Data & Instructions — Lab 10

In this lab you explore the ideal gas law under five different conditions. Read the lab manual and familiarize yourself with the theory behind the ideal gas law. If you're interested in finding out where the equations in the lab manual come from, here are some decent resources:

- `wikipedia.org/Ideal_gas_law`
- `khanacademy.org/temp-kinetic-theory-ideal-gas-law`
- `feynmanlectures.caltech.edu/kinetic-theory-of-gases`

The experiment itself is pretty straightforward. We measure the temperature and pressure of a bulb of air when immersed in five different environments:

- room temperature (control)
- water and ice mixture
- alcohol and dry ice mixture
- boiling water
- liquid nitrogen

We can then plot the five pairs of data points to examine the relationship between temperature and pressure. In this doc I've taken the liberty of making this plot and computing the slope and y-intercept of the regression line, as outlined in section 5 of the manual. Your task is to answer the questions at the end of this doc. **Note you don't need to write a lab report for this experiment. All you have to do is answer the 8 questions at the end. Please turn in your answers by Sunday 6th December at 11.55pm.**

Load & Display Experimental Data

First, load the raw data and print a table:

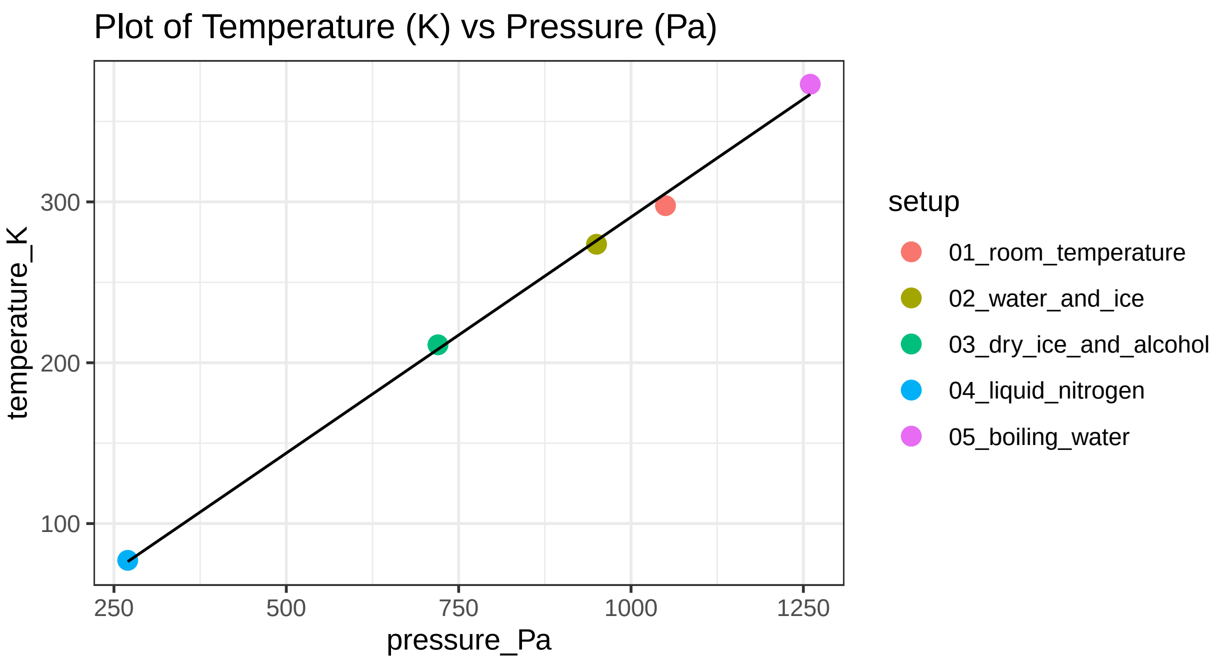
```r
## load raw data
data = read.csv("data.csv")

## add columns for Temperature (K) and Pressure (Pa)
data = data %>%
  mutate(temperature_K = temperature_C + 273.15) %>%
  mutate(pressure_Pa = pressure_kPa * 1000) %>%
  select(setup, temperature_C, temperature_K, pressure_Pa, pressure_kPa, pressure_psi)

## print dataset
kable(data)
```

setup	temperature_C	temperature_K	pressure_Pa	pressure_kPa	pressure_psi
01_room_temperature	24.5	297.65	1050	1.05	15.0
02_water_and_ice	0.5	273.65	950	0.95	13.5
03_dry_ice_and_alcohol	-62.0	211.15	720	0.72	10.5
04_liquid_nitrogen	-196.0	77.15	270	0.27	4.0
05_boiling_water	100.0	373.15	1260	1.26	18.5

Linear Regression of Temperature on Pressure

Next, we want to determine the relationship between temperature and pressure using this data. Below we use the experimental data to make a plot of temperature (K) on pressure (Pa) with the regression line overlaid:

```
ggplot(data = data, aes(x = pressure_Pa, y = temperature_K, color = setup)) +
  geom_point(size = 3) +
  geom_smooth(method = "lm", color = "black", size = 0.5, se = F) +
  ggtitle("Plot of Temperature (K) vs Pressure (Pa)") +
  theme_bw()
```



As predicted by the ideal gas law, the relationship looks pretty linear. The equation of the line should be of the form

$$T = mP + b$$

where m is the slope of the line and b is the y-intercept. We can compute the slope and y-intercept of the line by using R's `lm()` function to perform a linear regression of temperature (K) on pressure (Pa), and printing the two regression coefficients:[1]

```
regression1 = lm(temperature_K ~ pressure_Pa, data = data)
b = coefficients(regression1)[1]
m = coefficients(regression1)[2]
cat(paste("b = ", b, "\nm =", m))
```

```
## b =  -2.90134756737844
## m = 0.29347217360868
```

There. The y-intercept is -2.901, and the slope is 0.293.

[1] Linear regression is a technique for fitting a straight line to a set of data points. It yields two values, the slope of the line and the y-intercept, which are known as the *regression coefficients*. If you're unfamiliar with regression, check out this link: `https://online.stat.psu.edu/stat462/node/91/`

Questions

1. What are the dimensions of temperature and pressure (base units)?

2. What are the dimensions of m and b?

3. What does the slope correspond to in the graph?

4. Based on the lab data, what is the predicted value of absolute zero in degrees Celsius?

Many properties of the low-density gases can be summarized in the **ideal gas law,**

$$PV = nRT$$

where P is the pressure in Pa, V is the volume in m^3, n is the number of moles of gas, T is the temperature in Kelvins, and R is the universal gas constant with the empirical value

$$R = 8.31\ \mathrm{J/mol \cdot K}$$

A **mole** of molecules is Avogadro's number of them, denoted N_A:

$$N_A = 6.02 \cdot 10^{23}$$

Often we will want to discuss the **number of molecules**, denoted N:

$$N = n \cdot N_A$$

If you plug in $\frac{N}{N_A}$ for n in the ideal gas law, and group together the combination $\frac{R}{N_A}$ and call it a new constant k, you get

$$PV = NkT$$

This is an alternate form of the ideal gas law. The constant k is called **Boltzmann's constant,**

$$k = \frac{R}{N_A} = 1.381 \cdot 10^{-23}\ \mathrm{J/K}$$

5. The diameter of the bulb used in the experiment is 10 cm. Can you work out approximately how many moles of air are in the bulb?

6. Equation (3) in the manual states $P = n_N kT$, which is yet another way of writing the ideal gas law, using a quantity known as the number density of molecules, n_N. Show how this expression is equivalent to $PV = nRT$.

7. One of the assumptions in this experiment is that the density of air in the bulb stays constant. Can you think of any reasons why this may not be so? What errors can occur if the density is not constant? *Hint:* the number of moles can be written $n = \frac{m}{M}$ where m is total mass of the gas (kg) and M is the molar mass (kg/mol). Density is given by $\rho = \frac{m}{V}$. You can substitute these expressions into the ideal gas law and derive an expression that contains ρ.

8. What would happen if you accidentally spilled some liquid nitrogen on your hand? *Hint:* look into the Leidenfrost Effect.

Comment: the ideal gas law is an example of an **equation of state,** since it describes the relationship between three state variables, P, V, and T. However it only *approximately* describes the behavior of a real gas, since it doesn't take into account molecular size and intermolecular attractions (Van der Waals forces), which both cause deviations from ideal behavior. The ideal gas law is most accurate for gases at low densities and high temperatures, since under these conditions the molecules are far apart and moving at high speeds, such that intermolecular forces have little effect on the motion. There are, in fact, more complex equations of state that take into account the deviations from ideal behaviour caused by intermolecular attractions.

Bonus Exercise

In the following simulation you can adjust temperature, pressure, volume, and gas density:

`https://phet.colorado.edu/en/simulation/gas-properties`

Possible things to do—

- Run the simulation for 100 heavy particles. Vary the temperature and make a plot of temperature vs. pressure. Do the same thing for 100 heavy *and* 100 light particles. What do you notice?
- What is your value of absolute zero?
- What occurs to pressure and temperature when you adjust the volume? Explain.

Thanks everyone. Enjoy your break. ⧫