PH1102: Physics Laboratory I

Dynamical Viscosity of Castor Oil Using Falling Sphere Viscometer

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Abstract

In this experiment we determine the *Coefficient of Dynamic Viscosity* (η) of *Castor Oil* using the falling sphere viscometer.

1 Theory

In 1851, George Gabriel Stokes derived an expression, now known as *Stokes law*, for the frictional force – also called drag force – exerted on spherical objects with very small Reynolds numbers in a viscous fluid.

The expression of force ($\mathbf{F}_v$) on a spherical ball was given by,

$$\mathbf{F}_v = -6\pi\eta r\mathbf{v}$$

where η is the coefficient of viscosity, r is the radius of the ball and $\mathbf{v}$ is the velocity of the ball in the liquid.

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1.1 Equation of motion of a vertically falling ball in a viscous media

Let's use the $\mathbf{F}_v$, to find the equation of motion of spherical ball of radius r and mass m in a liquid of viscosity η. Here as we are concered with only one degreee of freedom therefore

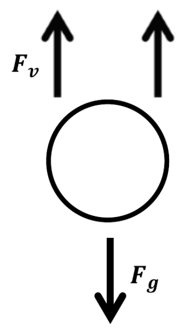


Figure 1: Forces acting on the ball

for our convenience I'll replace the bold face letters represeting vectors to scalar and use the '-' to represent the direction of force[1]

$$F_g - (F_v + F_b) = m\frac{dv}{dt}$$

$$\frac{4\pi r^3}{3}\sigma g - \left(6\pi\eta r v + \frac{4\pi r^3}{3}\rho g\right) = m\frac{dv}{dt}$$

$$V\sigma g - (6\pi\eta r v + V\rho g) = m\frac{dv}{dt}$$

where V is the volume of the ball. On solving the above equation we get,

$$v(t) = \frac{V(\rho - \sigma)g}{6\pi\eta r}\left(1 - \exp\left(\frac{-6\pi\eta r}{V\sigma}t\right)\right)$$

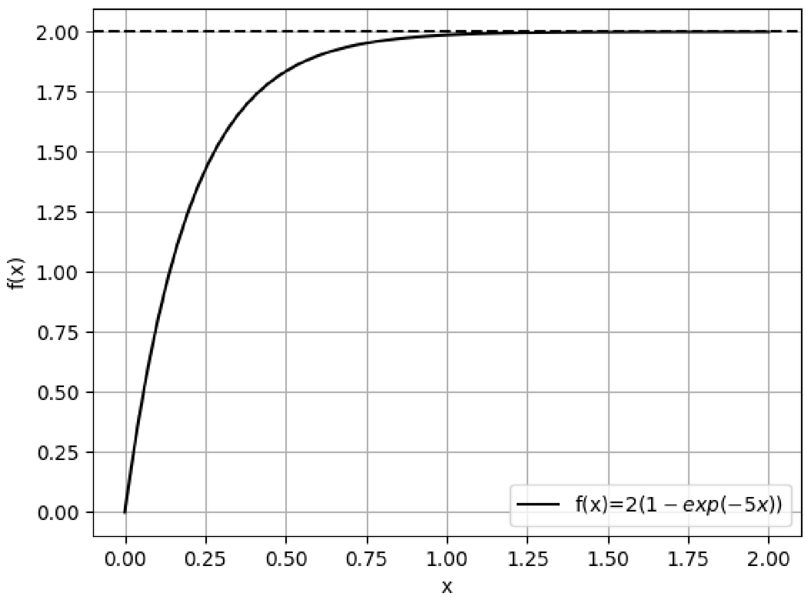
Here σ is the density of the ball and ρ is the density of the fluid.

[1]Here I am assuming downwards to be positive direction.

We can say that the above equation is of form,

$$f(x) = a\left(1 - \exp\left(-bx\right)\right)$$

If we take $a = 2$ and $b = 5$ the function will look like, we can clearly see that the value of



function approaches very close to the factor a.

In our velocity expression as the time passes the value of the velocity approaches the factor,

$$v_t = \frac{V\left(\rho - \sigma\right)g}{6\pi\eta r}$$

where v_t is the terminal velocity. v_t is what we will be using in this experiment to evaluate the value of η.

2 Experimental Setup

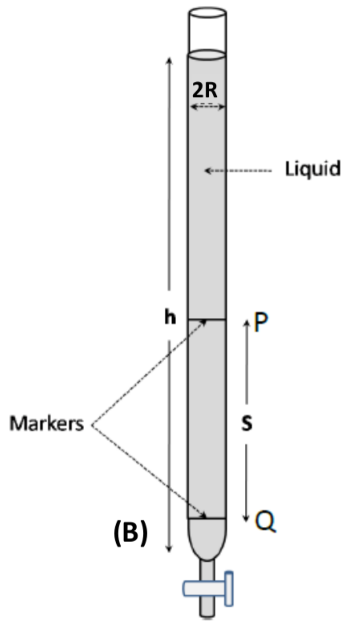


Figure 2: Schematic of the Experimental Setup

Here in the region PQ is chosen such that the ball is moving with the v_t terminal velocity, we will now modify the equation for v_t evaluated above to match our experimental, this is required because the above equation was for a ball moving in a viscous liquid contained in container of infinite physical dimension but here we have container of finite dimension therefore we can modify the equation as follows.

$$v_t = \frac{V(\rho - \sigma)g}{6\pi\eta r\left(1 + 2.4\frac{r}{R}\right)\left(1 + 3.3\frac{r}{h}\right)}$$

where R is the radius of the container and h is the height of the liquid column, for our experimental setup $\frac{r}{R} \approx 0.1$, $\frac{r}{h} \approx 0.001$, which means that we can ignore the factor $\left(1 + 3.3\frac{r}{h}\right) \approx 1$.

$$\boxed{\eta = \frac{2r^2(\rho - \sigma)g}{9\left(1 + 2.4\frac{r}{R}\right)v_t}}$$

Which is our working formula.

3 Observation

Table 1: Diameter of Ball 1 (small)

Sl. No.	MSR (cm)	CSR	d_1 (cm)
1	0.3	15	0.315
2	0.3	18	0.318
3	0.3	17	0.317
4	0.3	15	0.315
5	0.3	16	0.316
6	0.3	19	0.319
7	0.3	16	0.316

the average value of diameter and radius of ball 1 is $d_1 = 0.317 \pm 0.001$ cm, $r_1 = 0.159 \pm 0.001$ cm and the observed mass of 10 such balls is 1.31 gm which means that mass of one such ball is $m_1 = 0.13 \pm 0.01$ gm.

Table 2: Diameter of Ball 2 (medium)

Sl. No.	MSR (cm)	CSR	d_2 (cm)
1	0.3	48	0.348
2	0.3	46	0.346
3	0.3	49	0.349
4	0.3	46	0.346
5	0.3	48	0.348
6	0.3	49	0.349
7	0.3	47	0.347

the average value of diamete and radius of ball 2 is $d_2 = 0.348 \pm 0.001$ cm, $r_2 = 0.174 \pm 0.001$ cm and the observed mass of 10 such balls is 2.05 gm which means that mass of one such ball is $m_2 = 0.21 \pm 0.01$ gm.

Table 3: Diameter of Ball 3 (large)

Sl. No.	MSR (cm)	CSR	d_3 (cm)
1	0.4	24	0.424
2	0.4	26	0.426
3	0.4	28	0.428
4	0.4	24	0.424
5	0.4	28	0.428
6	0.4	26	0.426
7	0.4	24	0.424

the average value of diameter and radius of ball 3 is $d_3 = 0.426 \pm 0.001$ cm, $r_3 = 0.213 \pm 0.001$ cm and the observed mass of 10 such balls is 4.47 gm which means that mass of one such ball is $m_3 = 0.45 \pm 0.01$ gm.

Table 4: Inner Diameter of Glass Cylinder

Sl. No.	MSR (cm)	VSR	d_i (cm)
1	4.6	5	4.610
2	4.6	5	4.610
3	4.6	4	4.608

the average value of inner diameter and radius of the glass cylinder is $d_i = 4.609 \pm 0.002$ cm, $r_i = 2.305 \pm 0.002$ cm

Table 5: Time taken by Spheres to traverse PQ

Sl. No.	t_1 (s)	t_2 (s)	t_3 (s)
1	15.72	10.66	7.68
2	15.93	10.65	7.81
3	15.88	10.69	7.72
4	15.85	10.57	7.81
5	15.75	10.66	7.82
6	16.03	10.63	7.69
7	15.78	10.59	7.90

The length of PQ was measured using a meter scale which means that the lenght is $S = 80.0 \pm 0.1$ cm, the average value of times for the three balls are, $t_1 = 15.85 \pm 0.01$ s, $t_2 = 10.64 \pm 0.01$ s, $t_3 = 7.78 \pm 0.01$ s. We can now use,

$$v = \frac{S}{t}$$

to calculate the v_t for the three balls which comes out to be, $v_{t,1} = 5.05 \pm 0.01$ cm s^{-1}, $v_{t,2} = 7.52 \pm 0.02$ cm s^{-1}, $v_{t,3} = 10.28 \pm 0.03$ cm s^{-1}.
We have now calculated all the experimental parameters now we can make a combined table.

Table 6: Combined table for all the Balls

Ball No.	r (cm)	m (gm)	ρ (gm cm^{-3})	t (s)	v_t (cm s^{-1})
1 (small)	0.159 ± 0.001	0.13 ± 0.01	7.72 ± 0.74	15.85 ± 0.01	5.05 ± 0.01
2 (medium)	0.174 ± 0.001	0.21 ± 0.01	9.52 ± 0.62	10.64 ± 0.01	7.52 ± 0.02
3 (large)	0.213 ± 0.001	0.45 ± 0.01	11.12 ± 0.40	7.78 ± 0.01	10.28 ± 0.03

Here σ is the density of the ball, t is the time taken to traverse the section PQ and v_t is the terminal velocity of the ball.
Other experimental parameters are,

- Density of Castor Oil (σ) = 0.961 gm cm^{-3}
- Inner Radius of Cylinder (R) = 2.305 ± 0.002 cm
- Acceleration due to gravity (g) = 980.665 cm s^{-2}

4 Data Analysis

$$v_t = \left(\frac{2\,(\rho - \sigma)\,g}{9\left(1 + 2.4\frac{r}{R}\right)\eta} \right) r^2$$

We can now make plot of v_t vs r^2 to get value of the preceding factor,

Table 7: v_t vs r^2

Ball No.	v_t (cm s^{-2})	r^2 (cm^2)
1	5.05	0.025
2	7.52	0.030
3	10.28	0.045

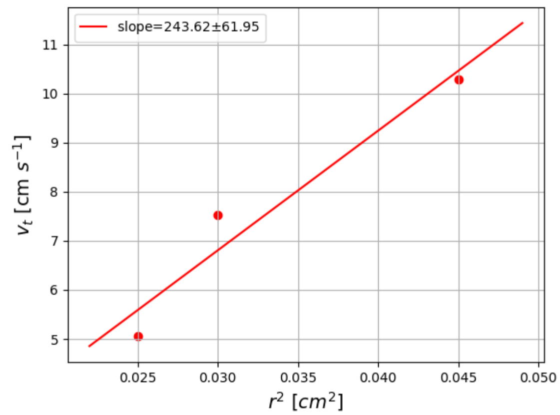


Figure 3: v_t vs r^2 (code can be found here here.)

We can clearly see that the plot is linear in nature.

$$\eta = \left(\frac{2\,(\rho - \sigma)\,g}{9\,\left(1 + 2.4\frac{r}{R}\right)\,v_t} \right) r^2$$

$$\frac{\Delta\eta}{\eta} = \frac{\Delta v_t}{v_t} + 2\frac{\Delta r}{r} + \frac{\Delta\rho}{\rho}$$

We can now use the above two equations to get the 3 values of η corresponding to each ball.[2]

Table 8: Table of calculated η

Ball No.	η (P)
1 (small)	5.88 ± 0.65
2 (medium)	6.00 ± 0.48
3 (large)	7.82 ± 0.38

$$\boxed{\bar{\eta} = 6.56 \pm 0.50 \text{ P}}$$

[2]We can substitute $\frac{r}{R} \approx 0.1$

5 Sources of Errors

Following are the most probable sources of error in this experiment.

- Theoretically ball never reaches the v_t and this might be a cause of imprecise measument of η.
- We have assumed that the liquid is not turbulent when the ball is moving through it, this might not be true for liquid touching the ball's surface.
- We have considered $\frac{r}{R} \approx 0.1$ and $\frac{r}{h}$ which is not exactly correct as we are have different value of r for each ball.
- Imprecise measurement from the instrument.

6 Conclusion

We know that the literature value of the coefficient of dynamical viscosity of *Castro Oil* is $6.50P$, calculated value is $6.56P$. We can see that the relative error is 0.92% which is very small. The propagation error due to the imprecise measument by the instrument is 7.62%.