PH1102: Physics Laboratory I

Young's Modulus

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February 14, 2022

Abstract

In this experiment, we determine the *Young's modulus* of a reactangular steel bar. We discuss the flexure method and it's advantage over the linear elongation method. Later discuss the sources of error in the experiment.

1 Theory

In 1678, British Scientist Robert Hooke came up with the Law of Elasticity, before talking about the law in detail let's define the few quantities which are essential for understanding this law:

- *Stress* - When an external deforming force acts on a body a reaction force (F) is also created which tries to balance out the the external deforming force. This reaction force per unit area (A) is known as *Stress*. For an isometric body[1] stress can be considered as a scalar quantity[2] having dimensions of *Pressure*[3].

$$\mathit{Stress} = \frac{F}{A}$$

- *Strain* - When a deforming force acts on a body is morphs the physical dimensions of the body, *Strain* is the ration between the change in the physical dimensions of the body to the original dimensions of the body.

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[1]A body with uniform dimensions

[2]In general stress is a *Tensor* quantity and it's only scalar for an isometric body.

[3]Unit of pressure is N m^{-2}

Now that we know what *Stress* and *Strain* are we can understand the Law of Elasticity given by Robert Hooke, according to this law the ratio of the *Stress* and *Strain* is a constant, this constant is know as the *modulus of elasticity* represented by ε.[4]

$$\boxed{\varepsilon = \frac{\text{Stress}}{\text{Strain}}}$$

Note that ε is strictly a material property.

2 Experimental Setup

2.1 Naive Setup

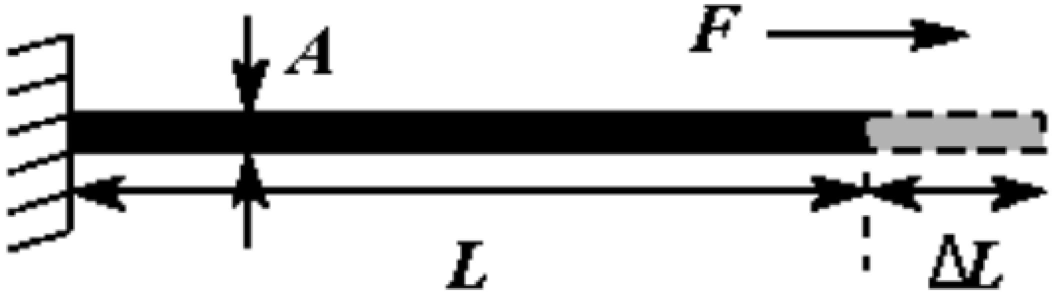

Figure 1: Linear elongation of the metal bar

We can have a setup as shown above where the metal bar is in horizontal position and a force is pulling the other end of the rod with a force F. Using the Hooke's law we can calculate the modulus of elasticity for the metal rod.

$$\text{Stress} = \frac{F}{A}$$

$$\text{Strain} = \frac{\Delta L}{L}$$

$$\boxed{\varepsilon = \frac{FL}{A\Delta L}}$$

For providing constant F we can use a pulley system as shown below.

[4]Hooke's Law is only valid in specific range called elastic range. We define a new quantity known as *Elastic Limit* it's the maximum limit where after removing *Stress* the body regains it's original dimensions beyond this limit Hooke's Law is not valid.

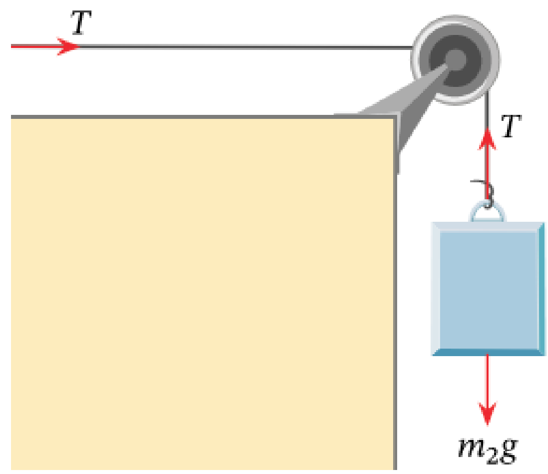

Figure 2: Constant F using pulley system

Now we can have constant F just by changing the mass m_2. We can now modify or equation for ε.

$$\boxed{\varepsilon = \frac{m_2 g L}{A \Delta L}}$$

On rearranging the above equation we can see that to have any significant change in length for a metal bar we would need really high mass, as the ε for the steel is really high.

$$\Delta L = \left(\frac{gL}{A\varepsilon}\right) m_2$$

As the change is really small for to be measured by the standard lab instrument[5] we will use a more practical setup discussed in the next section.

[5] Travelling microscope in this case, which has a least count (LC) is 0.001cm.

2.2 Practical Setup

As we know from the practical experience that it's much more easier to flex a rod by applying a force at it's centre as compared to elongating a rod by pulling it. We will exploit this exact thing and design our experiment such that we get depression in our rod when we apply a force at it's centre.

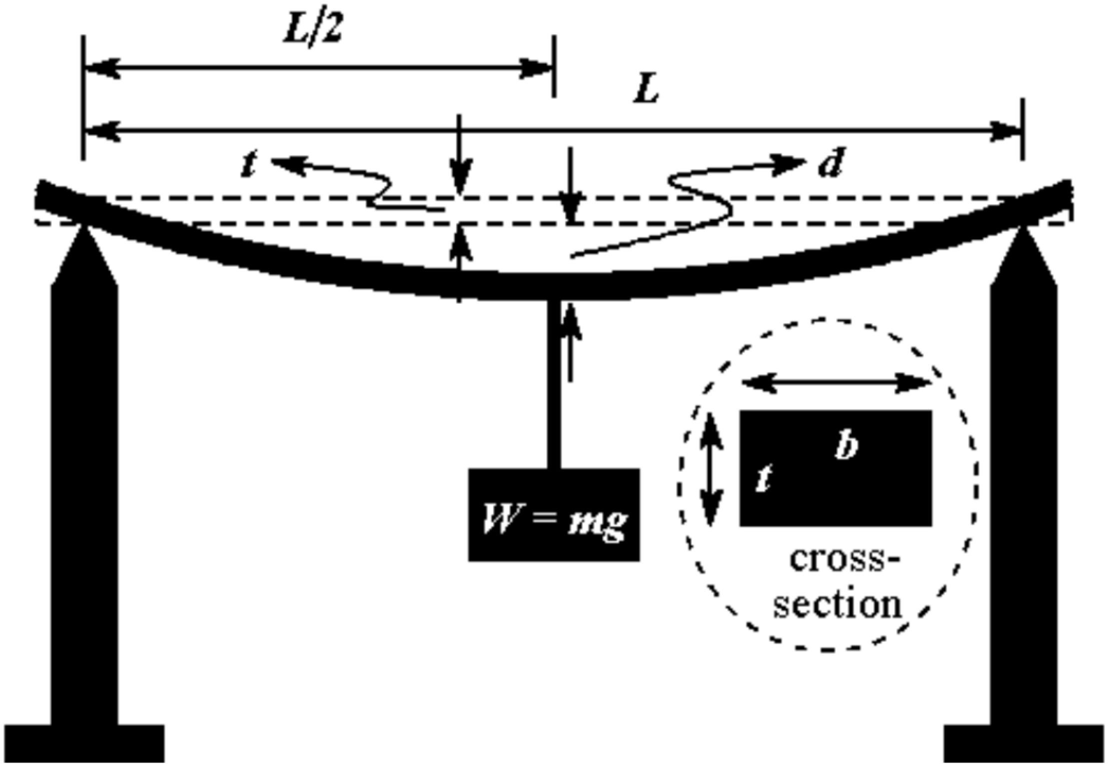

Figure 3: Depression(d) in metal rod due to the hanging weight at it's centre

2.2.1 Working principle

As we can see in the above figure we have a metal bar is placed between the two knife edges placed at a distance L from each other and at the centre we have hanger which is providing constant force $F = mg$ and thus creating the depression d.

Here we can clearly see that at the centre of the rod is horizontal as this part does not suffer and bending. If we take half the rod from centre to the edge and consider it to be upside down, it behaves like a cantilever supported at one end and loaded at the other end as shown below.

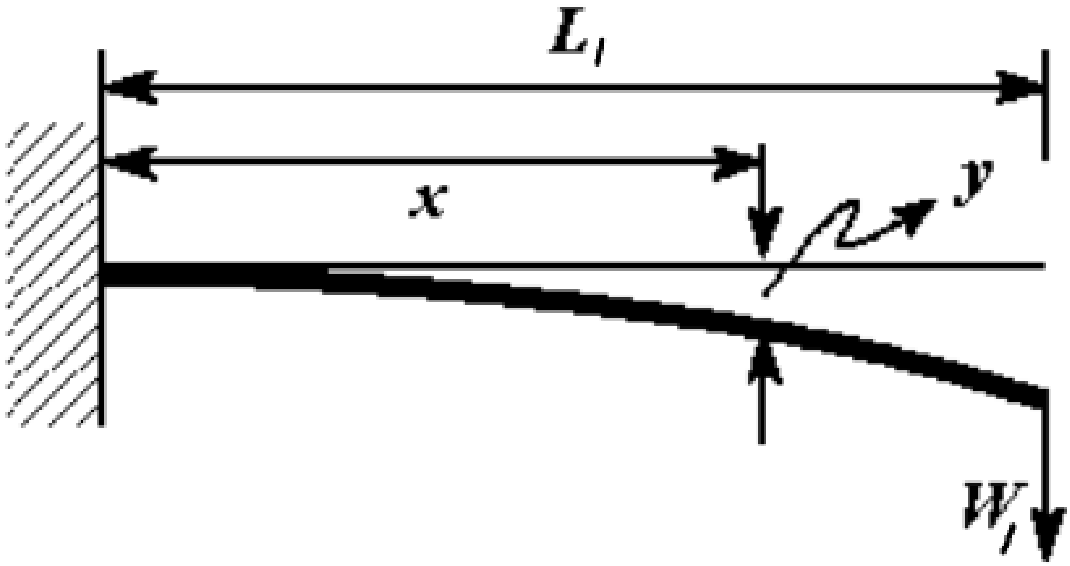

Figure 4: Cantilever loaded at one end and fixed at the other

Note that the length of the cantilever is $L_1 = \frac{L}{2}$ and the weight acting on it is $W_1 = \frac{W}{2}$.

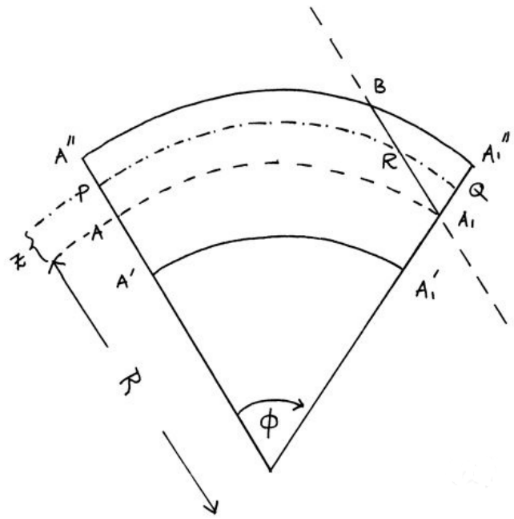

Figure 5: Left (a) Cantilever small element. Right (b) Area element dA between r and $r + dr$.

We consider a small section of this cantilever at distance x along the beam as shown in Figure 5, the element is bend and substends an angle ϕ at the radius of curvature R of the AA_1 which is the central line, due to this bending the upper half of the beam is stretched and the lower half of the beam is compressed, we also observe that the plane AA_1 at the centre which remains unstrained and this is known as the *Neutral Plane*. We can easily see that for a plane of the rod at a distance $R + z$ that's PQ due to bending is under strain and we can calculate this strain as,

$$\text{strain} = \frac{\text{change in length}}{\text{original length}} = \frac{(R+z)\phi - R\phi}{R\phi} = \frac{z}{R}$$

If we consider a region from z to $z + dz$ the area element between these two will be bdz, assuming that the force generated due to strain is constant dF in this region and we can use the Hooke's law to calculate this force,

$$\frac{dF}{bdz} = Y\frac{z}{R}$$

Here we are using Y in place ε cause here we have elongation/compression along one direction so it's common to represent this using Y, which also stands for *Youngs Modulus*. We can see that this force will act at some distance from the neutral layer so we can find the moment of this force about the neutral layer as,

$$d\mathcal{M} = zdF = \frac{Yb}{R}z^2dz$$

To calculate the total moment $\mathcal{M}$ acting on this element we will integrate the above equation.

$$\mathcal{M} = \frac{Yb}{R}\int_{-t/2}^{t/2} z^2 dz = \frac{Ybt^3}{12R}$$

As we know that the above calculated $\mathcal{M}$ is for the element at distance x along the bar so we can modify the equation,

$$\mathcal{M}(x) = \frac{Ybt^3}{12R(x)}$$

$R(x)$ here is the radius of curvature as x which we can approximate to $\frac{1}{R(x)} = \frac{d^2y}{dx^2}$ as the value of $\frac{dy}{dx}$ is very smalll when compared to 1 as we are having really small bend.

$$\mathcal{M}(x) = \frac{Ybt^3}{12}\frac{d^2y}{dx^2}$$

As the rod isn't rotating then this moment ($\mathcal{M}(x)$) should be balanced by the external weight we can write the torque balance equation as,

$$W_1(L_1 - x) = \frac{Ybt^3}{12}\frac{d^2y}{dx^2}$$

we know that at $x = 0$ both $y = 0$ and $\frac{dy}{dx} = 0$, so we can integrate the above equation twice with these boundary condition to get $y(x)$ and we know that at $x = L_1$ we will have maximum deflection so we can write $y(L_1)$ as,

$$y(L_1) = \frac{4W_1L_1^3}{Ybt^3}$$

Now we know that $y(L_1) = d$, $W_1 = \frac{W}{2}$ and $L_1 = \frac{L_1}{2}$ we can put these in the above equation to get,

$$d = \frac{WL^3}{4Ybt^3}$$

We can also write $W = mg$ and this final substitution will give us the working formula of our experiment.

$$\boxed{d = \frac{mgL^3}{4Ybt^3}}$$

Here the variables have the following meaning:

- d = Depression of the metal bar
- m = Mass hanging at the centre
- g = Acceleration due to gravity at the surface of the earth
- L = Length of rod between the knife edge
- Y = Youngs modulus of the metal bar
- b = Breadth of the metal bar
- t = Thickness of the metal bar

2.2.2 Laboratory Setup

Figure 6: Photograph of the experimental setup

3 Observation

Table 1: Breadth of bar using Vernier Calliper of LC=0.002 cm

Sl.No.	MSR (cm)	VSR	b (cm)
1	2.5	40	2.580
2	2.5	16	2.532
3	2.5	25	2.550

Using the above table we can easily calculate the average value of b.

$$\boxed{b = 2.554 \pm 0.002 \text{ cm}}$$

Table 2: Thickness of bar using Screw Gauge of LC=0.01 mm

Sl.No.	LSR (mm)	CSR	t (mm)
1	4.5	41	4.91
2	4.5	46	4.96
3	4.5	38	4.88

Using the above table we can easily calculate the average value of t.

$$\boxed{t = 4.91 \pm 0.01 \text{ mm} = 0.491 \pm 0.001 \text{ cm}}$$

Table 3: Measurment of Depression (d_1) in Loading of mass (m) using travelling microscope of LC=0.01 mm

Sl.No.	m (gm)	MSR (mm)	VSR	x_1 (mm)	d_1 (mm)
1	0	113	20	113.20	0.00
2	426	112	33	112.33	0.87
3	908	110.5	25	110.75	2.45
4	1398	109	10	109.10	4.10
5	1883	108	5	108.05	5.15
6	2843	105	38	105.38	7.82

Table 4: Measurment of Depression (d_2) in Unloading of mass (m) using travelling microscope of LC=0.01 mm

Sl.No.	m (gm)	MSR (mm)	VSR	x_2 (mm)	d_2 (mm)
1	0	112.5	40	112.90	0.00
2	426	112	5	112.05	0.85
3	908	110.5	15	110.65	2.25
4	1398	109	23	109.23	3.67
5	1883	107.5	30	107.80	5.10
6	2843	105	38	105.38	7.52

Table 5: Calculation of average depression d using d_1 and d_2

Sl.No.	d_1 (mm)	d_2 (mm)	d (mm)
1	0	0	0
2	0.87	0.85	0.86
3	2.45	2.25	2.35
4	4.10	3.67	3.89
5	5.15	5.10	5.13
6	7.82	7.52	7.67

4 Data Analysis

We now know the value of d for different m, so we can tabulate that.

Table 6: Value of depression d for each mass value m

m (gm)	d (mm)
0	0
426	0.86
908	2.35
1398	3.89
1883	5.13
2843	7.67

As we can see from our working formula below that the if we plot m vs d then we can use the slope of the best fit to calculate the value of the *Youngs Modulus* Y.

$$d = \left(\frac{gL^3}{4Ybt^3}\right) m$$

4.1 Data Fitting

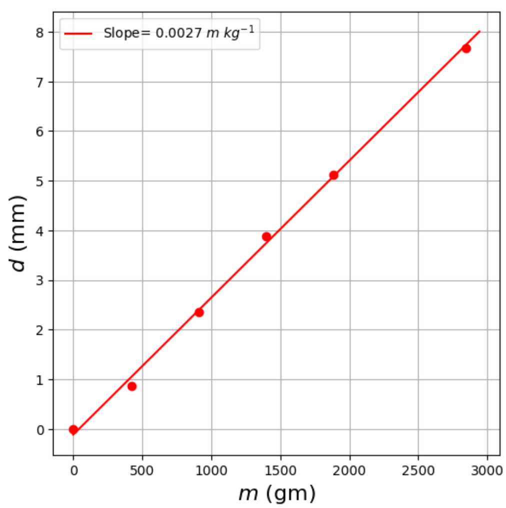

Figure 7: Mass m vs Depression d[6]

[6]Link of the code

4.2 Calculation

We can see that the slope s which we got using the curve fitting is equal to,

$$s = \frac{gL^3}{4Ybt^3}$$

$$\boxed{Y = \frac{gL^3}{4sbt^3}}$$

- g= 9.81 m s^{-2}
- L= 86 $\times 10^{-2}$ m
- s= 0.0027 m kg^{-1}
- b= 2.554 cm = 2.554 $\times 10^{-2}$ m
- t= 0.491 cm = 0.491 $\times 10^{-2}$ m

We can substitue the above values to get the value of Y.

$$\boxed{Y = 1.91 \times 10^{11} \frac{\mathrm{N}}{\mathrm{m}^2}}$$

4.3 Error Analysis

From the working formula we can get the formula for calculation of relative error in Y as,

$$\frac{\delta Y}{Y} = 3\frac{\delta L}{L} + \frac{\delta b}{b} + 3\frac{\delta t}{t} + \frac{\delta m}{m} + \frac{\delta d}{d}$$

The least count values are $\delta L = 0.01$ m, $\delta b = 0.00002$ m, $\delta t = 0.00001$ m, $\delta m = 0.001$ kg, $\delta d = 0.00001$ m, on substituting this we get.

$$\frac{\delta Y}{Y} = 0.0447$$

$$\boxed{\text{error percent} = 4.47\%}$$

5 Conclusion

The standard value of Young's modulus of steel is $2 \times 10^{11} \frac{N}{m^2}$, the experimental value which we got is $1.91 \times 10^{11} \frac{N}{m^2}$, we can see that the experimental value has an error of 4.50%. This can be due to many sources,

- In the measurement of the length, breadth and thickness of bar, there will be some errors. We cannot measure the length of the bar with accuracy more than 0.1 cm. Similarly, it is not possible to measure the breadth and thickness of bar with accuracy more than 0.002 cm and 0.01 mm respectively.

- Again for depression measurement, we cannot measure it with accuracy more than 0.01 mm. And for the mass measurement, the limit is 1 gm.