

# Ballistic Pendulum

Equipment- ballistic pendulum apparatus, 2 meter ruler, 30 cm ruler, blank paper, carbon paper, masking tape, scale, stop watch

## PRECAUTION

In this experiment a brass ball is projected horizontally across the room with sufficient speed, enough to injure a person. Be sure the “line of fire” is clear before firing the ball, and be aware of other students who are preparing to fire as well. **Do not aim at your lab partner.**

## 1 Purpose

To measure the speed of a projectile by a kinematic method and by the use of a ballistic pendulum. The latter two methods illustrates the use of conservation of energy and momentum, first for a simple pendulum and then a physical pendulum.

## 2 Description

A spring gun fires a brass ball horizontally from a bench top. The brass ball is either allowed to fly free and strike the floor, or embeds itself in a pendulum. In the first experiment, the initial velocity of the ball can be obtained by using basic kinematics, while in the second, the initial velocity of the ball can be obtained by conservation laws derived from Newton’s 2nd law. Finally, for the third we will take inertia and angular velocity into account.

The ballistic pendulum was invented in 1742 to measure the speed of bullets. As you can see from this experiment, it is not necessary to use a ballistic pendulum to measure the speed of a slowly moving object. It does however, illustrate the use of several important conservation laws in physics.

## 3 Kinematic Measurement Of Speed

### 3.1 Theory

See Fig. 1. Let the vertical distance that the ball drops before hitting the floor be  $d$  and the horizontal distance be  $D$ . Let the time the ball is in flight be  $t$  and the horizontal speed of the ball be  $v$ . We need to list the x and y components of the vertical and horizontal motions of the ball. The vertical and horizontal motions of the ball are uncoupled. There is no horizontal acceleration and  $D = vt$ . The vertical acceleration is  $g$ , and  $d = \frac{1}{2}gt^2$ . Combining these 2 equations,

$$v = D \left( \frac{g}{2d} \right)^{\frac{1}{2}}. \quad (1)$$

$D$  and  $d$  are the measured quantities.

## 3.2 Procedures

Swing the pendulum out of the way so the latching mechanism holds the pendulum to the side. Put the ball, which has a hole in it, on the end of the rod.

Push the ball into the gun so that the spring compresses. To make it easier, hold the trigger down while you push the ball on the rod in. Once the ball and rod are pushed back as far as possible, let go of the trigger and the rod will be locked in place. Obtain  $d$  by measuring the distance from the bottom of the ball to the floor with a 2 meter ruler. Aim the gun across a clear spot in the lab room. **Making sure that no one is in the way, depress the firing lever on top of the gun and observe where the ball lands on the floor.** Tape 2 pieces of white paper together along the  $8\frac{1}{2}$  in edge and lay the middle of this double sheet where the ball landed. Orient the long direction of the double sheet in the direction of the traveling ball and tape the double sheet to the floor. Place 2 sheets of carbon paper over the white sheets with the carbon side down. When the ball lands on the carbon paper it will leave a mark on the white paper. Fire the gun a number of times to build up some error statistics for the distance  $D$ . Use Eq.(1) to calculate  $v$ , the speed of the ball as it leaves the gun. At what point is the acceleration of the ball zero?

## 4 Ballistic Pendulum Measurement Of Speed I

### 4.1 Theory Assuming A Simple Pendulum

In this section you will unlatch the pendulum and allow it to hang vertically at rest. We will assume that the ballistic pendulum apparatus is a simple pendulum, which is a point mass suspended by a massless cord. Let the mass of the ball be  $m$  and the mass of the pendulum be  $M$ . Let the horizontal speed of the ball before the collision be  $v$  and that of the pendulum plus ball right after the collision to be  $V$ . When the ball strikes the pendulum and sticks, it is a completely inelastic collision and energy is not conserved.

Define the system as the pendulum mass and the ball. For the system, horizontal momentum is conserved for a very short time from just before the collision to just after the collision. For this very short period there are no horizontal forces acting on the system. The system is the pendulum mass and the ball. (When the pendulum has swung to the side a bit the cord of the pendulum exerts a horizontal force on the pendulum mass and horizontal momentum is no longer conserved.) The horizontal linear momentum of the system just before the collision is  $mv$  and just after the collision is  $(M + m)V$ . Conservation of horizontal linear momentum gives  $mv = (M + m)V$ .

Right after collision the pendulum mass and ball swing up against the force of gravity and eventually come to rest. Assuming no friction, energy is conserved during this part of the motion as the the force of gravity which is conservative and the force of the suspending cord does no work. Let the gravitational potential energy immediately after the collision be zero. At this time the total energy is then kinetic and equal to  $\frac{1}{2}(M + m)V^2$ . When the pendulum has stopped the kinetic energy is zero but the gravitational potential energy is  $(M + m)gh$ , where  $h$  is the vertical distance that the pendulum mass and ball have gone up. See figures 2 and 3. Equating the total energies at these two times gives  $\frac{1}{2}(M + m)V^2 = (M + m)gh$ . Eliminating  $V$  between these equations gives

$$v = \frac{M + m}{m} \sqrt{2gh}. \quad (2)$$

## 4.2 Procedures

No measure the mass  $m$  of the ball. Once your done with your measurements pull the pendulum to the side, place the ball onto the rod, and compress and latch the spring gun. Measure  $h_1$ , the distance from the center of the ball on the rod to the pendulum platform. Refer to figure 2. Release the pendulum so that it hangs vertically, try to make it as still as possible. Once still, fire the gun. The pendulum will latch near the highest point of its swing. Look at figure 3 and measure  $h_2$ , the vertical distance from the pendulum platform to the center of the ball when the pendulum is at the highest point in its swing. Then find  $h$  using  $h = h_2 - h_1$ . Repeat the measurement several times to get some error statistics. Use Eq.(2) to calculate the speed of the ball  $v$ .

Question A fraction of kinetic energy is lost during the collision. Derive an expression for it and compare it to your experimental result?

Comments There are a number of approximations that have been made in the analysis for this section. The pendulum used is **not a simple pendulum**, it is a physical pendulum which has a distribution of mass. Unless the physical pendulum is struck by the ball at one specific spot, the center of percussion, the pendulum support will impart a brief force or impulse during the collision. Horizontal momentum will not strictly be conserved. The kinetic energy immediately after the collision is not that of a moving point mass but of a solid body rotating about an axis. In calculating the change in gravitational potential energy of the pendulum plus ball, the center of mass should be used rather than the position of the ball. The analysis in the following section addresses these difficulties.

## 5 Ballistic Pendulum Measurement Of Speed II

### 5.1 Theory Assuming A Physical Pendulum

Assume the pendulum is a physical pendulum free to swing in a vertical plane about a horizontal axis. We use the previous definitions for  $M$ ,  $m$ ,  $V$ ,  $v$ , along with the following symbols. Refer to figure 4 and 5 for the following items.

- $a$  is the vertical distance from the axis to the initial trajectory of the ball.
- $b$  is the distance from the axis to the center of mass of the pendulum *with the ball in it*.
- $\omega$  is the angular velocity of the pendulum with ball immediately after the collision.
- $I$  is the moment of inertia of the pendulum plus ball about the axis.
- $H$  is the maximum vertical distance that the center of mass of the pendulum with ball rises after the collision.
- $T$  is the pendulum's period of oscillation with the ball in it.

As before, the system is the pendulum and the ball. We invoke the conservation of angular momentum about the pendulum axis of the system for the time immediately before the collision to the time immediately after the collision (before the pendulum has swung upward). There are no external torques on the system for this period of time. Any impulse from the pendulum support during the collision contributes zero torque about the pendulum axis as there is no lever arm. The angular momentum of the pendulum does change as the pendulum swings upward due to the

torque exerted by gravity about the pivot point, but there will be no torque due to gravity before the pendulum swings away from the vertical. The angular momentum of the system just before the collision is  $mva$ , and the angular momentum right after the collision is  $I\omega$ . Conservation of angular momentum gives  $mva = I\omega$ .

Let the gravitational potential energy of the system be zero right after the collision. Immediately after the collision, the total energy of the system is kinetic and given by  $\frac{1}{2}I\omega^2$ . When the pendulum has risen to its maximum height the kinetic energy is zero and the potential energy is  $(M+m)gH$ . Conservation of energy gives  $\frac{1}{2}I\omega^2 = (M+m)gH$ . Why? Explain.

The period of the physical pendulum is given by  $T = 2\pi\sqrt{\frac{I}{(M+m)gb}}$ . Combining the equations for conservation of angular momentum and energy, and the equation for the period, the initial velocity of the ball becomes

$$v = \frac{(M+m)gT\sqrt{bH}}{\sqrt{2}\pi ma}. \quad (3)$$

The quantities needed, in addition to  $m$ ,  $M$ , and  $g$ , are  $a$ ,  $b$ ,  $T$ , and  $H$ .

Exercise assignment: Explain and show how the equations in section 5.1 combine to become equation 3.

## 5.2 Procedures

Observe figures 4 and 5 for the following. Measure the value of  $a$ , the vertical distance from the pivot point to the center of the ball on the rod. The value of  $b$ , which is the distance from the pivot to the combined center of mass of the pendulum and ball, has been measured for you by disassembling the pendulum and balancing it on an edge.  $b = 25.3 \pm 0.2\text{cm}$ . The center of mass is marked purple along the pendulum arm. Set up the apparatus as you did in the previous section and fire the ball into the pendulum a number of times for error statistics. Measure the values of  $H$ , the vertical height that the combined center of mass of the pendulum and ball rises during the collisions, using the steps and equation outlined in figures 4 and 5.

Now you will determine the pendulum's period when the ball is inside. Set the pendulum swinging and measure the time for 15 or 30 periods. **Divide the number of swings by time to obtain the value of one period  $T$ .**

Use Eq.(3) to calculate values of the initial speed  $v$  of the ball.

## 6 Summing Up

Compare your three different kinds of velocity measurements and associated uncertainties. Are they consistent? What is the most important random error in this experiment? Explain. Out of all three methods which one gives the best experimental velocity values? Which one gives the worst velocity value?

## 7 Question

For the steel ball of this experiment the kinematic method for measuring the speed works very well. What difficulties would you anticipate in using this method for measuring the speed of a bullet?

## 8 Finishing Up

Please leave your bench in a ship-shape fashion. Thank you.

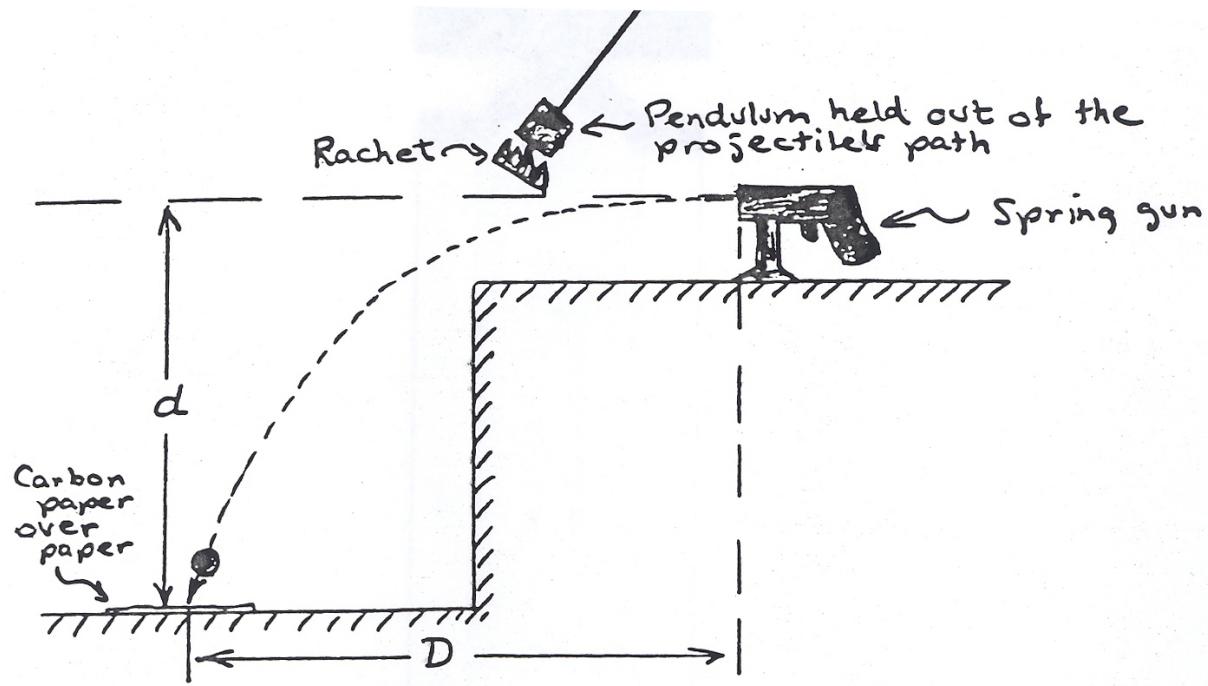


Figure 1: Arrangement for studying a projectile in free fall.

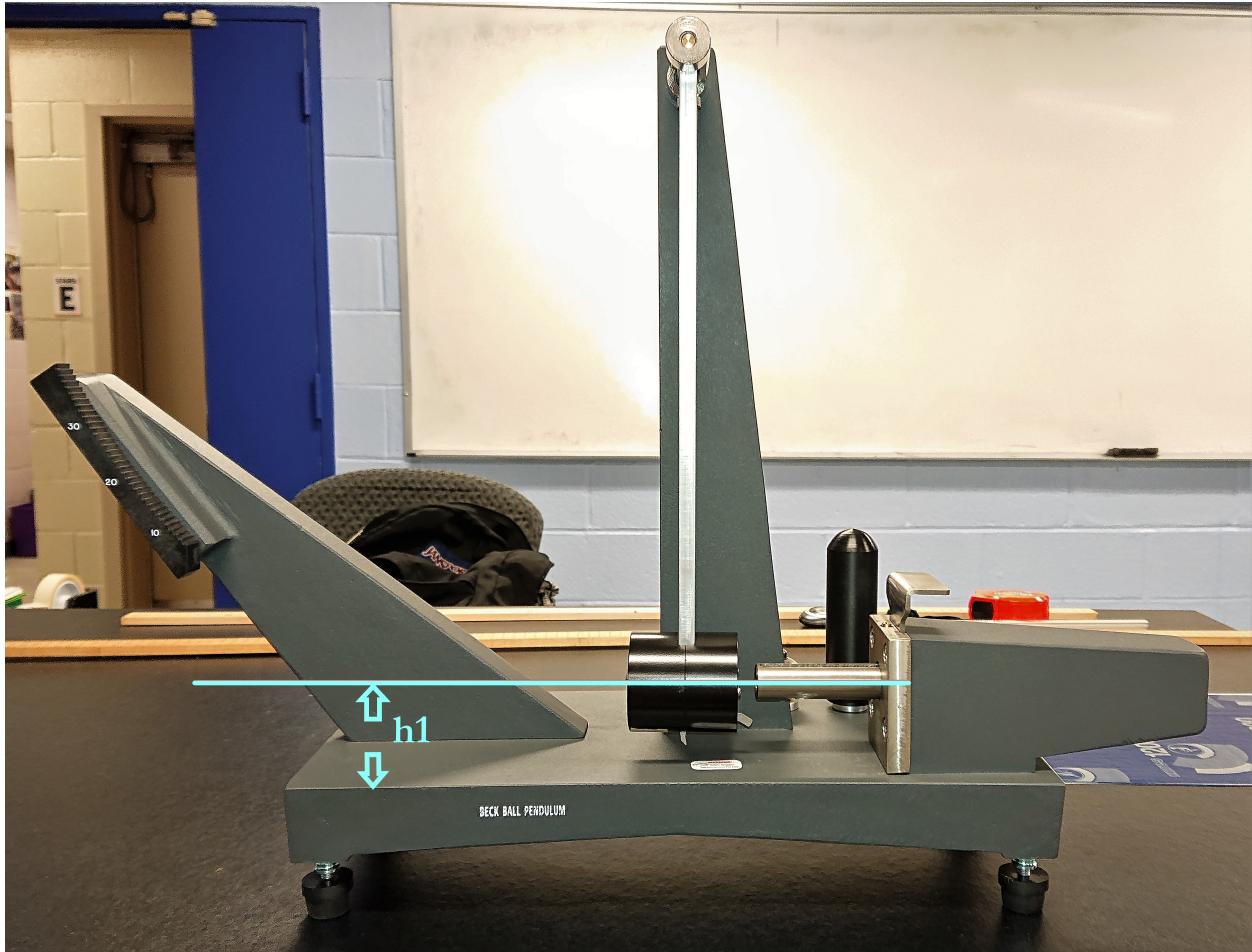


Figure 2: Ballistic pendulum at height  $h_1$ , before impact.

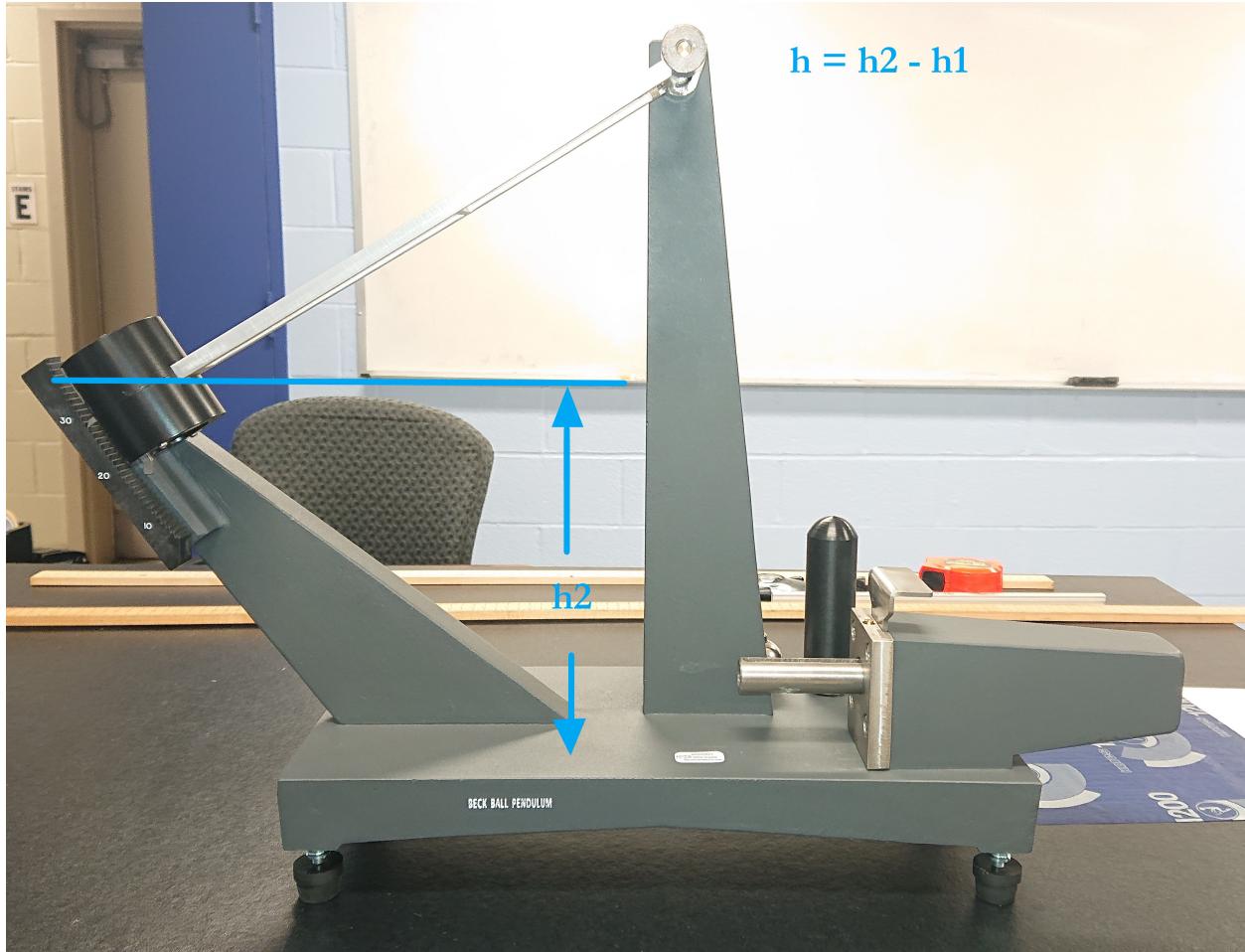


Figure 3: Ballistic pendulum at a height  $h_2$  on reaching its point of maximum swing after impact.

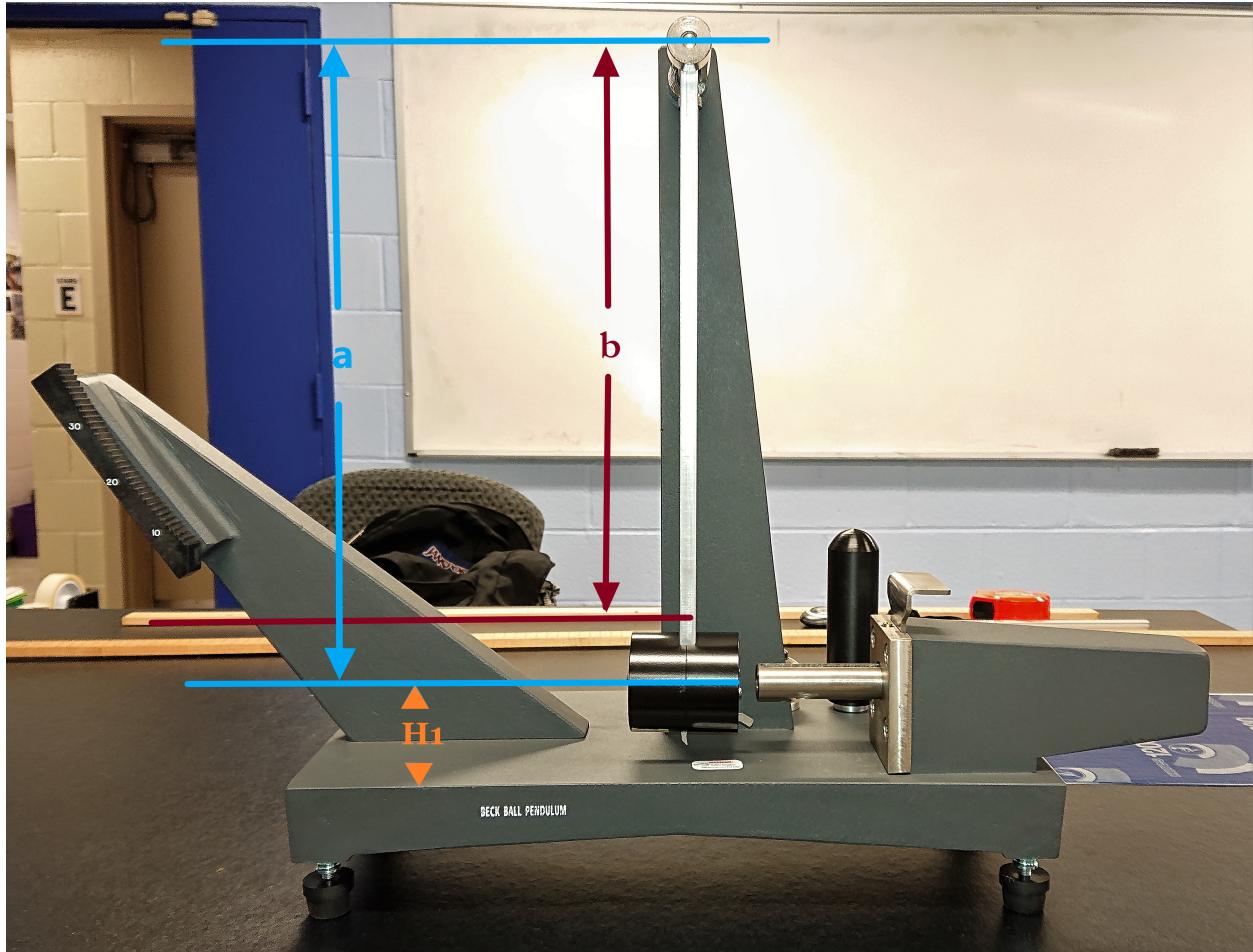


Figure 4: Ballistic pendulum at height  $H_1$ , before impact.

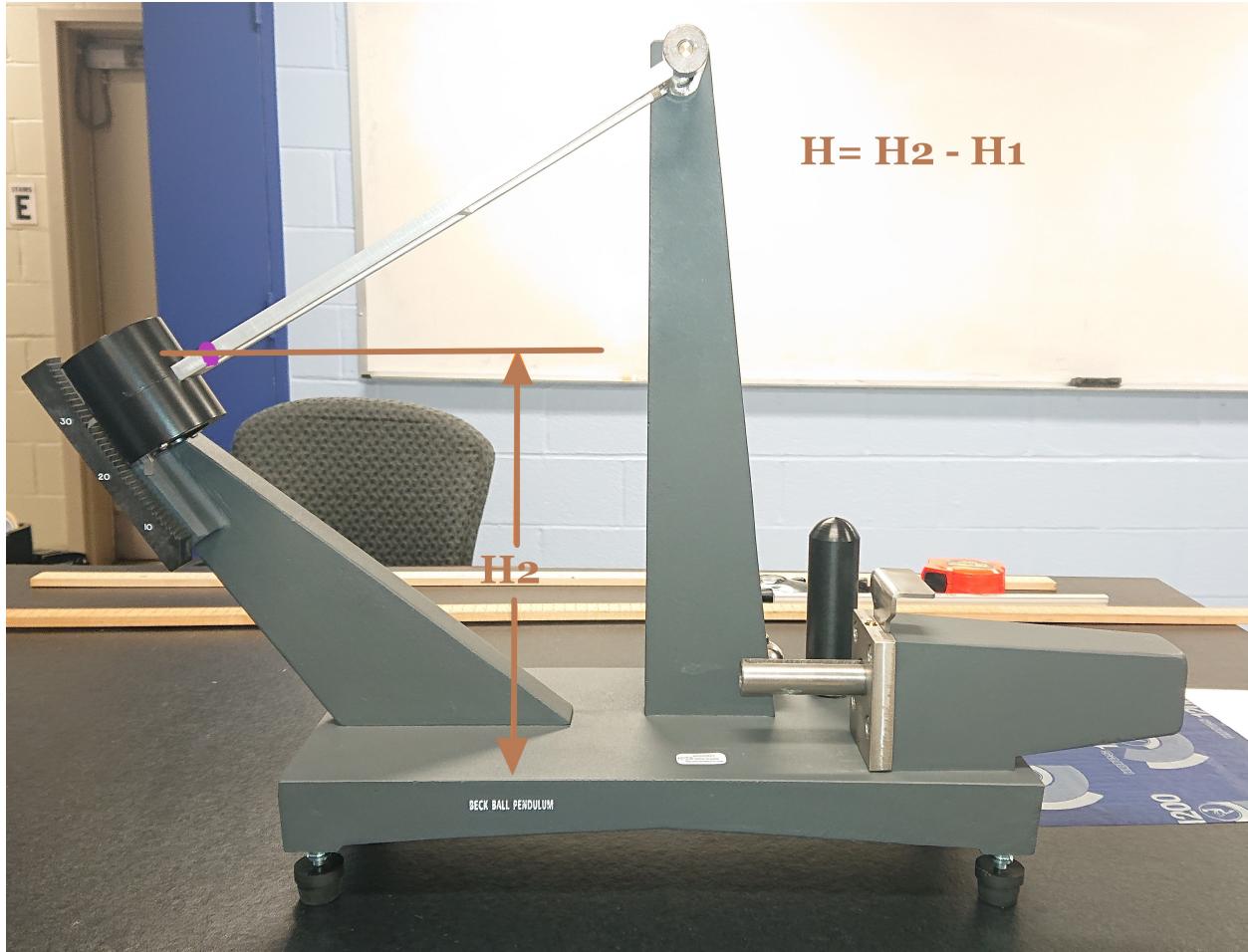


Figure 5: Ballistic pendulum at height  $H_2$ , after impact.