PH1102: Physics Laboratory I

Verification of the Newton's Second Law of Motion

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Abstract

In this experiment we verify the *Newton's Second Law* using an air track setup.

1 Theory

In 1687 Issac Newton published *Philosophiæ Naturalis Principia Mathematica (Mathematical Principles of Natural Philosophy)* where he talked about the laws of classical mechanics that describe the relationship between the motion of an object and the forces acting on it. These laws can be paraphrased as follows:

- **Law 1**: A body remains at rest, or in motion at a constant speed in a straight line, unless acted upon by a force.
- **Law 2**: When a body is acted upon by a force, the time rate of change of its momentum equals the force.
$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$
- **Law 3**: If two bodies exert forces on each other, these forces have the same magnitude but opposite directions.

In this experiment we will be focusing on the *Law 2*.

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2 Experimental Setup

2.1 Equation of Motion given by *Law 2*

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

If we assume that the mass of the system is constant which means that $d\mathbf{p} = md\mathbf{v}$,

$$\mathbf{F} = m\frac{d\mathbf{v}}{dt} = m\mathbf{a}$$

$$\boxed{\mathbf{a} = \frac{\mathbf{F}}{m}}$$

If assume that $\mathbf{F}$ is constant we will get $\mathbf{v}$ as,

$$\mathbf{F}\int_{t_i}^{t_f} dt = m\int_{t_i}^{t_f} d\mathbf{v}$$

$$\mathbf{F}\,(t_f - t_i) = m(\mathbf{v}_f - \mathbf{v}_i)$$

$$\boxed{\mathbf{v}_f = \frac{\mathbf{F}}{m}\,(t_f - t_i) + \mathbf{v}_i}$$

We can now write expression for $\mathbf{x}$ as follows.

$$\mathbf{F}\int_{t_i}^{t_f}\int_{t_i}^{t_f} dt^2 = m\int_{t_i}^{t_f}\int_{t_i}^{t_f} d^2\mathbf{x}$$

$$\frac{\mathbf{F}}{2m}\,(t_f - t_i)^2 = \mathbf{x}_f - \mathbf{x}_i$$

$$\boxed{\mathbf{x}_f = \frac{\mathbf{F}}{2m}\,(t_f - t_i)^2 + \mathbf{x}_i}$$

2.2 Schematic of the Experiment

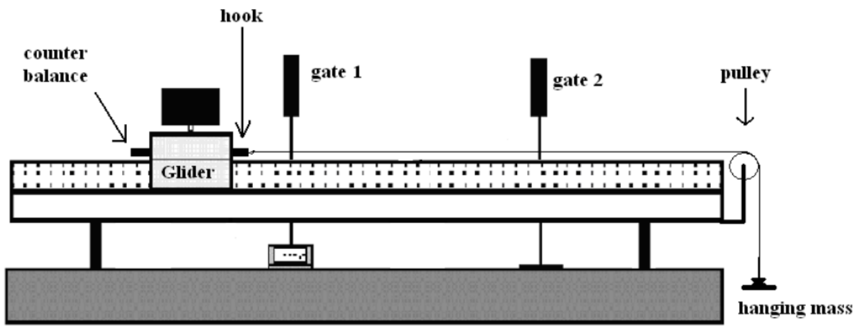

Figure 1: Experimental setup

2.3 Equation of Motion for the *Glider*

We can make use of the above experimental setup and simplify the above equations of motion to get the simplified equation of motion.

Our time starts from 0 which means that $t_i = 0$ and we will take position at the start as 0 which means $\mathbf{x}_i = 0$ we can see that the *glider* starts from rest which means that $\mathbf{v}_i = 0$, we also know note that the motion has only degree of freedom which means that we can omit the use of boldface letters as they represents a vector and here we can just work with the magnitudes of these vectors as there is no change in direction for the *glider*. I will also substitute t for t_f, x for x_f and v for v_f[1]. Here mass of hanging mass is taken to be m and mass of *glider* as M, the friction coefficient as μ and a as the acceleration of the system, now we can write the force balance equation where T is the tension in the string,

$$mg - T = ma$$

$$T - \mu M g = Ma$$

[1]This purely done for convenience.

As we are using the air track which means that $\mu \approx 0$, we can solve the above equation to get the value of a and then use this value to get v, x as a function of t.

$$\boxed{a = \frac{mg}{M+m}} \tag{1}$$

$$\boxed{v = at = \frac{mg}{M+m}t} \tag{2}$$

$$\boxed{x = \frac{1}{2}at^2 = \frac{1}{2}\frac{mg}{M+m}t^2} \tag{3}$$

We now have to verify these equation of motion by doing the experiment.

3 Data Analysis

We have the data of x at different values of t where $dt = 0.1s$, we can use this to get the value of $v(t)$ and $a(t)$ by the following expression,

$$v\left(t + \frac{dt}{2}\right) = \frac{x(t+dt) - x(t)}{dt}$$

$$a(t+dt) = \frac{x(t+2dt) + x(t) - 2x(t+dt)}{dt}$$

3.1 Fixed Force

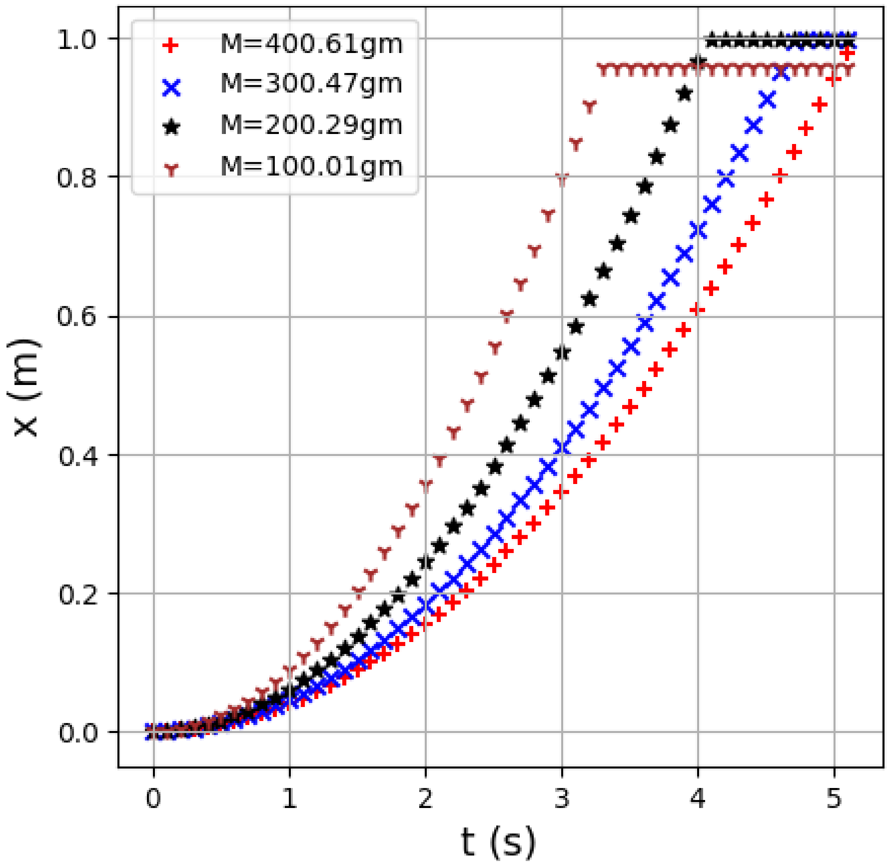

Figure 2: x vs t

Here we can see that for $M < 400.61gm$ the position settles at a constant x which is what we expect as we know that the force is kept constant that means that means lighter mass will reach the end faster which explains the above feature in the graph.

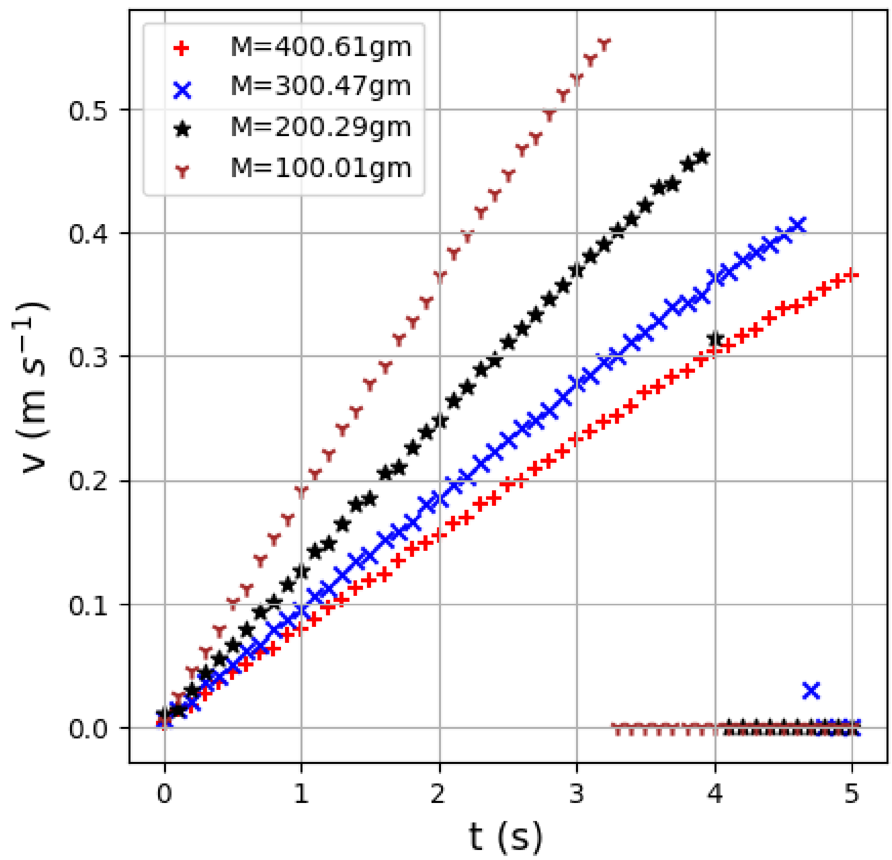

Figure 3: v vs t

Here we can see that for $M < 400.61gm$ the value of v becomes 0 near the end which is because of the same reason which was mentioned above.

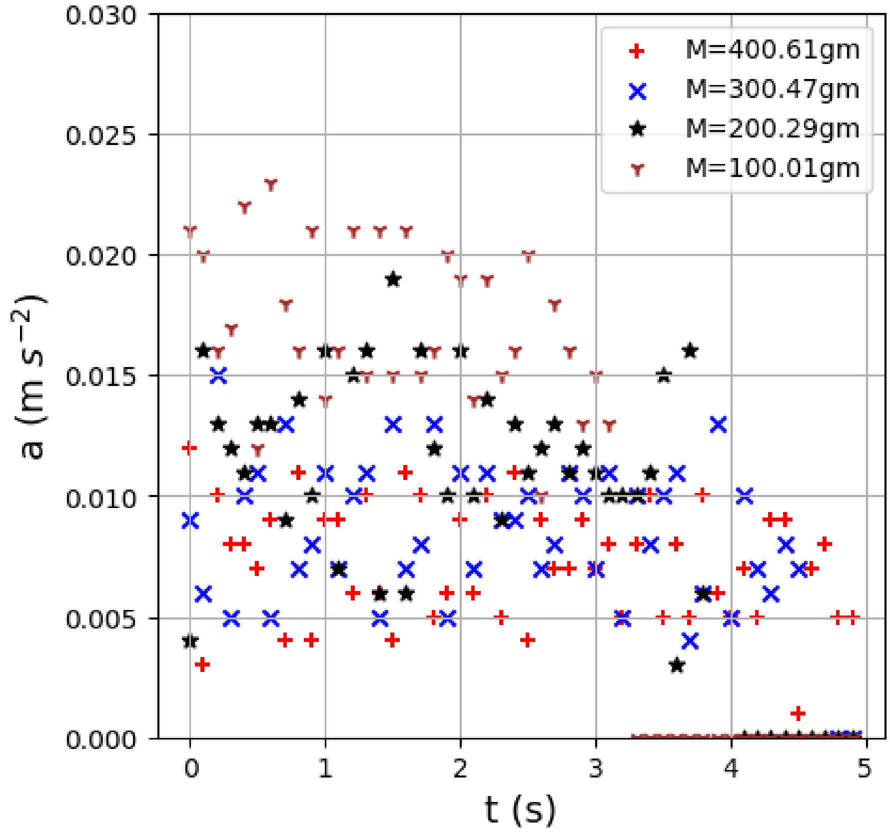

Figure 4: a vs t

Here we notice the same trend for $M < 400.61gm$ that the a is zero near the end, this is because the glider reaches the end faster as compared to the heavier mass case.

We will now do the fitting of the x vs t graph using a quadratic fit as we want to verify the *Law 2* and then compare the value with the *Law 2*. Here I have ignored the last few

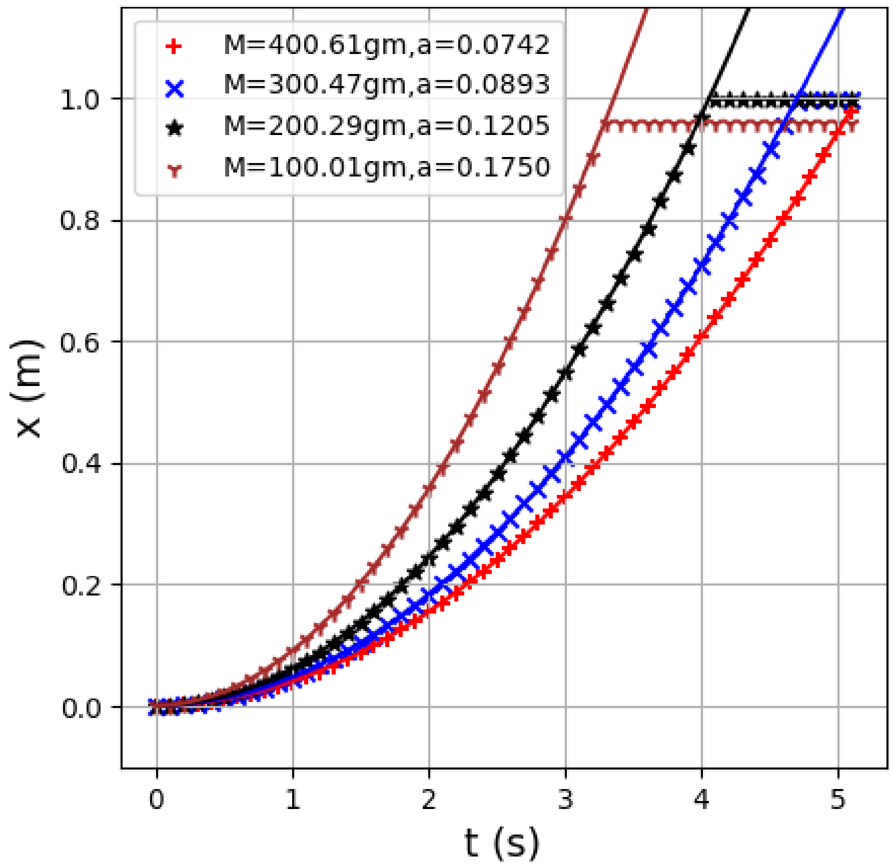

Figure 5: Fitted curve for x vs t

points for the lesses mass case as we know that in these cases the *glider* reaches the end faster as compared to the other cases which explains the flattening near the end.

Table 1: a from fitting the x vs t

M+m (gm)	a (m s^{-2})	a^{-1} (m^{-1} s^2)
404.74	0.0742	13.4770
304.60	0.0893	11.1982
204.42	0.1205	8.2988
104.14	0.1750	5.7143

We will now plot and fit $M + m$ vs a^{-1} and then compare it the value of the slope to the value of external force F which we know is equal to $0.0405N$, as we know that this is constant.

$$M + m = F\frac{1}{a}$$

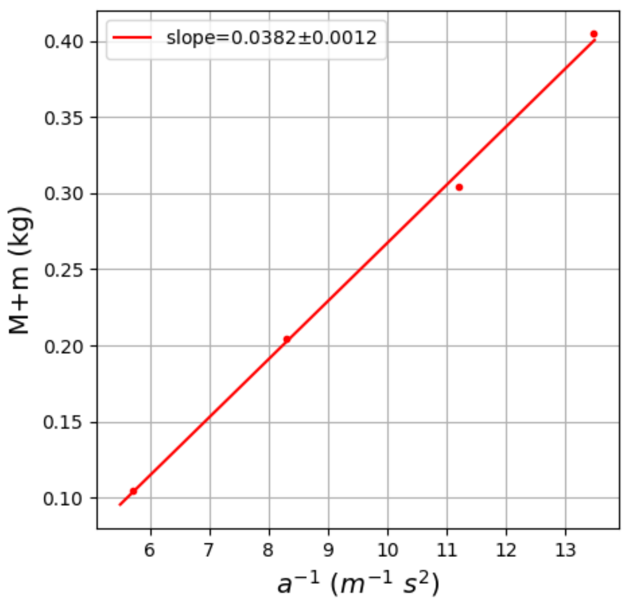

Figure 6: $M + m$ vs a^{-1}

The value of F which we get from the fit is $F = 0.0382 \pm 0.0012N$ which is very close to the actual value of force that is $F = 0.0405N$.

$$\text{error percentage} = \frac{|0.0382 - 0.0405|}{0.0405} \times 100$$

$$\boxed{\text{error percentage} = 5.61\%}$$

3.2 Fixed Mass

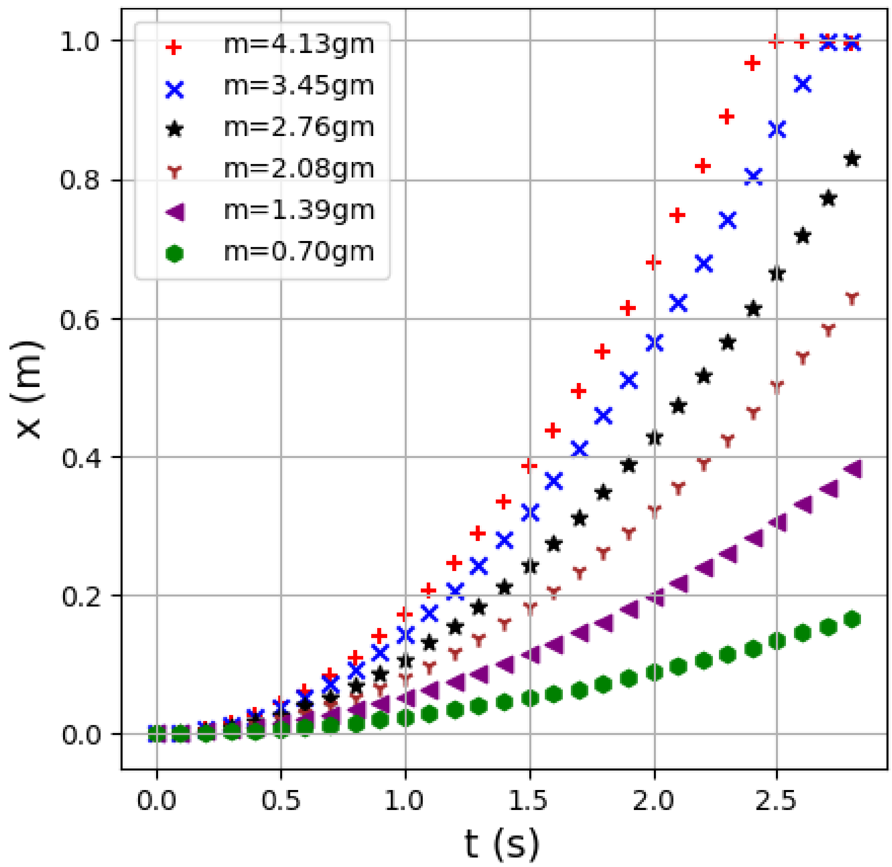

Figure 7: x vs t

Here observe that the for $m = 4.13gm$ and $m = 3.45gm$ there is flattening at the right end which is because as the applied force here is mg thus for greater m the acceleration is more which is why the glider reaches the end faster. Here the mass of glider is $M = 92.52gm$.

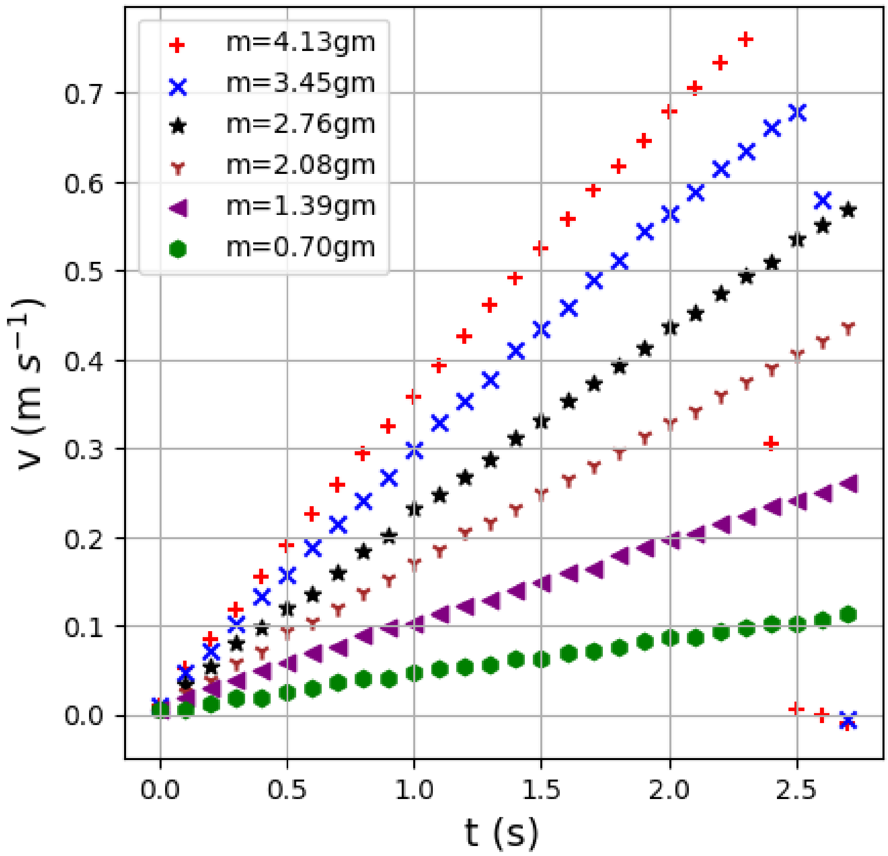

Figure 8: v vs t

Here we see that the velocity decreases for greater m which is because of the same reason as mentioned above.

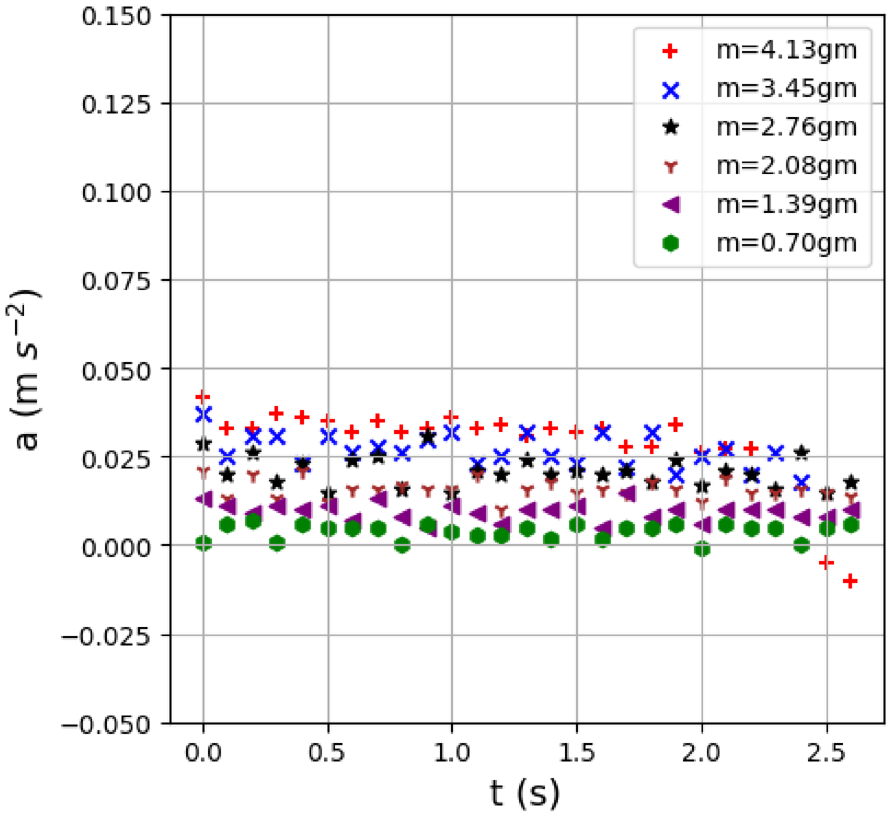

Figure 9: a vs t

In acceleration also we see the change the same deviation form the rest of the value near the end.

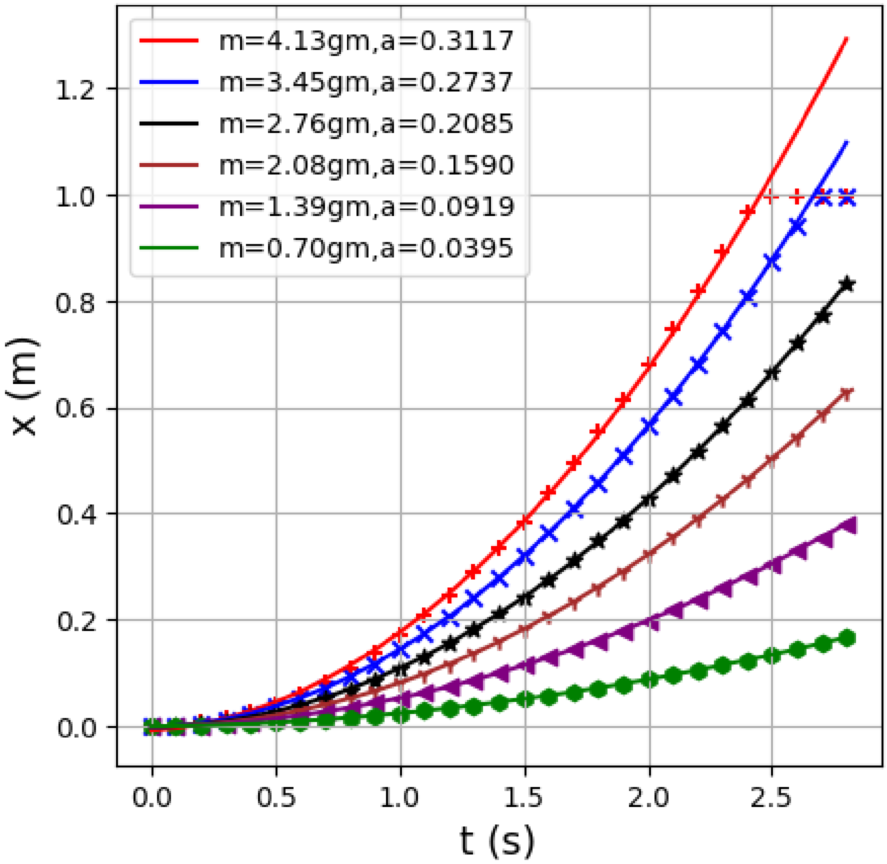

Figure 10: Fitted curve for x vs t

While fitting I have deliberately ignored the last few points for $m = 4.13gm$ and $3.45gm$ because we know that in these cases the glider reaches the end faster as compared to the other cases which explains the flattening of the curve near the end.

Table 2: a from fitting the x vs t

m (gm)	mg (N)	a (m s^{-2})
4.13	0.0405	0.3117
3.45	0.0338	0.2737
2.76	0.0270	0.2085
2.08	0.0204	0.1590
1.39	0.0136	0.0919
0.70	0.0069	0.0395

We will now plot the mg vs a and then do the fitting on this data, from the equation given below we expect that the value of slope should match $M + m = 96.65gm$.

$$a(M + m) = mg$$

The value of $M + m$ we get from the fitting is $0.1198 \pm 0.0037N$, we can now calculate the

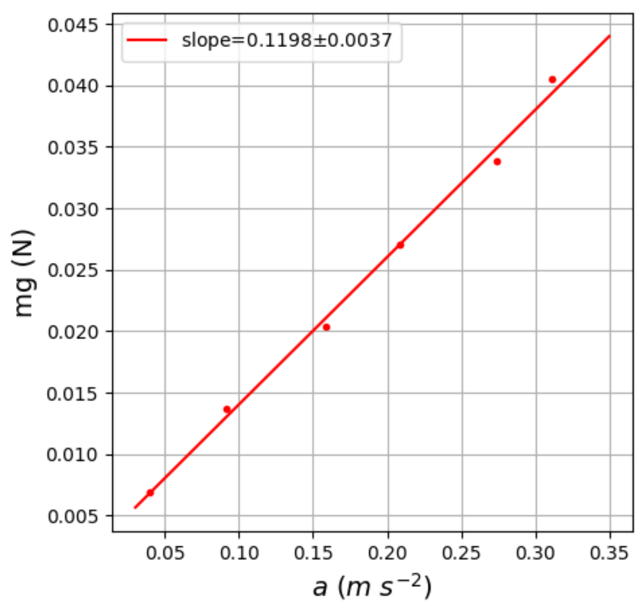

Figure 11: mg vs a

deviation from the observed value of $M + m$.

$$\text{error percentage} = \frac{|0.1198 - 0.09665|}{0.09665} \times 100$$

$$\boxed{\text{error percentage} = 23.95\%}$$

4 Conclusion

From Figure 6 we can clearly see that in case of fixed force we have,

$$(M+m) \propto \frac{1}{a}$$

where $M+m$ is the total mass of the system.
From Figure 11 we can see that in case of fixed mass case we have, here $F=mg$,

$$a \propto F$$

We can combine the two equations to get,

$$(M+m)a = kF$$

$$\boxed{(M+m)a = kmg}$$

where k is a constanst.
We will now use the value from the fit which we got in Figure 6 and Figure 11 and calculte the value of k.

$$k_{\text{fixed force}} = \frac{(mg)_{\text{obs}}}{(mg)_{\text{fit}}} = \frac{0.0405}{0.0382} = 1.06$$

$$k_{\text{fixed mass}} = \frac{(M+m)_{\text{obs}}}{(M+m)_{\text{fit}}} = \frac{0.1198}{0.0967} = 0.80$$

We can clearly see that the constant $k \approx 1$ which means that we can write the equation by substituting $k=1$.

$$\boxed{(M+m)a = mg}$$

Which is exactly what's predicted by *Law 2*, therefore we have succesfully verified that the *Law 2* is certainly **correct**.