Questão 1

$$\begin{cases} R^\circ \cdot R \subseteq id \\ R^\circ \cdot S \subseteq \perp \\ S^\circ \cdot S \subseteq id \end{cases}$$

$\Leftarrow$ { (5.116) e i_1, i_2 são funções, logo são simples $\Rightarrow$ lowering upper side }

$$\begin{cases} R^\circ \cdot R \subseteq i_1^\circ \cdot i_1 \\ R^\circ \cdot S \subseteq i_1^\circ \cdot i_2 \\ S^\circ \cdot S \subseteq i_2^\circ \cdot i_2 \end{cases}$$

$\equiv$ { (5.46) e (5.47) }

$$\begin{cases} i_1 \cdot R^\circ \cdot R \cdot i_1^\circ \subseteq id \\ i_1 \cdot R^\circ \cdot S \cdot i_2^\circ \subseteq id \\ i_2 \cdot S^\circ \cdot S \cdot i_2^\circ \subseteq id \end{cases}$$

$\equiv$ { (5.16) }

$$\begin{cases} (R \cdot i_1^\circ)^\circ \cdot (R \cdot i_1^\circ) \subseteq id \\ (R \cdot i_1^\circ)^\circ \cdot (S \cdot i_2^\circ) \subseteq id \\ (S \cdot i_2^\circ)^\circ \cdot (S \cdot i_2^\circ) \subseteq id \end{cases}$$

$\equiv$ { (5.32) e (5.36) e (5.85) }

$$\begin{cases} (R \cdot i_1^\circ) \text{ é simples} \\ (R \cdot i_1^\circ)^\circ \cdot (S \cdot i_2^\circ) \subseteq id \\ (S \cdot i_2^\circ) \text{ é simples} \end{cases}$$

$\equiv \{(5.70)\}$

$(R \cdot i_1^\circ) \cup (S \cdot i_2^\circ)$ é simples

$\equiv \{(5.117)$ e def $M^\circ\}$

M° é simples

$\equiv \{(5.34) ; (5.36)\}$

M é sobrejetivo

(b) o programador obriga que todas as lojas tenham pelo menos um colaborador a trabalhar num turno. (notemos que é essa a definição de sobrejetividade)

<u>Questão 2</u>

$f \cdot g^\circ$ é simétrica $\equiv \frac{f}{g} \cdot \frac{f}{g}$ é reflexiva

$f \cdot g^\circ$ é simétrica

$\equiv \{(5.87)\}$

$f \cdot g^\circ \subseteq (f \cdot g^\circ)^\circ$

$\equiv \{(5.16)$ e $(5.15)\}$

$f \cdot g^\circ \subseteq g \cdot f^\circ$

$\equiv \{(5.46)$ e $(5.47)\}$

$id \subseteq f^\circ \cdot g \cdot f^\circ \cdot g$ (*)

(*)

$\equiv \{(5.49)\}$

$id \subseteq \frac{f}{g} \cdot \frac{f}{g}$

$\equiv \{(5.84)\}$

$\frac{f}{g} \cdot \frac{f}{g}$ é reflexivo

Questão 3

$$\pi_1 = \langle id, T\rangle^\circ$$

$:: \{ \mathrm{II} \}$

$$X \subseteq \langle id, T\rangle^\circ$$

$\equiv \{ (5.136) \}$

$$X^\circ \subseteq \langle id, T\rangle$$

$\equiv \{ (5.103) \}$

$$\begin{cases} \pi_1 \cdot X^\circ \subseteq id \\ \pi_2 \cdot X^\circ \subseteq T \end{cases}$$ ← universalmente verdadeira

$\equiv \{ (5.46) \text{ e } (5.25) \}$

$$X^\circ \subseteq \pi_1^\circ$$

$\equiv \{ (5.137) \}$

$$X \subseteq \pi_1$$

$$\pi_1 \cdot \langle R, S\rangle = R \cdot \delta S$$

$:: \{ \mathrm{II} \}$

$$X \subseteq \pi_1 \cdot \langle R, S\rangle$$

$\equiv \{ (F2) \}$

$$X \subseteq \langle id, T\rangle^\circ \cdot \langle R, S\rangle$$

$\equiv \{ (5.108) \}$

$$X \subseteq (id^\circ \cdot R) \cap (T^\circ \cdot S)$$

(*)

(*) $\equiv \{ id^\circ = id \,;\, (5.13) \,;\, T^\circ = T \}$

$$X \subseteq R \cap T \cdot S$$

$\equiv \{ S := \Phi_q = \rho S ; \text{pos restriction of } R \}$

$$X \subseteq \rho S \cdot R$$

Questão 4

$$R \subseteq R \cdot R^{\circ} \cdot R$$

$\Leftarrow$ { lowering upper bound }

$$R \subseteq (R \cdot R^{\circ} \cap id) \cdot R$$

$\equiv$ { $\rho R = img\ R \cap id$; (5.33) }

$$R \subseteq \rho R \cdot R$$

$\equiv$ { }

true

Questão 5

$$R \dagger S = R$$

:: { II }

$$R \dagger S \subseteq X$$

$\equiv$ { def $\dagger$ }

$$S \cup R \cap \bot / S^{\circ} \subseteq X$$

$\Leftarrow$ { raise lower side: $S \subseteq R$ }

$$R \cup R \cap \bot / S^{\circ} \subseteq X$$

$\equiv$ { $R \cup (R \cap S) \equiv R$ }

$$R \subseteq X$$

Ou seja, $R = R \dagger S \Leftarrow S \subseteq R$

Questão 6

$$t = (a^* \times a^* \longleftarrow a^*) \longleftarrow (2 \longleftarrow a)$$

$$R_t = R_{(a^* \times a^* \longleftarrow a^*) \longleftarrow (2 \longleftarrow a)}$$

$\equiv \{ (179) \}$

$$R_t = R_{a^* \times a^* \longleftarrow a^*} \longleftarrow R_{2 \longleftarrow a}$$

$\equiv \{ (179) \}$

$$R_t = (R_{a^* \times a^*} \leftarrow R_{a^*}) \longleftarrow (R_2 \longleftarrow R_a)$$

$\equiv \{ (178) \}$

$$R_t = (R_a^* \times R_a^* \leftarrow R_a^*) \longleftarrow (id \longleftarrow R_a)$$

$\equiv \{ R_a := R \}$

$$R_t = (R^* \times R^* \leftarrow R^*) \longleftarrow (id \longleftarrow R)$$

—— // ——

$$break \; R_t \; break$$

$\equiv \{ R_t \text{ calculado} \}$

$$break ((R^* \times R^* \leftarrow R^*) \leftarrow (id \leftarrow R)) \, break$$

$\equiv \{ \text{Reynolds - arrow} \}$

$$break \cdot (id \leftarrow R) \subseteq (R^* \times R^* \leftarrow R^*) \cdot break$$

$\equiv \{ (5.46) \}$

$$(id \leftarrow R) \subseteq break^\circ \cdot (R^* \times R^* \longleftarrow R^*) \cdot break$$

$\equiv$ { Vars ; reynolds arrow ; guardanapo }

$$\langle \forall_{p,q} :: p \cdot R \subseteq q \Rightarrow (\text{break } p)(R^* \times R^* \leftarrow R^*)(\text{break } q) \rangle$$

$\equiv$ { reynolds arrow }

$$\langle \forall_{p,q} :: p \cdot R \subseteq q \Rightarrow (\text{break } p) \cdot R^* \subseteq (R^* \times R^*) \cdot (\text{break } q) \rangle$$

——— " ———

$R := f$

$$p \cdot f = q \Rightarrow (\text{break } p) \cdot f^* = (f^* \times f^*) \cdot (\text{break } q)$$

$\equiv$

$$(\text{break } p) \cdot f^* = (f^* \times f^*) \cdot (\text{break } (p \cdot f))$$

$\equiv$ { $f^* = \text{map } f$ }

$$\text{break } p \cdot \text{map } f = (\text{map } f \times \text{map } f) \cdot \text{break } (p \cdot f)$$

$\equiv$ { vars }

$$\text{break } p \, (\text{map } f \, x) = (\text{map } f \times \text{map } f) \, (\text{break } (p \cdot f) \, x)$$

$\equiv$ { }

$$\begin{cases} (y_1, y_2) = \text{break } p \, (\text{map } f \, x) \\ (x_1, x_2) = \text{break } (p \cdot f) \, x \\ (y_1, y_2) = (\text{map } f \times \text{map } f)(x_1, x_2) \end{cases}$$

$\equiv$ { def $\times$ }

$$\begin{cases} \text{———} \\ \text{———} \\ y_1 = \text{map } f \, x_1 \wedge y_2 = \text{map } f \, x_2 \end{cases}$$

Questão 7

$$joinOK\ (link\ (m,n)\ (R,S))$$

$\equiv \{$ def link $\}$

$$joinOK\ (R \cup \underline{n} \cdot \underline{m}^{\circ}, S)$$

$\equiv \{$ def joinOK; (5.138) $\}$

$$\ker (R \cup \underline{n} \cdot \underline{m}^{\circ}) \subseteq S^{\circ} \cdot T \cdot S \cup id$$

$\equiv \{$ 5.32; (5.16); (5.15); (5.65) $\}$

$$(R^{\circ} \cup \underline{m} \cdot \underline{n}^{\circ}) \cdot (R \cup \underline{n} \cdot \underline{m}^{\circ}) \subseteq S^{\circ} \cdot T \cdot S \cup id$$

$\equiv \{$ (5.60); (5.61) $\}$

$$(R^{\circ} \cdot R \cup \underline{m} \cdot \underline{n}^{\circ} \cdot R) \cup (R^{\circ} \cdot \underline{n} \cdot \underline{m}^{\circ} \cup \underline{m} \cdot \underline{n}^{\circ} \cdot \underline{n} \cdot \underline{m}^{\circ}) \subseteq S^{\circ} \cdot T \cdot S \cup id$$

$\equiv \{$ (5.59) $\}$

$$\begin{cases} R^{\circ} \cdot R \subseteq S^{\circ} \cdot T \cdot S \cup id \\ \underline{m} \cdot \underline{n}^{\circ} \cdot R \cup R^{\circ} \cdot \underline{n} \cdot \underline{m}^{\circ} \cup \underline{m} \cdot \underline{n}^{\circ} \cdot \underline{n} \cdot \underline{m}^{\circ} \subseteq S^{\circ} \cdot T \cdot S \cup id \end{cases}$$

$\equiv \{$ (5.32); (5.138); def joinOK $\}$

$$\begin{cases} joinOK\ (R,S) \\ \underline{m} \cdot \underline{n}^{\circ} \cdot R \cup R^{\circ} \cdot \underline{n} \cdot \underline{m}^{\circ} \cup \underline{m} \cdot \underline{n}^{\circ} \cdot \underline{n} \cdot \underline{m}^{\circ} \subseteq S^{\circ} \cdot T \cdot S \cup id \end{cases}$$

$\Leftarrow \{$ monotonia (5.39); $A \cup A = A \}$

$$\begin{cases} joinOK(R,S) \\ \underline{m} \cup R^{\circ} \cdot \underline{n} \cdot \underline{m}^{\circ} \subseteq S^{\circ} \cdot T \cdot S \cup id \end{cases}$$

$\equiv \{ (5.67); (5.152) \}$

$$\begin{cases} joinOK(R,S) \\ (\underline{m} \cup R^{\circ} \cdot \underline{n} \cdot \underline{m}^{\circ}) \cap (\underline{m} \cup \neg \underline{m}) \subseteq S^{\circ} \cdot T \cdot S \cup id \end{cases}$$

$\equiv \{ (A \cup B) \cap (A \cup C) \equiv A \cup (B \cap C) \}$

$$\begin{cases} joinOK(R,S) \\ \underline{m} \cup (R^{\circ} \cdot \underline{n} \cdot \underline{m}^{\circ} \cap \neg \underline{m}) \subseteq S^{\circ} \cdot T \cdot S \cup id \end{cases}$$

$\Leftarrow \{ (5.81) \}$

$$\begin{cases} joinOK(R,S) \\ \underline{m} \subseteq id \equiv True \\ R^{\circ} \cdot \underline{n} \cdot \underline{m}^{\circ} \cap \neg \underline{m} \subseteq S^{\circ} \cdot T \cdot S \end{cases}$$

$\equiv \{$ Pointwise; $\underline{n}\ a = n$; (5.14) $\}$

$$\begin{cases} joinOK(R,S) \\ \langle \forall k,k' : \langle \exists i :: i R k : \langle \exists j :: i = n : k' = m \rangle\rangle \wedge k \neq m : \langle \exists s : s S k : \langle \exists s' : s T s' : s' S k' \rangle\rangle \end{cases}$$

$\equiv$ { One point }

$$\left\langle \forall k : nRk \wedge k \neq m : \left\langle \exists s', s : sSk : s' S m \right\rangle \right\rangle$$