SYSTEMS SCIENCE & CONTROL ENGINEERING: AN OPEN ACCESS JOURNAL
2023, VOL. 11, NO. 1, 2207602
https://doi.org/10.1080/21642583.2023.2207602

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Consensus tracking for a class of fractional-order non-linear multi-agent systems via an adaptive dynamic surface controller

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ABSTRACT
In this paper we investigate bottlenecks in adaptive dynamic surface control (DSC) and unveil an innovative consensus tracking controller to track the desired trajectory for a class of fractional-order multi-agent systems with non-linear dynamics. The study derives an algorithm by implementing graph theory and the DSC method. The main approaches in the control of fractional-order systems are the DSC and the adaptive DSC techniques to avoid the computational complexity of fractional-order virtual control law. According to these techniques, a virtual control law is formulated and the proposed controller is passed through a fractional-order dynamic surface. By employing the DSC and adaptive DSC laws, we demonstrate that the desired consensus tracking between agents can be ensured. To verify the performance of the new approach, we simulate the desirable scenarios and evaluate the results against a popular adaptive sliding mode technique.

ARTICLE HISTORY
Received 2 August 2022
Accepted 22 April 2023

KEYWORDS
Multi-agent systems; fractional-order systems; consensus; dynamic surface control

Introduction

In recent years, research on multi-agent systems has attracted the attention and deployment of this technology in different fields such as engineering, physics and biology (Li & Tan, 2019; Shang, 2019) is gaining greater momentum. Agents in multi-agent systems exchange information via communication graphs or topology.

Our survey of recent literature shows that control of vehicles and target tracking are topics that are focused on (Karimi & Lu, 2021; Lu et al., 2022). Multi-agent systems are also emerging as important research fields due to their many desirable applications and works on these systems include a consensus analysis by (Cui et al., 2016; Han et al., 2020; Shang, 2018; Yaghoubi, 2020; Yaghoubi & Talebi, 2020a, 2020b), formation (Liu et al., 2021; Wang et al., 2020; Xiong & Gu, 2021; Zhao et al., 2022), flocking (Amirian & Shamaghdari, 2021), swarming (Wang et al., 2019) and synchronization (Wen et al., 2021).

Consensus means that all the agents reach a common value or a common path and this procedure is seen as an important and interesting topic for further study. In general, the consensus problem is divided into leader-following consensus and leaderless consensus (Ni & Cheng, 2010). The neural network methods (Wu et al., 2019, 2021) are examined for leader-following consensus by (G. Wen et al., 2016). In this paper the leader-following consensus is investigated for fractional-order non-linear multi-agent systems, as we have noted is currently a subject of intense exploration.

The fractional-order systems are controlled more problematic and difficult than integer-order systems and this system has an operational point. Among the many important properties of fractional-order systems (Gao & Liao, 2013), their versatile memory makes them suitable for applications such as robotics, bioengineering and economic systems modelling (Podlubny, 1998; Shen & Lam, 2014).

One of the most popular techniques for controlling non-linear systems is called the backstepping control method which is investigated for integer-order systems in (Liu et al., 2015; Yu et al., 2021). This technique is extended to fractional-order systems by implementing the fractional filter in (H. Liu et al., 2020; Wei et al., 2015). The drawback with this approach is that it gives rise to a major problem by fractional derivatives repetition which is called the 'explosion of complexity'.

To deal with the 'explosion of complexity', Dynamic Surface Control was introduced in (Yip & Hedrick, 1998), which exhibited the potential to control a non-linear system without requiring the derivative of the previous step virtual input, as disclosed in (Yang & Yue, 2017; Zhao et al., 2021). So, the fractional-order DSC method is used to avoid the computational complexity of fractional-order virtual control law.

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In short, in this paper, a dynamic surface controller (DSC) and an adaptive dynamic surface controller (Adaptive DSC) are proposed for fractional-order multi-agent systems to obtain consensus and track the desired trajectory.

The main contributions of our proposal are as follows:

(1) The dynamic surface control was extended and applied to fractional-order multi-agent systems. Thus, this provided the basis for our stability analysis proposal.
(2) The formulated consensus algorithm for tracking fractional-order multi-agent non-linear systems via DSC.
(3) The formulated consensus algorithm for tracking fractional-order multi-agent non-linear systems with unknown parameters via adaptive DSC.
(4) Agents with unknown parameters can track the desired trajectory and achieve consensus with adaptive DSC in less time.

The rest of the paper is organized as follows: In section II, we define graph theory and examine the structure and application of this important tool. Next, we present some lemmas and include definitions of fractional-order calculus in Section III. We move on to Section IV, in which we lay out our problem formulation and follow it up by providing the proof of stability in Section V. Section VI is devoted to the simulation of our proposed schemes and the comparison of results against competing approaches. The paper concludes in Section VII, highlighting its findings.

Graph theory

Let us begin this section by providing definitions of graph theory (Gu & Tian, 2019). The communication graph describes the connection between agents and is expressed as $G = (V, E, A)$, where $E \subseteq V \times V$ denotes the edge set and the node-set with n agents denoted as $V = \{\nu_1, \nu_2, \ldots, \nu_n\}$. $\mathcal{N}_i = \{\nu_j \in V : (\nu_j, \nu_i) \in E\}$ is the neighbour set of the agent i. One of the essential matrices in graph theory is the adjacency matrix which is defined as $A = [a_{ij}] \in \mathbb{R}^{n\times n}$, with $a_{ij} > 0$ if $(\nu_j, \nu_i) \in E$, otherwise $a_{ij} = 0$. Another essential matrix is Laplacian which is formulated as $L = D - A$, where $D = \operatorname{diag}\left\{\sum_{j=1}^{n} a_{ij}\right\} \in \mathbb{R}^{n\times n}$. There is still another matrix which is related to information exchange between leader and followers, and is represented as $B = \operatorname{diag}(b_1, b_2, \ldots, b_n)$, where $b_i = 1$ if the agent i receives information from the leader, otherwise $b_i = 0$.

Fractional calculus

Let us first present some fractional-order definitions. The Caputo fractional derivative for $m - 1 < \alpha < m, m \in \mathbb{N}^+$ is shown as ${}_0^c\mathcal{D}_t^\alpha x(t)$ or for simplicity $x^\alpha(t)$ which is defined as follows (Yin et al., 2013):

$$ {}_0^c\mathcal{D}_t^\alpha x(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-\tau)^{m-\alpha-1} x^{(m)}(\tau)\mathrm{d}\tau, \quad (1) $$

The Gamma function is defined as: $\Gamma(\alpha) = \int_0^{+\infty} \tau^{\alpha-1} e^{-\tau}\mathrm{d}\tau$.

The following function is the Mittag-Leffler function which is formulated for $\alpha, \beta \in R$ as follows (Wang & Yang, 2017):

$$ E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(k\alpha + \beta)} \quad (2) $$

The Mittag-Leffler function with $\alpha = 1$ is converted to the exponential function. In this section, some operative lemmas are presented as follows.

Lemma 1.1: *(Podlubny, 1998)**.** For the Mittag-Leffler function, the following inequality holds for $0 < \alpha < 2$ and $\beta \in R$ with $C \in R^+$: $|E_{\alpha,\beta}(z)| \le \frac{C}{1+|z|}$.*

Lemma 1.2: *For a continuous and derivable function $x(t) \in \mathbb{R}$, the following inequality is defined for $t \ge t_0$ (Aguila-Camacho et al., 2014; Li et al., 2010; Zhang et al., 2017):*

$$ \frac{1}{2} {}_{t_0}^c\mathcal{D}_t^\alpha x^2(t) \le x(t) {}_{t_0}^c\mathcal{D}_t^\alpha x(t), \quad \alpha \in (0, 1) \quad (3) $$

Lemma 1.3: *The following inequality for $x(t) \in \mathbb{R}^n$ is established:*

$$ {}_{t_0}^c\mathcal{D}_t^\alpha (x^T(t)Px(t)) \le 2x^T(t)P {}_{t_0}^c\mathcal{D}_t^\alpha x(t), \quad \alpha \in (0, 1) \quad (4) $$

with a positive definite symmetric matrix, $P \in \mathbb{R}^{n\times n}$.

Lemma 1.4: *If for the continuous function $V(t)$, the following inequality is established for $\alpha \in (0, 1]$:*

$$ V^\alpha(t) \le -\beta V(t) \quad (5) $$

then

$$ V(t) \le E_\alpha(-\beta(t - t_0)^\alpha)V(t_0), \quad t \ge t_0 \quad (6) $$

where $\beta > 0$ (Chen et al., 2014).

Problem formulation

A class of fractional-order multi-agent systems are considered in this paper in which the topology has N agents. The

ith agent is modelled as follows (Yang & Yue, 2017):

$$\begin{cases} x_{i1}^{\alpha} = f_{i1}(x_{i1}, x_{i2}) \\ x_{i2}^{\alpha} = f_{i2}(x_{i1}, x_{i2}, x_{i3}) \\ \vdots \\ x_{i(n_i-1)}^{\alpha} = f_{i(n_i-1)}(x_{i1}, \ldots, x_{i(n_i-1)}, x_{i(n_i)}) \\ x_{i(n_i)}^{\alpha} = f_{i(n_i)}(x_{i1}, \ldots, x_{i(n_i-1)}, x_{i(n_i)}, u_i) \\ y_i = x_{i1} \end{cases} \tag{7}$$

where the state of the agents is x_{ij}, $i, = 1, \ldots, N$, $j = 1, \ldots, n_i$ and $f_{i,}(.)$ are non-linear functions that satisfy Assumption 1. The input and output signals are denoted as $u_i \in \mathbb{R}$ and $y_i \in \mathbb{R}$ for the ith agent.

Assumption 1.1: For function $f(x)$, the Lipschitz condition is represented as follows with the Lipschitz constant l: $|f(x_2) - f(x_1)| \le l|x_2 - x_1|$, $\forall x_1, x_2 \in \mathbb{R}, \forall t \ge 0$.

As stated above, our objective is to achieve consensus among agents and synchronize the output of the system (7), y_i with y_d, which is the desired trajectory.

Definition 1.1: For multi-agent systems (7), consensus is achieved if the following conditions hold:

$$\lim_{t \to \infty} (y_i - y_d) = 0,$$
$$\lim_{t \to \infty} (y_i - y_j) = 0$$

where $i, = 1, \ldots, N$, $j = 1, \ldots, n_i$.

Controller design

DSC design

In this section, by exploiting the DSC technology a consensus for system (7) will be designed and the Lyapunov stability analysis will be investigated.

The system (7) can be rewritten as follows:

$$\begin{cases} x_{i1}^{\alpha} = x_{i2} + F_{i1}(x_{i1}, x_{i2}) \\ x_{i2}^{\alpha} = x_{i3} + F_{i2}(x_{i1}, x_{i2}, x_{i3}) \\ \vdots \\ x_{i(n_i-1)}^{\alpha} = x_{i(n_i)} + F_{i(n_i-1)}(x_{i1}, \ldots, x_{i(n_i-1)}, x_{i(n_i)}) \\ x_{i(n_i)}^{\alpha} = u_i + F_{i(n_i)}(x_{i1}, \ldots, x_{i(n_i-1)}, x_{i(n_i)}, u_i) \\ y_i = x_{i1} \end{cases} \tag{8}$$

where $F_{ij}(x_{i1}, \ldots, x_{i(j+1)}) = f_{ij}(x_{i1}, \ldots, x_{i(j+1)}) - x_{i(j+1)}$, $F_{i(n_i)}(x_{i1}, \ldots, x_{i(n_i-1)}, x_{i(n_i)}, u_i) = f_{i(n_i)}(x_{i1}, \ldots, x_{i(n_i-1)}, x_{i(n_i)}, u_i) - u_i$.

Next, let us define a surface error as follows:

$$s_{i1} = \sum_{j \in \mathcal{N}_i} a_{ij}(y_i(t) - y_j(t)) + b_i(y_i(t) - y_d(t)) \tag{9}$$

$$s_{ij} = x_{ij} - x_{d_{ij}} \tag{10}$$

where y_d is the desired trajectory and $x_{d_{ij}}$ is defined later.

Step $i1$: The fractional-order derivation of s_{i1} is given as follows:

$$s_{i1}^{\alpha} = l_i(x_{i2} + F_{i1}) - \sum_{j \in \mathcal{N}_i} a_{ij}(x_{j2} + F_{j1}) - b_i y_d^{\alpha} \tag{11}$$

where $l_i = d_i + b_i$. The virtual control law is formulated as follows:

$$\tilde{x}_{i2} = \frac{\sum_{j \in \mathcal{N}_i} a_{ij}(x_{j2} + F_{j1})}{l_i} + \frac{b_i}{l_i} y_d^{\alpha} - F_{i1} - k_{i1} s_{i1} \tag{12}$$

where $k_{i1} > 0$. $x_{d_{i2}}$ is obtained by applying the first-order filter.

$$\tau_{i2} x_{d_{i2}}^{\alpha} + x_{d_{i2}} = \tilde{x}_{i2}, \quad x_{d_{i2}}(0) = \tilde{x}_{i2}(0) \tag{13}$$

where τ_{i2} is the time constant of the filter.

Generally, other steps are obtained as follows.

Step ij: The fractional-order derivation of s_{ij} (14) is defined as follows:

$$\begin{aligned} s_{ij}^{\alpha} &= x_{ij}^{\alpha} - x_{d_{ij}}^{\alpha} \\ &= x_{i(j+1)} + F_{ij} - x_{d_{ij}}^{\alpha} \end{aligned} \tag{14}$$

The virtual control law is expressed as follows:

$$\tilde{x}_{i(j+1)} = x_{d_{ij}}^{\alpha} - F_{ij} - k_{ij} s_{ij} \tag{15}$$

where $k_{ij} > 0$. $x_{d_{i(j+1)}}$ is computed by applying the first-order filter.

$$\tau_{i(j+1)} x_{d_{i(j+1)}}^{\alpha} + x_{d_{i(j+1)}} = \tilde{x}_{i(j+1)}, \quad x_{d_{i(j+1)}}(0) = \tilde{x}_{i(j+1)}(0) \tag{16}$$

Step $i(n_i)$: The fractional-order derivation of $s_{i(n_i)}$ is given as follows:

$$\begin{aligned} s_{i(n_i)}^{\alpha} &= x_{i(n_i)}^{\alpha} - x_{d_{i(n_i)}}^{\alpha} \\ &= u_i + F_{i(n_i)} - x_{d_{i(n_i)}}^{\alpha} \end{aligned} \tag{17}$$

Hence, the control law is chosen as follows:

$$u_i = x_{d_{i(n_i)}}^{\alpha} - F_{i(n_i)} - k_{i(n_i)} s_{i(n_i)} \tag{18}$$

Theorem 1.1: *The group of followers with system (8) track leader and the consensus problem is achieved with controller (18).*

Proof: The ith Lyapunov function is considered as follows:

$$\boldsymbol{V_i} = \frac{1}{2} \sum_{\boldsymbol{j}=1}^{\boldsymbol{n_i}} \boldsymbol{s_{ij}^2} + \frac{1}{2} \sum_{\boldsymbol{j}=1}^{\boldsymbol{n_i}-1} \boldsymbol{e_{i(j+1)}^2} \tag{19}$$

where $e_{i(j+1)} = x_{d_{i(j+1)}} - \tilde{x}_{i(j+1)}$.

The fractional-order derivative of V is obtained as follows:

$$\begin{aligned} V_i^\alpha &\le \sum_{j=1}^{n_i} s_{ij}s_{ij}^\alpha + \sum_{j=1}^{n_i-1} e_{i(j+1)}e_{i(j+1)}^\alpha \\ &= s_{i1}s_{i1}^\alpha + \sum_{j=2}^{n_i} s_{ij}s_{ij}^\alpha + \sum_{j=1}^{n_i-1} e_{i(j+1)}e_{i(j+1)}^\alpha \\ &= (l_i(x_{i2}+F_{i1}) - \sum_{j\in\mathcal{N}_i} a_{ij}(x_{j2}+F_{j1}) - b_i y_d^\alpha)s_{i1} \\ &\quad + \sum_{\boldsymbol{j=2}}^{\boldsymbol{n_i}} \boldsymbol{s_{ij}s_{ij}^\alpha} + \sum_{\boldsymbol{j=1}}^{\boldsymbol{n_i-1}} \boldsymbol{e_{i(j+1)}e_{i(j+1)}^\alpha} \end{aligned} \tag{20}$$

The fractional-order derivation of $e_{i(j+1)}$ by applying (15) is obtained as follows:

$$\begin{aligned} e_{i(j+1)}^\alpha &= x_{d_{i(j+1)}}^\alpha - \tilde{x}_{i(j+1)}^\alpha = -\frac{e_{i(j+1)}^2}{\tau_{i(j+1)}} \\ &\quad + \underbrace{k_{ij}s_{ij}^\alpha + F_{ij}^\alpha - (x_{d_{i(j+1)}}^\alpha)^\alpha}_{B_{i(j+1)}} \end{aligned} \tag{21}$$

By applying (21), (20) is derived as follows:

$$\begin{aligned} V_i^\alpha &\le (l_i x_{i2} - k_{i1}s_{i1} - l_i\tilde{x}_{i2})s_{i1} + \sum_{j=2}^{n_i} s_{ij}s_{ij}^\alpha \\ &\quad + \sum_{j=1}^{n_i-1} e_{i(j+1)}e_{i(j+1)}^\alpha \\ &= l_i(s_{i1}s_{i2} + e_{i2}s_{i1} - k_{i1}s_{i1}^2) \\ &\quad + \sum_{j=2}^{n_i-1} (s_{ij}s_{i(j+1)} + s_{ij}e_{i(j+1)} - k_{ij}s_{ij}^2) \\ &\quad - k_{i(n_i)}s_{i(n_i)}^2 + \sum_{j=1}^{n_i-1}\left(-\frac{e_{i(j+1)}^2}{\tau_{i(j+1)}} + e_{i(j+1)}B_{i(j+1)}\right) \end{aligned} \tag{22}$$

The following inequalities are established:

$$s_{ij}s_{i(j+1)} \le \frac{s_{ij}^2}{4} + s_{i(j+1)}^2 \tag{23}$$

$$e_{i(j+1)}B_{i(j+1)} \le \frac{M_{i(j+1)}^2}{2\beta_{i(j+1)}} + \frac{\beta_{i(j+1)}e_{i(j+1)}^2}{2} \tag{24}$$

where β is the positive constant and $\max(B) = M$.

By applying (23) and (24), (22) can be rewritten as follows:

$$\begin{aligned} V_i^\alpha &\le l_i\left(\frac{s_{i1}^2}{4} + s_{i2}^2 + \frac{s_{i1}^2}{4} + e_{i2}^2 - k_{i1}s_{i1}^2\right) - k_{i2}s_{i2}^2 + \frac{s_{i2}^2}{4} \\ &\quad + s_{i3}^2 + \frac{s_{i2}^2}{4} + e_{i3}^2 \\ &\quad + \sum_{j=3}^{n_i-1}\left(\frac{s_{ij}^2}{4} + s_{i(j+1)}^2 + \frac{s_{ij}^2}{4} + e_{i(j+1)}^2 - k_{ij}s_{ij}^2\right) \\ &\quad - k_{i(n_i)}s_{i(n_i)}^2 \\ &\quad + \sum_{j=1}^{n_i-1}\left(-\frac{e_{i(j+1)}^2}{\tau_{i(j+1)}} + \frac{M_{i(j+1)}^2}{2\beta_{i(j+1)}} + \frac{\beta_{i(j+1)}e_{i(j+1)}^2}{2}\right) \\ &= l_i\left(\frac{1}{2} - k_{i1}\right)s_{i1}^2 + \left(l_i - k_{i2} + \frac{1}{2}\right)s_{i2}^2 \\ &\quad + \sum_{j=3}^{n_i-1}\left(\frac{3}{2} - k_{ij}\right)s_{ij}^2 + (-k_{i(n_i)}+1)s_{i(n_i)}^2 + l_i e_{i2}^2 \\ &\quad + \sum_{j=2}^{n_i-1} e_{i(j+1)}^2 \qquad (1) \\ &\quad + \sum_{j=1}^{n_i-1}\left(-\frac{e_{i(j+1)}^2}{\tau_{i(j+1)}} + \frac{M_{i(j+1)}^2}{2\beta_{i(j+1)}} + \frac{\beta_{i(j+1)}e_{i(j+1)}^2}{2}\right) \end{aligned} \tag{25}$$

Equation (25) can be rewritten as follows:

$$\begin{aligned} V_i^\alpha &\le l_i\left(\tfrac{1}{2} - k_{i1}\right)s_{i1}^2 + \left(l_i - k_{i2} + \tfrac{1}{2}\right)s_{i2}^2 \\ &\quad + \sum_{j=3}^{n_i-1}\left(\tfrac{3}{2} - k_{ij}\right)s_{ij}^2 + (-k_{i(n_i)}+1)s_{i(n_i)}^2 \\ &\quad - \sum_{j=1}^{n_i-1}\xi_{i(j+1)}e_{i,j+1}^2 + \sum_{j=1}^{n_i-1}\frac{M_{i(j+1)}^2}{2\beta_{i(j+1)}} \end{aligned} \tag{26}$$

where

$$\xi_{i(j+1)} + 1 + \frac{\beta_{i(j+1)}}{2} = \frac{1}{\tau_{i(j+1)}},\ j = 2, \ldots, n_i - 1,\ \xi_{i(j+1)} > 0$$

and $\xi_{i2} + l_i + \frac{\beta_{i2}}{2} = \frac{1}{\tau_{i2}}$, $\xi_{i2} > 0$.

Choose $V = \sum_{i=1}^N V_i$ then its fractional-order derivative becomes:

$$\begin{aligned} V^\alpha &\le \sum_{i=1}^N l_i\left(\tfrac{1}{2} - k_{i1}\right)s_{i1}^2 + \sum_{i=1}^N\left(l_i - k_{i2} + \tfrac{1}{2}\right)s_{i2}^2 \\ &\quad + \sum_{i=1}^N\sum_{j=3}^{n_i-1}\left(\tfrac{3}{2} - k_{ij}\right)s_{ij}^2 + \sum_{i=1}^N(-k_{i(n_i)}+1)s_{i(n_i)}^2 \\ &\quad - \sum_{i=1}^N\sum_{j=1}^{n_i-1}\xi_{i(j+1)}e_{i(j+1)}^2 + \sum_{i=1}^N\sum_{j=1}^{n_i-1}\frac{M_{i(j+1)}^2}{2\beta_{i(j+1)}} \\ &\le -\gamma V + Q \end{aligned} \tag{27}$$

where

$$\gamma = \min\{\gamma_1, \ldots, \mu_N\},$$

$$\begin{aligned} \gamma_i &= \{2l_ik_{i1} - l_i, 2k_{i2} - 2l_i - 1, 2k_{i(n_i)} - 2, \\ &\quad 2\min\left\{k_{i3} - \tfrac{3}{2}, \ldots, k_{i(n_i-1)} - \tfrac{3}{2}\right\} \\ &\quad 2\min\{\xi_{i2}, \ldots, \xi_{i(n_i)}\}\} \end{aligned}$$

and $Q = \sum_{i=1}^N\sum_{j=1}^{n_i-1}\frac{M_{i(j+1)}^2}{2\beta_{i(j+1)}}$.

By applying Lemma 3.1 and (Liu et al., 2019; Liu et al., 2016), (28) can be obtained as follows:

$$\begin{aligned} V &\le V(0)E_{\alpha,1}(-\gamma t^\alpha) + Qt^\alpha E_{\alpha,\alpha+1}(-\gamma t^\alpha) \\ &\le \tfrac{V(0)C_1}{1+\gamma t^\alpha} + \tfrac{QC_2 t^\alpha}{1+\gamma t^\alpha} \le \tfrac{QC_2}{\gamma} \end{aligned} \tag{28}$$

where $C_1 > 0, C_2 > 0$. Hence $V(t)$ is bounded and $|s_{i1}| \le \sqrt{\frac{2QC_2}{\gamma}}$. Under this condition the error converges to a small value by designing suitable parameters; therefore;

the consensus errors reach a point in the vicinity of zero and the systems are asymptotically stable. This proves that the consensus problem is solvable for the system (8) by applying the controller (18). ■

Adaptive DSC design

In this section, the adaptive DSC controller is applied to the system (8) to achieve consensus then the Lyapunov stability analysis will be investigated for systems with unknown parameters.

The system (8) can be rewritten with unknown parameters as follows:

$$\begin{cases} x_{i1}^{\alpha} = x_{i2} + \theta_{i1}F_{i1}(x_{i1}, x_{i2}) \\ x_{i2}^{\alpha} = x_{i3} + \theta_{i2}F_{i2}(x_{i1}, x_{i2}, x_{i3}) \\ \vdots \\ x_{i(n_i-1)}^{\alpha} = x_{i(n_i)} + \theta_{i(n_i-1)}F_{i(n_i-1)}(x_{i1}, \ldots, x_{i(n_i-1)}, x_{in_i}) \\ x_{in_i}^{\alpha} = u_i + \theta_{in_i}F_{in_i}(x_{i1}, \ldots, x_{i(n_i)}, , u_i) \\ y_i = x_{i1} \end{cases} \tag{29}$$

where $\theta_{ij}, j = 1, \ldots, n_i$ are unknown parameters. The surface error is defined as (9) and (10).

Step $i1$: The fractional-order derivation of s_{i1} is given as follows:

$$s_{i1}^{\alpha} = l_i(x_{i2} + \hat{\theta}_{i1}F_{i1}) - \sum_{j\in\mathcal{N}_i} a_{ij}(x_{j2} + \hat{\theta}_{j1}F_{j1}) - b_i y_d^{\alpha} \tag{30}$$

where $l_i = d_i + b_i$and $\hat{\theta}_{ij}, j = 1, \ldots, n_i$ are estimates for constant parameters θ_{ij}. The virtual control law is selected as follows:

$$\tilde{x}_{i2} = \frac{\sum_{j\in\mathcal{N}_i} a_{ij}(x_{j2} + \hat{\theta}_{j1}F_{j1})}{l_i} + \frac{b_i}{l_i}y_d^{\alpha} - \hat{\theta}_{i1}F_{i1} - k_{i1}s_{i1} \tag{31}$$

where $k_{i1} > 0$. $x_{d_{i2}}$ is obtained by applying the first-order filter as (13).

The parameter update law is chosen as follows:

$$\hat{\theta}_{i1}^{\alpha} = \rho_{i1}s_{i1}\left(l_iF_{i1} - \sum_{j\in\mathcal{N}_i} a_{ij}F_{j1}\right) \tag{32}$$

where ρ_{i1} are positive design parameters.

Generally, other steps are obtained as follows:

Step i, j: The fractional-order derivation of s_{ij} is expressed as follows:

$$\begin{aligned} s_{ij}^{\alpha} &= x_{ij}^{\alpha} - x_{d_{ij}}^{\alpha} \\ &= x_{i(j+1)} + \hat{\theta}_{ij}F_{ij} - x_{d_{ij}}^{\alpha} \end{aligned} \tag{33}$$

The virtual control law is selected as follows:

$$\tilde{x}_{i(j+1)} = x_{d_{ij}}^{\alpha} - \hat{\theta}_{ij}F_{ij} - k_{ij}s_{ij} \tag{34}$$

where $k_{ij} > 0$. $x_{d_{i(j+1)}}$ is computed by implementing the first-order filter as (16).

The parameter update law is chosen as follows:

$$\hat{\theta}_{ij}^{\alpha} = \rho_{ij}s_{ij}F_{ij} \tag{35}$$

where ρ_{ij} are positive design parameters.

Step $i(n_i)$: The fractional-order derivation of $s_{i(n_i)}$ is represented as follows:

$$\begin{aligned} s_{i(n_i)}^{\alpha} &= x_{i(n_i)}^{\alpha} - x_{d_{i(n_i)}}^{\alpha} \\ &= u_i + \hat{\theta}_{i(n_i)}F_{i(n_i)} - x_{d_{i(n_i)}}^{\alpha} \end{aligned} \tag{36}$$

Hence, the control law is chosen as follows:

$$u_i = x_{d_{i(n_i)}}^{\alpha} - \hat{\theta}_{i(n_i)}F_{i(n_i)} - k_{i(n_i)}s_{i(n_i)} \tag{37}$$

The parameter update law is chosen as follows:

$$\hat{\theta}_{i(n_i)}^{\alpha} = \rho_{i(n_i)}s_{i(n_i)}F_{i(n_i)} \tag{38}$$

where $\rho_{i(n_i)}$ are positive design parameters.

Theorem 1.2: *The group of followers with the system* (29) *track the desired trajectory and the consensus problem is achieved by the controller* (37) *and the adaptive laws* (32), (35) *and* (38) *with suitable design parameters.*

Proof: The ith Lyapunov function is considered as follows:

$$V_i = \frac{1}{2}\sum_{j=1}^{n_i} s_{ij}^2 + \frac{1}{2}\sum_{j=1}^{n_i-1} e_{i(j+1)}^2 + \frac{1}{2}\sum_{j=1}^{n_i}\frac{1}{\rho_{ij}}\tilde{\theta}_{ij}^2 \tag{39}$$

where $e_{i(j+1)} = x_{d_{i(j+1)}} - \tilde{x}_{i(j+1)}$ and $\tilde{\theta}_{ij} = \theta_{ij} - \hat{\theta}_{ij}$.

The fractional-order derivative of V is obtained as follows:

$$\begin{aligned} V_i^{\alpha} &\leq \sum_{j=1}^{n_i} s_{ij}s_{ij}^{\alpha} + \sum_{j=1}^{n_i-1} e_{i(j+1)}e_{i(j+1)}^{\alpha} + \sum_{j=1}^{n_i}\tfrac{1}{\rho_{ij}}\tilde{\theta}_{ij}\tilde{\theta}_{ij}^{\alpha} \\ &= s_{i1}s_{i1}^{\alpha} + \sum_{j=2}^{n_i} s_{ij}s_{ij}^{\alpha} + \sum_{j=1}^{n_i-1} e_{i(j+1)}e_{i(j+1)}^{\alpha} \\ &\quad + \tfrac{1}{\rho_{i1}}\tilde{\theta}_{i1}\tilde{\theta}_{i1}^{\alpha} + \sum_{j=2}^{n_i}\tfrac{1}{\rho_{ij}}\tilde{\theta}_{ij}\tilde{\theta}_{ij}^{\alpha} \\ &= (l_i(x_{i2} + \hat{\theta}_{i1}F_{i1}) - \sum_{j\in\mathcal{N}_i} a_{ij}(x_{j2} + \hat{\theta}_{j1}F_{j1}) - b_iy_d^{\alpha})s_{i1} \\ &\quad + \sum_{j=2}^{n_i} s_{ij}s_{ij}^{\alpha} + \sum_{j=1}^{n_i-1} e_{i(j+1)}e_{i(j+1)}^{\alpha} + \tfrac{1}{\rho_{i1}}\tilde{\theta}_{i1}\tilde{\theta}_{i1}^{\alpha} \\ &\quad + \sum_{j=2}^{n_i}\tfrac{1}{\rho_{ij}}\tilde{\theta}_{ij}\tilde{\theta}_{ij}^{\alpha} \end{aligned} \tag{40}$$

The fractional-order derivation of $e_{i(j+1)}$ can be computed by employing (34) as follows:

$$\begin{aligned} e_{i(j+1)}^{\alpha} &= x_{d_{i(j+1)}}^{\alpha} - \tilde{x}_{i(j+1)}^{\alpha} \\ &= -\tfrac{e_{i(j+1)}^2}{\tau_{i(j+1)}} + \underbrace{k_{ij}s_{ij}^{\alpha} + \hat{\theta}_{ij}F_{ij}^{\alpha} + \hat{\theta}_{ij}^{\alpha}F_{ij} - (x_{d_{i(j+1)}}^{\alpha})^{\alpha}}_{B_{i(j+1)}} \end{aligned} \tag{41}$$

By applying (41), (32) and (35), (40) can be expressed as follows:

$$
\begin{aligned}
V_i^{\alpha} &\leq (l_i x_{i2} - k_{i1} s_{i1} - l_i \tilde{x}_{i2}) s_{i1} + \sum_{j=2}^{n_i} s_{ij} s_{ij}^{\alpha} \\
&\quad + \sum_{j=1}^{n_i-1} e_{i(j+1)} e_{i(j+1)}^{\alpha} \\
&= l_i (s_{i1} s_{i2} + e_{i2} s_{i1} - k_{i1} s_{i1}^2) \\
&\quad + \sum_{j=2}^{n_i-1} (s_{ij} s_{i(j+1)} + s_{ij} e_{i(j+1)} - k_{ij} s_{ij}^2) \\
&\quad - k_{i(n_i)} s_{i(n_i)}^2 + \sum_{j=1}^{n_i-1} \left(-\frac{e_{i(j+1)}^2}{\tau_{i(j+1)}} + e_{i(j+1)} B_{i(j+1)} \right)
\end{aligned} \tag{42}
$$

By applying the adaptive laws (32), (35) and (38), it can be shown that (42) is the same as (22). Hence, $V(t)$ is bounded and $|s_{i1}| \leq \sqrt{\frac{2QC_2}{\gamma}}$. In this case the error converges to a small value by designing suitable parameters, and, therefore, the consensus errors approach a value in the vicinity of zero and the systems being asymptotically stable. This shows that the consensus problem is solvable for the system (29) by applying the controller (37) and the adaptive laws (32), (35) and (38).

To summarize, a dynamic surface controller (DSC) was proposed for fractional-order multi-agent systems to obtain consensus and track the desired trajectory. Next, an adaptive dynamic surface controller (Adaptive DSC) was proposed for these systems with unknown parameters, so consensus and tracking the desired trajectory were proved. ■

Simulation results

In this section, we provide simulation results of scenarios and demonstrate the validity of our theoretical work regarding the DCS controller.

A topology of agents with four followers and one leader who exchange information with the first agent is shown in Figure 1. Note that the dynamics of followers are as defined in (2) and parameters are set as $\alpha = 0.95$, $n_i = 3$, $F_{i1}(.) = x_{i2}(t)$, $F_{i2}(.) = x_{i3}(t)$, $F_{i3}(.) = x_{i3}(t)$ $\sin(t)$, $i = 1, \ldots, 4$. A desired trajectory is $y_d = \sin(t)$. for which the initial conditions are set as $x_1(0) = \begin{bmatrix} 6 & 4 & 0 \end{bmatrix}^T$, $x_2(0) = \begin{bmatrix} 3 & 2 & 6 \end{bmatrix}^T$, $x_3(0) = \begin{bmatrix} 1 & 0 & 3 \end{bmatrix}^T$, $x_4(0) = \begin{bmatrix} -3 & -3 & 1 \end{bmatrix}^T$. Next, the design parameters are defined as $k_{ij} = 10, \tau_{i(j+1)} = 0.1,\ \rho_{ij} = 2$ for $i = 1, 2, 3, 4$ and $j = 1, 2, 3$.

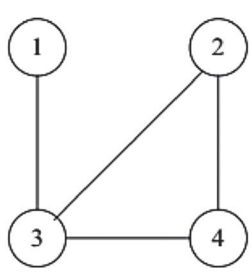

Figure 1. The graph of the agents.

Figures 2–4 display the results which are the basis for comparing the performance of the proposed scheme against that of the competing model. Let us begin with Figure 2, which depicts the behaviour of the output trajectories of the agents and the desired trajectory for the proposed DSC controller. Now, consider Figures 3 and 4, which show the states of the agents. It is clear that the output trajectories of the agents track the desired trajectory and, the consensus is obtained after about 0.2 s.

For more comparison and to show the efficiency of the proposed controller, the simulation result of agents with an adaptive sliding mode controller (Yaghoubi & Talebi, 2019) is depicted in Figure 5. Agents with known parameters are controlled to achieve consensus with an adaptive sliding mode controller in which the parameters of controllers are unknown so the adaptive law of these parameters is also designed. The adaptive sliding mode controller and adaptive law are given as follows:

$$
\begin{aligned}
u_i(t) &= \dddot{y}_d + (\ddot{y}_d - \ddot{x}_i) + (\dot{y}_d - \dot{x}_i) \\
&\quad - \theta_i k \left(\sum_{j=1}^{n} a_{ij} [x_i - x_j] \right) \\
&\quad - \omega.\mathrm{sgn} \left(\sum_{j=1}^{n} a_{ij} [x_i - x_j] \right), \\
&\quad i, j = 1, 2, \ldots, 4,\ i \neq j
\end{aligned} \tag{43}
$$

$$
\theta_i^{\alpha} = \beta_i \left(k \sum_{j=1}^{n} a_{ij} [x_i - x_j] \right)^T \left(k \sum_{j=1}^{n} a_{ij} [x_i - x_j] \right) \tag{44}
$$

Evidently, in this case consensus takes 2.5 s which is longer compared to the new method, thus confirming the latter's outperformance.

Further simulation results from the adaptive DSC controller are shown in Figures 6–9. The design parameters are defined as $k_{ij} = 20, \tau_{i(j+1)} = 0.02,\ \rho_{ij} = 1$ for $i = 1, 2, 3, 4$ and $j = 1, 2, 3$. Beginning with Figure 6, the output trajectories of the agents and the desired trajectory are shown. Moving on to Figure 7, in which the tracking errors, denoted by $error_i = y_i - y_d, i = 1, 2, 3, 4$, charted and then in FIGURES 8 and 9 states of agents are depicted. As can be seen, the output trajectories of the agents track the desired trajectory and the errors converge to a value in the vicinity of zero.

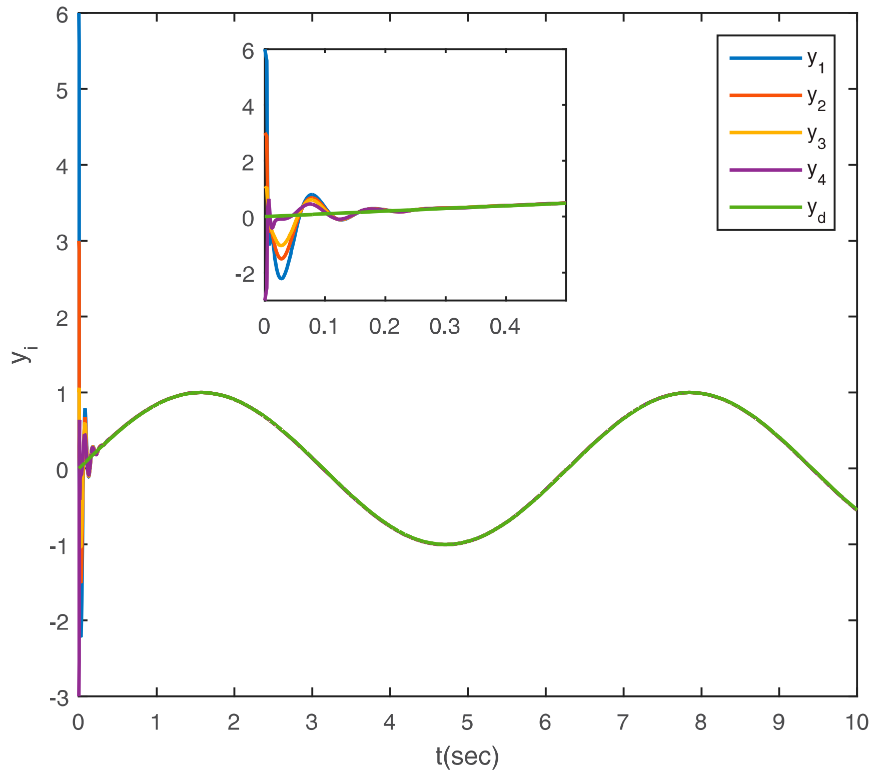

Figure 2. Output trajectories of the four agents and desired trajectory ($\boldsymbol{y_d}$) via the DSC controller.

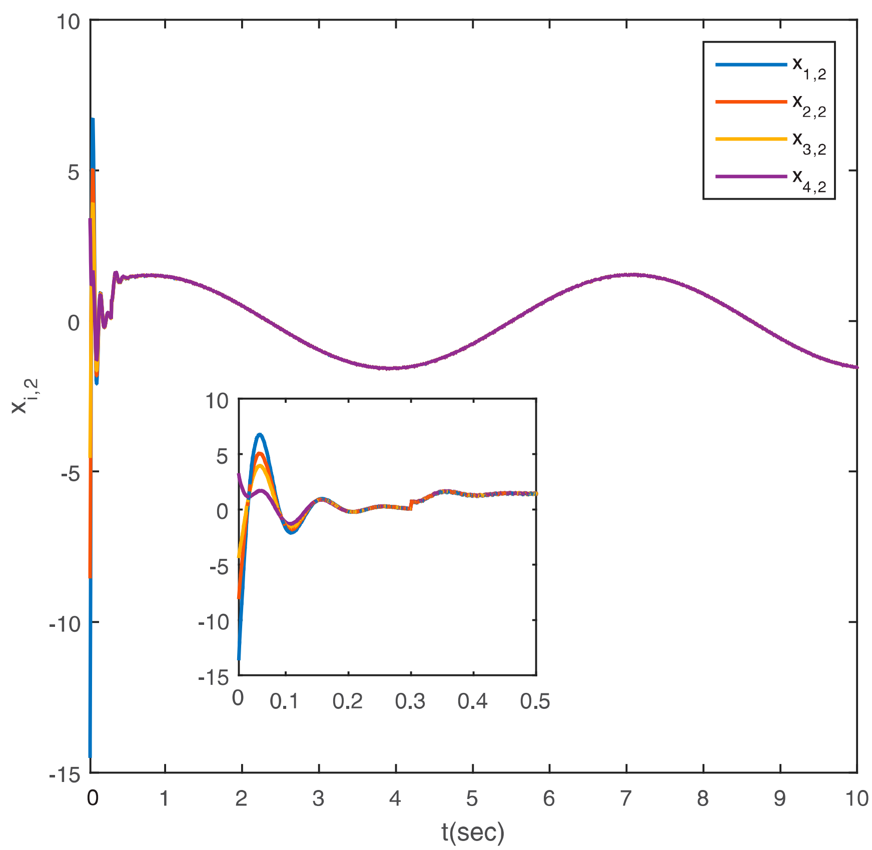

Figure 3. Consensus for $\boldsymbol{x_{i2}(t)}$, $\boldsymbol{i} = 1, \ldots, 4$ via the DSC controller.

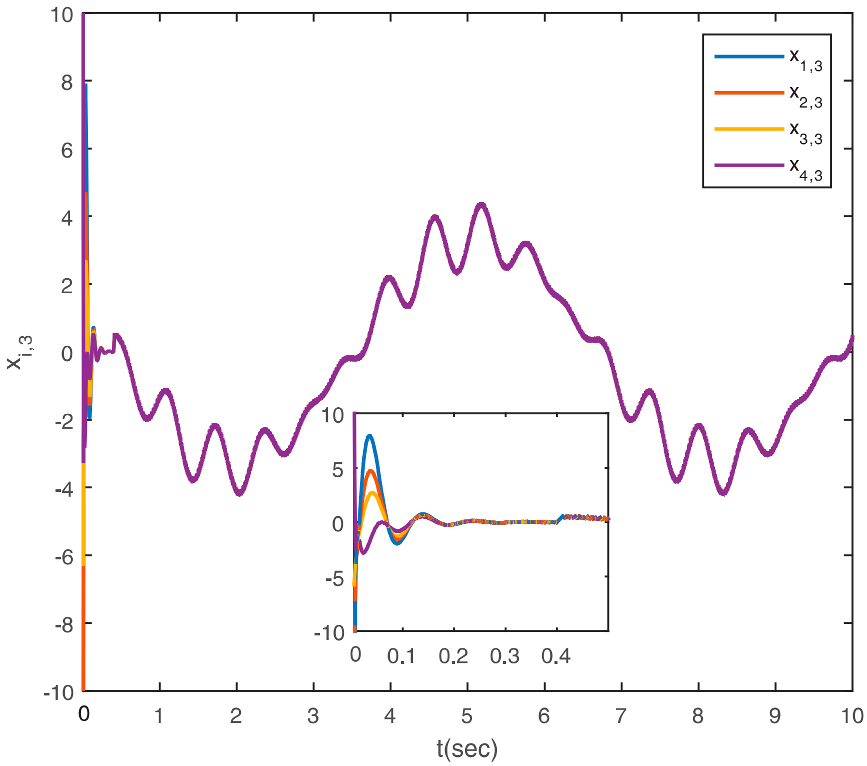

Figure 4. Consensus for $\boldsymbol{x}_{i3}(\boldsymbol{t})$, $\boldsymbol{i} = 1, \ldots, 4$ via the DSC controller.

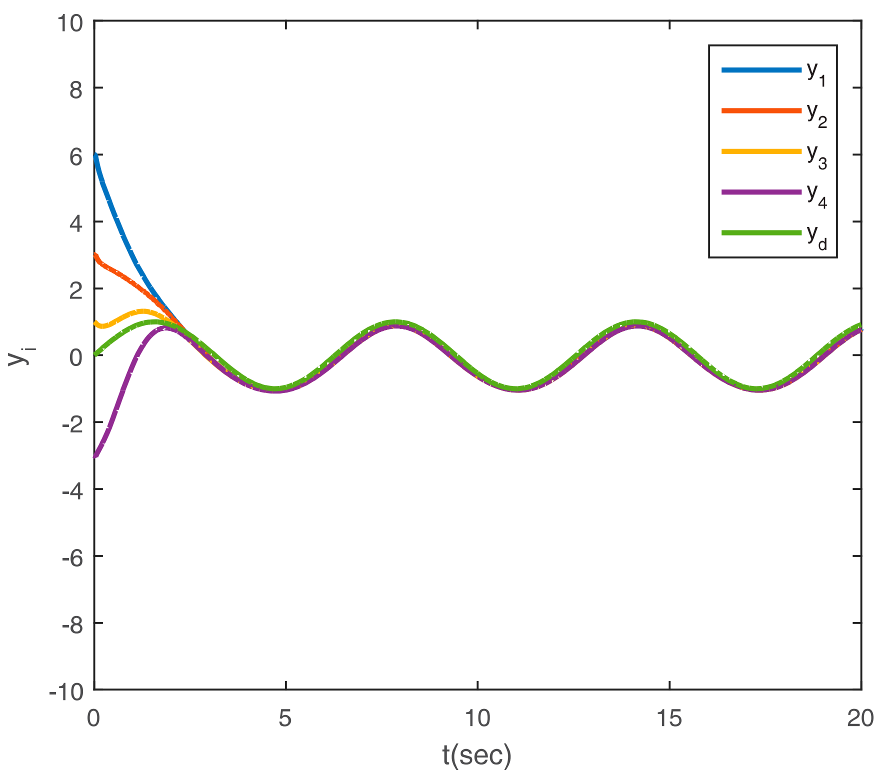

Figure 5. Output trajectories of four agents and desired trajectory ($\boldsymbol{y_d}$) via the adaptive sliding mode controller.

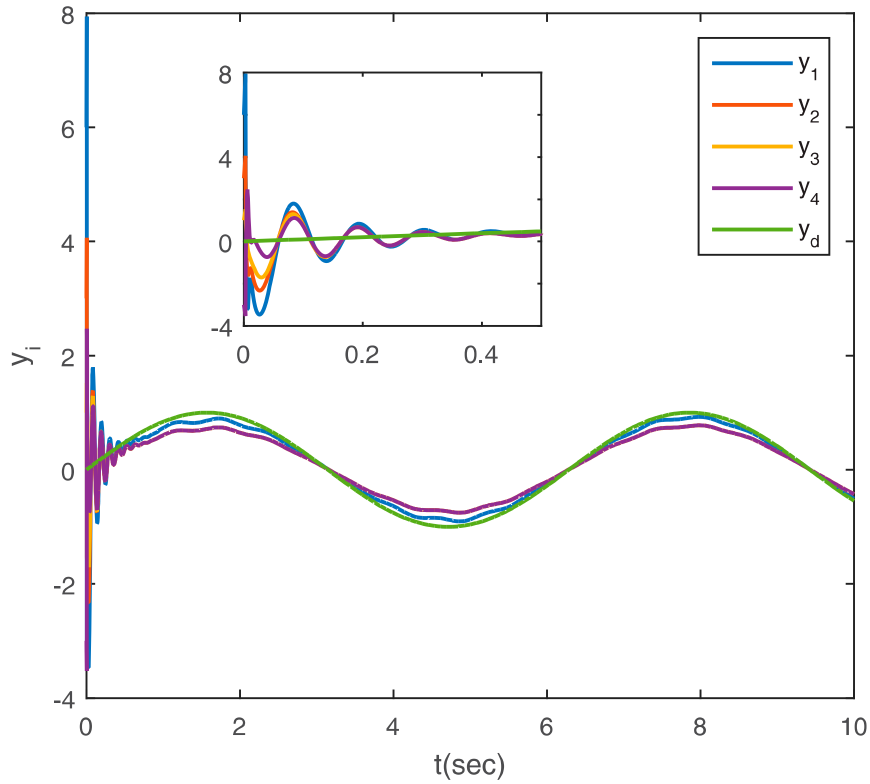

Figure 6. Output trajectories of four agents and desired trajectory ($\boldsymbol{y_d}$) via the adaptive DSC controller.

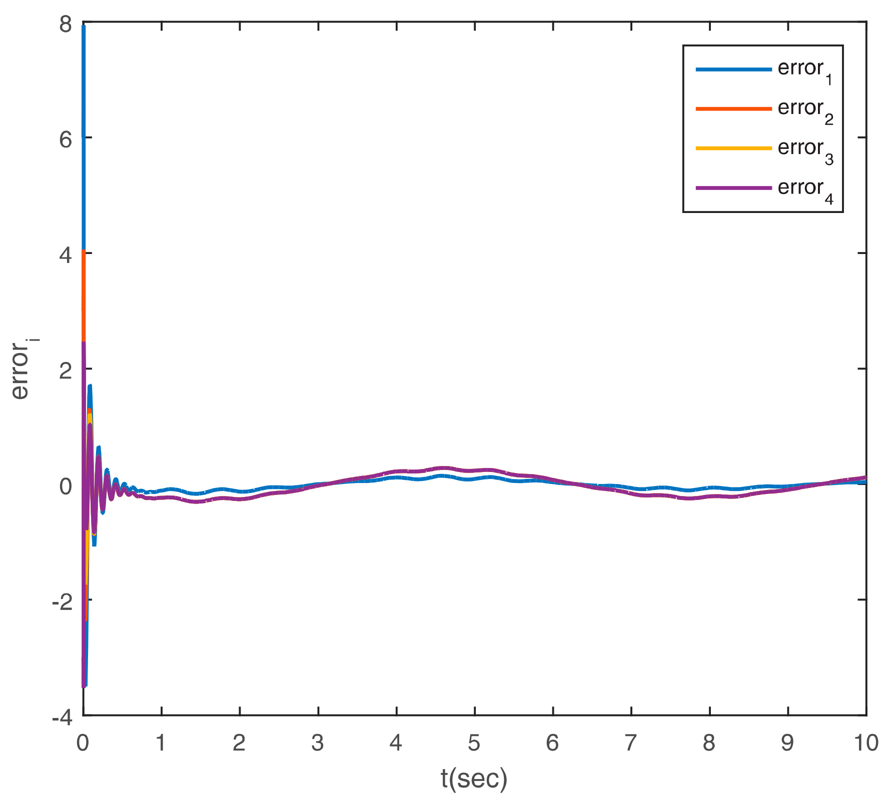

Figure 7. Tracking errors of agents via adaptive DSC.

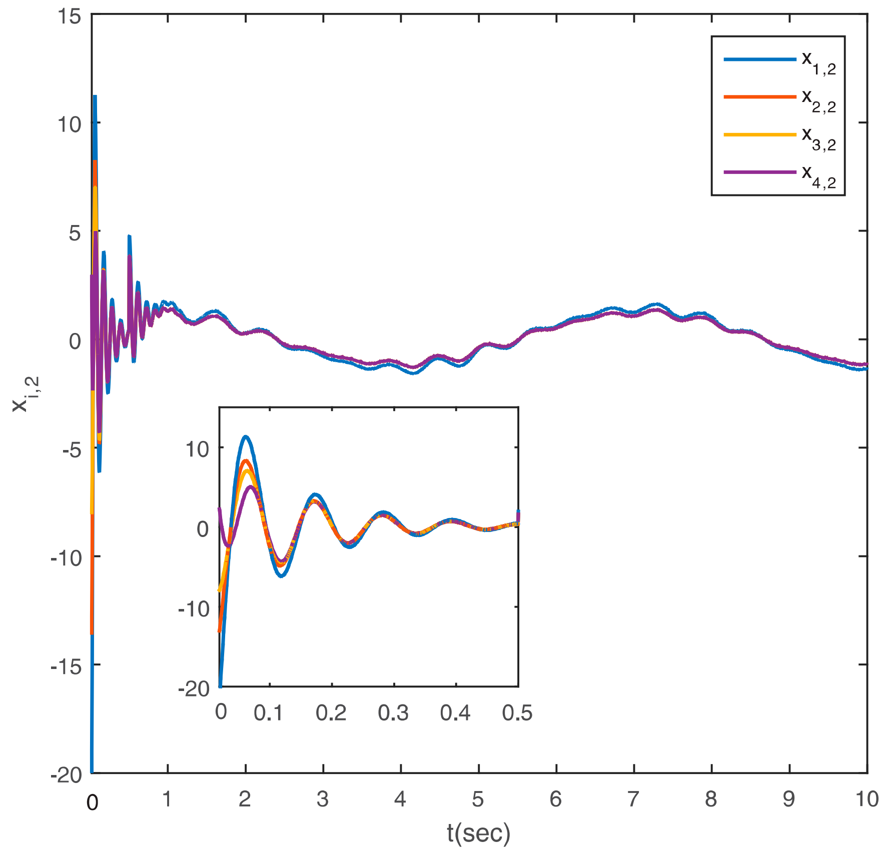

Figure 8. Consensus for $\boldsymbol{x}_{i2}(\boldsymbol{t})$, $\boldsymbol{i} = 1, \ldots, 4$ via the adaptive DSC controller.

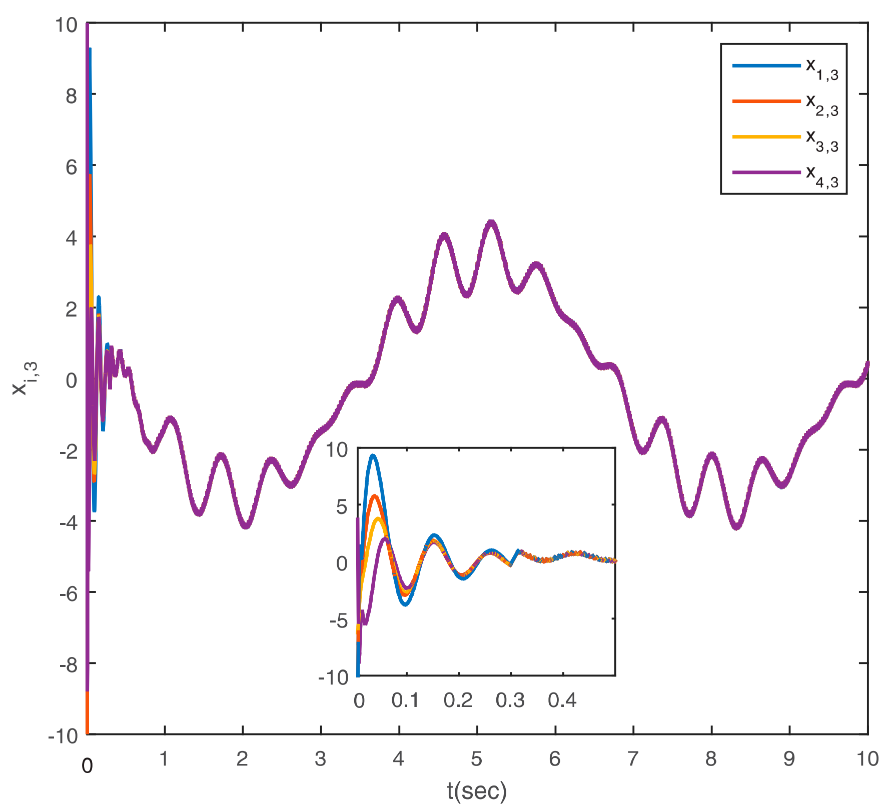

Figure 9. Consensus for $\boldsymbol{x}_{i3}(\boldsymbol{t})$, $\boldsymbol{i} = 1, \ldots, 4$ via the adaptive DSC controller.

Conclusion

In this paper, we explored current DSC technologies and attempted to improve the drawbacks with some of the studied propositions. We designed a novel DSC algorithm together with an adaptive DSC method to tackle the consensus problems for fractional-order non-linear multi-agent systems. By implementing these new solutions, a sufficient number of different conditions were examined to ensure that consensus of fractional-order non-linear multi-agent systems is achieved and the desired trajectory is tracked by output trajectories of the agents.

Evaluation of simulation results shows our proposals offer superiority over the tested models in terms of efficiency. In the near future, our next focus will be the study of cluster consensus for fractional-order non-linear multi-agent systems via DSC for switching topology.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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