Electrostatics

VA

Contents

1 Firstly

1.1 The point of electrodynamics

Given some **source charges** $q_1, ..., q_n$, and a **test charge** Q,

1. find the force exerted on the test charge by the source charges
2. determine the trajectory of the test charge

In general the source charges and test charge can move. It's assumed that we know the positions and velocities of the source charges.

1.2 Principle of superposition

The interaction between any two charges isn't affected by the presence of other charges. So the net force on the test charge is just the sum of its interaction forces with each source charge.

1.3 Electrostatics. . .

. . . is the specific case when all source charges are stationary. Note this isn't actually a thing irl.

2 Coulomb's Law

Describes the force between two charges at rest:

$$\mathbf{F}_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}^2}\hat{\mathbf{r}}_{12} = -\mathbf{F}_2 \tag{1}$$

where:

- $\mathbf{F}_1$ is the force exerted on q_1 (by q_2)
- $\hat{\mathbf{r}}_{12}$ is the unit vector pointing from $\mathbf{r}_1$ to $\mathbf{r}_2$
- ϵ_0 is the permittivity of free space, $\epsilon_0 = 8.85\text{e}^{-12}$ C^2/Nm^2

In a system with N charges, the Coulomb force on a particular charge is:

$$\mathbf{F}_i = \frac{q_i}{4\pi\epsilon_0} \sum_{j,j\neq i}^{N} \frac{q_j}{r_{ij}^2} \hat{\mathbf{r}}_{ij} \tag{2}$$

(via the principle of superposition).

3 Continuous Charge Distributions

Equations (1) and (2) assume the system is made of discrete point charges. You can also describe the system as a continuous distribution of charge (provided it's sufficiently big).

Notation: unprimed variables ($\mathbf{r}$, q, etc) denote the test charge. Primed variables ($\mathbf{r}'$, q', etc) denote the source charge distribution. This is the convention the literature seems to follow.

3.1 Line charge density, λ

Defined as the charge per unit length:

$$\lambda = \lim_{\Delta l \to 0} \frac{\Delta q}{\Delta l} \tag{3}$$

Let $\mathbf{r}'$ be the position of an arbitrary point on the line charge distribution. Then $\lambda = \lambda(\mathbf{r}')$. Let $\mathbf{r}$ be the position of the test charge q. The force exerted on q by the line charge distribution is:

$$\mathbf{F}(\mathbf{r}) = \frac{q}{4\pi\epsilon_0} \int_l \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} \lambda(\mathbf{r}') \mathrm{d}l' \tag{4}$$

3.2 Surface charge density, σ

Defined as the charge per unit area:

$$\sigma = \lim_{\Delta S \to 0} \frac{\Delta q}{\Delta S} \tag{5}$$

where $\sigma = \sigma(\mathbf{r}')$ and the force exerted on q by the surface charge distribution is:

$$\mathbf{F}(\mathbf{r}) = \frac{q}{4\pi\epsilon_0} \int_S \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} \sigma(\mathbf{r}') \mathrm{d}S' \tag{6}$$

where $\mathrm{d}S' = \mathrm{d}^2 r'$.

3.3 Volume charge density, ρ

Defined as the charge per unit volume:

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta q}{\Delta V} \tag{7}$$

where $\rho = \rho(\mathbf{r}')$ and the force exerted on q by the volume charge distribution is:

$$\mathbf{F}(\mathbf{r}) = \frac{q}{4\pi\epsilon_0}\int_V \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3}\rho(\mathbf{r}')\mathrm{d}V' \tag{8}$$

where $\mathrm{d}V' = \mathrm{d}^3 r'$. This is the most general form of Coulomb's law.

4 The Electric Field

5 References

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