\sigma^2)[I_{3\times3}]\cdot\sigma\sigma^T) + 4(\sigma\sigma^T)^2$$

$$= (1-\sigma^2)^2[I_{3\times3}] - 4(\boxed{\sigma\sigma^T - \sigma^2[I_{3\times3}]})$$
$$+ 4((1-\sigma^2)[I_{3\times3}]\cdot\sigma\sigma^T) + 4(\sigma\sigma^T)^2$$

$$= (1-\sigma^2)^2[I_{3\times3}] - 4\sigma\sigma^T + 4\sigma^2[I_{3\times3}] + 4((1-\sigma^2[I_{3\times3}]\sigma\sigma^T) + 4(\sigma\sigma^T)^2$$

$$= (1-\sigma^2)^2[I_{3\times3}] - \cancel{4\sigma\sigma^T} + 4\sigma^2[I_{3\times3}] + \cancel{4\sigma\sigma^T} - 4\sigma^2[I_{3\times3}]\sigma\sigma^T + 4(\sigma\sigma^T)^2$$

$$= (1-\sigma^2)^2[I_{3\times3}] + 4\sigma^2[I_{3\times3}] - 4\sigma^2[I_{3\times3}]\sigma\sigma^T + 4(\sigma\sigma^T)^2$$

$$= (1-2\sigma^2+\sigma^4)[I_{3\times3}] + 4\sigma^2[I_{3\times3}] - 4\sigma^2[I_{3\times3}]\sigma\sigma^T + 4(\sigma\sigma^T)^2$$

$$= (1+2\sigma^2+\sigma^4)[I_{3\times3}] - 4\sigma^2[I_{3\times3}]\sigma\sigma^T + 4(\sigma\sigma^T)^2$$

$$= (1+\sigma^2)^2[I_{3\times3}] - 4\sigma^2[I_{3\times3}]\sigma\sigma^T + 4(\sigma\sigma^T)^2$$

$$= (1+\sigma^2)^2[I_{3\times3}] + 4\sigma\sigma^T(\underbrace{\boxed{-\sigma^2[I_{3\times3}] + \sigma\sigma^T}}_{\text{IDENTITY}})$$

$$\left\{\begin{array}{l}(\sigma\sigma^T) = \sigma^2\\ (\sigma^T\sigma) = \sigma^2\end{array}\right.$$

$$= (1+\sigma^2)^2[I_{3\times3}] + 4\sigma\sigma^T\boxed{[\tilde{\sigma}]}^2$$

$$[B][B]^T = (1+\sigma^2)^2[I_{3\times3}]$$

$\Rightarrow$ PROPERTIES: $I\cdot A = A$; $A\cdot I = A$

$A\cdot A^{-1} = A^{-1}A = I$

$$\frac{1}{(1+\sigma^2)^2}[B][B]^T = [I_{3\times3}]$$

$$\frac{1}{(1+\sigma^2)^2}[B][B]^T[B]^{-1} = [I_{3\times3}][B]^{-1}$$

$$\frac{1}{(1+\sigma^2)^2}[B]^T[I_{3\times3}] = [B^{-1}][I_{3\times3}]$$

$$\therefore [B]^{-1} = \frac{1}{(1+\sigma^2)^2}[B]^T$$

$\square$