

Conservation of Energy

Equipment: Capstone, motion sensor II, photogate sensor, small bench clamp, double clamp, 90 cm rod (clamped vertically with bench clamp), 40 cm rod (clamped horizontally to 90 cm rod), brass spring (taped to horizontal rod 25 cm from vertical rod), hook masses, index cards, 30 cm string with loops at both ends, calipers, rubber tube, paper tube about 2.5 cm in diameter, 2 meter ruler, 6 inch ruler, 12 inch ruler, and tape.

Comments: There are three parts in this lab. In the third part be aware how you set off the motion of the hook mass on the spring. The mass can disengage from the spring and fall on your foot or motion sensor!

1 Purpose

The total energy E of a simple mechanical system is the sum of the potential energy PE and the kinetic energy KE, $E = KE + PE$. In the absence of friction, the total energy E is a conserved quantity; so if the KE and PE change, they must change so that their sum is still equal to the total energy E. The total energy in several simple mechanical systems will be examined for this property.

2 Theory

The work-energy theorem is obtained from a spatial integration of Newton's Second Law. Let \vec{F} be the force on a point mass m with position \vec{r} and velocity \vec{v} . If the mass moves from an initial position (i) to a final position (f) the work-energy theory states that

$$\int_i^f \vec{F} \cdot d\vec{r} = \left(\frac{1}{2}mv^2 \right)_f - \left(\frac{1}{2}mv^2 \right)_i.$$

The left hand side of this equation is defined as the work W and the right hand side is defined as the change in the kinetic energy ΔKE . For work, also called a line integral, there are 2 possibilities.

1. It **depends on** the path or route taken by the mass as it moves from (i) to (f). In this case the force is called non-conservative. An example is the force of friction. A consequence of this is that there is no function whose differential equals the integrand of the line integral and a definite path for the mass m must be specified to evaluate the work.

2. It **does not** depend on the path or route taken by the mass as it moves from (*i*) to (*f*). In this case the force is called conservative. Examples are the force exerted by a linear spring and the uniform gravitational force. In this case there is a function called $-U$ whose differential $-dU$ is equal $\vec{F} \cdot d\vec{r}$. The function U is called the potential energy (PE).

For case 2 the work-energy theorem can now be written as the potential energy equal to kinetic energy.

$$-\int_i^f dU = -(U)_f + (U)_i = \left(\frac{1}{2}mv^2\right)_f - \left(\frac{1}{2}mv^2\right)_i = (KE)_f - (KE)_i.$$

As U is evaluated only at the points (*i*) and (*f*) it is clear that for conservative forces the work depends only on the end points and not on the particular path traversed by the mass (m). This equation can be written as

$$(U)_f + (KE)_f = (U)_i + (KE)_i.$$

Each side of this equation is called the total (mechanical) energy E of the mass. On the left, E has been evaluated at point (*f*) and on the right at point (*i*). The quantity E has been conserved and $E_f = E_i$. This statement is called the conservation of energy. **Energy is not conserved if friction or other non-conservative forces are present.** Why?

It is convenient to refer the PE at any point to a fixed reference point (*o*). For a conservative force the work integral from (*i*) to (*f*) is independent of the path. If we let that path go through the reference point (*o*). The work integral becomes

$$W = \int_i^f \vec{F} \cdot d\vec{r} = \int_i^o \vec{F} \cdot d\vec{r} + \int_o^f \vec{F} \cdot d\vec{r} = - \int_o^i \vec{F} \cdot d\vec{r} + \int_o^f \vec{F} \cdot d\vec{r} = +U_{io} - U_{fo}, \text{ where}$$

$$U_{io} = - \int_o^i \vec{F} \cdot d\vec{r} \quad \text{and} \quad U_{fo} = - \int_o^f \vec{F} \cdot d\vec{r}.$$

U_{fo} is called the final potential at (*f*) relative to (*o*) and U_{io} is called the initial potential at (*i*) relative to (*o*). The conservation of energy statement may be written

$$U_{fo} + (KE)_f = U_{io} + (KE)_i.$$

From this last equation it is evident that any value can be given to the PE at the reference point. This value will appear on both sides of the equation and effectively cancels out. For simplicity, the value of the PE at the reference point is usually taken as zero and we assume this to be the case. The subscript (*o*) referring to the reference point is often omitted but the PE is always taken to be with respect to a reference point. The location of the reference point should be clearly stated.

In one dimension, if that dimension is taken to be x , $dU = -Fdx$, and

$$F = -\frac{dU}{dx}.$$

2.1 Linear Spring

Let a spring lie on the x axis and assume one end of the spring is fixed. We will define x_o as the position of the unstretched free end of the linear spring. When a mass is attached to the spring, the spring will experience a restoring force $F = -k(x - x_o)$ where x is the position of the free end of the stretched spring, $(x - x_o)$ is the length the spring has been stretched and k is the spring constant. Take the reference point for the PE at x_o . The PE for a mass attached to the spring will be

$$U(x) = - \int_{x_0}^x F dx' = - \int_{x_0}^x -k(x' - x_o) dx' = \frac{1}{2}k(x - x_o)^2,$$

Often, x_o is chosen as the origin then where the potential energy at $x = x_0$ will be taken as zero. So, $U(x) = \frac{1}{2}kx^2$.

2.2 Uniform Gravitational Field

For a mass m in the uniform gravitational field on the surface of the earth the force of gravity is $F = -mg$ where g is the acceleration of gravity and up is considered the positive direction. The positive y coordinate is taken as the up direction and y_o as the PE reference height. The PE at any height y is given by

$$U(y) = - \int_{y_0}^y F dy' = - \int_{y_0}^y -mg dy' = mg(y - y_o),$$

where the potential energy at $y = y_0$ has been taken as zero. If y_0 is taken at the origin of the coordinate, the PE becomes $U = mgy$. This PE is often written as mgh where h is the height above the reference point. (h is negative if the mass is below the reference point.) The PE depends only on y and not how far the mass moves horizontally. Horizontal motion does not contribute to the PE. Why?

2.3 Multiple Masses and Conservative Forces

For a system consisting of a number of masses and forces, the analysis is easily extended. If all the forces, both internal and external, are conservative, the work done by all the forces can be represented by potential energies. The total energy is the sum of all the PE's and KE's of all the masses. This quantity is conserved.

3 Free Fall

3.1 Description

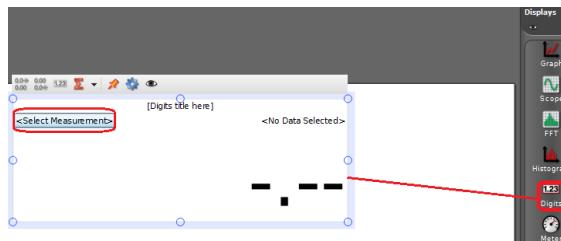
A tube with mass m is held horizontally a distance h above the level beam of a photogate sensor. The tube is dropped (without rotation) and its velocity is measured as it passes through the photogate. The total energy at the time of dropping is compared to the total energy as the tube passes through the photogate.

3.2 Theory

Take the reference point for the gravitational PE as the level of the photogate beam. From here is where you measure h . At the time of dropping the total energy is mgh . If v is the velocity of the tube as it passes through the photogate the total energy at the photogate beam is $\frac{1}{2}mv^2$. Equating the energy at the time of dropping to the energy as the tube passes through the photogate, $mgh = \frac{1}{2}mv^2$, which gives $v = \sqrt{2gh}$.

3.3 Setting Up and Programming

Measure the diameter of the tube with the calipers, and check for uniformity. If the diameter of the tube is not uniform, discuss it in your error analysis. Adjust the photogate so its sensor plane is horizontal and not over the base plate. Also adjust its height of the sensor so the beam is 25 to 35 cm above the table. Place the photogate near the edge of the table, making sure the sensor arm is out past the table edge. Plug the photogate into the 850 interface and program Capstone for the digital **One Photogate (single flag)** sensor. In the Hardware setup window click on properties (the gear icon in the lower right corner). In the properties window change the **Flag Width** to the diameter of the tube. Click ok. Set up a digits display by dragging the Digits icon from the displays column to the white screen, In the digits box click on **Select Measurements** and select **speed**.



3.4 Taking Data and Analysis

Drop the rubber tube so that it is horizontal, does not rotate, and is perpendicular to the photogate beam. Why is it critical for the rubber tube to be perpendicular to the photogate beam? Use the vertical rod next to the photogate as a guide for dropping the tube through the photogate without hitting it. Click Record, and using the meter stick drop the tube onto bubble wrap from a height $h = 15$ cm above the beam. Measure height to the middle of the tube. Record photogate velocity through the photogate. You will want to take some practice runs. Repeat for $h = 25$ and 35 cm.

Compare your measured velocities with that predicted by the theory. Is there any friction in this experiment? If so, how would it affect your data? Next, try dropping the paper tube from a height of 50 cm above the photogate beam and analyze the data as before. Note: Adjust the flag length again to the width of the paper tube. Is the conservation of energy observed? Why or why not? Explain.

4 Pendulum

4.1 Description

A mass of 100 g is hung from a 30 cm string and used as a pendulum. The weight is pulled to one side and let go. At the lowest point the weight passes through the beam of a photogate sensor and its velocity is measured.

4.2 Theory

As the pendulum swings down, PE is converted into KE. The change in PE is given by mgh where h is the vertical distance that the weight has moved. The weight travels the arc of a circle, and this distance is longer than h . The velocity is given by exactly the same expression as in the previous experiment, $v = \sqrt{2gh}$. As the pendulum swings down the string exerts a force on the mass. Does this force contribute to PE? Why or why not?

4.3 Setting Up and Programming

Measure the diameter of the 100 g weight with the calipers. hook the weight onto one end of the 30 cm string. Loop the other end around the horizontal rod. Restart the Capstone software. Adjust the photogate so that the legs point up and the beam is 10 to 20 cm above the bench. Adjust the height of the suspension point of the pendulum so that the middle of the weight swings through the center of the photogate beam. Refer to the following image.



Click on the channel that the photogate sensor is plugged and select **Pendulum Timer**. In the Hardware setup window click on properties (the gear icon in the lower right corner). In the properties window set the **Pendulum Width** to the diameter of the 100 g mass. Click ok.

4.4 Taking Data and Analysis

Pull the weight to one side so that the middle of the weight is 15 cm above the level of the photogate beam. Let go and record the velocity of the weight at its lowest point. Repeat for $h = 12, 7$, and 5 cm.

Compare your measured velocities to the theoretical values. What contributes to the change in PE for the pendulum?

5 Gravity + Spring

5.1 Description

You will hang a 0.5 kg mass (m) on a vertical spring. The mass will be set into vertical oscillatory motion and its position will be measured as a function of time using a motion sensor. The velocity and acceleration are calculated by Capstone. The total energy of the mass is compared at different points in the motion. The PE of the spring-mass system is due to both gravitational PE and the PE of the spring.

5.2 Theory

The mass m performs simple harmonic motion with an angular frequency given by $\omega = \sqrt{k/m}$, where k is the spring constant. What are the units of angular frequency?

Let the y axis be the vertical coordinate (up is positive). The y value will give the position *at the end of the spring* from which m is hung. The coordinate origin will be defined to be the direction *at the end of the spring* when there is no mass on the spring (spring is unstretched) $y = 0$. When the spring is stretched downward by hanging a mass (y will be negative). The PE of the spring will be $\frac{1}{2}ky^2$ where k is the force constant of the spring. Let the gravitational PE of the mass m be zero when $h = 0$ (spring stretched). The gravitational PE for m is then mgh , which will be negative. (Keep in mind the end of the spring and the mass move the same amount.) When the end of the spring is at a point given by y and has a velocity $v = dy/dt$ (also the velocity of the mass), The total energy E of the spring-mass system is given by

$$E = \frac{1}{2}mv^2 + \frac{1}{2}ky^2 + mgh.$$

Assuming no friction, E should be the same at any point in the motion. In this case h is defined as the height of the center of mass of the hanging weight. **Keep in mind $h_0 = 0$ when the mass is hanging on the spring without motion.**

5.3 Setting Up

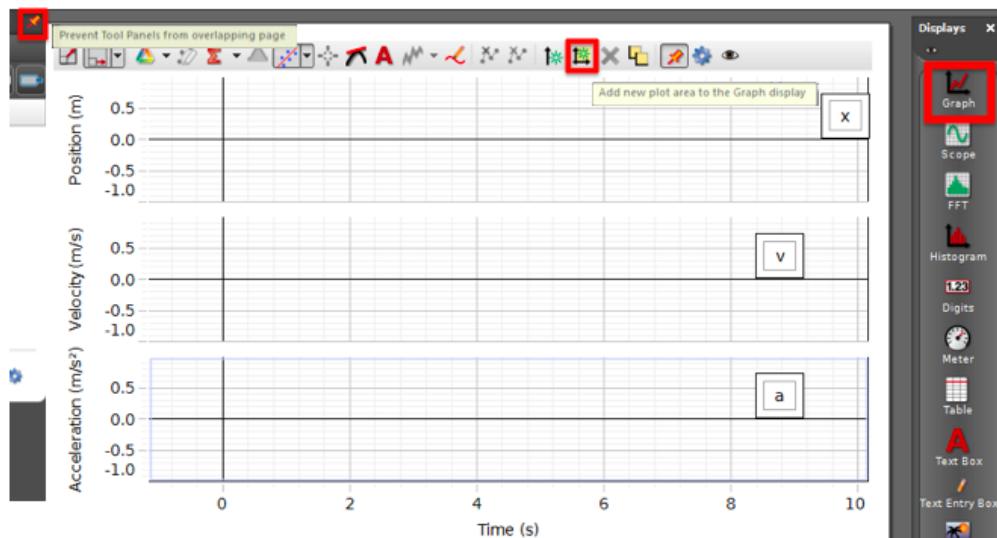
The spring should be hanging over the edge of the bench. Attach the 0.5 kg mass and adjust the height of the spring so that the bottom of the mass is approximately 65 cm from the floor. Then remove the mass and measure the distance from the unclamped end of the spring to the floor using the two meter stick, oriented vertically, and the 6 in ruler, oriented horizontally.

Reattach the mass to the spring and measure the distance from the same end of the spring to the floor. Determine k for the spring. Let the amount the spring stretches be y_0 , a negative number, making $y = y_0$ the position at the end of the spring with the mass at rest.

Place the motion sensor on the floor directly beneath the the mass and check that the sensor face is horizontal. There is a switch on the motion sensor for narrow or wide which sets the width of the acoustic beam of the sensor; you will probably find that wide works the best but feel free to experiment. The motion sensor should be plugged into the interface so that position increases as the mass goes higher.

5.4 Programming

Restart Capstone. Program Capstone for the motion sensor, and use the default values. Drag and drop the graph icon from the displays column onto the white screen. Add two additional plots by using "Add a new plot area to the graph display." Set up position, velocity, and acceleration on the graph display. Click on the orange tack if the pages are overlapping. You will probably find the default sampling rate of 20 Hz to be satisfactory, but feel free to experiment. Look at the following illustration for guidance.



5.5 Experiment and Analysis

Set the mass into motion by pulling it down a reasonable amount and letting go (the motion sensor does not record distances less than 0.15 m from its face). Do not set the mass in motion by lifting it up. If you lift it too far it will crash into the motion sensor. After the motion settles down, click Record and do the following.

- Simultaneously observe the motion of the mass and the graphs of position, velocity, and acceleration. Does your intuition about the motion correspond to what the graphs are displaying? For example, is the acceleration maximum or minimum when the velocity is zero? When the velocity is maximum is the acceleration maximum or minimum?

- Observe the spring-mass system. Is there kinetic energy that is not given by $\frac{1}{2}mv^2$? If so, this would be worth while mentioning in your error analysis. (Hint: There are at least two items one might notice here.)

Click STOP after 4 or 5 cycles of the motion. Using the Add a Coordinates Tool on the position graph, determine the distance the mass has traveled from a chosen highest position (H) to the following lowest position (L) on the position graph. Also, you need to obtain the maximum down velocity after the highest position and the maximum up velocity after the lowest position on the velocity graph display.



This distance will be $2A$, where A is the amplitude of the motion and is a positive number. Determine the total energy E for the following 4 positions of m during the motion. The highest and lowest positions. The two positions of maximum velocity following these highest and lowest positions of the mass. Recall that the coordinate y gives the position of the end of the spring, not the position of the mass. We will define y_M as the center of mass of the hanging weight. You will find the y_M value by measuring the distance of the center of the 0.5kg hanging weight to the end of the spring when there is no mass on the spring (spring is unstretched). The highest position of the mass occurs when $y = y_M + A$ and the lowest position of the mass when $y = y_M - A$. Maximum velocity occurs when $y = y_M$. At that point the mass is at a height of h_0 . **Keep in mind h_0 was defined in the theory section.**

1. When the mass is at the highest chosen position and the KE is zero, the energy E_H will be

$$E_H = \frac{1}{2}k(y_M + A)^2 + mg(h_0 + A). \quad (1)$$

2. When the mass has the maximum down velocity following the highest chosen position of the mass the energy E_{01} is given

$$E_{01} = \frac{1}{2}mv^2 + \frac{1}{2}ky_M^2 + mgh_0. \quad (2)$$

3. When the mass is at the lowest chosen point and the KE is zero, the energy is given by

$$E_L = \frac{1}{2}k(y_M - A)^2 + mg(h_0 - A). \quad (3)$$

4. When the mass has the maximum up velocity following the lowest chosen position of the mass the energy E_{02} is given by

$$E_{02} = \frac{1}{2}mv^2 + \frac{1}{2}ky_M^2 + mgh_0. \quad (4)$$

Obtain the velocity from the velocity curve of the graph display.

Compare the total energy at these 4 points. Why would energy vary at each point? Is energy conserved? What factors might introduce error into your measurements and calculations?

5.6 Question

1. Initially we assumed no friction in this experiment. Will friction affect your experimental results? Is there any evidence of friction in the curve of position vs time graph? Explain.

6 Finishing Up

Return the bench to the condition in which you found it. Be considerate for your fellow mates. Thank you.