Shri G. S. Institute of Technology and Science
Department Of Computer Engineering
CO 3463: Design and Analysis of Algorithms
Lab Assignment # 04 (Greedy Algorithms)
Marks: 10 points

Submission Date: 5 March 2017@23:59
Demo Date: 6 March 2017 - 10 March 2017
Late Submission: Not allowed
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Note: For the each program/algorithm, prove its correctness.

Q. 23. *Assume that we have **N** workers and **N** jobs that should be done. For each pair (worker, job) we know salary that should be paid to worker for him to perform the job. Our goal is to complete all jobs minimizing total inputs, while assigning each worker to exactly one job and vice versa.*

Converting this problem to a formal mathematical definition we can form the following equations:

$\{c_{ij}\}_{N \times N}$ – cost matrix, where c_{ij} – cost of worker i to perform job j.

$\{x_{ij}\}_{N \times N}$ – resulting binary matrix, where x_{ij} = 1 if and only if i^{th} worker is assigned to j^{th} job.

$$\sum_{j=1}^{N} x_{ij} = 1, \qquad \forall i \epsilon \overline{1,N}$$

– one worker to one job assignment.

$$\sum_{i=1}^{N} x_{ij} = 1, \qquad \forall j \epsilon \overline{1,N}$$

– one job to one worker assignment.

$$\sum_{i=1}^{N} \sum_{j=1}^{N} c_{ij} x_{ij} \rightarrow min$$

– total cost function.

We can also rephrase this problem in terms of graph theory. Let's look at the job and workers as if they were a bipartite graph, where each edge between the i^{th} worker and j^{th} job has weight of c_{ij}.

Write a program to find minimum-weight matching in the graph (the matching will *consists* of **N** edges, because our bipartite graph is complete).

Q. 24. Write a program for the following problem:

INPUT: A set S = { (x_i; y_i) $1 \le i \le n$ } of intervals over the real line.
OUTPUT: A maximum cardinality subset S' of S such that no pair of intervals in S' overlap.

Consider the following algorithm:

Repeat until S is empty
1. Select the interval I that overlaps the least number of other intervals.
2. Add I to final solution set S'.
3. Remove all intervals from S that overlap with I.

Prove or disprove that this algorithm solves the problem.

Q. 25. Write a program for the following problem:

INPUT: Positive integers r_1;r_2; : : : ; r_n and c_1; : : : ; c_n.
OUTPUT: An n by n matrix A with 0/1 entries such that for all i the sum of the ith row in A is r_i and the sum of the ith column in A is c_i, if such a matrix exists.
Think of the problem this way. You want to put pawns on an n by n chessboard so that the ith row has r_i pawns and the ith column has c_i pawns.

Q. 26. Write a program(s) for the following problem and compare the results:

INPUT: A set S = { (xi; yi) $1 \le i \_\le n$} of intervals over the real line. Think of interval (xi; yi) as being a request for a room for a class that meets from time xi to time yi.
OUTPUT: Find an assignment of classes to rooms that uses the fewest number of rooms.
Note that every room request must be honored and that no two classes can use a room at the same time.

(a) Consider the following iterative algorithm. Assign as many classes as possible to the first room (we can do this using the greedy algorithm discussed in class, and in the class notes), then assign as many classes as possible to the second room, then assign as many classes as possible to the third room, etc. Does this algorithm solve the Problem? Justify your answer.

(b) Consider the following algorithm. Process the classes in increasing order of start times. Assume that you are processing class C. If there is a room R such that R has been assigned to an earlier class, and C can be assigned to R without overlapping previously assigned classes, then assign C to R. Otherwise, put C in a new room. Does this algorithm solve the Problem?

Q. 27. You wish to drive from point A to point B along a highway minimizing the time that you are stopped for gas. You are told beforehand the capacity C of you gas tank in liters, your rate F of fuel consumption in liters/kilometer, the rate r in liters/minute at which you can fill your tank at a gas station, and the locations A=x1,x2, … , B=xn of the gas stations along the highway. So if you stop to fill your tank from 2 liters to 8 liters, you would have to stop for 6/r minutes. Consider the following two algorithms:

(a) Stop at every gas station, and fill the tank with just enough gas to make it to the next gas station.
(b) Stop if and only if you don't have enough gas to make it to the next gas station, and if you stop, fill the tank up all the way.

For each algorithm write a program and also either prove or disprove that this algorithm correctly solves the problem. Your proof of correctness must use an exchange argument.

Q. 28. The setting for this problem is storage system with a fast memory consisting of k pages and a slow memory consisting of n pages. At any time, the fast memory can hold copies of up to k of the pages in slow memory. The input consists of a sequence of pages from slow memory, think of these as being accesses to memory. If an accessed page is not in fast memory, then it must be swapped into fast memory, and if the fast memory was full, some page must selected to be evicted from fast memory. The goal is to determine the pages to evict so as to minimize the total number of evictions. Consider for example that k = 2, n = 4 pages are named A, B, C and D, and the access sequence is A B C A. Then after the first two pages, the fast memory contains A and B. When C is accessed then either A or B must be evicted. If B is evicted then no further evictions are necessary, and the total number of evictions is 1. If A was evicted, then either B or C must be evicted when A is accessed again, and the total number of evictions would be 2. Give a greedy for this problem and prove that it is correct using an exchange argument.

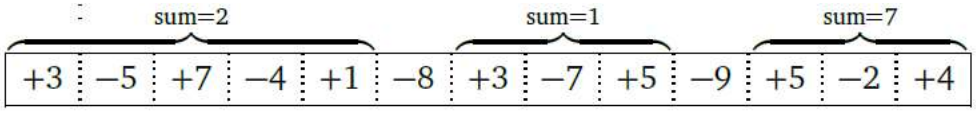
Q. 29. Write a program for the following problem:

Consider the following bridge crossing problem where n people with speeds s1; : : : ; sn wish to cross the bridge as quickly as possible.
The rules remain:

- It is nighttime and you only have one ash light.
- A maximum of two people can cross at any one time
- Any party who crosses, either 1 or 2 people must have the ash light with them.
- The ash light must be walked back and forth, it cannot be thrown, etc.
- A pair must walk together at the rate of the slower person's pace.

Give an efficient algorithm to find the fastest way to get a group of people across the bridge. You must have a proof of correctness for your method.

Q. 30. Suppose you are given an array A[1 .. n] of integers, each of which may be positive, negative, or zero. A contiguous subarray A[i .. j] is called a positive interval if the sum of its entries is greater than zero. Describe and analyze an algorithm to compute the minimum number of positive intervals that cover every positive entry in A. For example, given the following array as input, your algorithm should output the number 3.



Q. 31. Write a program using greedy techniques for yours own two problems and prove the correctness of yours own greedy algorithm.