

Reversible Data Hiding Using Controlled Contrast Enhancement and Integer Wavelet Transform

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Abstract—The conventional reversible data hiding (RDH) algorithms pursue high Peak-Signal-to-Noise-Ratio (PSNR) at the certain amount of embedding bits. Recently, Wu *et al.* deemed that the improvement of image visual quality is more important than keeping high PSNR. Based on this viewpoint, they presented a novel RDH scheme, utilizing contrast enhancement to replace the PSNR. However, when a large number of bits are embedded, image contrast is over-enhanced, which introduces obvious distortion for human visual perception. Motivated by this issue, a new RDH scheme is proposed using the controlled contrast enhancement (CCE) and Haar integer wavelet transform (IWT). The proposed scheme has large embedding capacity while maintaining satisfactory visual perception. Experimental results have demonstrated the effectiveness of the proposed scheme.

Index Terms—Controlled contrast enhancement, integer wavelet transform, reversible data hiding.

I. INTRODUCTION

DATA hiding is applied extensively to the fields of ownership protection, fingerprinting, authentication and secret communication [1], [2]. The most classical data hiding leads to permanent distortions. Recently, a new data hiding technique, i.e., reversible data hiding (RDH), is proposed, which can not only extract the embedded bits, but also restore the original cover image without any error [3]–[12].

Up to now, RDH algorithms mainly adopt three techniques, i.e., difference expansion (DE) [3]–[5], histogram shifting (HS) [6], [7], and prediction-error (PE) based [8]–[11]. The performance of these algorithms is generally evaluated by embedding capacity and quality of marked image in terms of Peak-Signal-to-Noise-Ratio (PSNR).

Recently, Wu *et al.* presented a new RDH algorithm with contrast enhancement [12]. They deemed that the improvement of visual quality is more important than keeping the image's

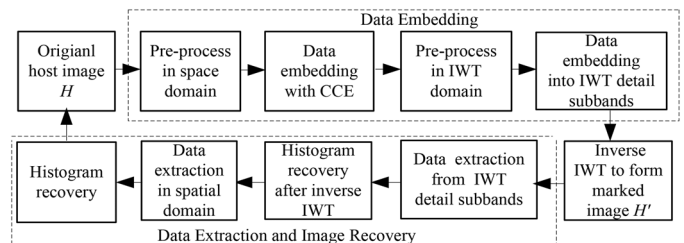


Fig. 1. Diagram of proposed scheme.

PSNR high. By using some pairs of peaks, the histogram of pixel values is modified to achieve histogram equalization, thus leading to the image contrast enhancement while RDH is realized. The method provides a new direction for the research of RDH.

It is observed that as more data needs to be embedded Wu *et al.*'s method needs to use more peak-pairs (for example 50 pairs), consequently the image contrast may be over-enhanced, which can introduce annoying perception distortion. This observation motivates us to propose a new reversible data hiding scheme with the so-called controlled contrast enhancement (CCE) and integer wavelet transform (IWT). Firstly, a proper image contrast enhancement is achieved during the data embedding into the image in spatial domain by monitoring the contrast enhancement indicator. Secondly, it is shown that the modification of the detail subband coefficients in IWT domain only produces very small contrast change. This leads to embedding more data into the detail subbands without lowering the visual quality significantly. The experimental results demonstrate that the proposed scheme performs better than Wu *et al.*'s scheme as well as some of state-of-the-art schemes with keeping image's PSNR high as criterion for RDH.

The rest of the paper is organized as follows. In Section II, the proposed algorithm is presented in detail. The experimental results and performance analysis are presented in Section III. Finally, Section IV concludes the presentation.

II. PROPOSED ALGORITHM

The diagram of the proposed scheme is illustrated in Fig. 1. There are two main parts in the message embedding process, including data embedding with CCE in spatial domain and more data embedding in Haar IWT domain. The data extraction and image recovery are the reverse procedure of data embedding process.

A. Data Embedding in Spatial Domain

As data embedding in spatial domain, a pre-processing needs to be used to prevent the overflow/underflow. For a given 8-bit gray-level image H , supposing that the amount of adopted peak-pairs is L , all pixel values from 0 to $L - 1$, excluding the first 16 pixels in the first column, are added by L . Also, those from

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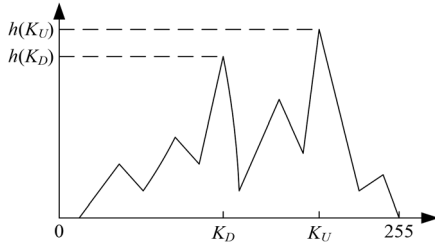


Fig. 2. Histogram with a pair of peaks.

256 - L to 255 are subtracted by L . The position information of modified pixels in the operation is saved in a location map.

Then, the image histogram is calculated, and $h(k)$ is adopted to indicate the number of pixels assuming gray level value k . We can find a pair of peaks denoted by K_D and K_U , K_D is the smaller pixel value, and K_U is the larger one as shown in Fig. 2. It is noted that if there are three or more peaks with the same height, then the two-side peaks are selected as K_D , K_U , leading to the least amount of changes. The data embedding can be conducted by

$$k' = \begin{cases} k - 1, & \text{for } k < K_D \\ K_D - w_i, & \text{for } k = K_D \\ k, & \text{for } K_D < k < K_U \\ K_U + w_i, & \text{for } k = K_U \\ k + 1, & \text{for } k > K_U \end{cases}, \quad (1)$$

where w_i is the i -th secret bit to be embedded, and k' is the modified pixel value. We can embed $(h(K_D) + h(K_U))$ bits into the image by Eq. (1). Meanwhile, the histogram equalization is achieved to some extent, enhancing the image contrast [12].

For further increasing the embedding capacity and image contrast, the same process can be repeated by utilizing more peak-pairs in the modified histogram. The location map compressed by arithmetic encoding (AE) is embedded into the image together with the secret message according to Eq. (1). The side information, including the value of L , the length of compressed location map, the least significant bits (LSBs) of the 16 excluded pixels, and the previous peak values, is embedded using the last pair of peaks. Finally, the LSBs of the 16 excluded pixels are substituted with the last pair of peak values to get the marked image.

B. Controlled Contrast Enhancement

The image contrast enhancement is evaluated by the relative contrast error (RCE) [13], and the equation to compute RCE is given by

$$\text{RCE} = \frac{\text{Std}_e - \text{Std}_o}{R - 1} + 0.5 \quad (2)$$

where Std_o , Std_e indicate the standard deviations of original and enhanced images, respectively, and $R = 256$ for a given 8-bit gray level image. The RCE indicates the degree of contrast enhancement between original and enhanced images, and is in the range of $[0, 1]$.

For avoiding the visual distortion caused by contrast enhanced excessively, a threshold denoted by T_{rce} is used to control the degree of contrast enhancement. In our experiments, T_{rce} is set to 0.55, which can guarantee the enhanced image has satisfactory visual quality. An initial value of L is defined firstly, then according to the computed RCE between original and embedded images, we adjust the value of L . If $\text{RCE} < T_{rce}$, L is added by 1, else it is subtracted by 1. The same operation can be repeated until a suitable RCE close to T_{rce} is achieved.

C. Pre-processing in IWT Domain

The Haar IWT of an image can be realized by two dimensional S transform as shown in Eqs. (3)–(4) at the bottom of the page, where $r_{i,j}$ is the original pixel value, $i \in [0, M - 1]$, $j \in [0, N - 1]$. $A_{p,q}$ is the approximation subband coefficient of first level Haar IWT, $H_{p,q}$, $V_{p,q}$, $D_{p,q}$ are the corresponding coefficients of horizontal, vertical and diagonal subbands, respectively, with $p \in [0, M/2 - 1]$, $q \in [0, N/2 - 1]$. $M \times N$ is the image size.

The approximation subband (LL) is highly sensitive to coefficient changes, thus the coefficient modifications on LL for embedding bits can lead to obvious influence on image contrast which can produce severe visual distortion. The horizontal subband (LH), diagonal subband (HH) and vertical subband (HL) are detail subbands where coefficients accord with Laplacian-like distribution. The coefficient changes on LH , HH and HL only have very small effect on image contrast. For example, the two RCEs are 0.5476 with only embedding data in spatial domain and 0.5463 with simultaneously embedding data in both spatial domain and detail subbands for *Boat* image, respectively. The difference of two RCEs is only 0.0014, thus it is a good choice to embed bits into the detail subbands.

The maximum absolute change of single detail subband coefficient is denoted by Δ . From (4), it is easily known that the maximum absolute change of single image pixel value induced by the changes of detail subband coefficients is not more than $\lceil 1.25 \times (\Delta + 1) \rceil$, where $\lceil x \rceil$ means taking the top integral of x . Because the reconstruction of wavelet coefficients after data embedding may cause overflow or underflow of image pixel values, the image pixel values should firstly be pre-processed into the region of $[\lceil 1.25 \times (\Delta + 1) \rceil, 255 - \lceil 1.25 \times (\Delta + 1) \rceil]$, namely, all pixel values, from 0 to $\lceil 1.25 \times (\Delta + 1) \rceil - 1$, are added by $\lceil 1.25 \times (\Delta + 1) \rceil$, also, those from $256 - \lceil 1.25 \times (\Delta + 1) \rceil$ to

$$\begin{cases} A_{p,q} = \lfloor ((r_{2p,2q} + r_{2p,2q+1})/2) + \lfloor (r_{2p+1,2q} + r_{2p+1,2q+1})/2 \rfloor / 2 \rfloor \\ V_{p,q} = \lfloor (r_{2p,2q+1} - r_{2p,2q} + r_{2p+1,2q+1} - r_{2p+1,2q})/2 \rfloor \\ H_{p,q} = \lfloor (r_{2p+1,2q} + r_{2p+1,2q+1})/2 \rfloor - \lfloor (r_{2p,2q} + r_{2p,2q+1})/2 \rfloor \\ D_{p,q} = (r_{2p+1,2q+1} - r_{2p+1,2q}) - (r_{2p,2q+1} - r_{2p,2q}) \end{cases} \quad (3)$$

$$\begin{cases} r_{2p,2q} = A_{p,q} - \lfloor H_{p,q}/2 \rfloor - \lfloor (V_{p,q} - \lfloor D_{p,q}/2 \rfloor)/2 \rfloor \\ r_{2p+1,2q} = A_{p,q} + \lfloor (H_{p,q} + 1)/2 \rfloor - \lfloor (V_{p,q} + \lfloor (D_{p,q} + 1)/2 \rfloor)/2 \rfloor \\ r_{2p,2q+1} = A_{p,q} - \lfloor H_{p,q}/2 \rfloor + \lfloor (V_{p,q} - \lfloor D_{p,q}/2 \rfloor + 1)/2 \rfloor \\ r_{2p+1,2q+1} = A_{p,q} + \lfloor (H_{p,q} + 1)/2 \rfloor + \lfloor (V_{p,q} + \lfloor (D_{p,q} + 1)/2 \rfloor + 1)/2 \rfloor \end{cases} \quad (4)$$

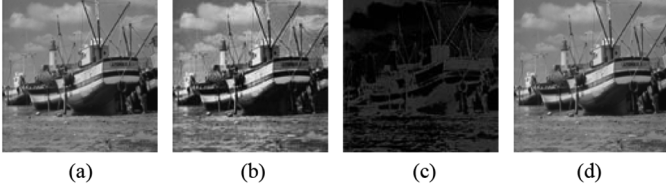


Fig. 3. Reversibility test of proposed scheme. (a) Original *Boat* image. (b) Marked image with 221,586 bits embedded inside. (c) Difference of (a) and (b). (d) Restored image (SSIM = 1).

255 are subtracted by $\lceil 1.25 \times (\Delta + 1) \rceil$. Then the location map of these pixels is generated and further compressed by using the AE.

D. Data Embedding in IWT Domain

Next, more message bits are embedded into detail subbands, using the companding technique [14]. For a bit b , we embed it through the extending of coefficient $x \in \{LH, HL, HH\}$, i.e., the extended version of coefficient x is denoted by x_b , and $x_b = 2x + b$. To avoid big change of pixel value brought by extension of x with a high value, the compression operation on x need to be implemented. The compression function f_c is given as follows.

$$x_c = f_c(x) = \begin{cases} x, & |x| < T \\ \rho(x) \cdot (\lfloor (|x| - T)/2 \rfloor + T), & |x| \geq T \end{cases} \quad (5)$$

where x, x_c are the original and compressed coefficients of IWT detail subbands, respectively. The threshold T is positive integer. And $\rho(x) = 1$ if $x \geq T$, otherwise, $\rho(x) = -1$. The embedded version of x_c is denoted by x_{cb} , and $x_{cb} = 2x_c + b$. The expanding function f_e corresponding to f_c is shown by

$$x' = f_e(x_c) = \begin{cases} x_c, & |x| < T \\ \rho(x_c) \cdot (2\lfloor |x_c| - T \rfloor), & |x| \geq T \end{cases} \quad (6)$$

For the coefficient x satisfying $|x| \geq T$, the companding error between x and x' is $e = x - f_e(f_c(x))$. Obviously, for some coefficients $x, e \neq 0$. We can get $e \in \{0, 1\}$ for $x \geq T$, and $e \in \{-1, 0\}$ for $x \leq -T$. The proof of $e \in \{0, 1\}$ for $x \geq T$ is shown in Appendix I, and that of $e \in \{-1, 0\}$ for $x \leq -T$ is similar and omitted. For the restoration of original coefficient x , the length of location map, the companding error e , the threshold T and the compressed location map, as side information, should be embedded into the cover image along with the message bits. It is noted that the bit '1' is embedded when $e = -1$.

When $x \geq T$, the upper limit of the difference x_{diff} between x_{cb} and x is calculated by

$$\begin{aligned} x_{diff} &\leq 2x_c + 1 - x \\ &= 2(\lfloor (|x| - T)/2 \rfloor + T) + 1 - x \\ &\leq 2((|x| - T)/2 + T) + 1 - x \\ &= T + 1 \end{aligned} \quad (7)$$

Similarly, we can also get $x_{diff} \geq T - 1$ for $x \geq T$, $-T \leq x_{diff} \leq -T + 2$ for $x \leq -T$, and $-T < x_{diff} < T + 1$ for $|x| < T$. Thus, the maximum absolute change of single detail subband coefficient after embedding bit is $T + 1$, corresponding to Δ of pre-processing phase.

E. Data Extraction and Image Recovery

On the receiving side, data extraction includes two phases. In the first phase, the embedded bits are extracted in IWT domain.

In the second phase, the peak-pairs are utilized to obtain the rest of embedded bits in the spatial domain.

Firstly, Haar IWT is applied to transform the marked image into wavelet domain, then the LSB of the detail subband coefficient x_{cb} is extracted, i.e., $b = \text{LSB}(x_{cb})$ and $x_c = (x_{cb} - b)/2$. From the extracted LSBs, we can obtain the secret message and the side information including the length of location map, the companding error e , the threshold T and the compressed location map. The expansion operation is implemented by applying T, x_c , and Eq. (6) to obtain x' , and the original coefficient x is generated by

$$x = \begin{cases} x' + 1 & \text{if } x' > 0 \text{ and } e = 1 \\ x' & \text{if } e = 0 \\ x' - 1 & \text{if } x' < 0 \text{ and } e = 1 \end{cases} \quad (8)$$

Next, the image is transformed back to the spatial domain by applying the inverse IWT. By using location map, the modified image histogram in pre-processing phase can be recovered. The location map from the extracted bits is firstly decompressed. Then through the decompressed map, we can label those pixels changed in pre-processing phase. If the identified pixel value is less than 128, it is subtracted by $\lceil 1.25 \times (T + 2) \rceil$, otherwise, the pixel value is added by $\lceil 1.25 \times (T + 2) \rceil$.

In the spatial domain, the peak-pairs are utilized to extract data. Firstly, we extract the last peak-pair from the LSBs of 16 excluded pixels in first column of the image. Then, by using the last peak-pair, the side information involving the value of L , the length of compressed location map, the LSBs of 16 excluded pixels, and the previous peak-pairs can be obtained by Eq. (9).

$$w'_i = \begin{cases} 1, & \text{if } k' = K_D - 1 \\ 0, & \text{if } k' = K_D \\ 0, & \text{if } k' = K_U \\ 1, & \text{if } k' = K_U + 1 \end{cases} \quad (9)$$

where w'_i is the i -th bit extracted from the marked image H' . The restoration of image histogram is executed on all pixels except the 16 excluded ones using Eq. (10).

$$k = \begin{cases} k' + 1, & \text{for } k' < K_D - 1 \\ K_D, & \text{for } k' = K_D - 1 \text{ or } k' = K_D \\ K_U, & \text{for } k' = K_U \text{ or } k' = K_U + 1 \\ k' - 1, & \text{for } k' > K_U + 1 \end{cases} \quad (10)$$

Then, we repeat the data extraction and image restoration by the extracted peak pairs until all the embedded data are obtained. The modified image pixel values in the pre-processing stage are recovered using the extracted location map, which is the same as the operation in the IWT domain. Finally, the LSBs of 16 excluded pixels are written back to restore the original image.

III. EXPERIMENTAL RESULTS

In this Section, the experimental results for six standard 512×512 test images, *Lena*, *Mandrill*, *Jet*, *Barbara*, *Tiffany* and *Boat* are presented with the experimental environment of MATLAB R2012b under windows 8. The experimental data for the amount of embedded bits, listed in Figs. 3–5 and Table I, are the number of pure secret bits, not including the side information.

Fig. 3 shows the reversibility test of proposed scheme for *Boat*. The original and marked images are listed in Fig. 3(a) and (b), respectively. Fig. 3(c) gives the difference of the original and marked images, which is not easily perceptible for human eyes, thus an enhancement measure is adopted through Matlab



Fig. 4. Comparisons of visual perception and embedding capacity of marked image among Wu et al.'s [12], Ou et al.'s [9] and proposed schemes for *Lena*. (a) Original image. (b) [12], 18 dB, 168,001 bits. (c) [9], 44 dB, 113,000 bits. (d) Proposed, 25 dB, 214,918 bits.

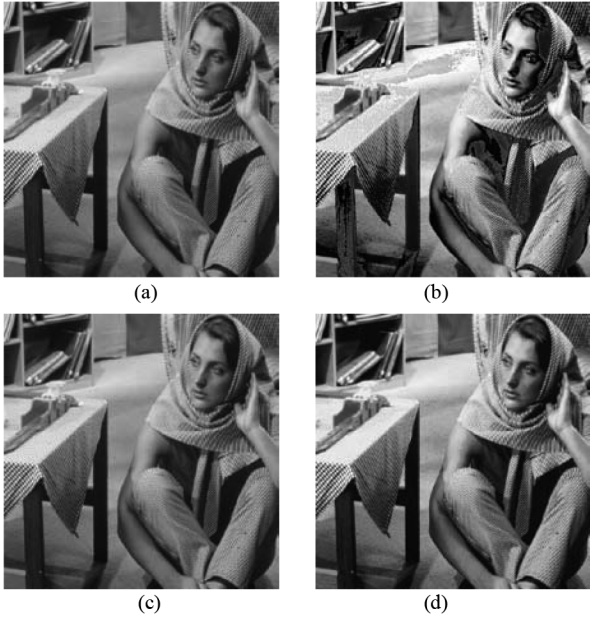


Fig. 5. Comparisons of visual perception and embedding capacity of marked image among Wu et al.'s [12], Ou et al.'s [9] and proposed schemes for *Barbara*. (a) Original image. (b) [12], 17 dB, 147,407 bits. (c) [9], 44 dB, 89,000 bits. (d) Proposed, 25 dB, 161,933 bits.

TABLE I
COMPARISONS OF AMOUNT OF EMBEDDED BITS AMONG
[9], [10], [14] AND PROPOSED SCHEME

	<i>Lena</i>	<i>Baboon</i>	<i>Jet</i>	<i>Barbara</i>	<i>Tiffany</i>	<i>Boat</i>
Ou et al.[9]	113,000	41,000	150,000	89,000	88,000	70,000
Dragoi et al. [10]	117,960	39,322	165,150	107,480	128,450	83,886
Xuan et al.[14]	73,400	10,485	104,857	39,321	47,185	34,078
Proposed	214,918	173,807	353,310	161,933	267,629	221,586
Mean gain	113,464	143,538	213,307	83,332	179,750	158,931

function *adapthisteq* to make the difference visible. The restored image is shown in Fig. 3(d) after the embedded bits are extracted. Then we computed the structure similarity (SSIM) between original and restored images, and got $SSIM = 1$, demonstrating that proposed scheme is fully reversible.

The experimental comparisons among Wu *et al.*'s [12], Ou *et al.*'s [9] and the proposed schemes are shown in Figs. 4, 5. Due to the space limitation, only the results of two test images are shown, other test images also have similar comparison results. It is observed from Figs. 4 and 5 that when a large number of bits are embedded, the proposed scheme with $T_{rce} = 0.55$ has the best visual quality and largest embedding capacity although the PSNR value is lower than that achieved by [9], but higher than that achieved by [12]. Wu *et al.*'s scheme has obvious distortion for human visual perception as embedding a large number of bits, here 168,001 bits, which is due to that the image contrast is over-enhanced while the 50 peak-pairs are used. Ou *et al.*'s scheme has high PSNR value, but the visual perception is not as good as that achieved by the proposed scheme, meanwhile, the embedding capacity is smaller than that achieved by the proposed scheme. On the contrary, owing to the satisfactory visual quality ensured by proper contrast enhancement, the proposed scheme can embed much more data. The amount of data that can be embedded by the schemes reported in [9], [10], [14] at the 44 dB PSNR and that by the proposed scheme with $T_{rce} = 0.55$ are listed in Table I. It is clear that the proposed scheme can embed more data while keeping higher visual quality.

IV. CONCLUSION

Based on Wu *et al.*'s RDH algorithm, a new RDH scheme utilizing controlled contrast enhancement and embedding more data into the detail subband coefficients of IWT is proposed. It avoid severe distortion suffered by Wu *et al.*'s scheme as data embedding rate is high. Furthermore, compared with other existing RDH algorithms, the proposed scheme can embed significantly larger amount of data and achieve better visual quality from human vision point of view. In future work, the research on adaptively determining REC threshold will be carried on to further improve the algorithm performance.

APPENDIX I

For $x \geq T$, the range of companding error e is given by

$$\begin{aligned}
 e &= x - x' \\
 &= x - \rho(x) \cdot (2 \lfloor \rho(x) \cdot (\lfloor (|x| - T)/2 \rfloor + T) \rfloor - T) \\
 &= x - 2 \lfloor (\lfloor (|x| - T)/2 \rfloor + T) \rfloor + T \\
 &\geq x - T - 2(|x| - T)/2 \\
 &= 0
 \end{aligned} \tag{A1}$$

$$\begin{aligned}
 e &= x - x' \\
 &= x - \rho(x) \cdot (2 \lfloor \rho(x) \cdot (\lfloor (|x| - T)/2 \rfloor + T) \rfloor - T) \\
 &= x - 2 \lfloor (\lfloor (|x| - T)/2 \rfloor + T) \rfloor + T \\
 &\leq x - T - 2((x - T)/2 - 1/2) \\
 &= 1
 \end{aligned} \tag{A2}$$

Due to that x, x' are all integers, hence, e is also integer. Combining with Eqs. ((A1), (A2)), we get $e \in \{0, 1\}$ for $x \geq T$.

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