

Data Structures

The Capacity Trick

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Root cause analysis (RCA)

- Root cause analysis (RCA) is the process of discovering the **root causes** of problems in order to identify appropriate solutions
 - Tip: Senior Software Engineers must be clever in that.
- We know our **append** is slow as it does linear number of steps!
 - Hence the whole program is quadratic steps!
- But why we ended with such solution?
- Because with **every** append
 - We create a new array. Move old data \Rightarrow Linear number of steps per an append!
- Now, we know the **design issue**. How to solve?
 - Intuitively: how can we make a single append takes a few steps (e.g. 2-5)?
 - Hence in total the loop is linear!

The Capacity Trick

- Assume the user asked for an array of length 10
- But, internally: we reserved an array of 3000 values!
 - Let's call this number the **capacity**
- Now, with every **append**, we can just add the value in $O(1)$
 - But after **2990 additions**, our whole **array** is filled again!
- Now we need to create a **new array** once again, and copy the data
 - New array size? 3001? NO. We will face the original problem again!
 - Let's **multiply** the **current size by 2** \Rightarrow reserves an array of 6000.
 - Now we have another 3000 values to use.
 - Filled? Create a new array of size $2 \times 6000 = 12000$ and so on
- We moved from being very slow in every step to being very fast most of the time except on **increasingly rare occasions** (exponentially with size doubling each time)

Capacity Trick: Data

- To implement this idea, we first need now 2 variables
 - Size = The actual elements size from the user
 - Capacity = The actual array size in the memory
 - Capacity \geq size, typically larger. Let's start with value like 16
- Try to finish the code :)

```
def __init__(self, size):  
    self.size = size                # user size  
    self._capacity = max(16, 2*size)  # actual memory size  
  
    array_data_type = ctypes.py_object * self._capacity  
    self.memory = array_data_type()  
  
    for i in range(self._capacity):  
        self.memory[i] = None
```

Capacity Trick: Improved append

- Now with every append, if the current array capacity is enough
 - Just add the element (total 3 steps)
- But what if the capacity is not enough?
 - Double the array capacity
 - Move old data
 - This will be a slow step, but it happens no very rare times
- Can you code the expand function?

```
def append(self, value):  
    if self.size == self._capacity:  
        self.expand_capacity()  
    self.memory[self.size] = value  
    self.size += 1
```

Capacity Trick: Expanding capacity

- The function logic is exactly what we did before
- The function
 - doubles the capacity
 - moves the old data

```
def expand_capacity(self):  
    # Double the actual array size  
    self._capacity *= 2  
    print(f'Expand capacity to {self._capacity}')  
    # create a new array of _capacity  
    array_data_type = ctypes.py_object * self._capacity  
    new_memory = array_data_type()  
  
    for i in range(self.size): # copy  
        new_memory[i] = self.memory[i]  
  
    # use the new memory and delete old one  
    del self.memory  
    self.memory = new_memory
```

Let's run again

- How fast?!
- Try to analyze
 - How many times approximately the expansion statement will be printed?

```
array = Array(3)

for i in range(len(array)):
    array[i] = i + 1

array.append('12')
array.append('hello')

print(array)

for i in range(10 ** 6):
    array.append(i)

print(len(array))
```

Output

- We will need around $\sim \log_2(1000000)$ expansions only for 1 million numbers
- This means, we rarely apply the expansion idea
- So for **approximately**
 - 1000000 - 16 steps \Rightarrow we were constant time
 - For 16 steps \Rightarrow we were linear time
 - If you thought from the **average** perspective, as if we do on average constant steps
 - This is called Amortized Analysis
 - Optionally, read after the complexity section

```
1, 2, 3, 12, hello,  
Expand capacity to 32  
Expand capacity to 64  
Expand capacity to 128  
Expand capacity to 256  
Expand capacity to 512  
Expand capacity to 1024  
Expand capacity to 2048  
Expand capacity to 4096  
Expand capacity to 8192  
Expand capacity to 16384  
Expand capacity to 32768  
Expand capacity to 65536  
Expand capacity to 131072  
Expand capacity to 262144  
Expand capacity to 524288  
Expand capacity to 1048576  
1000005
```


“Acquire knowledge and impart it to the people.”

“Seek knowledge from the Cradle to the Grave.”