Centripetal Force

<u>Equipment</u>: Centripetal Force apparatus, meter stick, ruler, timer, slotted weights, weight hanger, and digital scale.

1 Introduction

In classical mechanics, the dynamics of a point particle are described by Newton's 2nd law, $\vec{F} = m\vec{a}$, where $\vec{F}$ is the net force, m is the mass, and $\vec{a}$ is the acceleration. This equation guarantees that $\vec{F}$ and $\vec{a}$ are parallel to each other. The velocity $\vec{v}$ by definition is always parallel to path of the particle. For arbitrary motion, it is not necessary for $\vec{v}$ and $\vec{a}$ to have particular direction with respect to the other.

In the *special* case of a particle moving in a circle with constant speed, $\vec{F}$ and $\vec{a}$ are always perpendicular to $\vec{v}$ and point from the particle's position toward the center of the circle. The force $\vec{F}$ is called the centripetal force (centripetal means "center seeking"). In this experiment a mass m is rotated in a circle and the force on the mass is compared to the theoretical value.

Related terms, which are often misused, are "centrifugal force" and the velocity dependent "Coriolis force." These two "forces" are only relevant in a rotating frame of reference and arise because the reference frame is not inertial. You must add centrifugal force to $\vec{F} = m\vec{a}$ in a rotating frame of reference, and if a particle is moving in this frame, you must also add Coriolis force.

2 Theory

2.1 Angles In Radians

Mathematicians and Physicists often express angles in radians (rad). A full circle has 2π rad, so $360° = 2\pi$ rad. In Fig. 1, a circle of radius r has been drawn with the vertex of angle θ at its center. Let the length of the arc spanned by θ be $\widehat{ab}$. In radians, θ is defined by

$$\theta = \frac{\widehat{ab}}{r}.$$

This result is independent of the size of r.

2.2 Angular Velocity $\vec{\omega}$, Frequency f, and Period T

The speed of rotation or circular motion of an object is often expressed in terms of angular velocity $\vec{\omega}$, with units rad/s. In Fig. 2 let $\vec{r}$ be a rotating position vector with constant magnitude so that its tip traces out a circle. Suppose that $\vec{r}$ sweeps out a small angle $\Delta\theta$ in the small time Δt, the magnitude of $\vec{\omega}$ is defined by

$$\omega = \frac{d\theta}{dt} = \lim_{\Delta\theta \to 0} \frac{\Delta\theta}{\Delta t}.$$

The direction of $\vec{\omega}$ is perpendicular to the plane in which $\vec{r}$ is rotating and is given by the right hand rule. With the fingers pointing in the direction in which $\vec{r}$ rotates, the extended thumb points in the direction of $\vec{\omega}$. In general, ω does not have to be a constant. However, it is in the case of a particle rotating in a circle at a constant speed.

Rotation rates are also expressed in frequency (f or ν), which is the number of cycles or rotations per second. One cycle per second is called a hertz (Hz). The relationship between f and ω is $\omega = 2\pi f$, with ω in rad/s and f in Hz. The period T is the time it takes for one revolution, and is given by T = (1/f).

2.3   Velocity, Acceleration, and Force

The velocity $\vec{v}$ and acceleration $\vec{a}$ are defined by

$$\vec{v} = \frac{d\vec{r}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} \ and,$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t}.$$

In the case where a particle moves in a circle with constant speed, the magnitudes of $\vec{r}$ and $\vec{v}$ are constant but their directions change. This leads to the following equations for v and a:

$$v = r\omega, \ and \ a = r\omega^2.$$

Letting F_c be the centripetal force on a particle moving in a circle with constant speed, and substituting the above expression for a into Newton's 2nd law, we have

$$F_c = mr\omega^2. \tag{1}$$

This is the equation that will be examined experimentally.

3   Apparatus

Refer to Fig. 3. On a horizontal base, a vertical shaft is supported by a bearing that allows the shaft to rotate. At the top of the shaft there is a horizontal arm which supports a movable slider and a counter weight. A mass M_1 has a vertical pointer at its bottom. The mass is suspended by string from the arm and attached to the shaft by a spring whose tension can be varied. At the base there is a vertical movable pointer with two screws that projects upward toward the vertical pointer on M_1. The shaft can be rotated with the fingers of one hand by twirling the knurling near the bottom of the shaft. The shaft is twirled so that the pointer on the base and M_1 line up. The centripetal force can be supplied by both the spring and the horizontal component of the tension in the string holding M_1. To directly measure the centripetal force, the apparatus is brought to rest. A string attached to the side of M_1 will go horizontally across a pulley and then vertically suspends mass M_2. The value of M_2 is adjusted so that the pointers again line up. The centripetal force is $M_2 g$, where g is the acceleration of gravity.

4   Procedures

You will now determine the centripetal force using angular velocity.

1. Level the base by using the thumb screws on both ends and watching the small bubble level near the center.

2. Adjust the base pointer so that the screws are at about the middle of the track and measure the distance from center of the base pointer to the center of the axis of rotation of the shaft.

3. Detach M_1 from the spring and vertical string. Measure the mas of M_1.

4. Reattach M_1 to the vertical on the arm and adjust the horizontal arm so that the M_1 pointer aligns with the base pointer. You may need to adjust counter weight as well so the arm is approximately balanced. Then adjust the length of the vertical string holding mass M_1 until the pointers are within a few mm of each other.

   Note: You will probably not be able to fulfill these conditions exactly.

5. Reattach the spring to M_1. Now, run the horizontal string across the pulley, making sure the part of the string connected to M_1 is parallel with the base, and attach the weight hanger to the string. Add various slotted weights to the hanger until the pointers are aligned again. When both pointers are aligned the force needed to pull M_1 at that distance is the same as the force from the spring. The force on the spring acts as the centripetal force. The overall combined mass needed to keep both pointers aligned is M_2. Take note of the mass and then remove M_2. Don't forget to take into account the mass of the weight hanger.

6. With just the spring attached to M_1, rotate the shaft so that the pointers line up again and measure the time for either 10 or 20 revolutions. While rotating the shaft and looking at the pointers, **watch your head!** Don't get knocked by M_1. If needed, adjust the tension in the spring so that the pointers line up for a "reasonable" rate of rotation. The rate of rotation should not be so slow that the centripetal force will be hard to measure, but it should also not be so fast that it is dangerous, or that the apparatus does not stay quietly on the bench. Calculate the period T, which is the time for one revolution or cycle. Repeat a few more times to build up some statistics.

7. Repeat the steps 3 through 5 with the base pointer at various radii. **Your teaching assistant will tell you the distance of the other two radii you will use.** Measure the rotation radii for these two positions.

4.1   Taking data and summarizing the previous steps.

With the string from M_2 not attached to M_1, rotate the shaft so that the pointers line up and measure the time for either 10 or 20 revolutions. Calculate the period T, which is the time for one revolution or cycle. Repeat a few more times to build up some statistics. With the apparatus at rest, attach the string from M_2 to M_1 and add weights to the weight holder so that the pointers line up. When the string attached to M_1 and M_2 is in place, the string connected to M1 should be parallel to the base. Also, don't forget to take into account the mass M_1, and of the weight holder. Repeat the above procedures with the base pointer at various radii. Measure the rotation radii for these two positions.

5 Analysis

Convert the periods you have measured to angular velocity, using

$$\omega = \frac{2\pi}{T}.$$

Then referring to Eq. (1), compare your values of $M_2 g$ with the corresponding values of $M_1 r \omega^2$.

6 Questions

1. What are the most significant causes of errors in this experiment?

2. Are the significant errors random or systematic?

3. Does the data enable you to estimate any of these errors? Explain.

4. Does the data support Newton's 2nd law when its applied to a mass going in a circular orbit at constant speed?

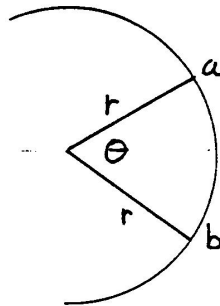5. What are all the forces that are occurring to M1 in rotation? You can draw a free body diagram.
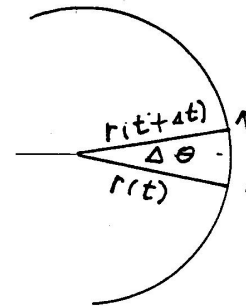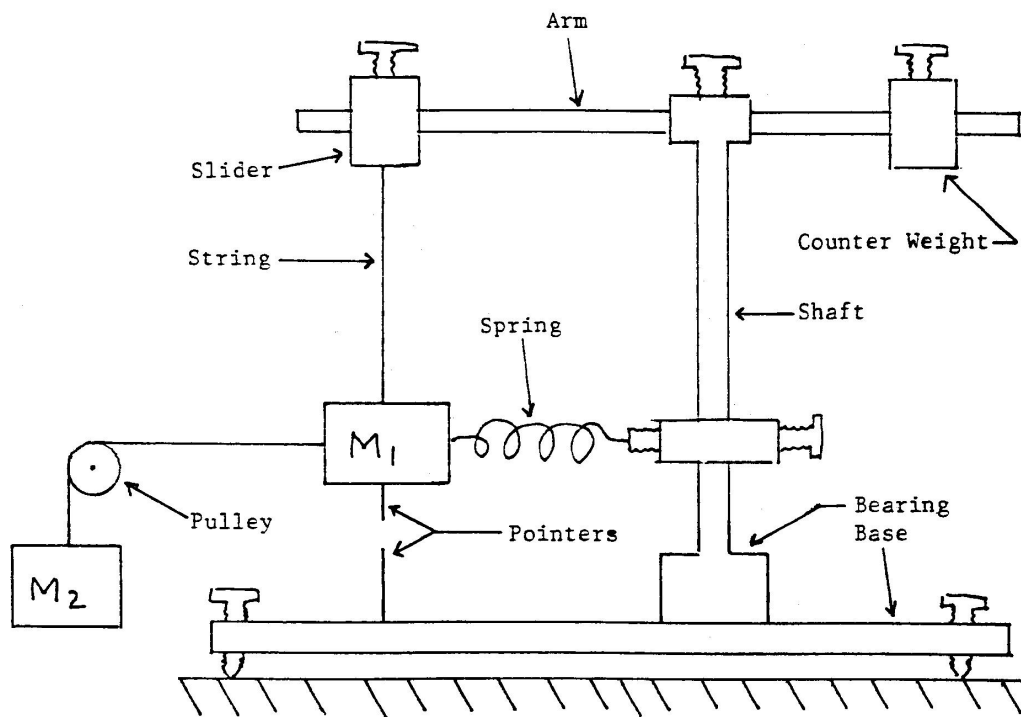
Fig. 1

Fig. 2

Figure. 3