

Example with three CST triangles:

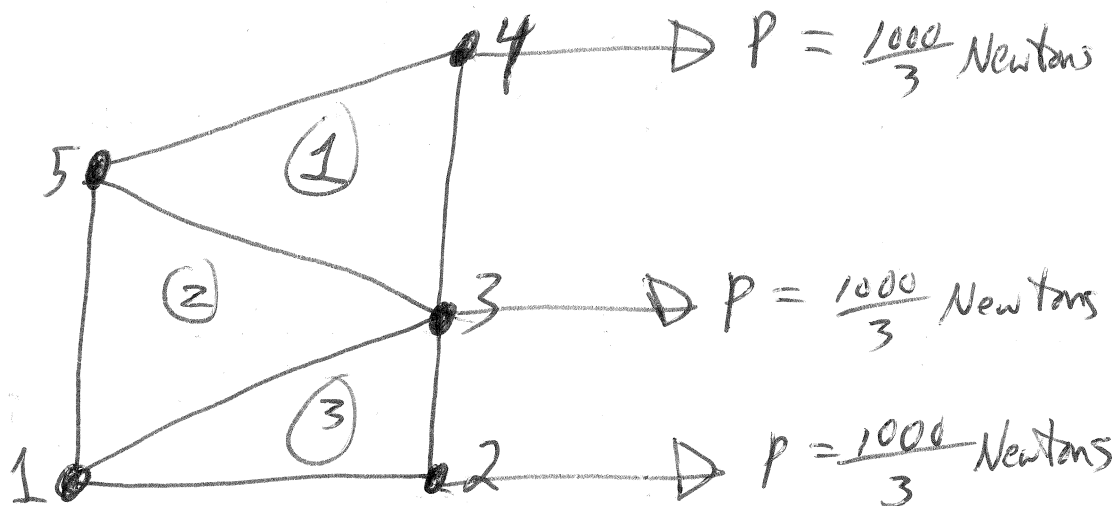
$$t = 0.001 \text{ ~~meters~~ meters}$$

$$E = 210 \times 10^9 \frac{N}{m^2}$$

$$\nu = 0.3 \text{ (Poisson's ratio)}$$

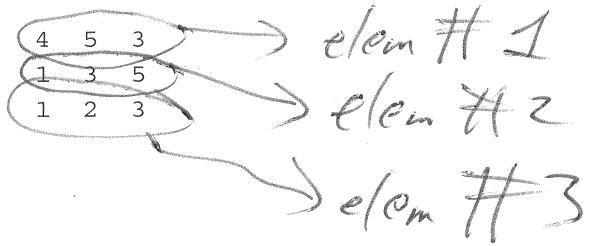
Plane-stress

sketch (not to scale):



Boundary conditions: $u_1 = u_5 = 0$, $\cancel{\nu_5 = 0}$

$$\nu_1 = 0$$



| | | |
|-------------------------|-------------------------|----------|
| 0.0000000000000000e+000 | 0.0000000000000000e+000 | — node 1 |
| 1.1000000000000001e+000 | 0.0000000000000000e+000 | — node 2 |
| 1.2000000000000000e+000 | 1.3000000000000000e+000 | — node 3 |
| 9.0000000000000002e-001 | 2.899999999999999e+000 | — node 4 |
| 1.0000000000000001e-001 | 2.1000000000000001e+000 | — node 5 |

(x,y) locations of nodes

1

5

4
3
2

which_elem =

1

$$\vec{d} = \{u_1, v_1, u_2, v_2, u_3, v_3, u_4, v_4, u_5, v_5\}^T$$

K =

| | 7 | 8 | 9 | 10 | 5 | 6 |
|----|--------------|--------------|--------------|--------------|--------------|--------------|
| 7 | 8.0731e+007 | 4.3421e+007 | -1.0593e+008 | -5.2227e+007 | 2.5202e+007 | 8.8057e+006 |
| 8 | 4.3421e+007 | 1.0886e+008 | -4.6457e+007 | -5.9059e+007 | 3.0364e+006 | -4.9798e+007 |
| 9 | -1.0593e+008 | -4.6457e+007 | 1.9672e+008 | 2.3684e+007 | -9.0789e+007 | 2.2773e+007 |
| 10 | -5.2227e+007 | -5.9059e+007 | 2.3684e+007 | 7.4848e+007 | 2.8543e+007 | -1.5789e+007 |
| 5 | 2.5202e+007 | 3.0364e+006 | -9.0789e+007 | 2.8543e+007 | 6.5587e+007 | -3.1579e+007 |
| 6 | 8.8057e+006 | -4.9798e+007 | 2.2773e+007 | -1.5789e+007 | -3.1579e+007 | 6.5587e+007 |

Ilist =

7

8

) stiffness matrix of elem #1.

Jlist =

9

10

Mlist =

5

6

list =

7 } dof list for elem #1
 8 }
 9 }
 10 }
 5 }
 6 }

which_elem =

2

K =

| | 1 | 2 | 5 | 6 | 9 | 10 |
|---|-------------|-------------|--------------|--------------|-------------|-------------|
| 1 | 5.1344e+007 | 2.7615e+007 | -7.9248e+007 | -3.7874e+007 | 2.7905e+007 | 1.0259e+007 |

| | 1 | 2 | 5 | 6 | 7 | 10 |
|----|--------------|--------------|--------------|--------------|--------------|--------------|
| 2 | 2.7615e+007 | 6.9231e+007 | -3.2105e+007 | -2.3077e+007 | 4.4899e+006 | -4.6154e+007 |
| 5 | -7.9248e+007 | -3.2105e+007 | 2.1308e+008 | -6.5900e+006 | -1.3383e+008 | 3.8695e+007 |
| 6 | -3.7874e+007 | -2.3077e+007 | -6.5900e+006 | 7.5000e+007 | 4.4464e+007 | -5.1923e+007 |
| 9 | 2.7905e+007 | 4.4899e+006 | -1.3383e+008 | 4.4464e+007 | 1.0592e+008 | -4.8954e+007 |
| 10 | 1.0259e+007 | -4.6154e+007 | 3.8695e+007 | -5.1923e+007 | -4.8954e+007 | 9.8077e+007 |

Ilist =

1
2

↓
stiffness
matrix of elem # 2

Jlist =

5
6

Mlist =

9
10

list =

1 } dof list for elem # 2
2
5
6
9
10

which_elem =

3

K =

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|--------------|--------------|--------------|--------------|--------------|--------------|
| 1 | 1.3665e+008 | -6.8182e+006 | -1.3975e+008 | 4.1434e+007 | 3.1065e+006 | -3.4615e+007 |
| 2 | -6.8182e+006 | 4.8534e+007 | 4.7203e+007 | -5.7410e+007 | -4.0385e+007 | 8.8757e+006 |
| 3 | -1.3975e+008 | 4.7203e+007 | 1.7703e+008 | -8.1818e+007 | -3.7278e+007 | 3.4615e+007 |
| 4 | 4.1434e+007 | -5.7410e+007 | -8.1818e+007 | 1.6392e+008 | 4.0385e+007 | -1.0651e+008 |
| 5 | 3.1065e+006 | -4.0385e+007 | -3.7278e+007 | 4.0385e+007 | 3.4172e+007 | 0 |
| 6 | -3.4615e+007 | 8.8757e+006 | 3.4615e+007 | -1.0651e+008 | 0 | 9.7633e+007 |

Ilist =

stiffness matrix of element # 3

1
2

Jlist =

3
4

Mlist =

5
6

list =

1 } dof list for element #3
2
3
4
5
6

Kglobal_columns_1thru5 =

| | 1 | 2 | 3 | 4 | 5 |
|----|--------------|--------------|--------------|--------------|--------------|
| 1 | 1.8799e+008 | 2.0797e+007 | -1.3975e+008 | 4.1434e+007 | -7.6142e+007 |
| 2 | 2.0797e+007 | 1.1776e+008 | 4.7203e+007 | -5.7410e+007 | -7.2490e+007 |
| 3 | -1.3975e+008 | 4.7203e+007 | 1.7703e+008 | -8.1818e+007 | -3.7278e+007 |
| 4 | 4.1434e+007 | -5.7410e+007 | -8.1818e+007 | 1.6392e+008 | 4.0385e+007 |
| 5 | -7.6142e+007 | -7.2490e+007 | -3.7278e+007 | 4.0385e+007 | 3.1283e+008 |
| 6 | -7.2490e+007 | -1.4201e+007 | 3.4615e+007 | -1.0651e+008 | -3.8169e+007 |
| 7 | 0 | 0 | 0 | 0 | 2.5202e+007 |
| 8 | 0 | 0 | 0 | 0 | 3.0364e+006 |
| 9 | 2.7905e+007 | 4.4899e+006 | 0 | 0 | -2.2462e+008 |
| 10 | 1.0259e+007 | -4.6154e+007 | 0 | 0 | 6.7237e+007 |

global stiffness
matrix

Kglobal_columns_6thru10 =

| | 6 | 7 | 8 | 9 | 10 |
|---|--------------|-------------|--------------|--------------|--------------|
| 1 | -7.2490e+007 | 0 | 0 | 2.7905e+007 | 1.0259e+007 |
| 2 | -1.4201e+007 | 0 | 0 | 4.4899e+006 | -4.6154e+007 |
| 3 | 3.4615e+007 | 0 | 0 | 0 | 0 |
| 4 | -1.0651e+008 | 0 | 0 | 0 | 0 |
| 5 | -3.8169e+007 | 2.5202e+007 | 3.0364e+006 | -2.2462e+008 | 6.7237e+007 |
| 6 | 2.3822e+008 | 8.8057e+006 | -4.9798e+007 | 6.7237e+007 | -6.7713e+007 |
| 7 | 8.8057e+006 | 8.0731e+007 | 4.3421e+007 | -1.0593e+008 | -5.2227e+007 |
| 8 | -4.9798e+007 | 4.3421e+007 | 1.0886e+008 | -4.6457e+007 | -5.9059e+007 |

| | | | | | |
|----|--------------|--------------|--------------|--------------|--------------|
| | 6 | 7 | 8 | 9 | 10 |
| 9 | 6.7237e+007 | -1.0593e+008 | -4.6457e+007 | 3.0265e+008 | -2.5270e+007 |
| 10 | -6.7713e+007 | -5.2227e+007 | -5.9059e+007 | -2.5270e+007 | 1.7293e+008 |

nodal_load =

| | | |
|-------------|---|----|
| | 0 | 1 |
| | 0 | 2 |
| 3.3333e+002 | 3 | |
| | 0 | 4 |
| 3.3333e+002 | 5 | |
| | 0 | 6 |
| 3.3333e+002 | 7 | |
| | 0 | 8 |
| | 0 | 9 |
| | 0 | 10 |

listdofremove =

1
9
2

} constrained dof's.
These are zero.

listkeep =

3
4
5
6
7
8
10

} list of unconstrained
degrees of
freedom

Kreduced_columns_1thru_5 =

| | | | | | |
|----|--------------|--------------|--------------|--------------|--------------|
| | 3 | 4 | 5 | 6 | 7 |
| 3 | 1.7703e+008 | -8.1818e+007 | -3.7278e+007 | 3.4615e+007 | 0 |
| 4 | -8.1818e+007 | 1.6392e+008 | 4.0385e+007 | -1.0651e+008 | 0 |
| 5 | -3.7278e+007 | 4.0385e+007 | 3.1283e+008 | -3.8169e+007 | 2.5202e+007 |
| 6 | 3.4615e+007 | -1.0651e+008 | -3.8169e+007 | 2.3822e+008 | 8.8057e+006 |
| 7 | 0 | 0 | 2.5202e+007 | 8.8057e+006 | 8.0731e+007 |
| 8 | 0 | 0 | 3.0364e+006 | -4.9798e+007 | 4.3421e+007 |
| 10 | 0 | 0 | 6.7237e+007 | -6.7713e+007 | -5.2227e+007 |

Kreduced_columns_6thru_7 =

| | | |
|---|---|----|
| | 8 | 10 |
| 3 | 0 | 0 |
| 4 | 0 | 0 |

5 3.0364e+006 6.7237e+007
 6 -4.9798e+007 -6.7713e+007
 7 4.3421e+007 -5.2227e+007
 8 1.0886e+008 -5.9059e+007
 10 -5.9059e+007 1.7293e+008

nodal_load_reduced =

3.3333e+002 3
 0 4
 3.3333e+002 5
 0 6
 3.3333e+002 7
 0 8
 0 10

disp_reduced =

2.4854e-006 3
 8.4473e-007 4
 6.8114e-007 5
 -3.5089e-007 6
 5.3969e-006 7
 -2.0451e-006 8
 5.2927e-007 10

disp =

0 1
 0 2
 2.4854e-006 3
 8.4473e-007 4
 6.8114e-007 5
 -3.5089e-007 6
 5.3969e-006 7
 -2.0451e-006 8
 0 9
 5.2927e-007 10

maxsx =

6.4132e+005

} maximum T_x over all three elements.