

# *Data Structures*

## Binary Tree Types

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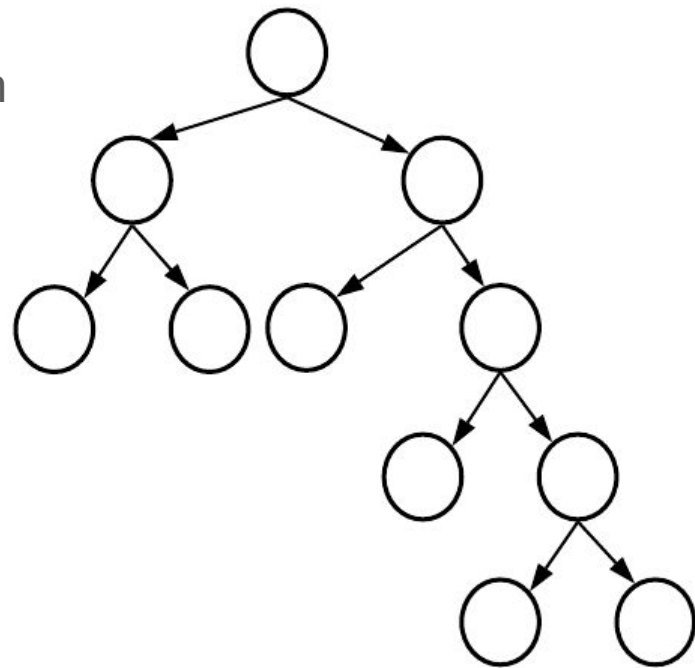
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# Full (or strict) Binary Tree

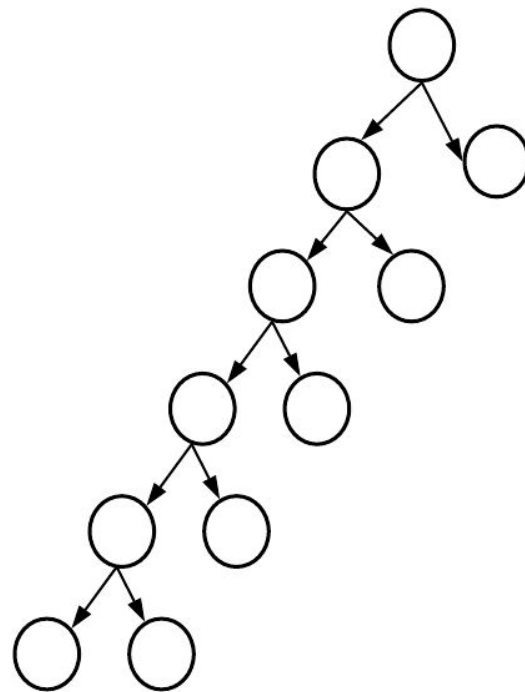
- Condition: Every node has either 0 or 2 children
  - Internal node (non-leaf) a node with 1+ child
  - $N - \text{leaf\_nodes}$
- In any **full** tree, always
$$\text{Leaf\_nodes} = \text{Internal\_nodes} + 1$$
- The general version is called a full **k-ary tree**
  - Each node has either 0 or exactly k children



# Full (or strict) Binary Tree

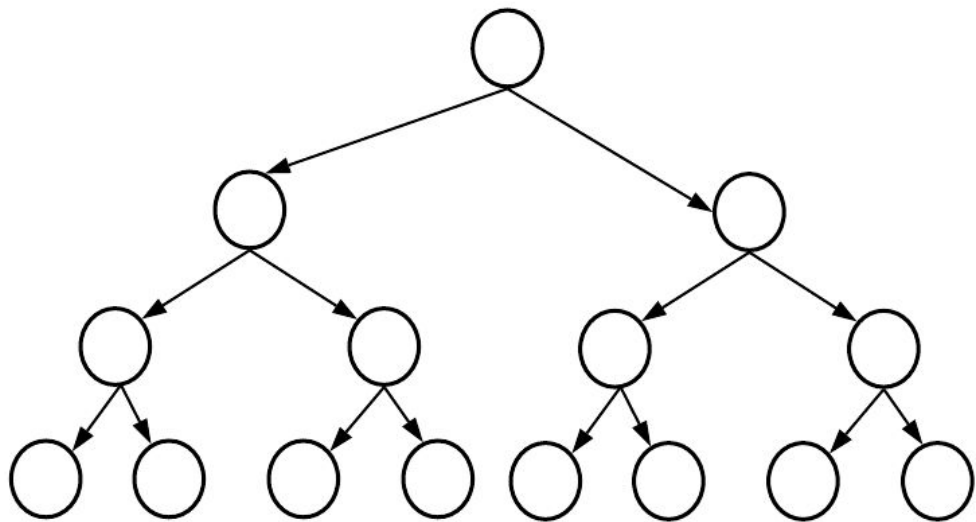
- Given height  $h$ , the min # of nodes in a full tree is:  **$2^{h+1}$**
- Can you create such a tree for  $h = 5$ ?
- Take 5 minutes

Just put the minimum number of nodes per level, which is 2, except for the root, which will have 1



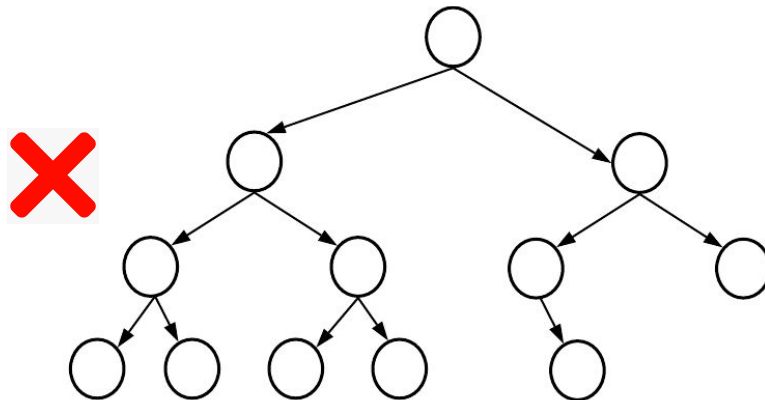
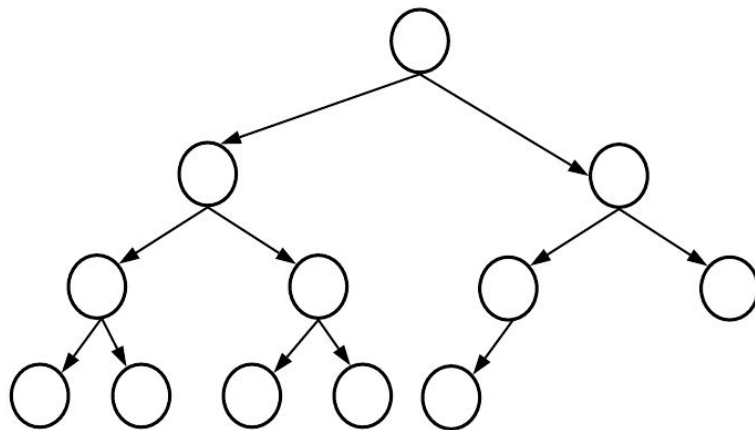
# Perfect Binary Tree

- 2 conditions
  - All leaf nodes have the **same level**
  - All other nodes have **2 children**
- This tree has 4 levels
  - All leaf nodes are at the 4th level
  - The first 3 levels have 2 children
- 4 **complete** levels
  - A complete level: **all possible** nodes **exist**
- Recall
  - Levels = 4 but Height = 3



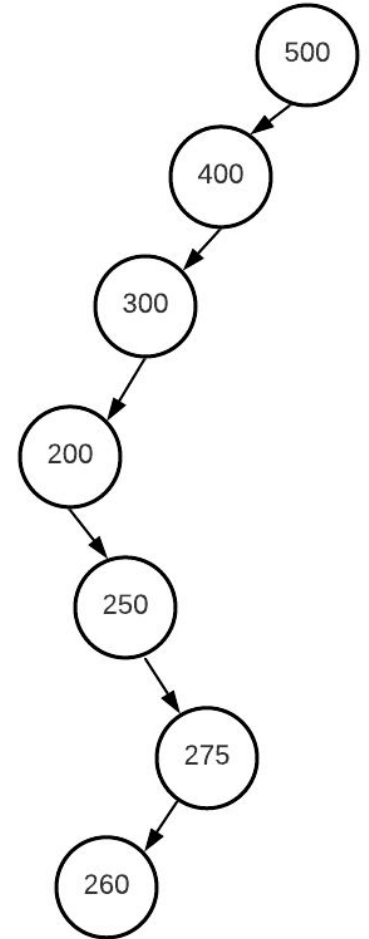
# Complete Binary Tree

- All levels are **complete**
  - **except possibly** the last one, which is filled from the **left**.
- Top tree
  - 4 levels
  - The first 3 are complete
  - The last one has left nodes
- Bottom tree: NOT complete
  - Has a right node before a left one
- A **perfect** Binary Tree is a **complete** tree / full tree



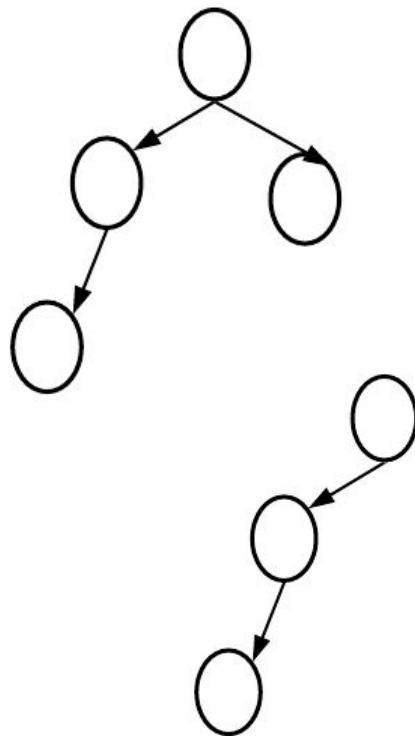
# A degenerate tree

- Each node has 1 child
- Given N nodes, we can construct a tree with the **largest depth** of N using a degenerate tree
- But what about the **smallest depth**?



# A balanced binary tree

- Degenerate subtrees make our code very slow.
- In a balanced tree, we try to make the height as small as possible (  $\sim \log n$  )
- We achieve that, we build a balanced binary tree, which satisfies the following conditions:
  - The difference in height between the left and right subtrees is **never greater than 1**
    - E.g. heights of (left, right): (6, 6) or (6, 5) or (5, 6)
  - The left subtree is balanced
  - The right subtree is balanced
- The tree above is balanced; the one below is not
- Complete/perfect trees are balanced



*“Acquire knowledge and impart it to the people.”*

*“Seek knowledge from the Cradle to the Grave.”*