

Work Energy

CAUTION For this lab, force sensors are mounted near the floor on rods that stick out into the aisle. Please be careful.

Equipment Capstone, air track, 28.7 cm glider, set of kg masses with hooks, photogate/smart pulley, small bench clamp, force sensor, 100 cm string with loops, 100 cm string with loops and 4 rubber bands, 130 cm string with loops, meter stick, and digital scale.

1 Purpose

To investigate the validity of the work-energy theory.

2 Theory

Consider a point mass m acted upon by a net force $\vec{F} = \vec{F}(\vec{r})$. The position, velocity, and acceleration of the mass are given by \vec{r} , \vec{v} , and \vec{a} , and the time by t . The force \vec{F} may be a function of \vec{r} . Newton's 2nd Law for the mass is $\vec{F} = m\vec{a}$. To find the work performed on the mass. This is integrated over position from an initial position (i) to a final position (f). The left hand side is defined as the work W done on m as it moves from \vec{r}_i to \vec{r}_f (distance). It is only the component of the force parallel or anti-parallel to the motion of m that contributes to the work.

$$\int_i^f \vec{F} \cdot d\vec{r} = m \int_i^f \vec{a} \cdot d\vec{r}.$$

The right hand side can be transformed to

$$m \int_i^f \frac{d\vec{v}}{dt} \cdot d\vec{r} = m \int_i^f \vec{v} \cdot d\vec{v} = \left(\frac{1}{2} m \vec{v} \cdot \vec{v} \right)_i^f = \left(\frac{1}{2} m v^2 \right)_f - \left(\frac{1}{2} m v^2 \right)_i.$$

The quantity $\frac{1}{2}mv^2$ is defined as the kinetic energy, or KE. In other words, when m moves from \vec{r}_i to \vec{r}_f , the net work W done on m between those two points is equal to the change in kinetic energy KE between those two points. More succinctly,

$$W = \Delta KE.$$

This is the most basic form of the work-energy theorem. To be used correctly, W must include all the work done by all of the forces between the two points under consideration.

The work-energy theorem can be applied to a system of particles. The total work done is equal to the total change in KE. Care must be taken with the work done by the forces between the particles of the system, or the “internal” work. The internal work may not cancel out. What are examples of internal work?

3 Does $W = \Delta KE$?

3.1 Description

A glider of mass m is on a horizontal air track. A horizontal string attached to the glider goes over a photogate with smart pulley. The end of the string that goes over the photogate with smart pulley is attached to 3 or 4 rubber bands that are connected in series, and the last rubber band is attached to a force sensor which is clamped to a rod near the floor. The glider is pulled back to stretch the rubber bands and let go. The force on the glider and velocity of the glider are measured as a function of position of the glider. Capstone integrates the force curve to give the work done (W). The velocity curve can be examined to give the velocities at the chosen beginning and end points. Be consistent with the points you pick. The change in KE is calculated and compared to W . Why is the force on the glider not constant?

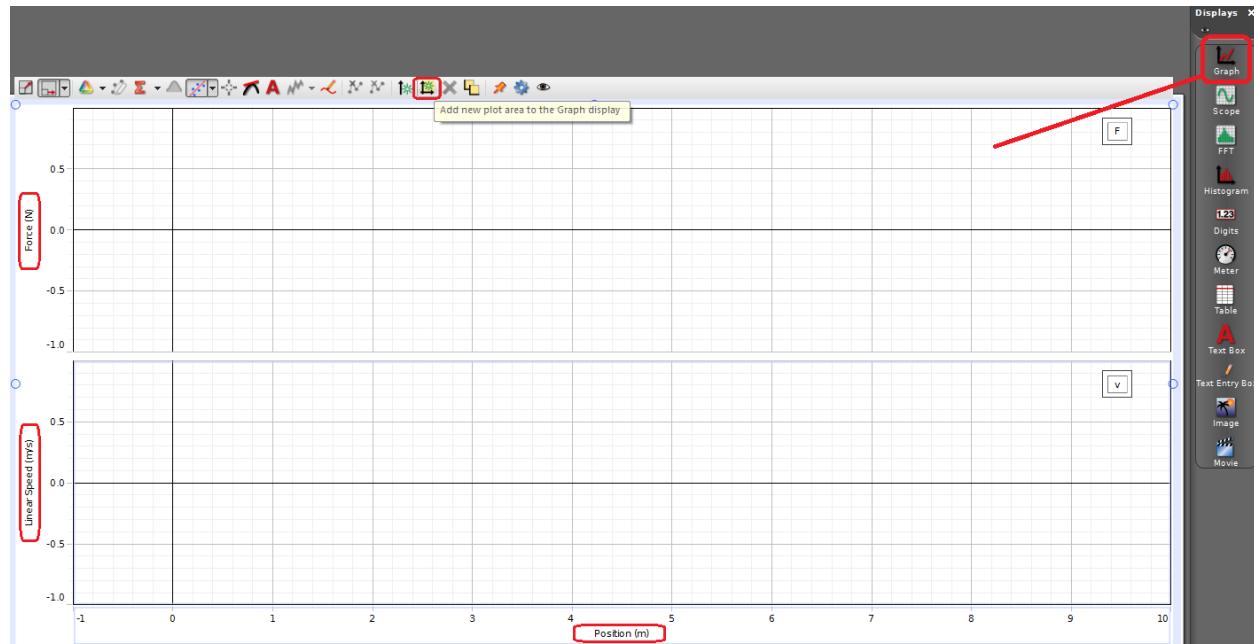
The tension T in the string will be the same on both sides of the pulley if the mass of the string and pulley can be neglected and if the pulley is frictionless. The magnitude of the force between the last rubber band and the force sensor will be the same as T if the rubber bands are massless.

3.2 Theory

The motion in this experiment is one dimensional so the vector notation used in defining work is not needed. If the one dimension is taken to be x , the work becomes $\int_i^f F dx$. This is equal to the area under the F vs x curve between the two vertical lines defined by x_i and x_f . Capstone will calculate this area for you.

3.3 Programming

Check that the (analog) force sensor and the (digital) photogate / smart pulley are plugged into the interface, and noting which channels they are plugged into, program Capstone for these two sensors. To program photogate/smart pulley click on the digital input that its plugged in and select **photogate with pulley**. Next, program the force sensor. Program Capstone for two graph displays. Drag the graph icon from the **Displays** column onto the Capstone page. Use add new **plot button** for the second graph.(The add a new plot option is located on the top bar above the plot) Program the graph so position is on the horizontal axis. Program the two vertical axes for **Force** and **Linear Speed**. The following figure illustrates setting up the graph.



3.4 Calibrating the Force Sensor

Click on **Calibration** in the tools column to open up the setup dialogue box. Calibrate the force sensor as follows.

- Leave the force sensor in the position in which it will be used.
- In Capstone, click the **Record** button. You need to run the program to calibrate the Force Sensor.
- With nothing touching the hook on the force sensor, push the **Tare** button on the side of the sensor.
- In the **Tools** column of Capstone click **Calibration**.
- Select force and click next.
- Pick "Two Standards (2 point)" and then next.
- The **Calibration Point 1** box will appear first. Enter 0 in the **Standard Value** box. Click **Set Current Value to Standard Value**.
- Take a 100 cm string with loops at both ends and attach one end to the hook of the force sensor. Pass the other end of the string over the pulley and down through the hole in the bracket holding the smart pulley. Hang a 100 g mass from that end of the string. Look at figure 1 on the next page.



Figure 1:

- In the box labeled **Calibration Point 2**, enter the **weight** of the 100 g mass in the box labeled Standard Value. Then click **Set Current Value to Standard Value**.
- Click **Finish**.
- Click **Stop**.
- Go to **Data Summary** in the Tools column and delete the data it recorded.
- Go to the bottom left center of the Capstone work page. Where it states Force Sensor next to "Recording Conditions", increase the periodic samples to 200 Hz.

3.5 Taking Data

Remove the string used for calibrating the force sensor. Turn the air track on. Level the air track. Take the string with the rubber bands at one end and hook the last rubber band to the force sensor. Pass the string over the smart pulley and hook the end to the 28.7 cm glider. Pull the glider back to stretch the rubber bands but do not pull the rubber bands into the photogate with smart pulley. Click Record, and let go of the glider. Click Stop just after the string goes slack. Note: Capstone will start recording when you let go of the glider.

3.6 Analysis

Click the scale to fit button, then click on the force curve. Use the highlight button and highlight the part of the force curve you wish to integrate. Your beginning integration point x_i should exclude perhaps the first two data points. The end of your integration x_f should be just before the velocity curve levels off and before the force curve goes to zero. Click on **Display area under the curve** icon. This is the value of the integral (area under the graph) which is the work W done between the limits chosen. Note: To increase the number of decimal points click on the **Data Summary** icon and then follow the instructions in figure 2.

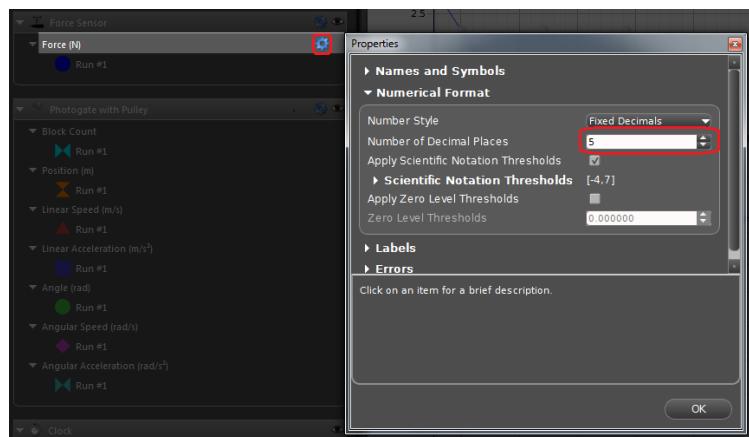
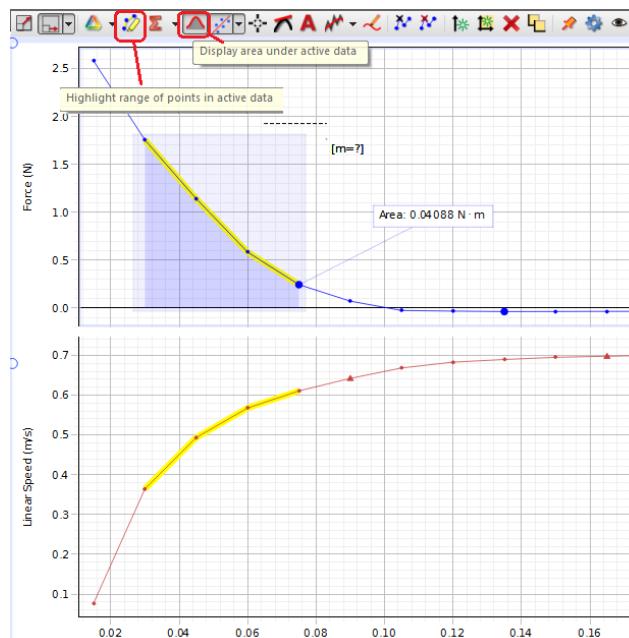


Figure 2:

To further add your analysis. Click on the velocity graph and then on the **Add a Coordinates tool**. Look at figure 3. Use the cross hair tool to determine the velocities at the end points of your integration and calculate the change in KE. Compare to the measured work done. Is there a difference in value between the work done and kinetic energy? Why or why not?

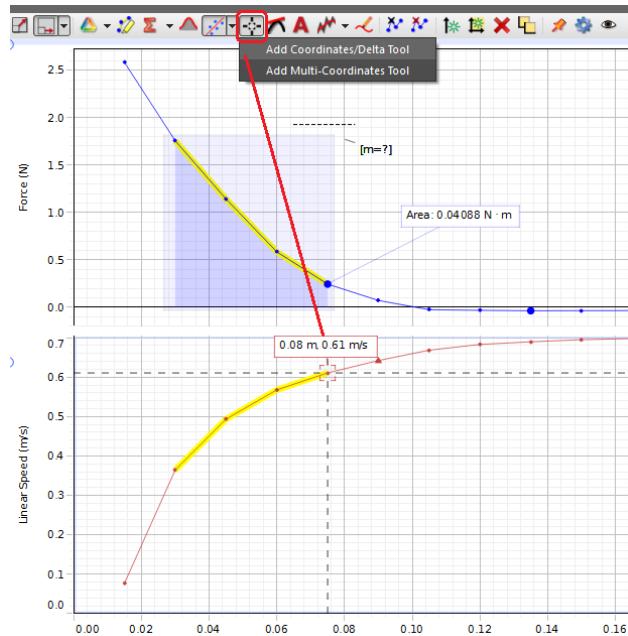


Figure 3:

3.7 Repeat

Take a few more data runs. Discuss random errors that contribute to the scatter of your results from run to run. Also, discuss the systematic errors which will effect all of your runs in the same way.

4 $W = \Delta KE$ Applied to a System

4.1 Description

The work-energy theorem is tested for a system of two masses. The theorem is applied to the two masses separately and to the entire system.

A horizontal string is attached to a glider on a horizontal air track. The string goes over the smart pulley and a mass is attached to the end. The glider is released and the velocity of the glider is measured as a function of distance. The velocity of the glider is measured at two points and the change of KE is calculated for each mass (glider and hanging mass) and for both masses together. The work done on each mass and for the system is calculated.

4.2 Theory

The hanging mass is denoted M_1 and the glider mass by M_2 . From the equations given in the Lab Newton's 2nd Law the tension T in the string can be calculated as

$$T = \frac{M_1 M_2 g}{M_1 + M_2}.$$

This is a constant and the work done by T simplifies to $T \times (\text{distance})$ with an appropriate sign.

The work done on M_1 is $(M_1 g - T)(x_f - x_i)$. Explain the minus sign before the T . The work done on M_2 is $T(x_f - x_i)$.

The work done on the system is $M_1 g(x_f - x_i)$. Explain clearly why T does not appear in this expression. (Due to Newton's 3rd law, internal forces cancel out. Keep in mind internal work does not always cancel out.) What are the external forces on the M_1 mass?

4.3 Programming

Restart the Capstone program and program for the photogate/ smart pulley. Reminder select **photogate with pulley**. Open a graph display and setup the graph display with linear speed on the vertical axis and position on the horizontal axis.

4.4 Taking Data

Move the air track and the photogate / smart pulley so that a string with weight hanging from the smart pulley will not hit the force sensor. You might have to remove the force sensor. Also, level the air track. Hook the string to the glider and pass the string over the smart pulley. Hang a 50 g mass from the string. Pull the glider back until the 50 g mass is close to the photogate / smart pulley, Click the Record button, and let the glider go. Click Stop just before the hanging mass hits the floor.

4.5 Analysis

Reminder use the scale to fit button and add a coordinate tool to pick suitable x_i and x_f . Determine the velocities at those points. For M_1 , M_2 and the system, calculate the work done and the change in KE. Compare the work done with the change in KE for these three items. Is there a difference in value? Explain.

Is the velocity vs distance curve a straight line? If not, what do you think it is?