Lab Demonstration 1

Fractals with Tensorflow

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In [3]:
```
import tensorflow as tf
tf.__version__
```

Out[3]: '1.14.0'

Part 1

In [2]:
```
import numpy as np

sess = tf.InteractiveSession()
```

Producing a 2D Gaussian image

In [3]:
```
X, Y = np.mgrid[-4.0:4:0.01, -4.0:4:0.01]
```

In [ ]:
```

```

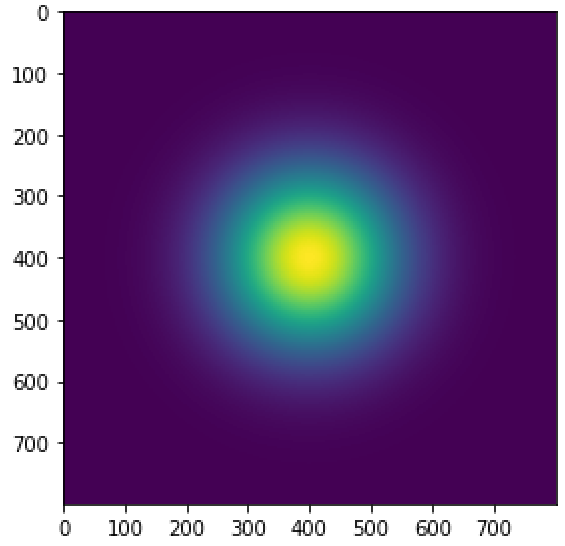
In [4]:
```
xs = tf.constant(X.astype(np.float32))
ys = tf.constant(Y.astype(np.float32))
```

In [5]:
```
tf.global_variables_initializer().run() #init variables
```

In [ ]:
```

```

In [6]:
```
zs = tf.exp(-(xs**2+ys**2)/2.0)
```

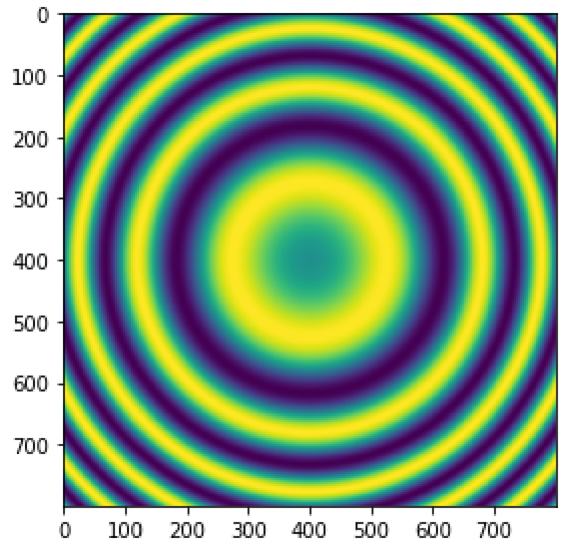
In [8]:
```
#plot
import matplotlib.pyplot as plt
plt.imshow(zs.eval())
plt.tight_layout()
plt.show()
```



Use $sin(x)$

In [9]:
```
zs_sin = tf.sin(xs**2 + ys**2)
```

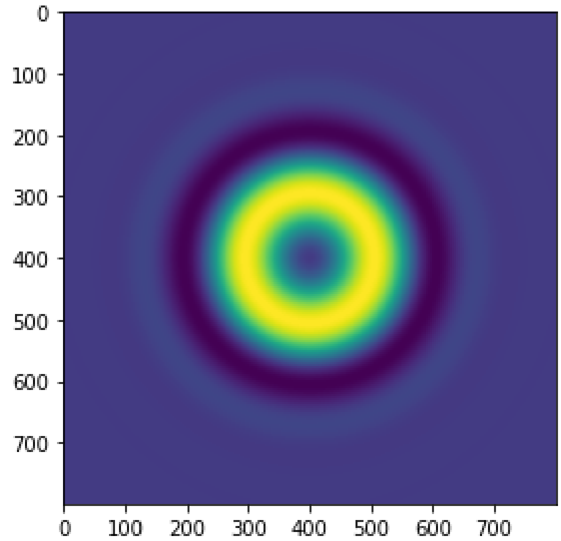
In [10]:
```
plt.imshow(zs_sin.eval())
plt.tight_layout()
plt.show()
```



Plot $e^{-\frac{r^2}{\sigma}} sin(r^2)$

In [11]:
```
zs_prod = zs * zs_sin
```

In [12]:
```
plt.imshow(zs_prod.eval())
plt.tight_layout()
plt.show()
```



Part 2

In [13]:
```
Y1, X1 = np.mgrid[-1.3:1.3:0.001, -2:1:0.001] # increasing resolution to 0.001
Z1 = X1+1j*Y1
```

In [ ]:

In [20]:
```
xs_const = tf.constant(Z1.astype(np.complex64))
zs_var = tf.Variable(xs_const)
ns_var = tf.Variable(tf.zeros_like(xs_const, tf.float32))
```

In [ ]:

In [21]:
```
tf.global_variables_initializer().run()
```

In [22]:
```
zs_ = zs_var*zs_var + xs_const
```

In [23]:
```
not_diverged = tf.abs(zs_) < 2
```

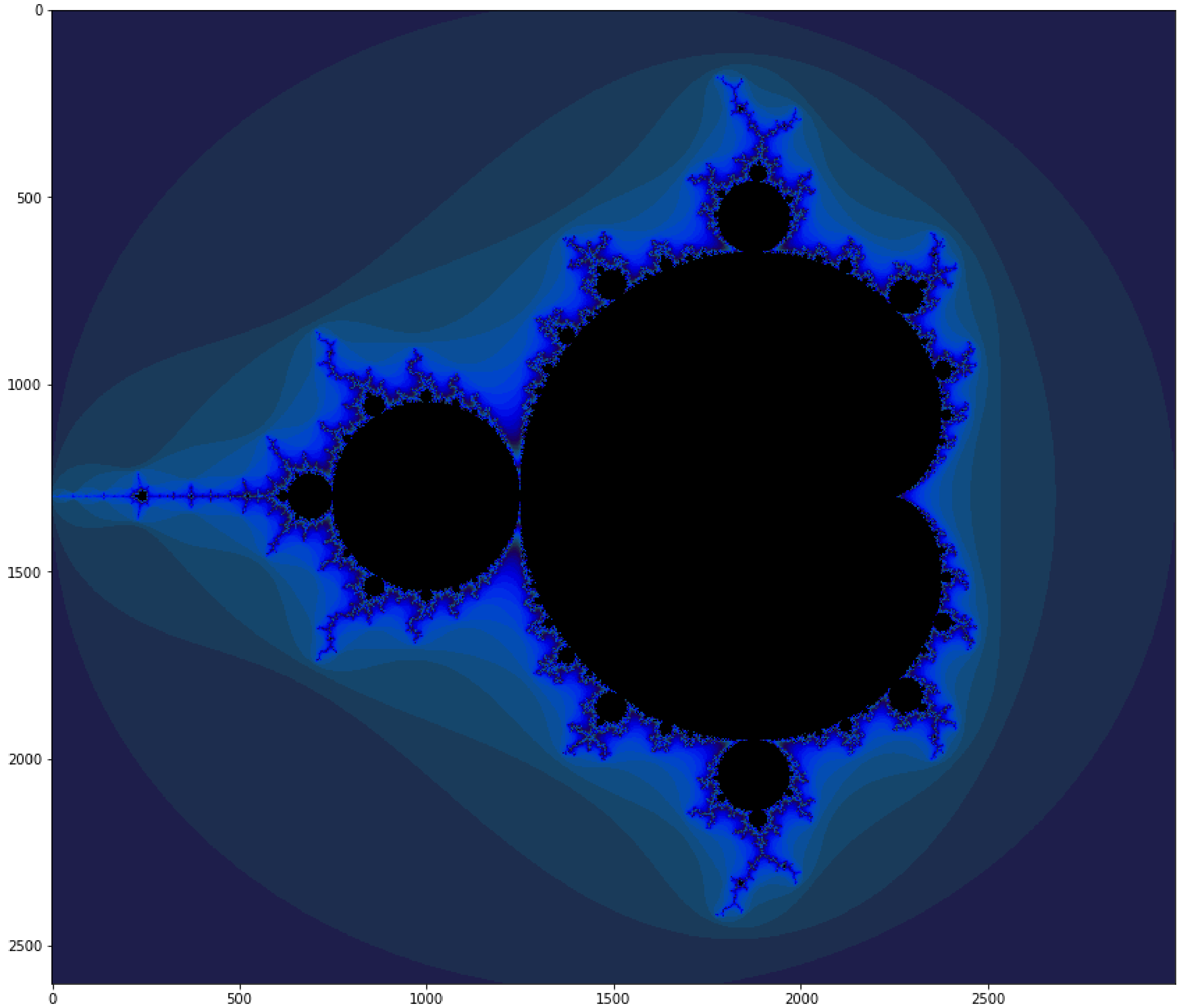
In [26]:
```
step = tf.group( zs_var.assign(zs_), ns_var.assign_add(tf.cast(not_diverged, t
f.float32)) )
```

In [27]:
```
for i in range(200):
    step.run()
```

In [28]:
```
def processFractal(a):
    """Display an array of iteration counts as a
    colorful picture of a fractal."""
    a_cyclic = (6.28*a/20.0).reshape(list(a.shape)+[1])
    img = np.concatenate([10+20*np.cos(a_cyclic), 30+50*np.sin(a_cyclic), 155-
80*np.cos(a_cyclic)], 2)
    img[a==a.max()] = 0
    a = img
    a = np.uint8(np.clip(a, 0, 255))
    return a
```

In [ ]:

In [29]:
```
fig = plt.figure(figsize=(16,10))
plt.imshow(processFractal(ns_var.eval()))
plt.tight_layout(pad=0)
plt.show()
```



Zooming in

In [30]:
```
Y2, X2 = np.mgrid[0.2:0.7:0.0001, 0:0.5:0.0001] # zooming in on smaller area a
nd increase resolution
Z2 = X2+1j*Y2
```

In [31]:
```
xs_const = tf.constant(Z2.astype(np.complex64))
zs_var = tf.Variable(xs_const)
ns_var = tf.Variable(tf.zeros_like(xs_const, tf.float32))
```

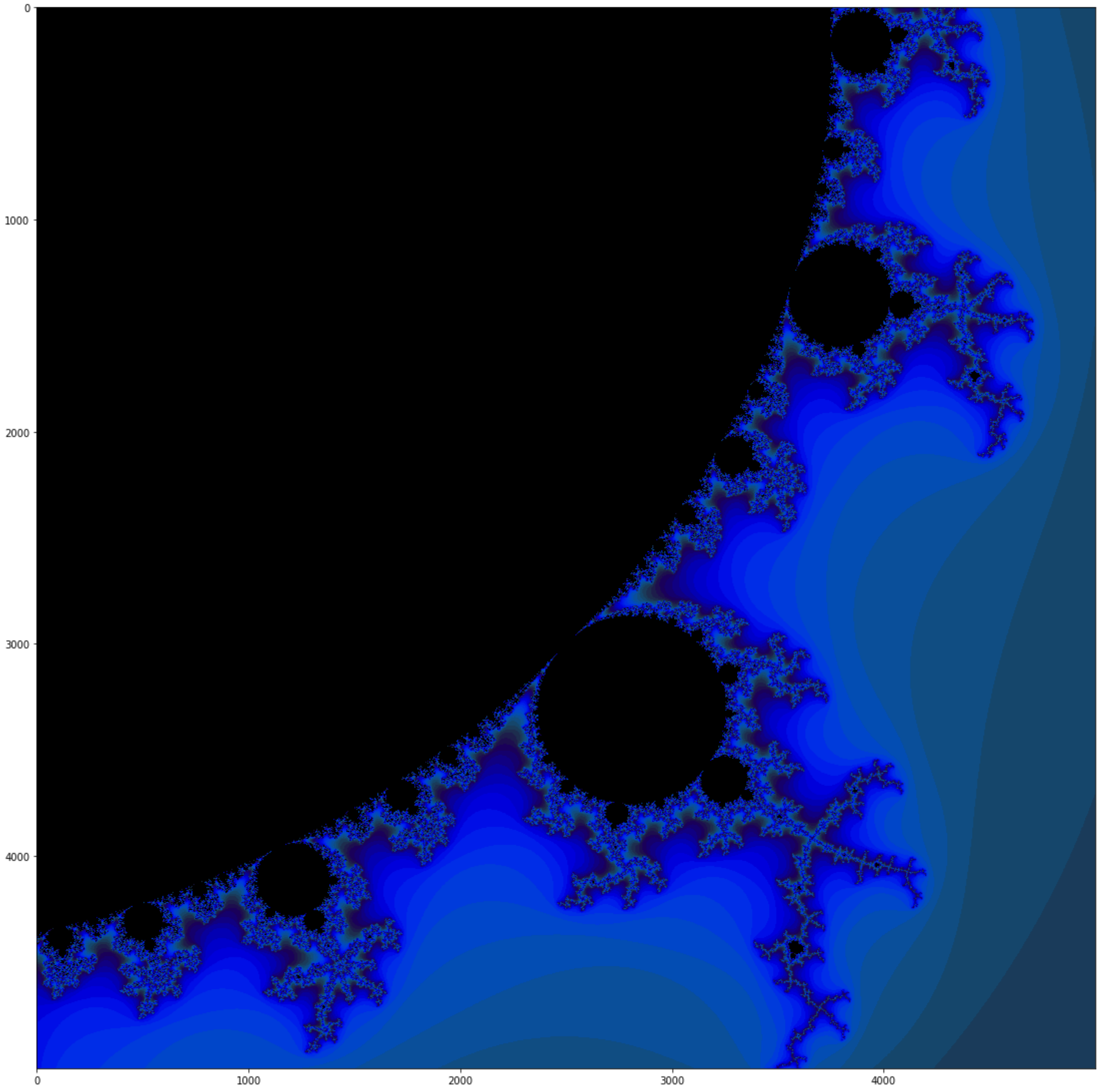
In [32]:
```
tf.global_variables_initializer().run()
```

In [33]:
```
zs_ = zs_var*zs_var + xs_const
not_diverged = tf.abs(zs_) < 2
step = tf.group( zs_var.assign(zs_), ns_var.assign_add(tf.cast(not_diverged, t
f.float32)) )
```

In [34]:
```
for i in range(200):
    step.run()
```

In [ ]:

In [35]:
```
fig = plt.figure(figsize=(15,15))
plt.imshow(processFractal(ns_var.eval()))
plt.tight_layout(pad=0)
plt.show()
```



Julia set

Set $c = (\Phi - 2) + (\Phi - 1)i = -0.4 + 0.6i$ where Φ is the Golden ratio

In [36]:
```
Y3, X3 = np.mgrid[-1.3:1.3:0.001, -2:2:0.001]
Z3 = X3+1j*Y3

c = -0.4 + 1j*0.6
```

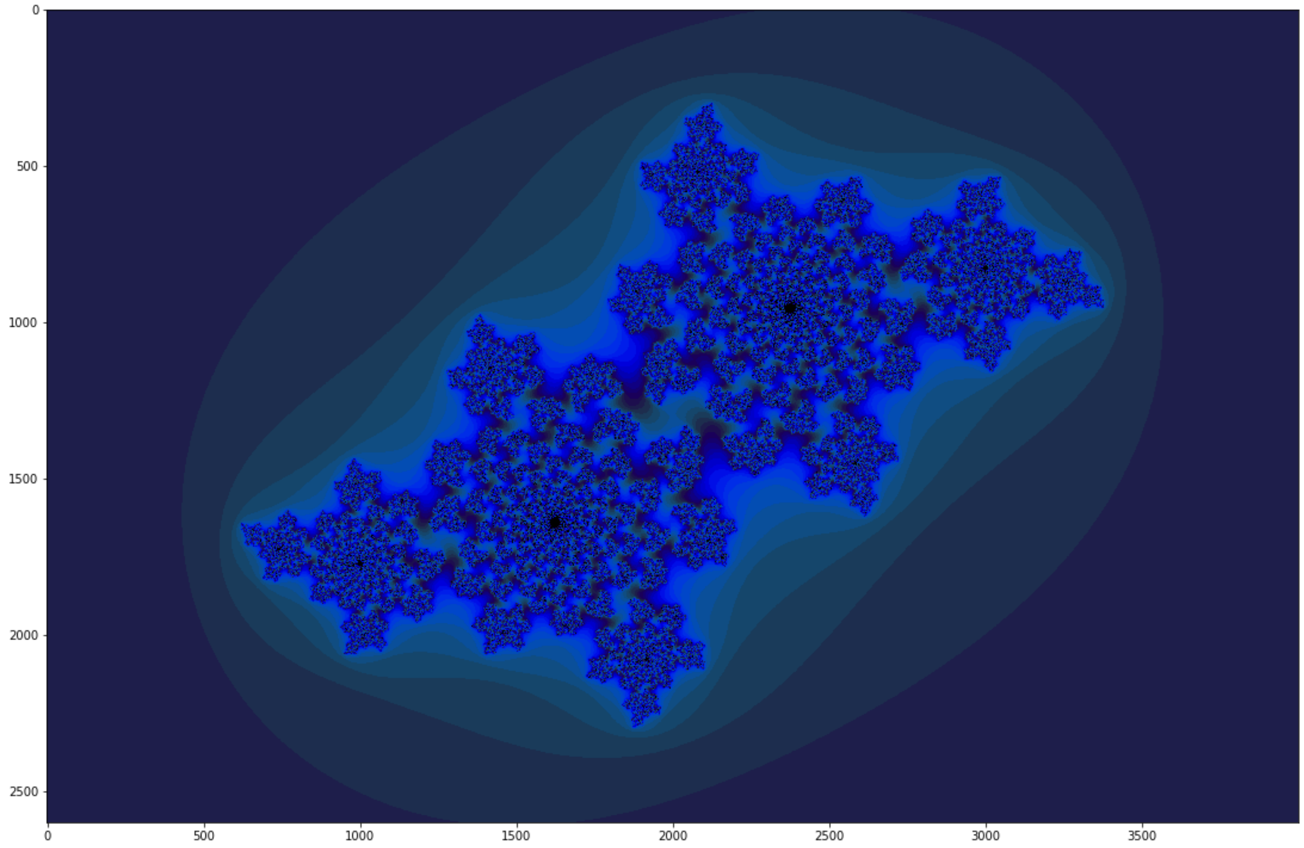
In [37]:
```
xs_const = tf.constant(c)
zs_var = tf.Variable(Z3)
ns_var = tf.Variable(tf.zeros_like(zs_var, tf.float32))
```

In [38]:
```
tf.global_variables_initializer().run()
```

In [39]:
```
zs_ = zs_var*zs_var + xs_const
not_diverged = tf.abs(zs_) < 2
step = tf.group( zs_var.assign(zs_), ns_var.assign_add(tf.cast(not_diverged, t
f.float32)) )
```

In [40]:
```
for i in range(200):
    step.run()
```

In [41]:
```
fig = plt.figure(figsize=(15,10))
plt.imshow(processFractal(ns_var.eval()))
plt.tight_layout(pad=0)
plt.show()
```



In [42]:
```
sess.close()
```

Part 3

Barnesley Fern using IFS (Iterated Function Systems)

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
import tensorflow as tf
```

In [2]:
```
sess2 = tf.InteractiveSession()
```

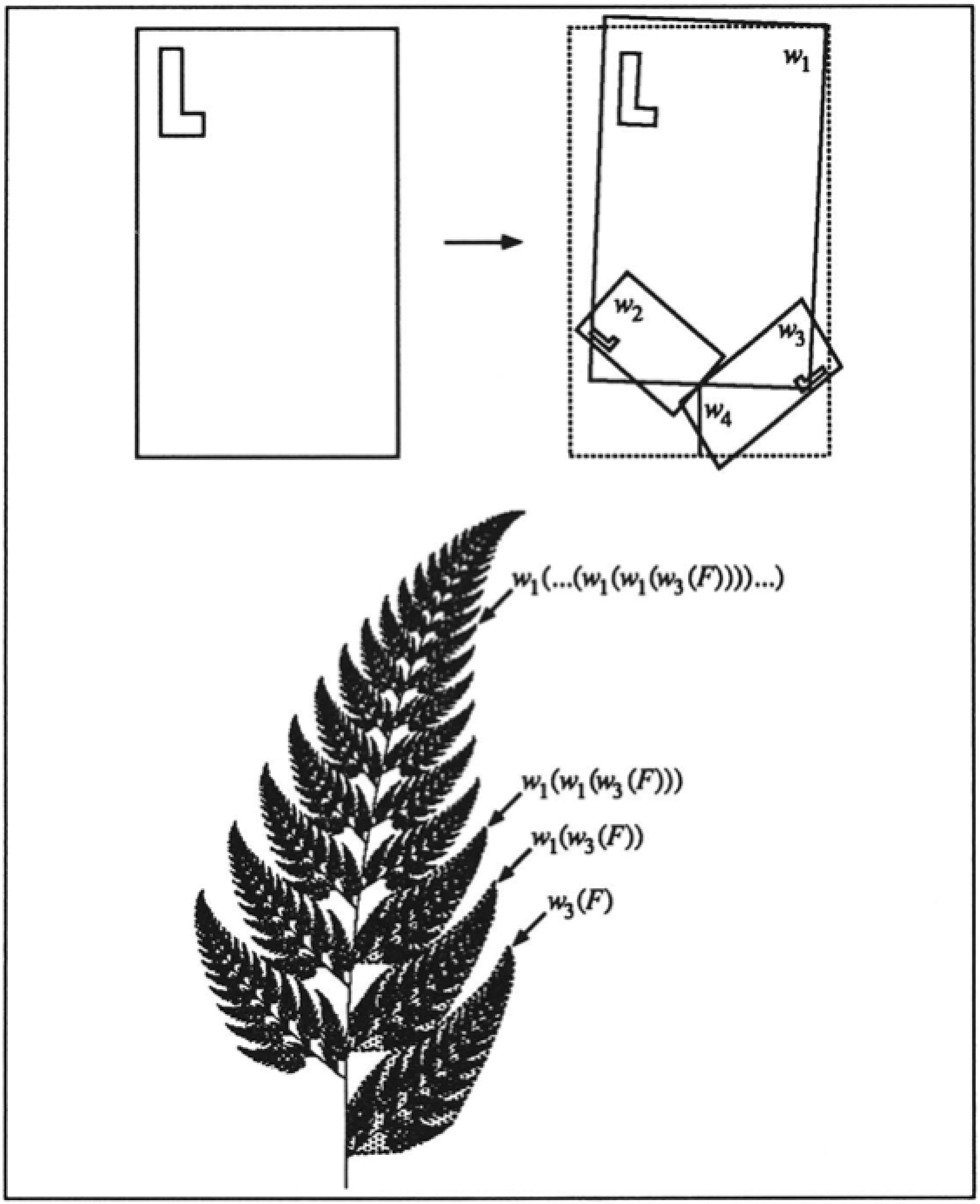
To produce each part of the fern, an affine transformation is chosen based on a probability distribution. Applying an affine transformation on a point $x_n = (x_1, x_2)^T$ is equivalent to the matrix operation below:

$x_{n+1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} x_n + \begin{bmatrix} e \\ f \end{bmatrix}$ where a, b, c, d, e, f parameters are taken from table 5.48 for Figure 5.25 on page 317 in the "Fractals For The Classroom" book.

The affine transformation above can be re-written with just one matrix/vector multiplication by augmenting x_n to be $(x_1, x_2, 1)^T$ and adding (e, f) as a third column and adding a third row of $(0, 0, 1)$ to keep the transformation matrix square with a 3x3 shape:

$$x_{n+1} = \begin{bmatrix} a & b & e \\ c & d & f \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_n \\ 1 \end{bmatrix}$$

To build the fern, we need to start from a point, could be $(0, 0)$, and iterate through the 4 different transformations, $\mathbf{w_i}$, with each is selected based on a probability p_i for $i \in 1, 2, 3, 4$ as shown in Figure 6.25 in the book, which was reproduced below:



In [3]:
```
tf_w1 = tf.constant([[0.849, 0.037, 0.075],
                     [-0.037, 0.849, 0.183],
                     [0.0, 0.0, 1.0]], dtype=tf.float32) # main body

tf_w2 = tf.constant([[0.197, -0.226, 0.4],
                     [0.226, 0.197, 0.049],
                     [0.0, 0.0, 1.0]], dtype=tf.float32) # left leaf

tf_w3 = tf.constant([[-0.15, 0.283, 0.575],
                     [0.26, 0.237, -0.084],
                     [0.0, 0.0, 1.0]], dtype=tf.float32) # right leaf

tf_w4 = tf.constant([[0.0, 0.0, 0.5],
                     [0.0, 0.16, 0.0],
                     [0.0, 0.0, 1.0]], dtype=tf.float32) # The stem

# pack them in an array of transforms
tf_transforms = [tf_w1, tf_w2, tf_w3, tf_w4]
```

In [4]:
```
tf_transforms
```

Out[4]:
```
[<tf.Tensor 'Const:0' shape=(3, 3) dtype=float32>,
 <tf.Tensor 'Const_1:0' shape=(3, 3) dtype=float32>,
 <tf.Tensor 'Const_2:0' shape=(3, 3) dtype=float32>,
 <tf.Tensor 'Const_3:0' shape=(3, 3) dtype=float32>]
```

The probabilities p_i are evaluated based on the formula on page 353 in the book:

$p_i = \frac{\max(\delta, |\det w_i|)}{\sum_{k=1}^{N} \max(\delta, |\det w_k|)}$ where $\delta > 0$ is a small constant that was set to 0.01 in this example to prevent p_i from being set to 0 if $\det w_i = 0$

It can be easily shown as well that: $\det \begin{bmatrix} a & b & e \\ c & d & f \\ 0 & 0 & 1 \end{bmatrix} = \det \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

In [5]:
```
delta = tf.constant(0.01, dtype=tf.float32) # set the delat constant

tf_dets = tf.linalg.det([tf_w1, tf_w2, tf_w3, tf_w4]) # calculate all determin
ants
tf_P = tf.Variable(tf.maximum(delta, tf.abs(tf_dets)))
tf_P = tf.assign(tf_P, tf.math.divide(tf_P, tf.reduce_sum(tf_P))) # normalize
 them to get p_i as a probability
```

In [6]:
```
tf.global_variables_initializer().run()
```

The probability distribution P of selecting different transform

In [7]:
```
P = tf_P.eval()
P
```

Out[7]:
```
array([0.7755387 , 0.09652755, 0.11719476, 0.01073901], dtype=float32)
```

Initializing the points array with a $x_0 = (0, 0, 1)^T$ as the first point

In [9]:
```
points = []
points.append(tf.constant([0.0,0.0, 1.0], dtype=tf.float32))
```

Run the transformations iteratively starting from $x_0 = (0, 0, 1)$:

$x_1 = w_i(x_0)$
$x_2 = w_i(x_1)$
$\ldots$
$x_n = w_i(x_{n-1})$

where $i \in 1, 2, 3, 4$ and on each iteration w_i is selected from the 4 different transformations based on the probability distribution P

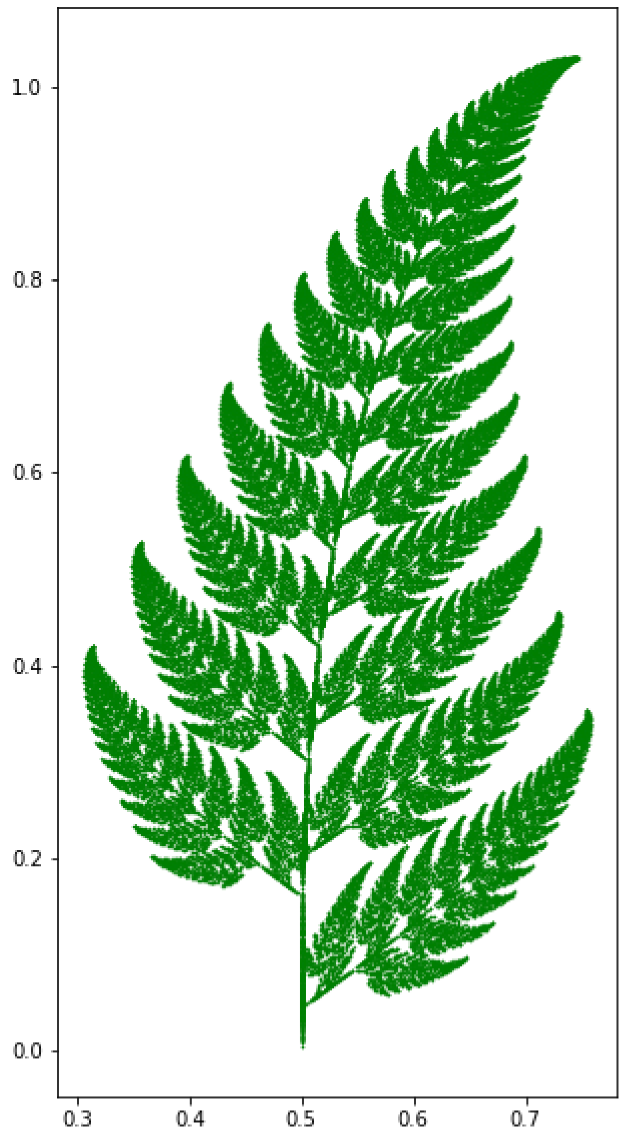
In [10]:
```
for i in range(100000): #generating 100,000 points
    tf_chosen_tran = np.random.choice(tf_transforms, p=P)
    nextP = tf.linalg.matvec(tf_chosen_tran, points[i])
    points.append(nextP)
```

In [11]:
```
tf_points = tf.Variable(points)
tf.global_variables_initializer().run()
points = tf_points.eval() #evaluating all the resulting tensors (takes very lo
ng)
```

Draw the resulting fractal after throwing away the first 10 points (burn in)

In [13]:
```
plt.figure(figsize=(5, 10))
plt.scatter(points[10:,0], points[10:,1], s=0.2, edgecolor='green')
```

Out[13]: <matplotlib.collections.PathCollection at 0x15325a780>



In [14]:
```
sess2.close()
```

TensorFlow is not suitable to this fractal generation using IFS. Points generation cannot be parallelised as each point is calculated based on the previous point, hence, the code is excuted serially. Performance is many times better when using only numpy library as below:

Part 3

(NumPy only)

In [1]:
```
import numpy as np
import matplotlib.pyplot as plt
```

In [24]:
```
np_w1 = np.array([[0.849, 0.037, 0.075],
                  [-0.037, 0.849, 0.183],
                  [0.0, 0.0, 1.0]])

np_w2 = np.array([[0.197, -0.226, 0.4],
                  [0.226, 0.197, 0.049],
                  [0.0, 0.0, 1.0]])

np_w3 = np.array([[-0.15, 0.283, 0.575],
                  [0.26, 0.237, -0.084],
                  [0.0, 0.0, 1.0]])

np_w4 = np.array([[0.0, 0.0, 0.5],
                  [0.0, 0.16, 0.0],
                  [0.0, 0.0, 1.0]])
```

In [30]:
```
# create a list of transform operations
frac_ops = [lambda p: np.dot(np_w1, p),
            lambda p: np.dot(np_w2, p),
            lambda p: np.dot(np_w3, p),
            lambda p: np.dot(np_w4, p)]
```

In [4]:
```
# Calculate probability distribution
np_delta = 0.01
np_P = np.array([max(np_delta, np.abs(np.linalg.det(np_w1))),
                 max(np_delta, np.abs(np.linalg.det(np_w2))),
                 max(np_delta, np.abs(np.linalg.det(np_w3))),
                 max(np_delta, np.abs(np.linalg.det(np_w4)))])

np_P = np_P / np_P.sum()
np_P
```

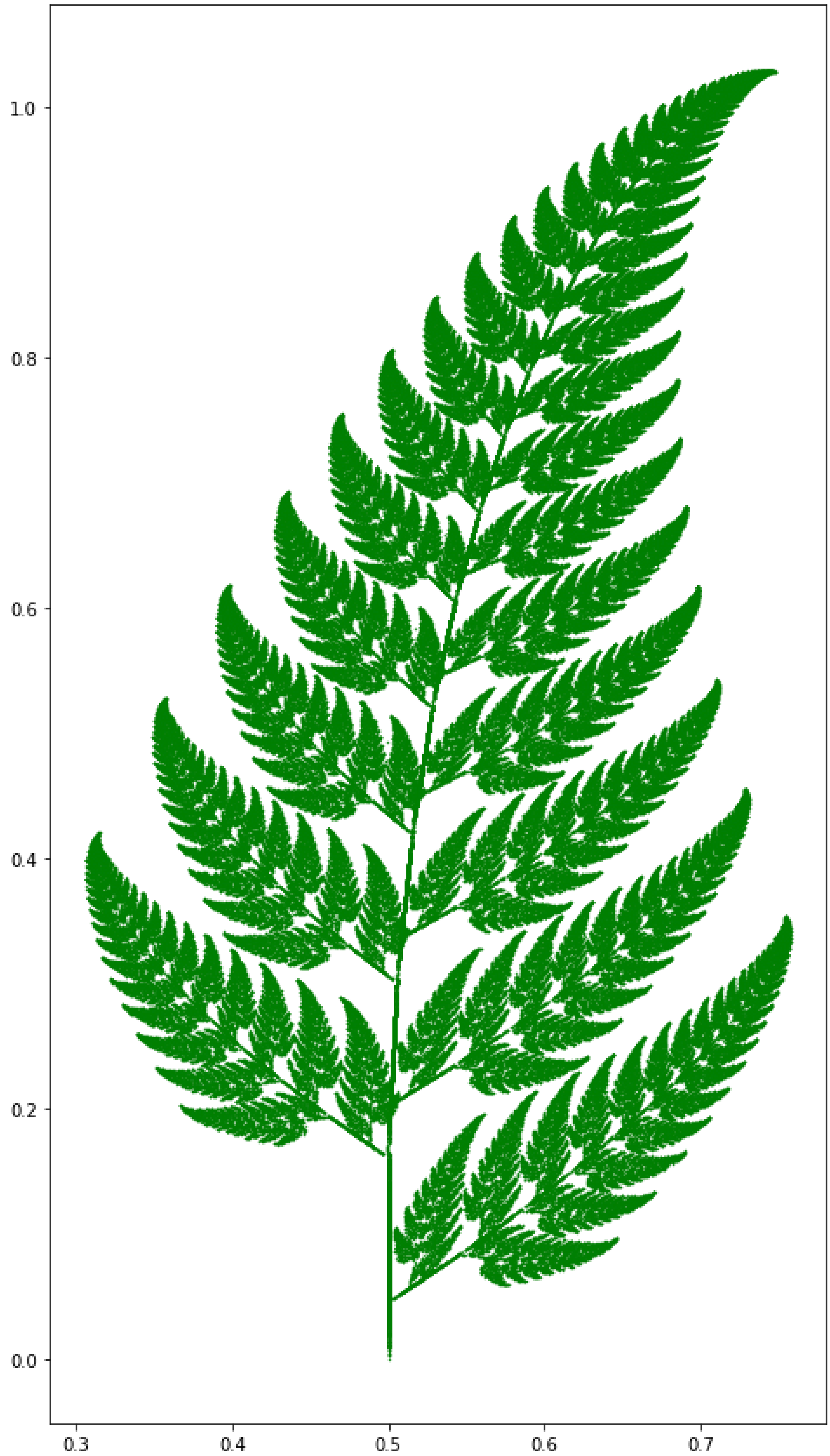
Out[4]:
```
array([0.7755387 , 0.09652754, 0.11719476, 0.010739  ])
```

In [47]:
```
# init points array
N = 300000
np_points = np.zeros((2,N))
np_points = np.row_stack((np_points, np.ones((1,N))))
```

In [56]:
```
# generate the points
for i in range(N-1):
    np_points.T[i+1] = np.random.choice(frac_ops, p=np_P)(np_points.T[i])
```

In [57]:
```
# Draw the fern
plt.figure(figsize=(8, 15))
plt.scatter(np_points[0,10:], np_points[1,10:], s=0.2, edgecolor='green')
```

Out[57]: <matplotlib.collections.PathCollection at 0x1153a70f0>



Generating A tree

In [50]:

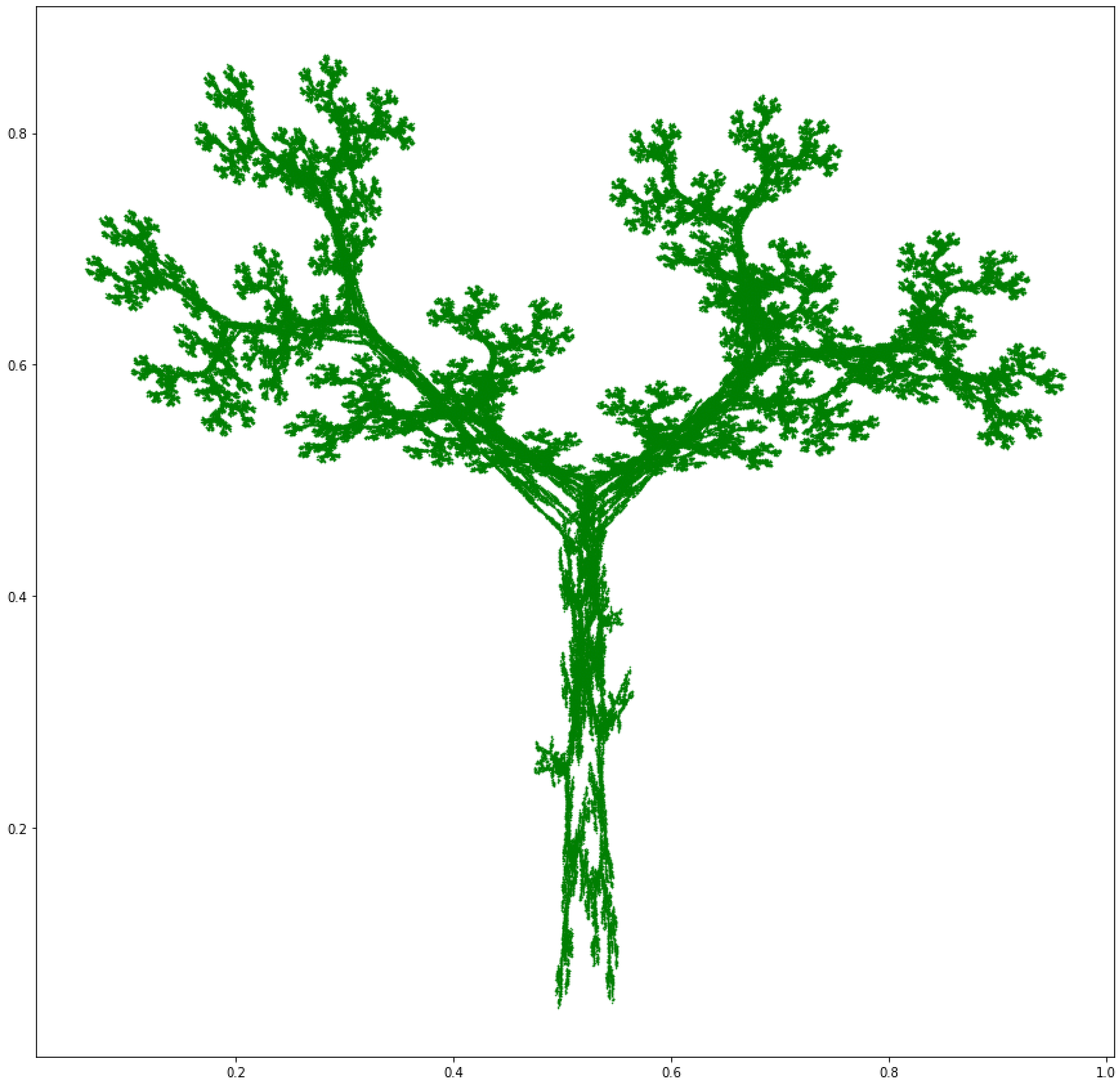
```
# initializing the affine transform
t1 = np.array([[0.195, -0.488, 0.4431], [0.344, 0.443, 0.2452], [0.0, 0.0, 1.0
]])
t2 = np.array([[0.462, 0.414, 0.2511], [-0.252, 0.361, 0.5692], [0.0, 0.0, 1.0
]])
t3 = np.array([[-0.058, -0.07, 0.5976], [0.453, -0.111, 0.0969], [0.0, 0.0, 1.
0]])
t4 = np.array([[-0.035, 0.07, 0.4884], [-0.469, -0.022, 0.5069], [0.0, 0.0, 1.
0]])
t5 = np.array([[-0.637, 0.0, 0.8562], [0.0, 0.501, 0.2513], [0.0, 0.0, 1.0]])

P_Atree = np.array([max(np_delta, np.abs(np.linalg.det(t1))),
                    max(np_delta, np.abs(np.linalg.det(t2))),
                    max(np_delta, np.abs(np.linalg.det(t3))),
                    max(np_delta, np.abs(np.linalg.det(t4))),
                    max(np_delta, np.abs(np.linalg.det(t5)))])

P_Atree = P_Atree / P_Atree.sum()

frac_ops_Atree = [lambda p: np.dot(t1, p),
                  lambda p: np.dot(t2, p),
                  lambda p: np.dot(t3, p),
                  lambda p: np.dot(t4, p),
                  lambda p: np.dot(t5, p)]
```

In [52]:

```
# generate the points
for i in range(N-1):
    np_points.T[i+1] = np.random.choice(frac_ops_Atree, p=P_Atree)(np_points.T
[i])
```

In [54]:
```
plt.figure(figsize=(15, 15))
plt.scatter(np_points[0,10:], np_points[1,10:], s=0.2, edgecolor='green')
```

Out[54]: <matplotlib.collections.PathCollection at 0x1153b5a20>



In [ ]: