Advanced Encryption Standard (AES)

The Advanced Encryption Standard (AES) is a symmetric-key block cipher published by the National Institute of Standards and Technology (NIST)inDecember2001.

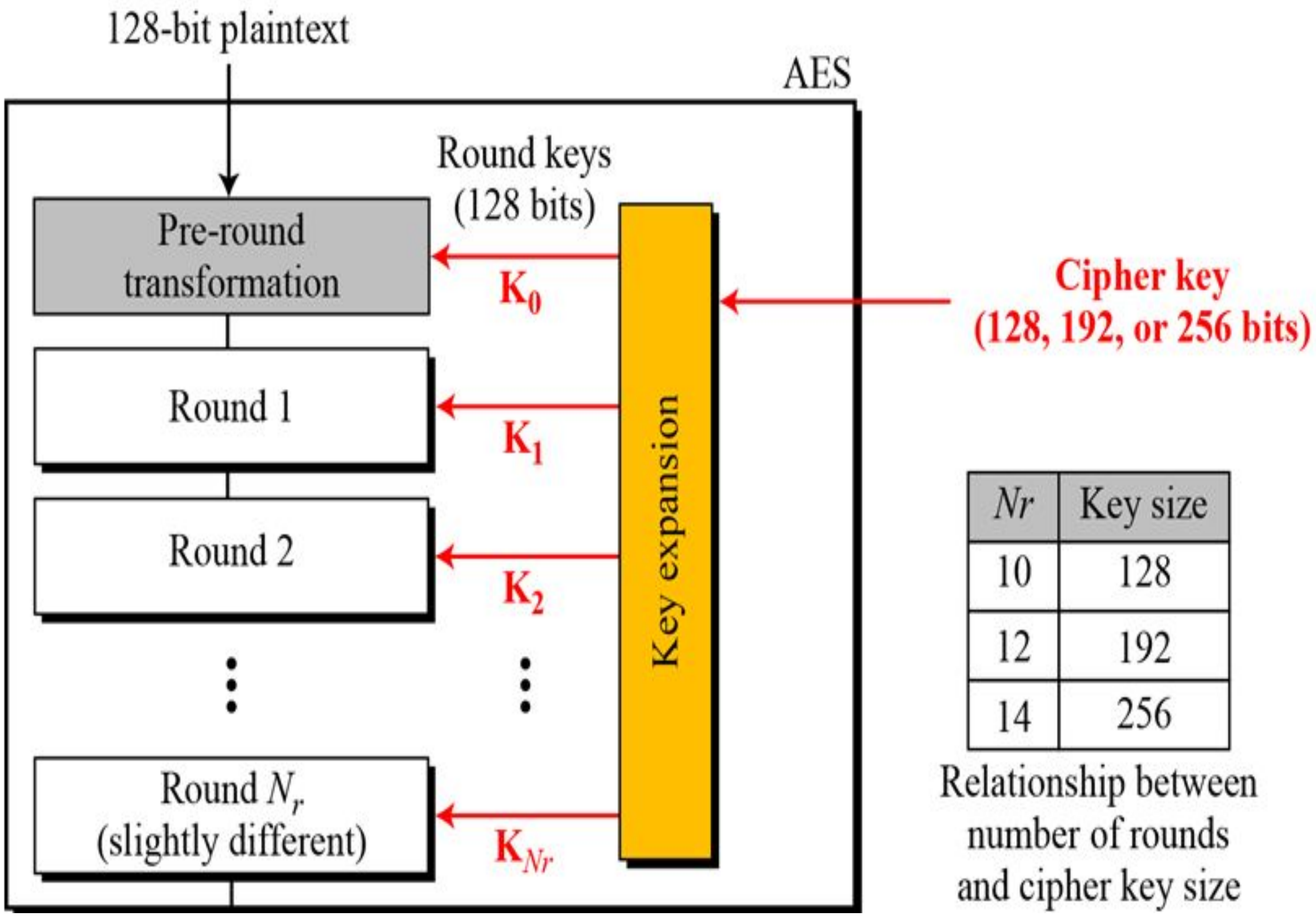
Objectives

- To define the basic structure of AES
- To define the transformations used by AES
- To define the key expansion process

Contd..

AES has defined three versions, with 10, 12, and14 rounds. Each version uses a different cipher key size (128, 192, or 256), but the **round keys are always 128 bits.**

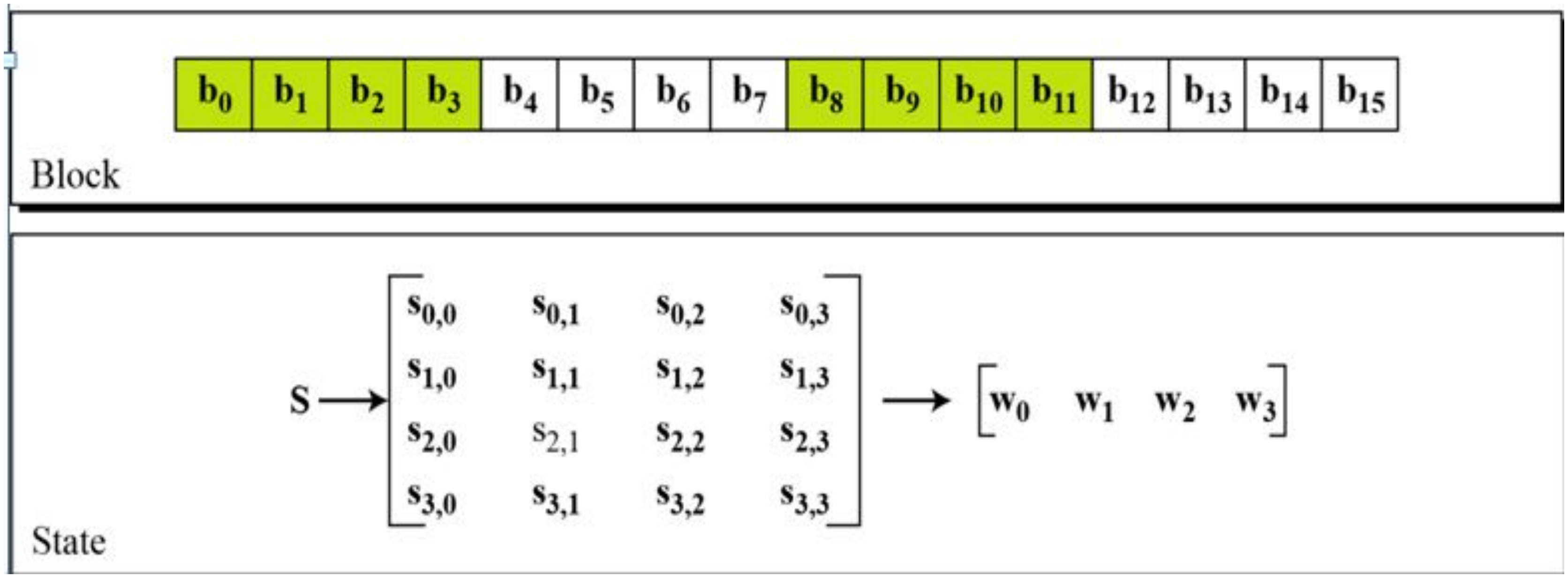
General design of AES encryption cipher ..



Nr	Key size
10	128
12	192
14	256

Relationship between number of rounds and cipher key size

Contd..



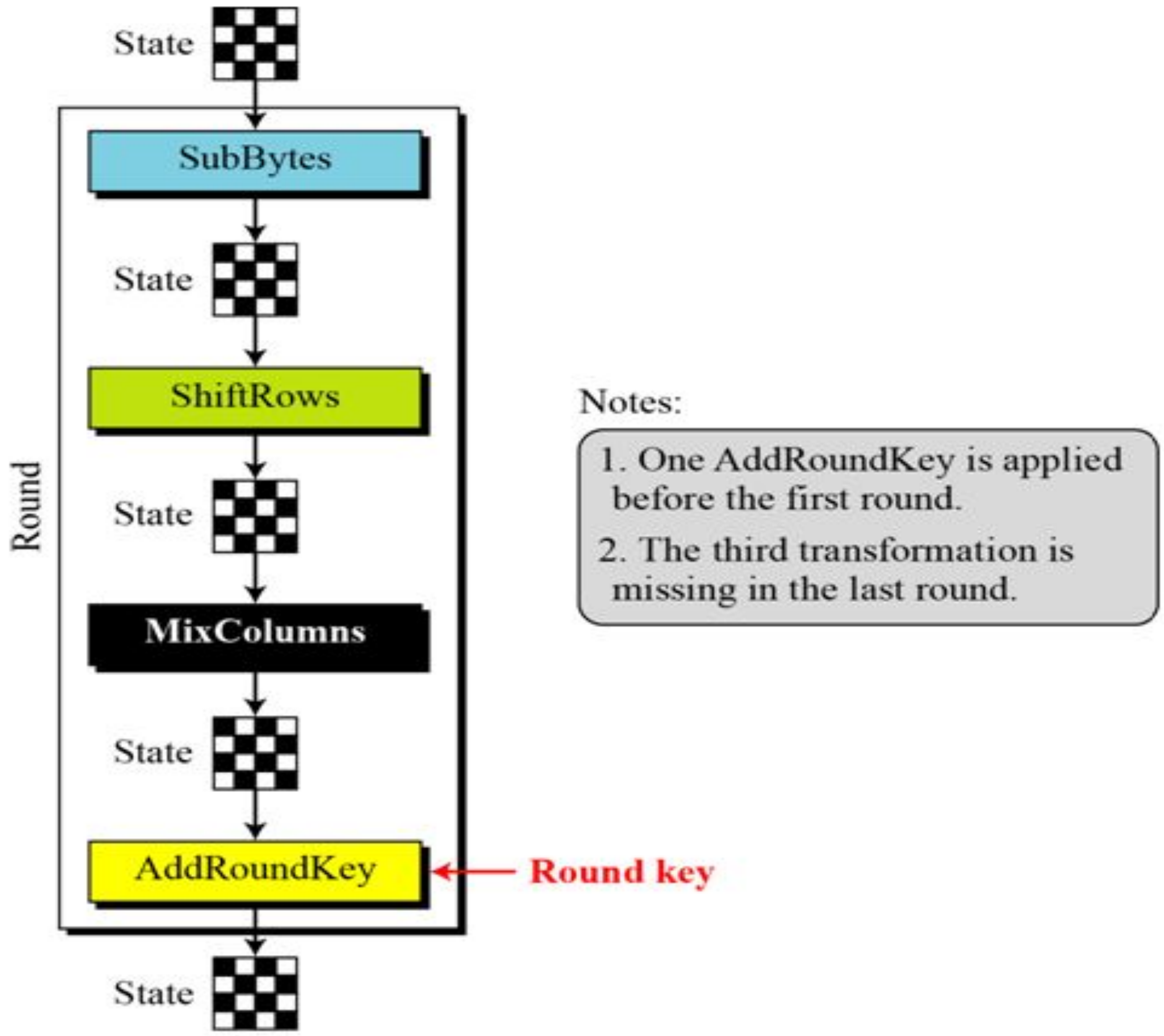
$$\text{State} \quad \begin{bmatrix} s_{0,0} = b_0 & s_{0,1} = b_4 & s_{0,2} = b_8 & s_{0,3} = b_{12} \\ s_{1,0} = b_1 & s_{1,1} = b_5 & s_{1,2} = b_9 & s_{1,3} = b_{13} \\ s_{2,0} = b_2 & s_{2,1} = b_6 & s_{2,2} = b_{10} & s_{2,3} = b_{14} \\ s_{3,0} = b_3 & s_{3,1} = b_7 & s_{3,2} = b_{11} & s_{3,3} = b_{15} \end{bmatrix}$$

Contd..

Text	A	E	S	U	S	E	S	A	M	A	T	R	I	X	Z	Z
Hexadecimal	00	04	12	14	12	04	12	00	0C	00	13	11	08	23	19	19

$$\begin{bmatrix} 00 & 12 & 0C & 08 \\ 04 & 04 & 00 & 23 \\ 12 & 12 & 13 & 19 \\ 14 & 00 & 11 & 19 \end{bmatrix} \text{State}$$

Structure of each round at the encryption site

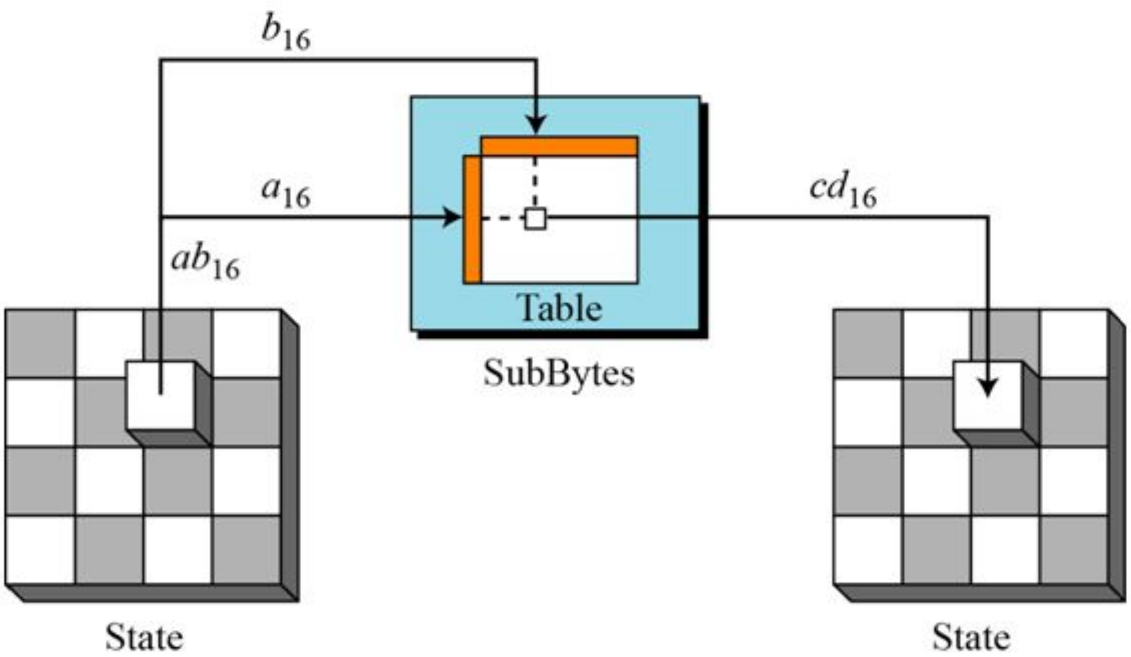


Contd..

To provide security, AES uses four types of transformations: **substitution, permutation, mixing, and key-adding.**

Contd..

To substitute a byte, we interpret the byte as two hexadecimal digits.

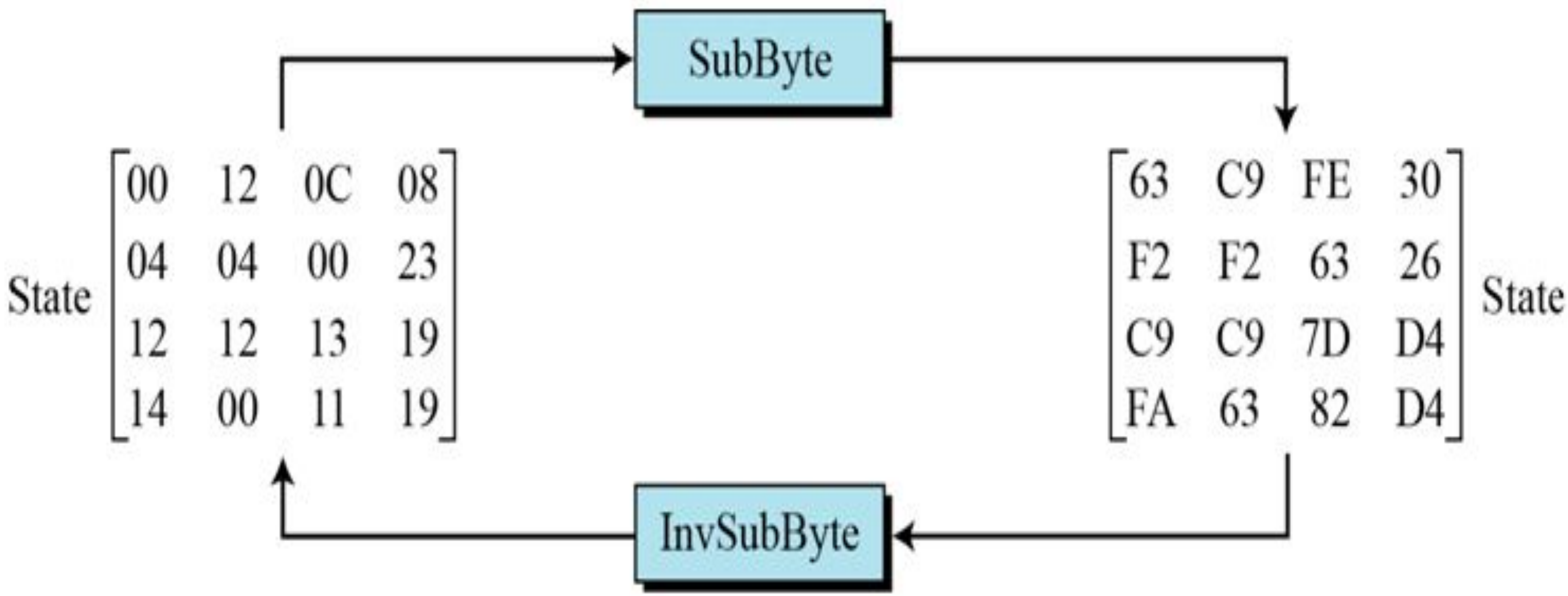


Contd..

SubBytes transformation table

	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8

InvSubBytes transformation creates the original one



Transformation Using the GF(2^8) Field

- *AES also defines the transformation algebraically using the GF(2^8) field with the irreducible polynomials*

$$(x^8 + x^4 + x^3 + x + 1)$$

Polynomials

A polynomial of degree $n - 1$ is an expression of the form

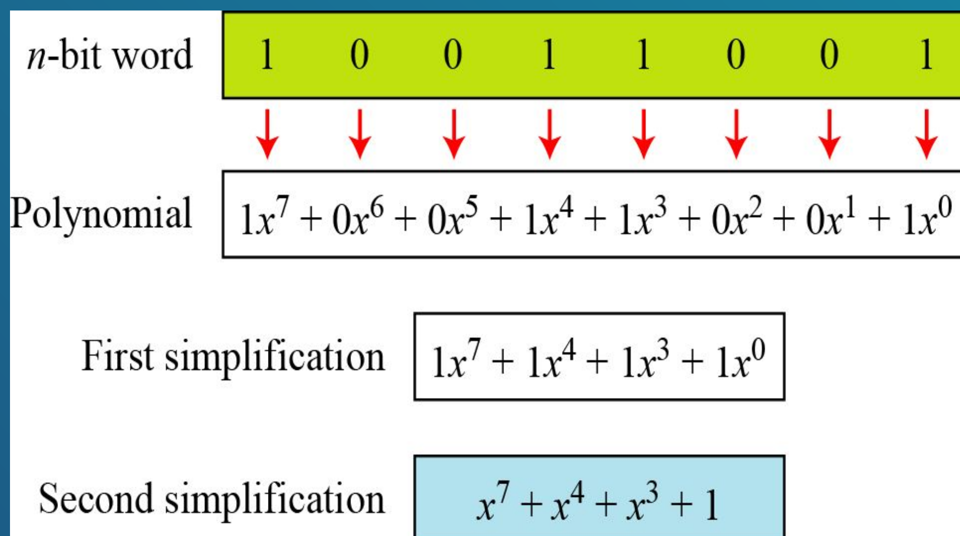
$$f(x) = a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_1x^1 + a_0x^0$$

where x^i is called the i^{th} term and a_i is called coefficient of the i^{th} term.

The **degree** of a **polynomial** is the highest **degree** of its terms. The degree is the value of the greatest exponent of its terms. The degree is the value of the greatest exponent of any expression (except the constant) in the polynomial.

Contd.

Below figure show how we can represent the 8-bit word (10011001) using a polynomials.



Contd.

To find the 8-bit word related to the polynomial $x^5 + x^2 + x$, we first supply the omitted terms. Since $n = 8$, it means the polynomial is of degree 7. The expanded polynomial is

$$0x^7 + 0x^6 + 1x^5 + 0x^4 + 0x^3 + 1x^2 + 1x^1 + 0x^0$$

This is related to the 8-bit word 00100110

Arithmetic in GF(2^n)

We may add, subtract polynomials in as we do for ordinary arithmetic . Even though the coefficients are elements of GF(2) instead of actual integers, it is easy to do the calculations so long as we remember to always reduce coefficients mod 2.

Contd.

Note: Addition and subtraction operations on polynomials are the same operation in (mod 2) arithmetic.

Let us do $(x^5 + x^2 + x) \oplus (x^3 + x^2 + 1)$ in $GF(2^8)$. We use the symbol $\oplus$ to show that we mean polynomial addition. The following shows the procedure:

$$
\begin{array}{ll}
0x^7 + 0x^6 + 1x^5 + 0x^4 + 0x^3 + 1x^2 + 1x^1 + 0x^0 & \oplus \\
0x^7 + 0x^6 + 0x^5 + 0x^4 + 1x^3 + 1x^2 + 0x^1 + 1x^0 & \\
\hline
0x^7 + 0x^6 + 1x^5 + 0x^4 + 1x^3 + 0x^2 + 1x^1 + 1x^0 & \rightarrow \quad x^5 + x^3 + x + 1
\end{array}
$$

Irreducible polynomials (modulus polynomials).

A polynomialA polynomial is said to be irreducible if it cannot be factored into polynomials of lower positive degrees over the same field.

Degree	*Irreducible Polynomials*
1	$(x+1), (x)$
2	(x^2+x+1)
3	$(x^3+x^2+1), (x^3+x+1)$
4	$(x^4+x^3+x^2+x+1), (x^4+x^3+1), (x^4+x+1)$
5	$(x^5+x^2+1), (x^5+x^3+x^2+x+1), (x^5+x^4+x^3+x+1),$ $(x^5+x^4+x^3+x^2+1), (x^5+x^4+x^2+x+1)$

Contd.

Find the result of $(x^5 + x^2 + x) \otimes (x^7 + x^4 + x^3 + x^2 + x)$ in $GF(2^8)$ with irreducible polynomial $(x^8 + x^4 + x^3 + x + 1)$. Note that we use the symbol $\otimes$ to show the multiplication of two polynomials.

$$P_1 \otimes P_2 = x^5(x^7 + x^4 + x^3 + x^2 + x) + x^2(x^7 + x^4 + x^3 + x^2 + x) + x(x^7 + x^4 + x^3 + x^2 + x)$$

$$P_1 \otimes P_2 = x^{12} + x^9 + x^8 + x^7 + x^6 + x^9 + x^6 + x^5 + x^4 + x^3 + x^8 + x^5 + x^4 + x^3 + x^2$$

$$P_1 \otimes P_2 = (x^{12} + x^7 + x^2) \bmod (x^8 + x^4 + x^3 + x + 1) = x^5 + x^3 + x^2 + x + 1$$

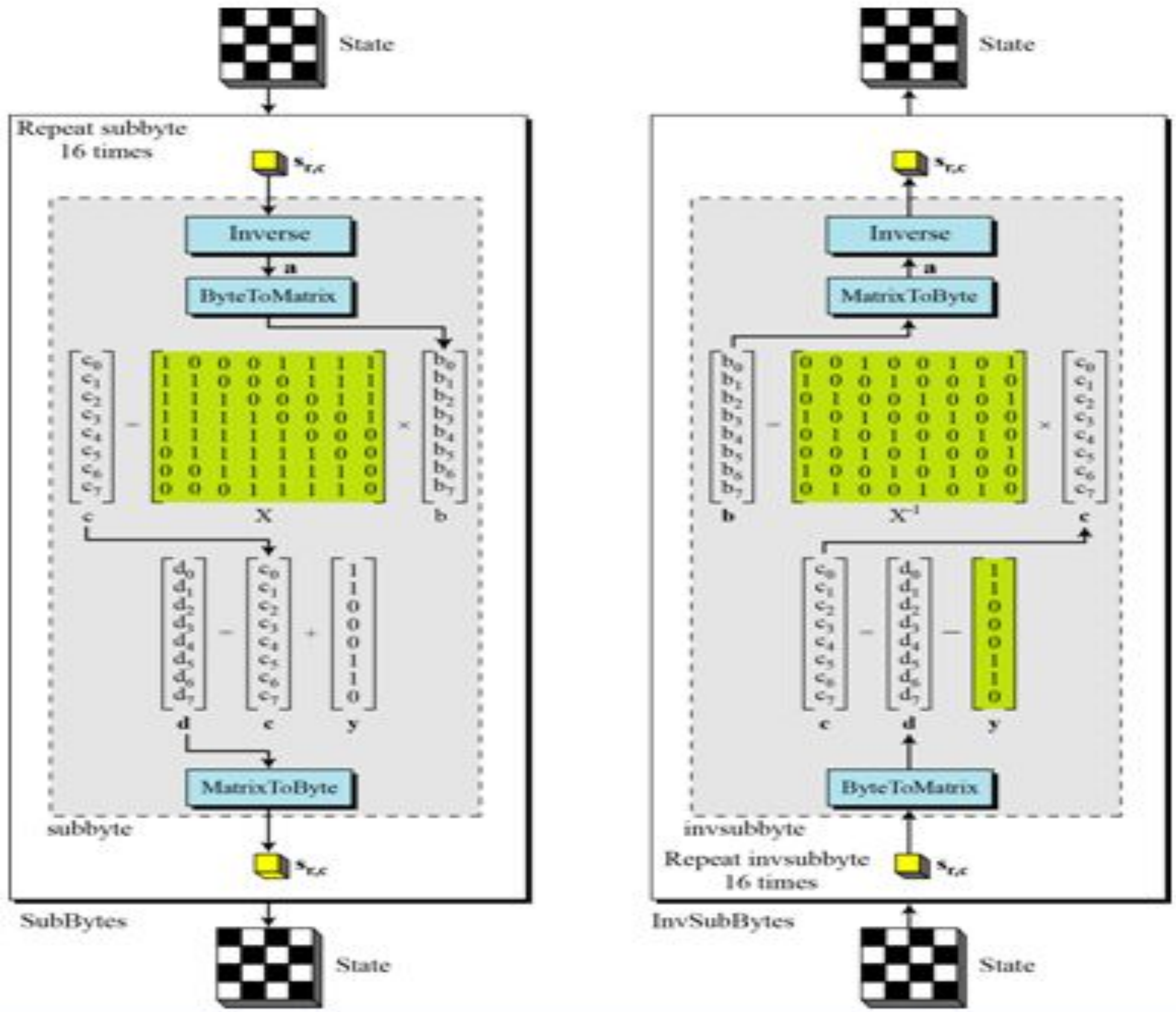
To find the final result, divide the polynomial of degree 12 by an irreducible polynomial of degree 8 (the modulus) and keep only the remainder. Next shows the process of division.

Contd.

Polynomial division with coefficients in GF(2)

$$
\begin{array}{r|l}
 & x^4 + 1 \\
\hline
x^8 + x^4 + x^3 + x + 1 & x^{12} + x^7 + x^2 \\
 & x^{12} + x^8 + x^7 + x^5 + x^4 \\
\cline{2-2}
 & x^8 + x^5 + x^4 + x^2 \\
 & x^8 + x^4 + x^3 + x + 1 \\
\cline{2-2}
\text{Remainder} & \boxed{x^5 + x^3 + x^2 + x + 1}
\end{array}
$$

SubBytes and InvSubBytes processes



Example

1. *subbyte:*
   a. The multiplicative inverse of 0C in GF(2^8) field is B0, which means **b** is (10110000).
   b. Multiplying matrix **X** by this matrix results in **c** = (10011101)
   c. The result of XOR operation is **d** = (11111110), which is FE in hexadecimal.

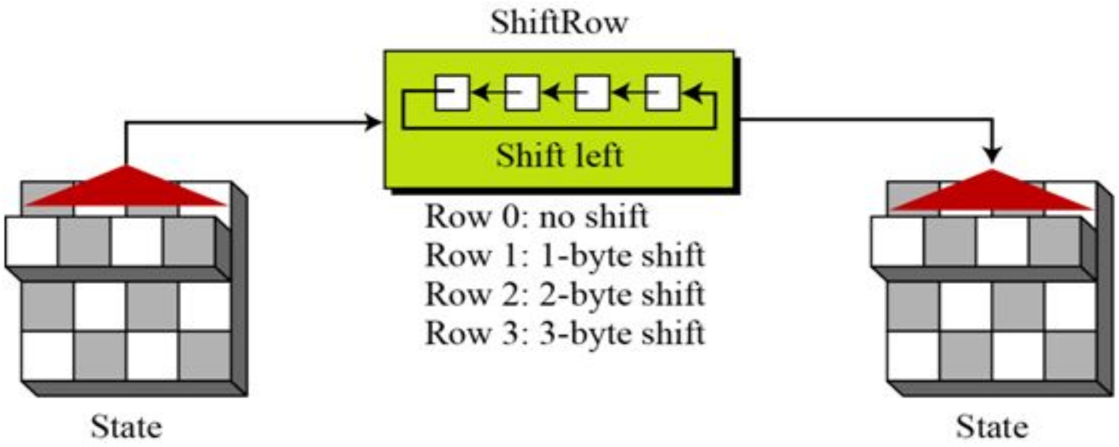
2. *invsubbyte:*
   a. The result of XOR operation is **c** = (10011101)
   b. The result of multiplying by matrix $\mathbf{X}^{-1}$ is (11010000) or B0
   c. The multiplicative inverse of B0 is 0C.

Permutation

Another transformation found in a round is shifting, which permutes the bytes.

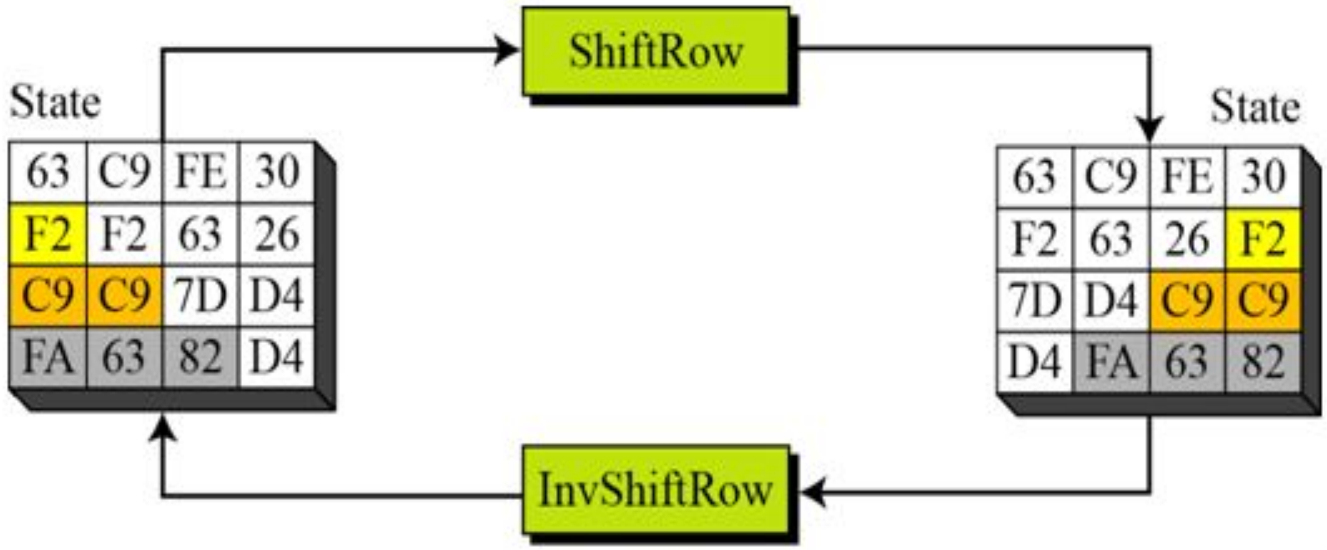
'Shift Rows' transformation

(Left circular shift)



Note: In the decryption, the transformation is called InvShiftRows and the shifting is to the right.

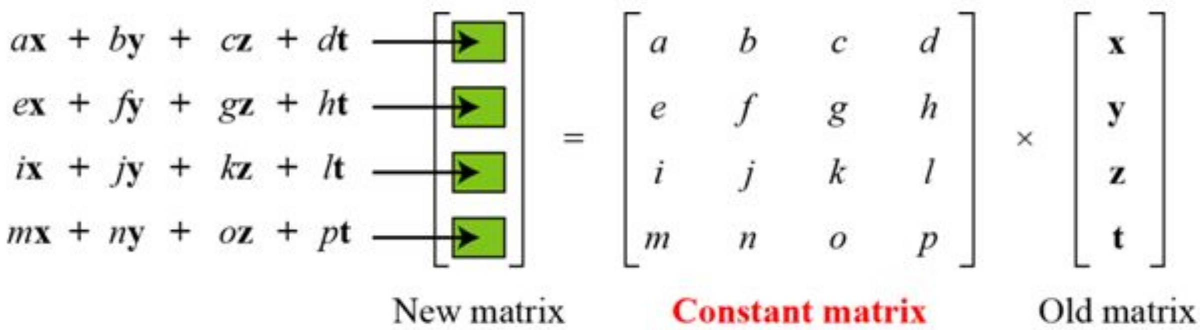
ShiftRow Transformation



Mixing

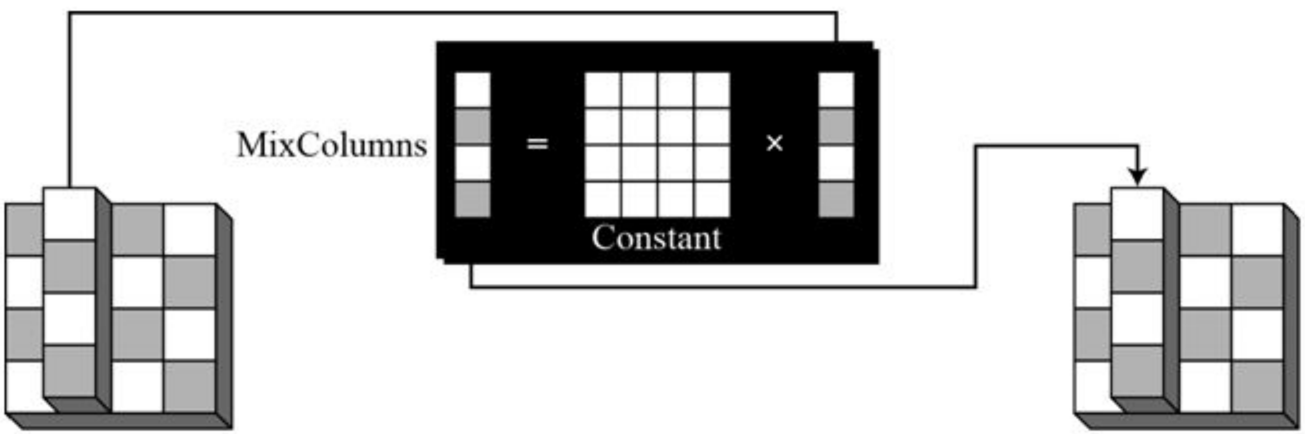
We need an interbyte transformation that changes the bits inside a byte, based on the bits inside the neighboring bytes. We need to mix bytes to provide diffusion at the bit level.

Mixing bytes using matrix multiplication



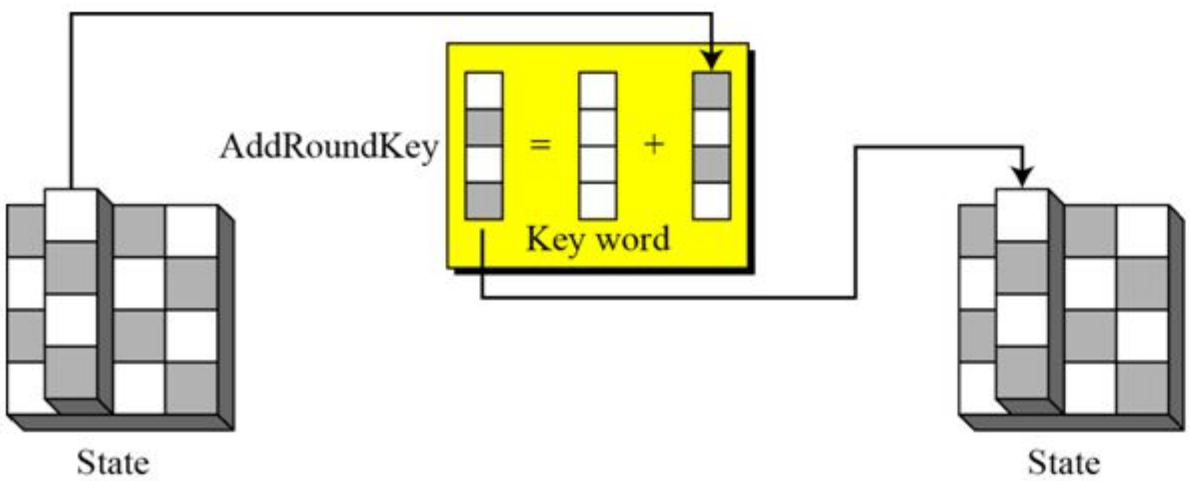
MixColumns

The MixColumns transformation operates at the column level; it transforms each column of the state to a new column.



Key Adding

- AddRoundKey proceeds one column at a time. AddRoundKey adds a round key word with each state column matrix; the operation in **AddRoundKey is matrix addition.**



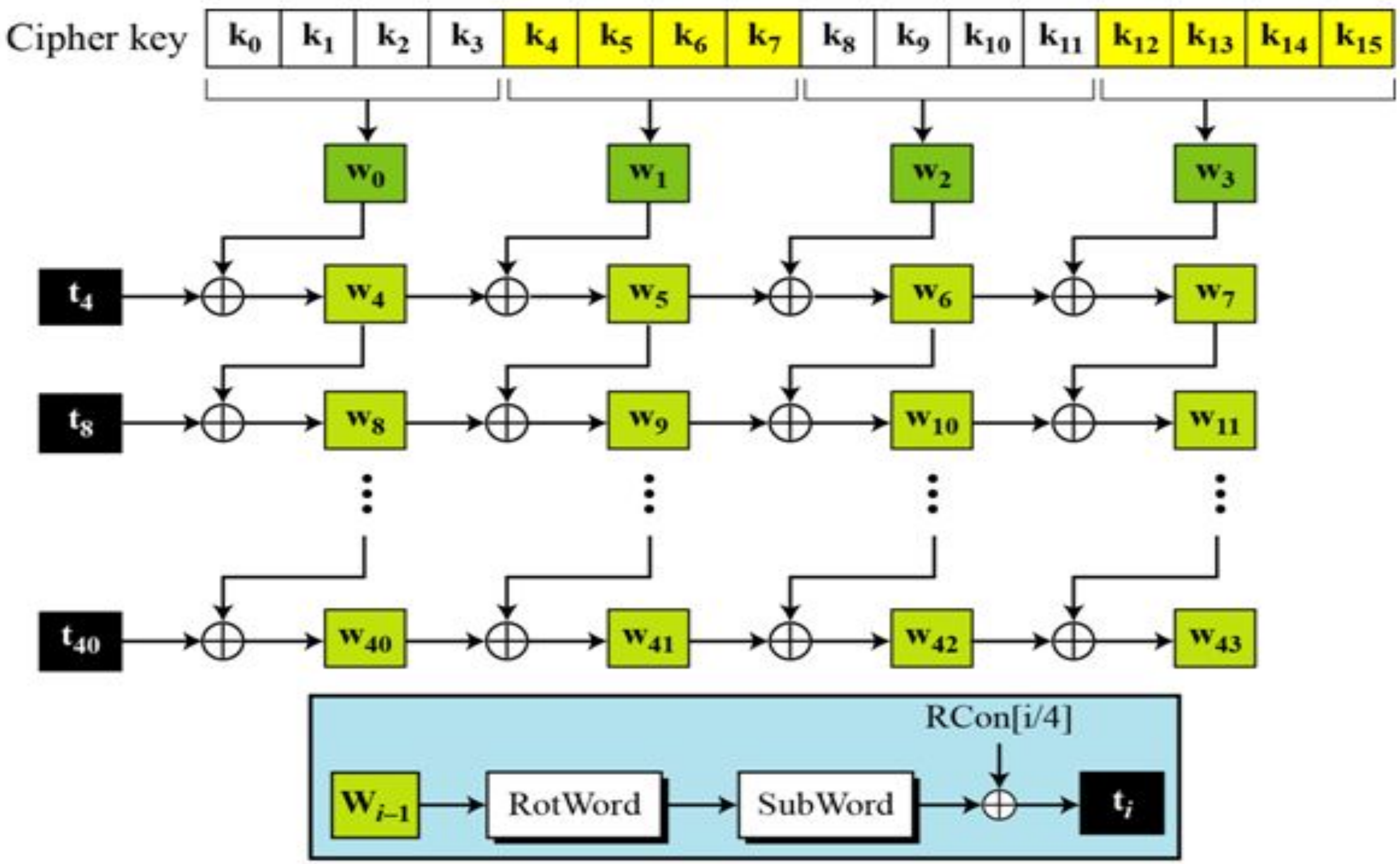
KEY EXPANSION

- To create round keys for each round, AES uses a key-expansion process. If the number of rounds is N_r , the key-expansion routine creates $N_r + 1$ 128-bit round keys from one single 128-bit cipher key.

Words for each round

Round	*Words*			
Pre-round	$\mathbf{w}_0$	$\mathbf{w}_1$	$\mathbf{w}_2$	$\mathbf{w}_3$
1	$\mathbf{w}_4$	$\mathbf{w}_5$	$\mathbf{w}_6$	$\mathbf{w}_7$
2	$\mathbf{w}_8$	$\mathbf{w}_9$	$\mathbf{w}_{10}$	$\mathbf{w}_{11}$
. . .	. . .			
N_r	$\mathbf{w}_{4N_r}$	$\mathbf{w}_{4N_r+1}$	$\mathbf{w}_{4N_r+2}$	$\mathbf{w}_{4N_r+3}$

Key Expansion in AES-128



Making of t_i (temporary) words $i = 4\ N_r$

Contd..

RCon constants

Round	*Constant (RCon)*	*Round*	*Constant (RCon)*
1	$(\underline{\mathbf{01}}\ 00\ 00\ 00)_{16}$	6	$(\underline{\mathbf{20}}\ 00\ 00\ 00)_{16}$
2	$(\underline{\mathbf{02}}\ 00\ 00\ 00)_{16}$	7	$(\underline{\mathbf{40}}\ 00\ 00\ 00)_{16}$
3	$(\underline{\mathbf{04}}\ 00\ 00\ 00)_{16}$	8	$(\underline{\mathbf{80}}\ 00\ 00\ 00)_{16}$
4	$(\underline{\mathbf{08}}\ 00\ 00\ 00)_{16}$	9	$(\underline{\mathbf{1B}}\ 00\ 00\ 00)_{16}$
5	$(\underline{\mathbf{10}}\ 00\ 00\ 00)_{16}$	10	$(\underline{\mathbf{36}}\ 00\ 00\ 00)_{16}$

Contd..

Each round key in AES depends on the previous round key. The dependency, however, is nonlinear because of SubWord transformation. The addition of the round constants also guarantees that each round key will be different from the previous one.

Contd..

The keys for each round are calculated assuming that the 128-bit cipher key agreed upon by Alice and Bob is (24 75 A2 B3 34 75 56 88 31 E2 12 00 13 AA 54 87)16.

Key expansion example

Round	*Values of t's*	*First word in the round*	*Second word in the round*	*Third word in the round*	*Fourth word in the round*
—		w_{00} = 2475A2B3	w_{01} = 34755688	w_{02} = 31E21200	w_{03} = 13AA5487
1	AD20177D	w_{04} = 8955B5CE	w_{05} = BD20E346	w_{06} = 8CC2F146	w_{07} = 9F68A5C1
2	470678DB	w_{08} = CE53CD15	w_{09} = 73732E53	w_{10} = FFB1DF15	w_{11} = 60D97AD4
3	31DA48D0	w_{12} = FF8985C5	w_{13} = 8CFAAB96	w_{14} = 734B7483	w_{15} = 2475A2B3
4	47AB5B7D	w_{16} = B822deb8	w_{17} = 34D8752E	w_{18} = 479301AD	w_{19} = 54010FFA
5	6C762D20	w_{20} = D454F398	w_{21} = E08C86B6	w_{22} = A71F871B	w_{23} = F31E88E1
6	52C4F80D	w_{24} = 86900B95	w_{25} = 661C8D23	w_{26} = C1030A38	w_{27} = 321D82D9
7	E4133523	w_{28} = 62833EB6	w_{29} = 049FB395	w_{30} = C59CB9AD	w_{31} = F7813B74
8	8CE29268	w_{32} = EE61ACDE	w_{33} = EAFE1F4B	w_{34} = 2F62A6E6	w_{35} = D8E39D92
9	0A5E4F61	w_{36} = E43FE3BF	w_{37} = 0EC1FCF4	w_{38} = 21A35A12	w_{39} = F940C780
10	3FC6CD99	w_{40} = DBF92E26	w_{41} = D538D2D2	w_{42} = F49B88C0	w_{43} = 0DDB4F40