S. Rinzivillo – rinzivillo@isti.cnr.it

DATA VISUALIZATION AND VISUAL ANALYTICS

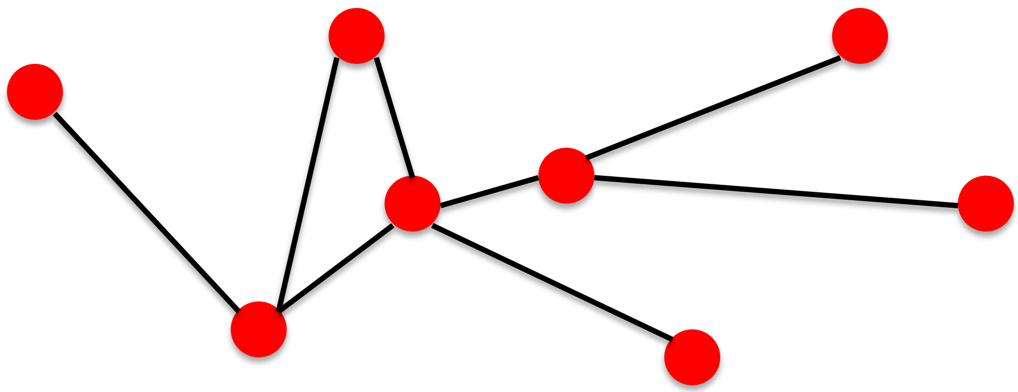
Relational Structures: Networks

Networks

- Data main focus is relationship
- Study the patterns of connection among different parts of a complex system
- Visualization has a key roles to add insights to numerical analysis

NETWORKS AND GRAPHS

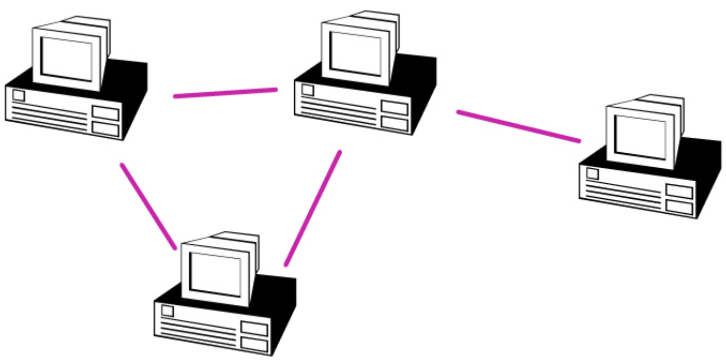
Basic Elements

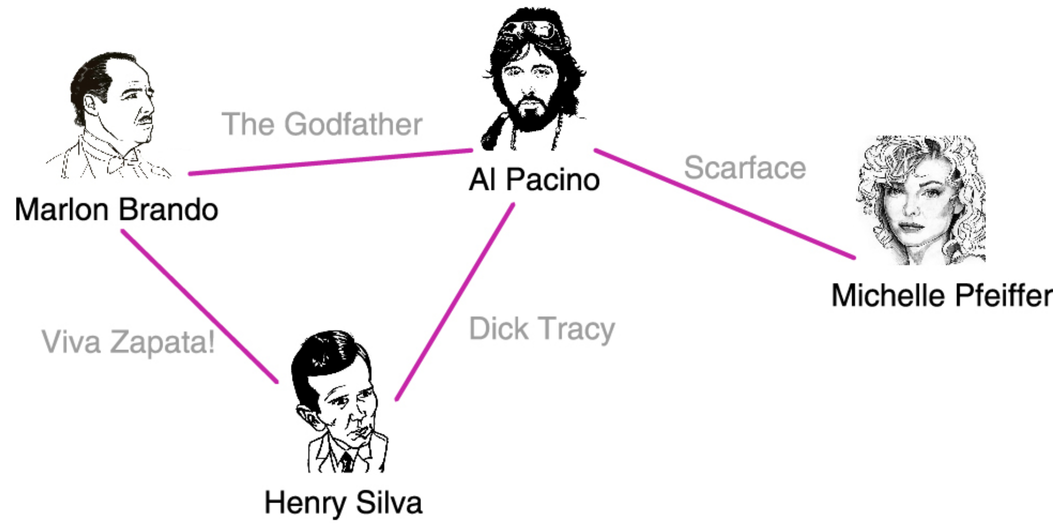


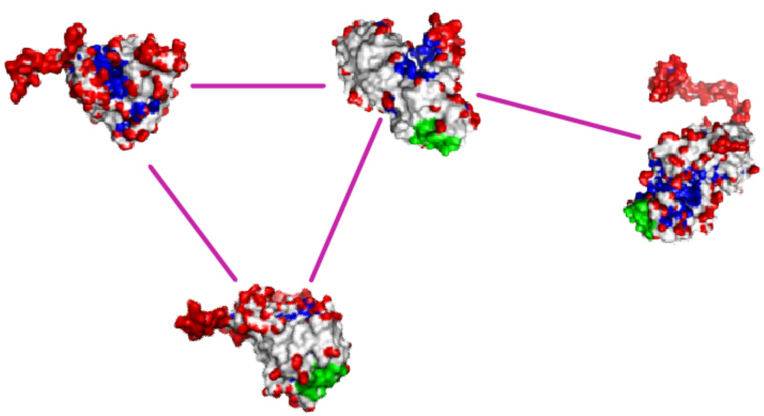
- **components**: nodes, vertices — N
- **interactions**: links, edges — L
- **system**: network, graph — (N,L)

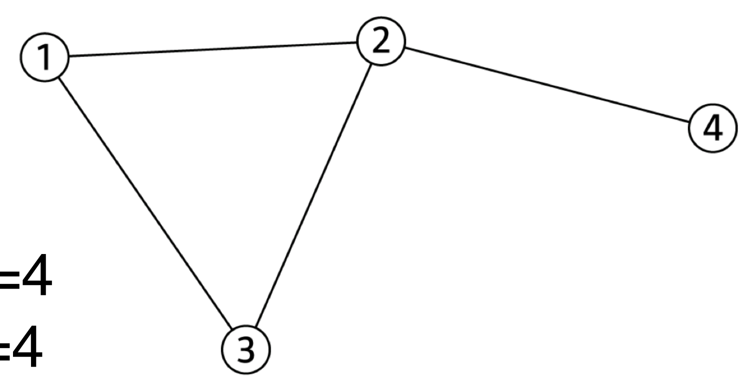
Networks or graphs? A common language

Network refer to a real system
Graph refers to mathematical representation of a network









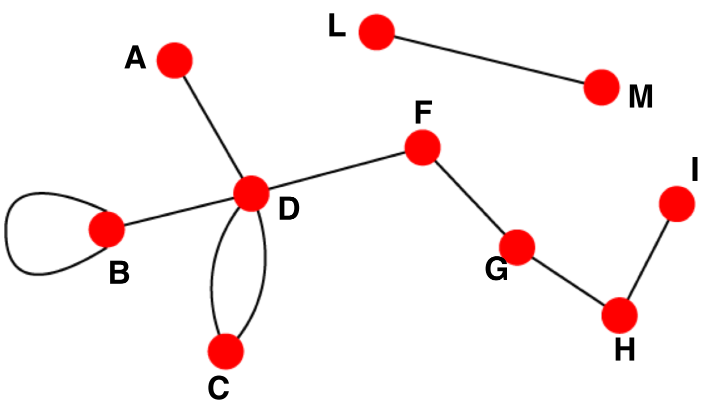
N=4
L=4

Undirected vs Directed Networks

Undirected

Links: undirected (*symmetrical*)

Graph:

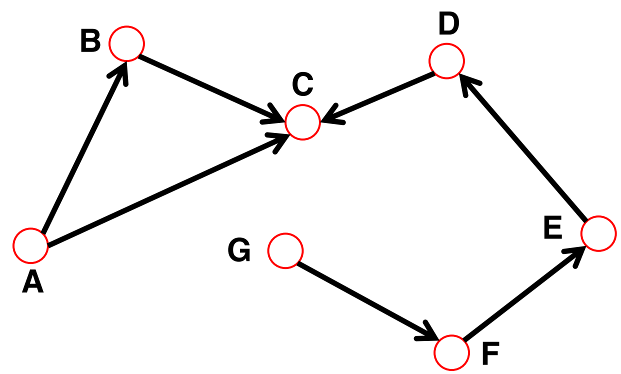


Undirected links :
coauthorship links
Actor network
protein interactions

Directed

Links: directed (*arcs*).

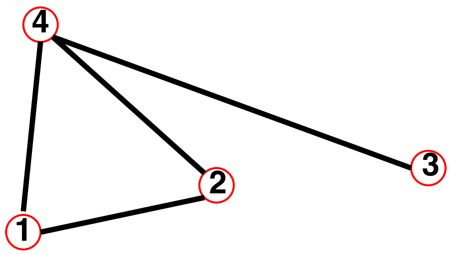
Digraph = directed graph:

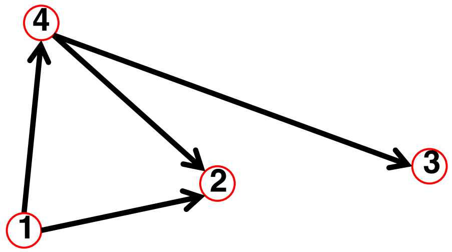


An undirected link is the superposition of two opposite directed links.

Directed links :
URLs on the www
phone calls
metabolic reactions

Adjacency Matrix





A_{ij}=1 if there is a link between node *i* and *j*

A_{ij}=0 if nodes *i* and *j* are not connected to each other.

$$A_{ij} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \qquad A_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

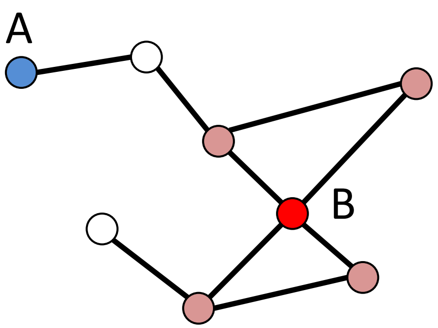
Note that for a directed graph (right) the matrix is not symmetric.

$A_{ij} = 1$ if there is a link pointing from node *j* and *i*

$A_{ij} = 0$ if there is no link pointing from *j* to *i*.

Node degree

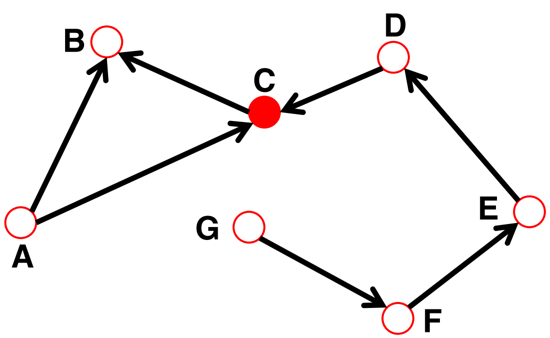
Undirected



Node degree: the number of links connected to the node.

$$k_A = 1 \qquad k_B = 4$$

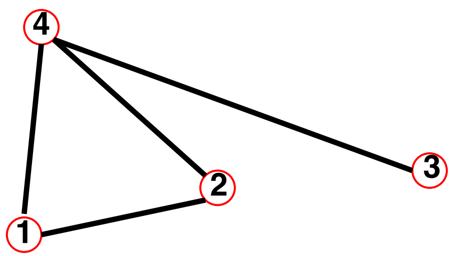
Directed



$$k_C^{in} = 2 \quad k_C^{out} = 1 \qquad k_C = 3$$

Adjacency Matrix and Node Degree

Undirected



$$A_{ij} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

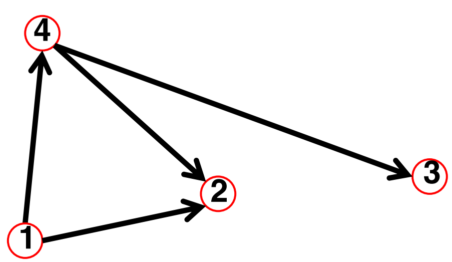
$$A_{ij} = A_{ji}$$
$$A_{ii} = 0$$

$$\boxed{k_i = \sum_{j=1}^{N} A_{ij}}$$

$$\boxed{k_j = \sum_{i=1}^{N} A_{ij}}$$

$$L = \frac{1}{2}\sum_{i=1}^{N} k_i = \frac{1}{2}\sum_{ij}^{N} A_{ij}$$

Directed



$$A_{ij} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$A_{ij} \neq A_{ji}$$
$$A_{ii} = 0$$

$$\boxed{k_i^{in} = \sum_{j=1}^{N} A_{ij}}$$

$$\boxed{k_j^{out} = \sum_{i=1}^{N} A_{ij}}$$

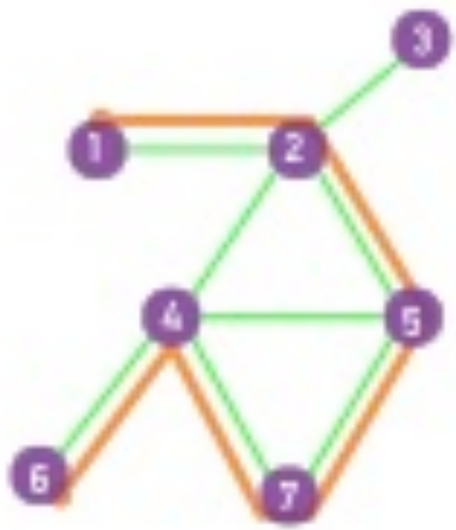
$$L = \sum_{i=1}^{N} k_i^{in} = \sum_{j=1}^{N} k_j^{out} = \sum_{i,j}^{N} A_{ij}$$

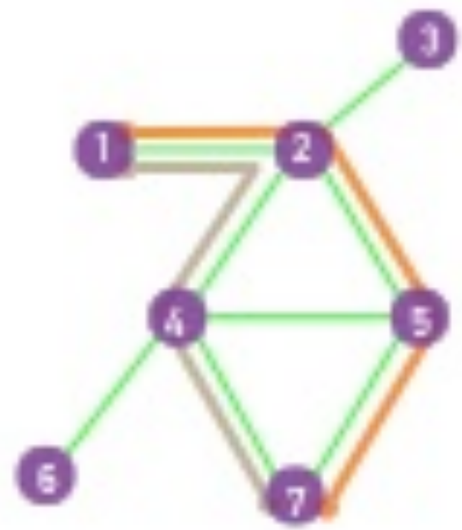
Paths

A *path is* a sequence of nodes in which each node is adjacent to the next one

$P_{i0,in}$ of length *n* between nodes i_0 and i_n is an ordered collection of *n+1* nodes and *n* links

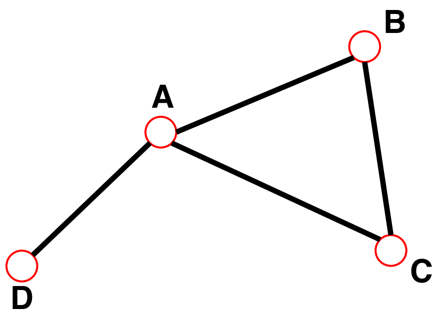
$$P_n = \{i_0, i_1, i_2, \ldots, i_n\} \qquad P_n = \{(i_0, i_1), (i_1, i_2), (i_2, i_3), \ldots, (i_{n-1}, i_n)\}$$





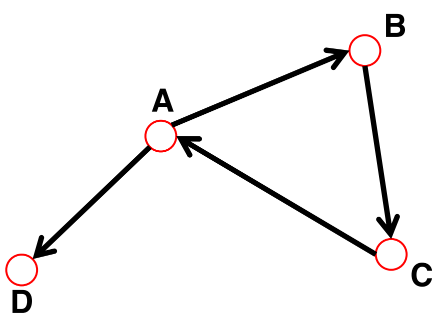
- In a directed network, the path can follow only the direction of an arrow.

Distance in a Graph



The *distance (shortest path, geodesic path)* between two nodes is defined as the number of edges along the shortest path connecting them.

*If the two nodes are disconnected, the distance is infinity.

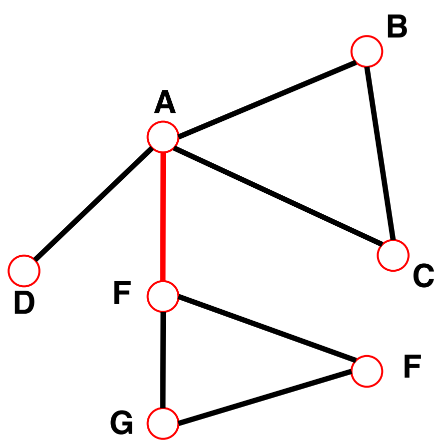


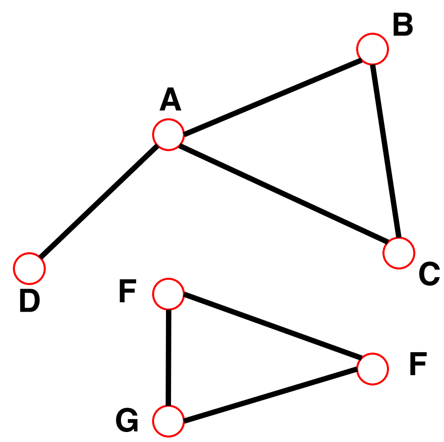
In directed graphs each path needs to follow the direction of the arrows.

Thus in a digraph the distance from node A to B (on an AB path) is generally different from the distance from node B to A (on a BCA path).

Connectedness

Connected (undirected) graph: any two vertices can be joined by a path.
A disconnected graph is made up by two or more connected components.





Largest Component:
Giant Component

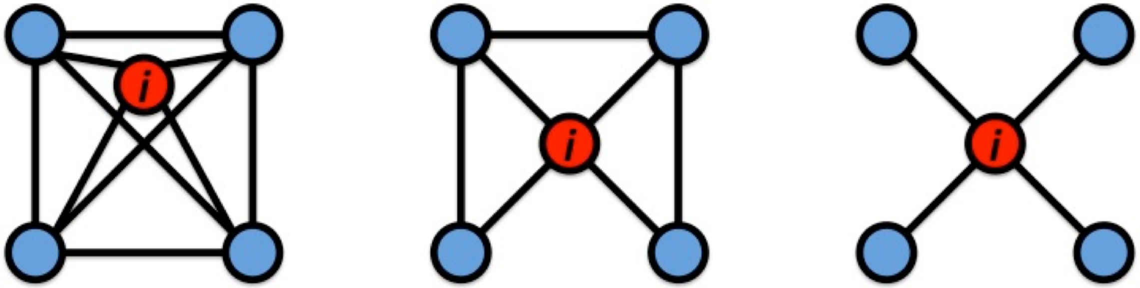
The rest: **Isolates**

Bridge: if we erase it, the graph becomes disconnected.

Clustering Coefficient

- Clustering coefficient:
  - what fraction of your neighbors are connected?
- Node i with degree k_i
- C_i in [0,1]

$$C_i = \frac{2e_i}{k_i(k_i - 1)}$$



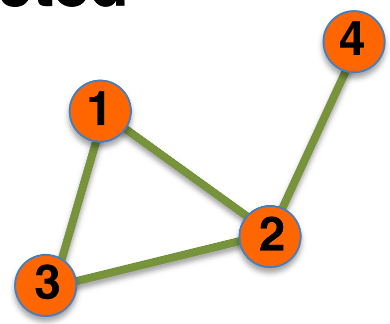
$C_i = 1$ $\quad$ $C_i = 1/2$ $\quad$ $C_i = 0$

Watts & Strogatz, Nature 1998.

SUMMARY

Undirected vs Directed

Undirected



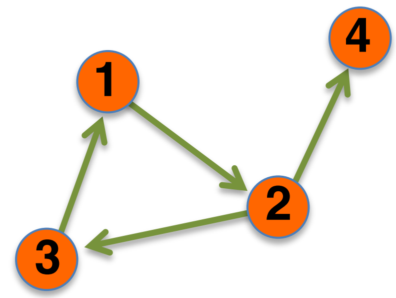
$$A_{ij} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$A_{ii} = 0 \qquad A_{ij} = A_{ji}$$

$$L = \frac{1}{2}\sum_{i,j=1}^{N} A_{ij} \qquad < k >= \frac{2L}{N}$$

Actor network, protein-protein interactions

Directed



$$A_{ij} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_{ii} = 0 \qquad A_{ij} \neq A_{ji}$$

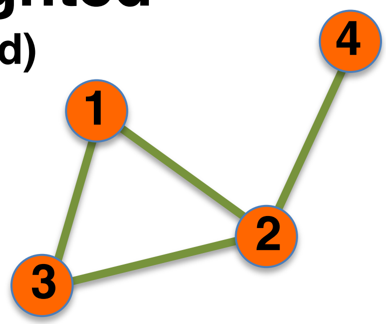
$$L = \sum_{i,j=1}^{N} A_{ij} \qquad < k >= \frac{L}{N}$$

WWW, citation networks

Unweighted vs Weighted

Unweighted

(undirected)



$$A_{ij} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

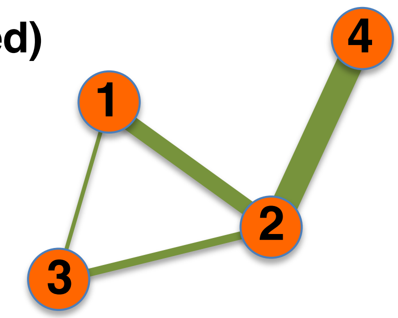
$$A_{ii} = 0 \qquad A_{ij} = A_{ji}$$

$$L = \frac{1}{2}\sum_{i,j=1}^{N} A_{ij} \qquad < k >= \frac{2L}{N}$$

protein-protein interactions, www

Weighted

(undirected)



$$A_{ij} = \begin{pmatrix} 0 & 2 & 0.5 & 0 \\ 2 & 0 & 1 & 4 \\ 0.5 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{pmatrix}$$

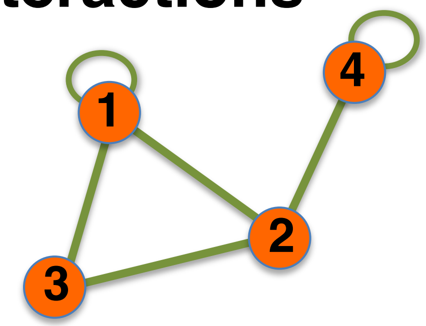
$$A_{ii} = 0 \qquad A_{ij} = A_{ji}$$

$$L = \frac{1}{2}\sum_{i,j=1}^{N} nonzero(A_{ij}) \qquad < k >= \frac{2L}{N}$$

Call Graph, metabolic networks

Link properties

Self-interactions



$$A_{ij} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

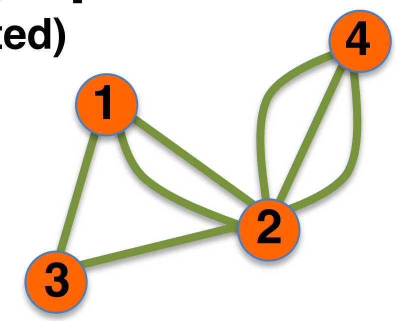
$$A_{ii} \neq 0 \qquad A_{ij} = A_{ji}$$

$$L = \frac{1}{2} \sum_{i,j=1, i \neq j}^{N} A_{ij} + \sum_{i=1}^{N} A_{ii} \qquad ?$$

Protein interaction network, www

Multigraph

(undirected)



$$A_{ij} = \begin{pmatrix} 0 & 2 & 1 & 0 \\ 2 & 0 & 1 & 3 \\ 1 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{pmatrix}$$

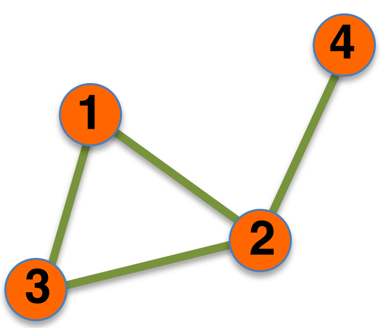
$$A_{ii} = 0 \qquad A_{ij} = A_{ji}$$

$$L = \frac{1}{2} \sum_{i,j=1}^{N} nonzero(A_{ij}) \qquad < k >= \frac{2L}{N}$$

Social networks, collaboration networks

Network Visual Representation

- Three main methods
  a) Adjacency Lists
  b) Matrices
  c) Node-link diagrams



1: 2,3
2: 3,1,4
3: 1,2
4: 2

a)

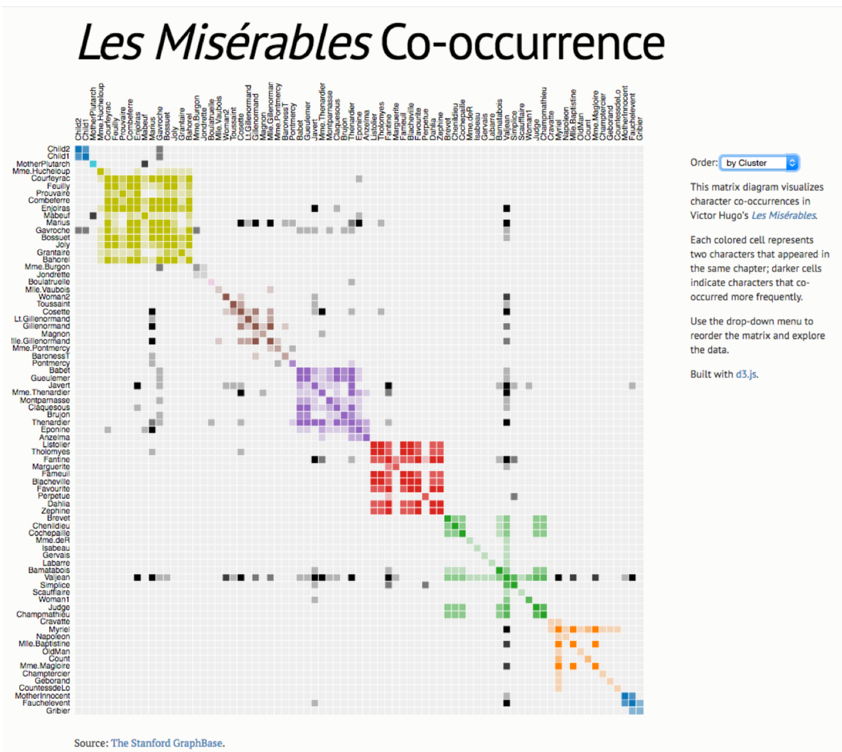
$$A_{ij} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

b)

1,2
1,3
2,3
2,4

c)

Adjacency Matrix



- Each cell ij represents an edge from vertex i to vertex j
- Effectiveness of visualization depens on rows/columns ordering
- First example by Jacques bertin (with paper strips rearranged by hand)
- Effective also for highly connected graphs

https://bost.ocks.org/mike/miserables/

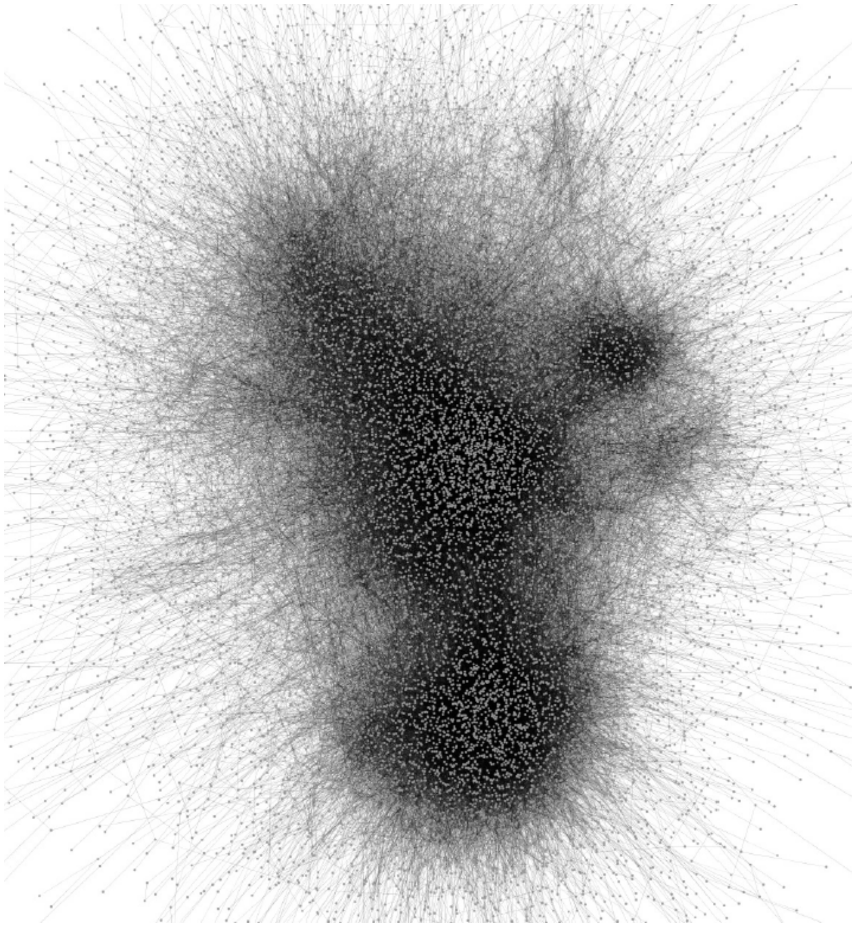
Node-link representation

http://bl.ocks.org/mbostock/4062045

- Symbolic elements for nodes
- Lines for connection among nodes
- Physical networks (roads, power grids) have a natural spatial encoding
- Abstract networks need layouts to infer a spatial position for nodes

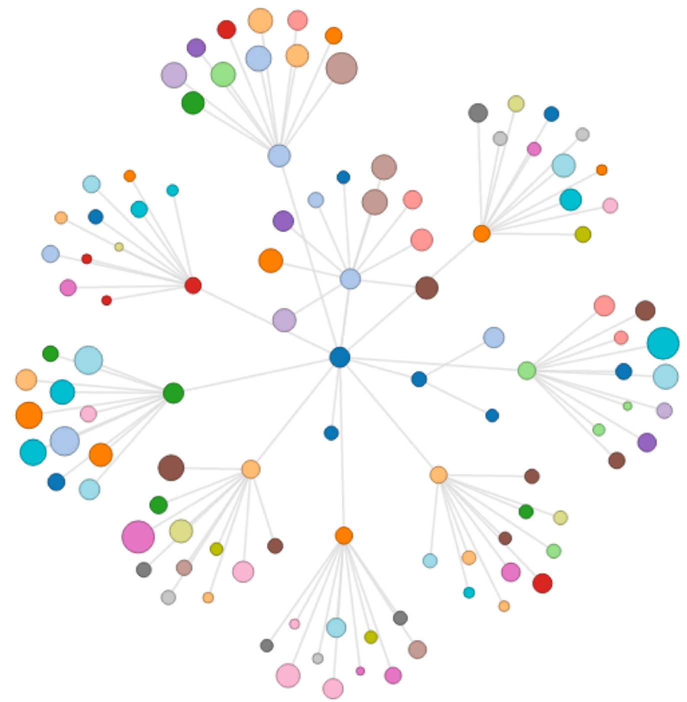
Problems of Node-Link diagrams

- Occlusion of node and link crossings
- Large networks may produce hairball like networks
- Many algorithms to produce effective layouts to reduce cluttering



Cluttering Reduction

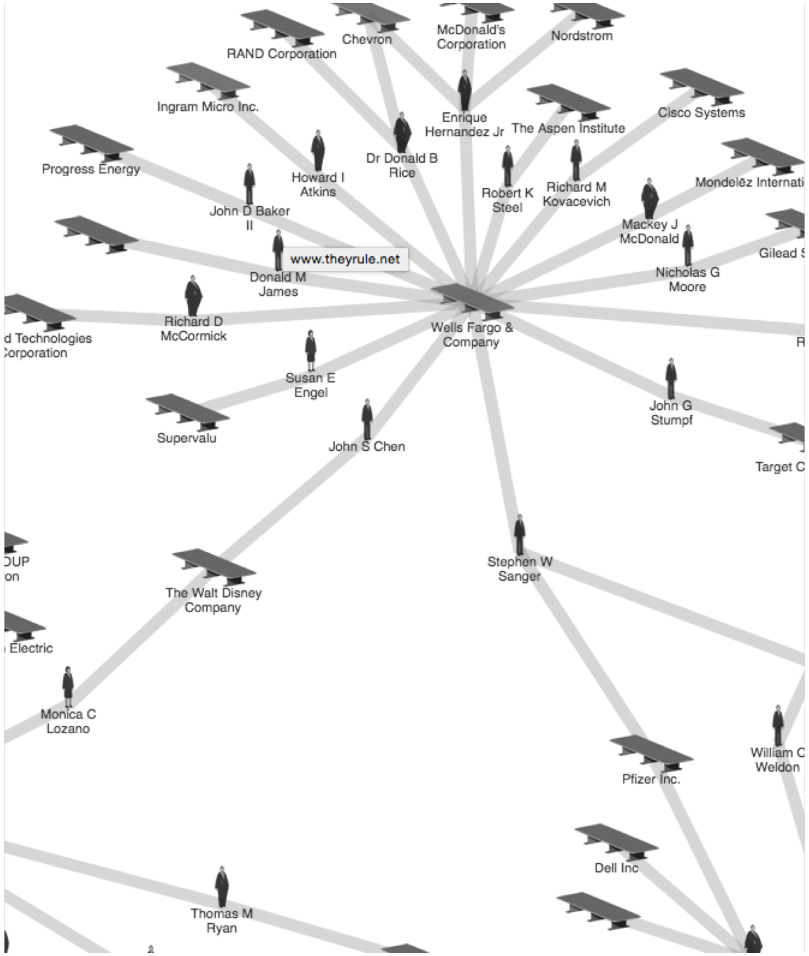
- Interaction to switch between different layouts
- Effective positioning of labels
  - Centered on nodes
  - Visualization based on interaction and mouse hover



http://projects.flowingdata.com/tut/interactive_network_demo/

Cluttering Reduction

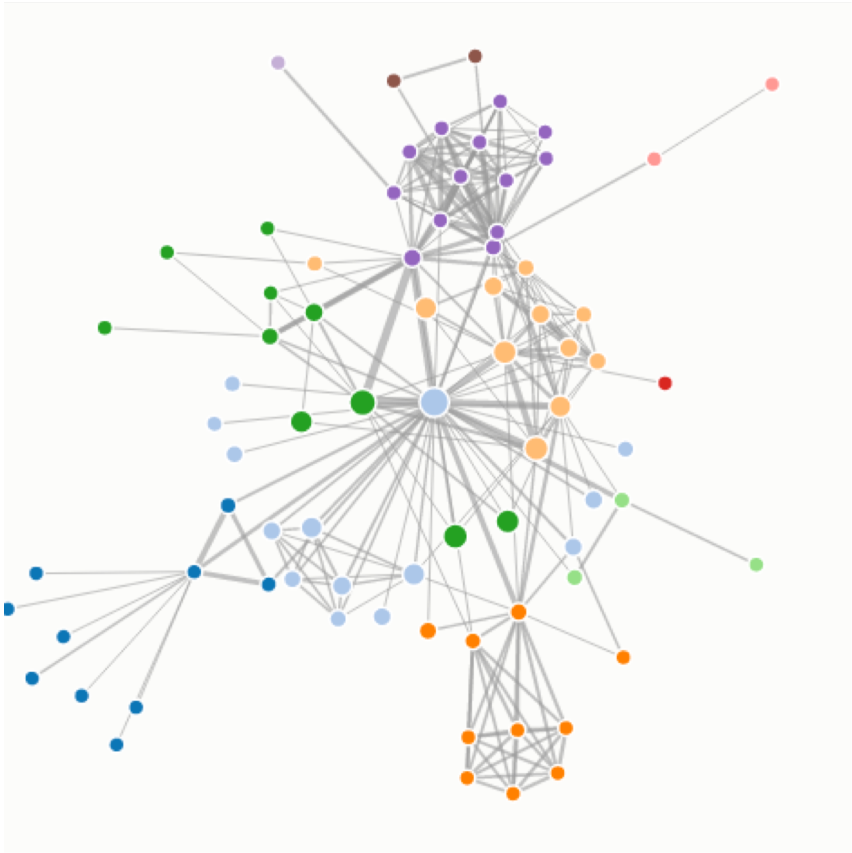
- Collapsing nodes into clusters



http://www.theyrule.net/

Cluttering Reduction

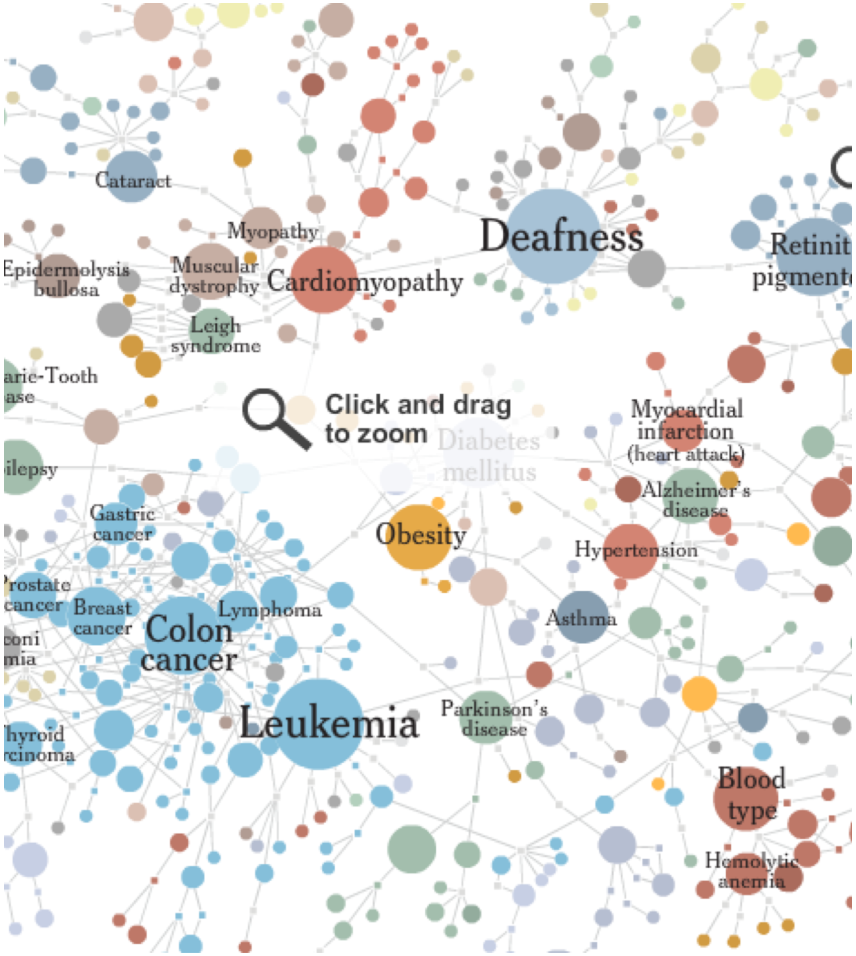
- Zooming and context distortion



https://bost.ocks.org/mike/fisheye/

Cluttering Reduction

- Zooming and context distortion



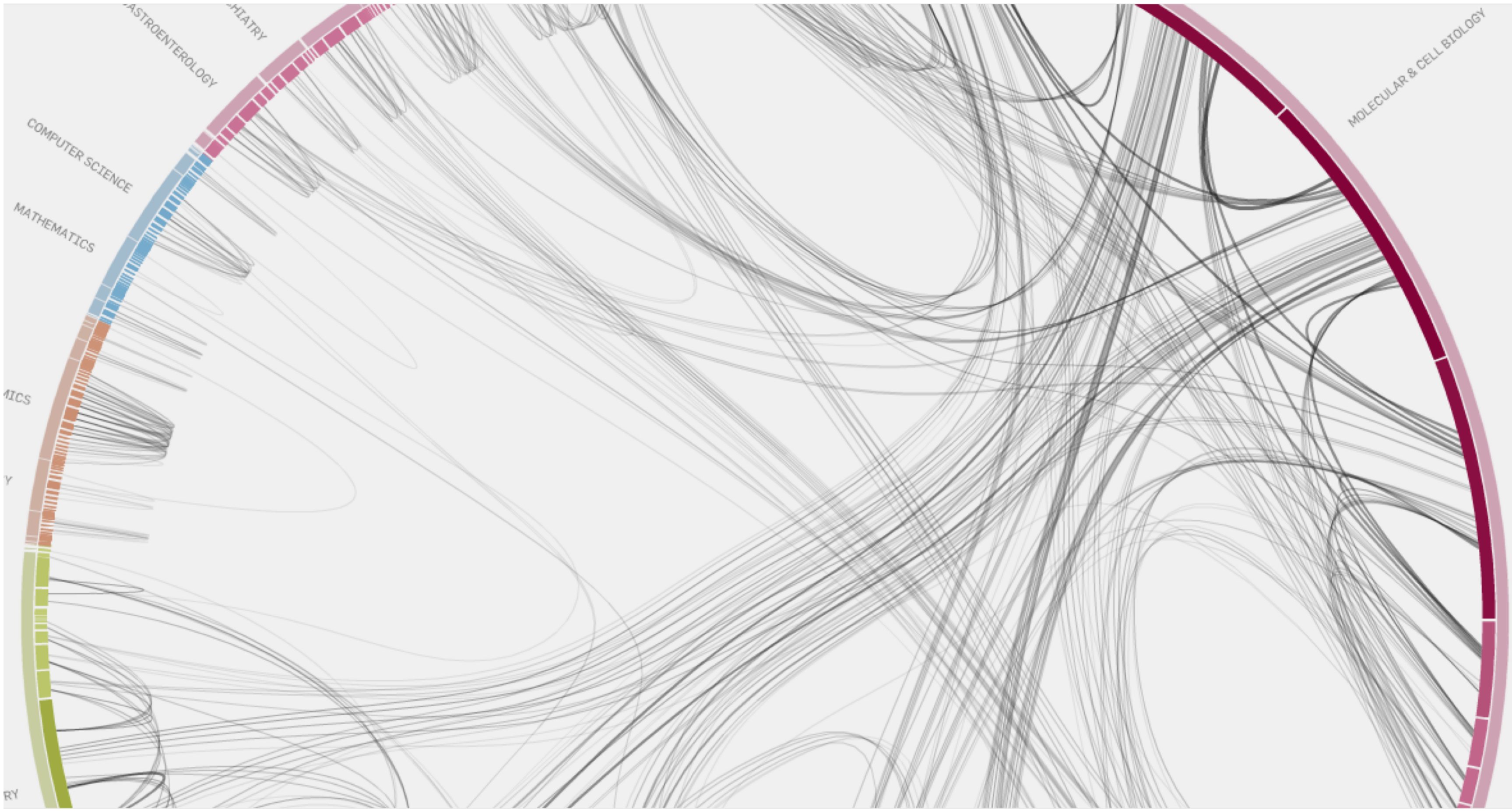
http://www.nytimes.com/interactive/2008/05/05/science/20080506_DISEASE.html?_r=0

Case Study: Force Directed

http://projects.flowingdata.com/tut/interactive_network_demo/

Case Study: Information Flow

Circular Layout



http://www.eigenfactor.org/projects/well-formed/

Case Study: Sankey type diagrams