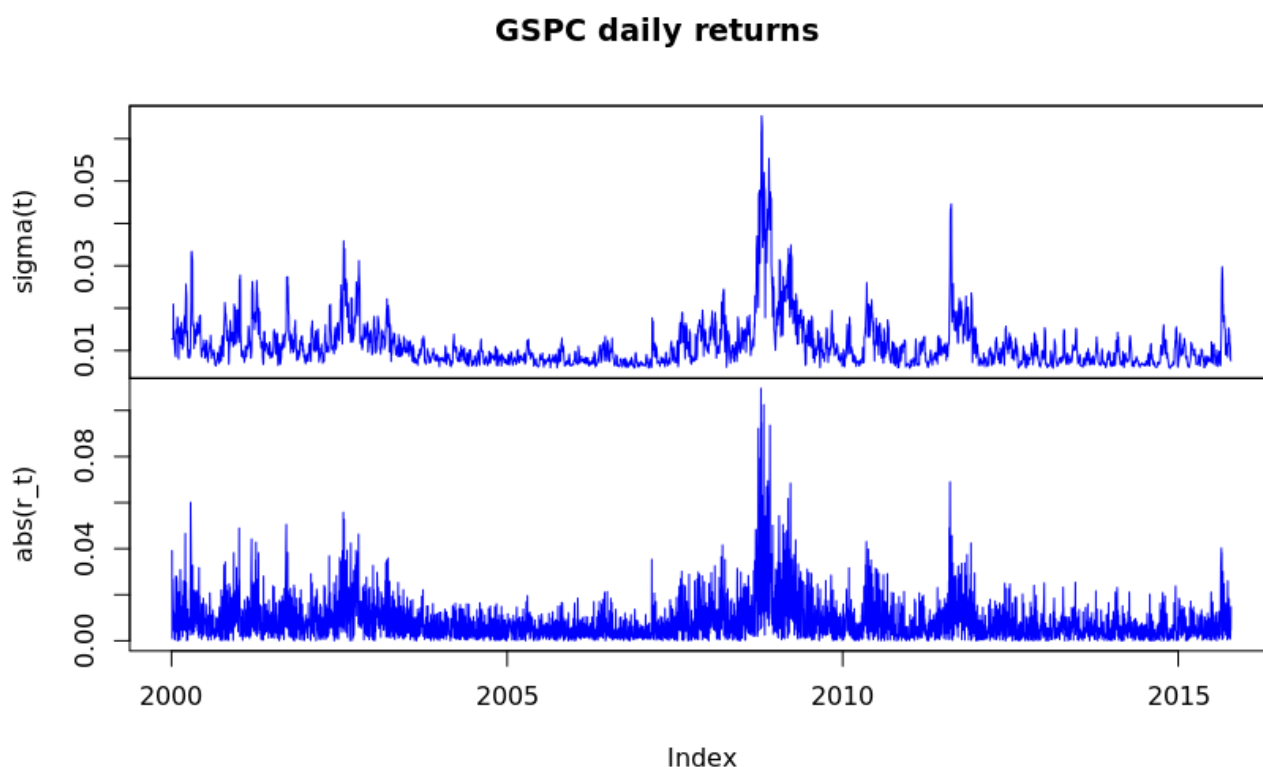


A5

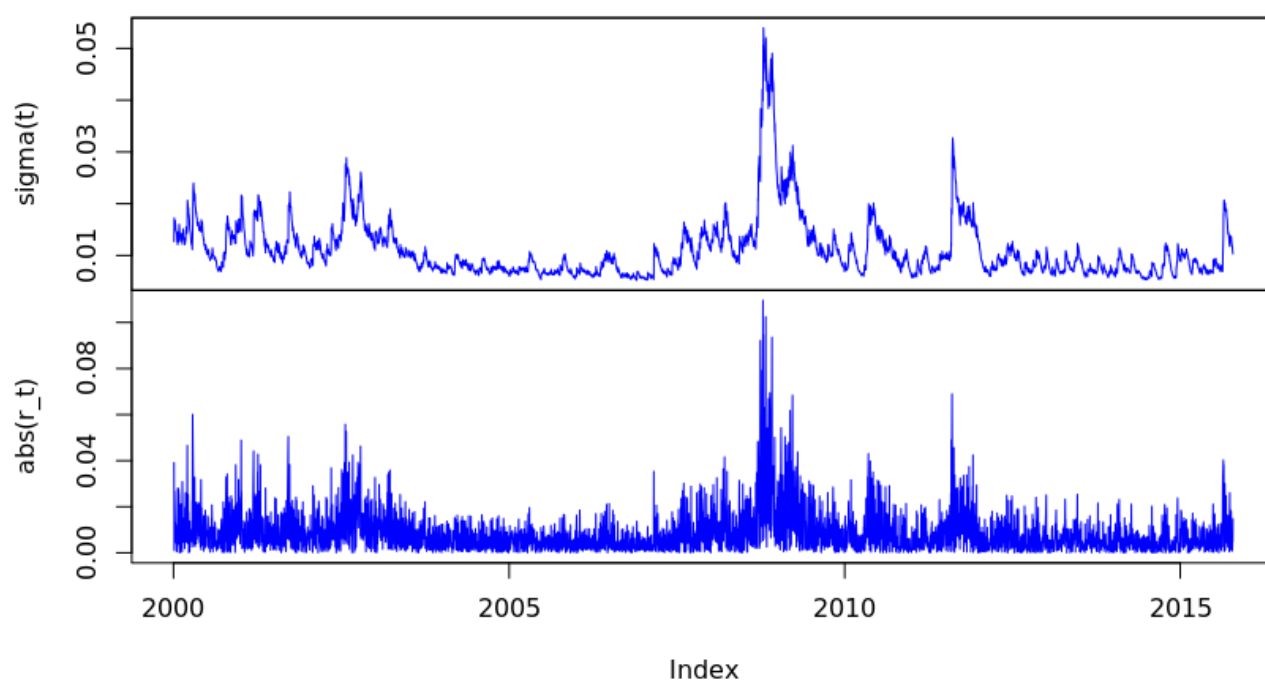
Q1

1. The sum of the ARCH coefficient estimates is **0.8024**.
2. The estimate of the unconditional volatility is **0.01303**, and the sample standard deviation of the returns is **0.01271**. These values are very close, suggesting that the estimate is very accurate, as it closely aligns with the sample's SD.

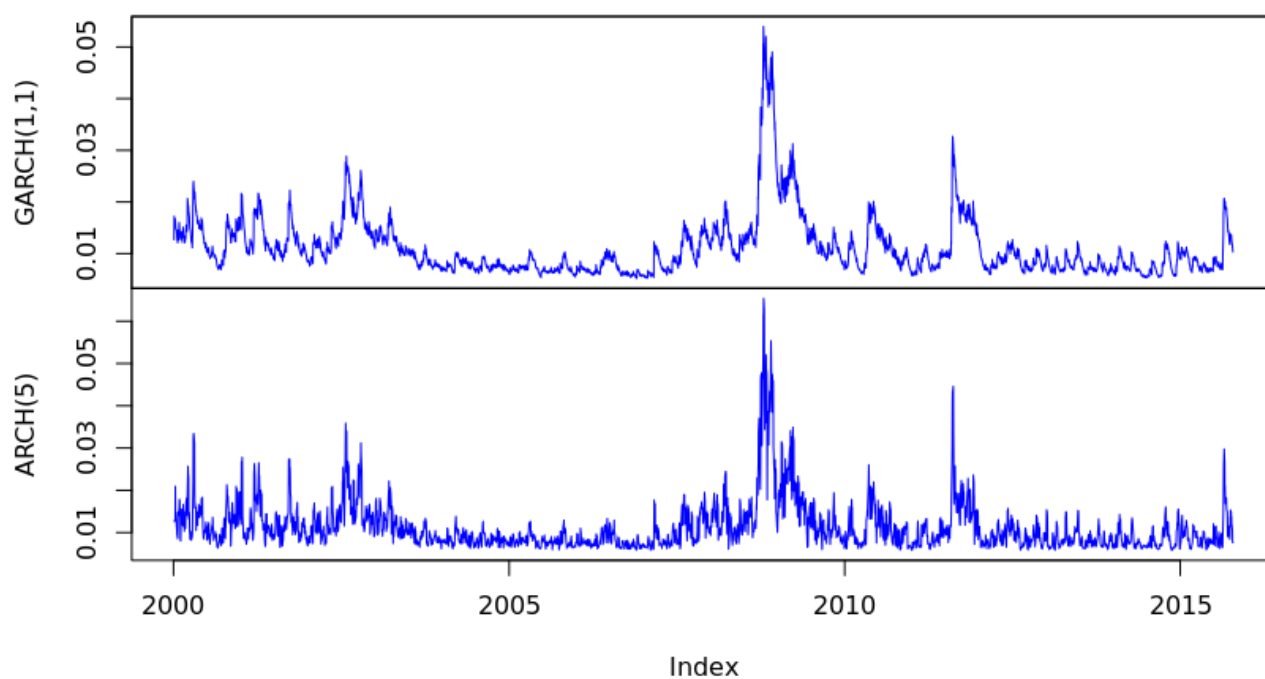


3. The sum of the GARCH(1,1) coefficient estimates is **0.987**.
4. The GARCH(1,1) estimates look very similar to those of the ARCH model, with the notable difference that the GARCH(1,1) model is much smoother. This is especially valuable because it means that models using this to trade would incur far fewer transaction costs.

GSPC daily returns

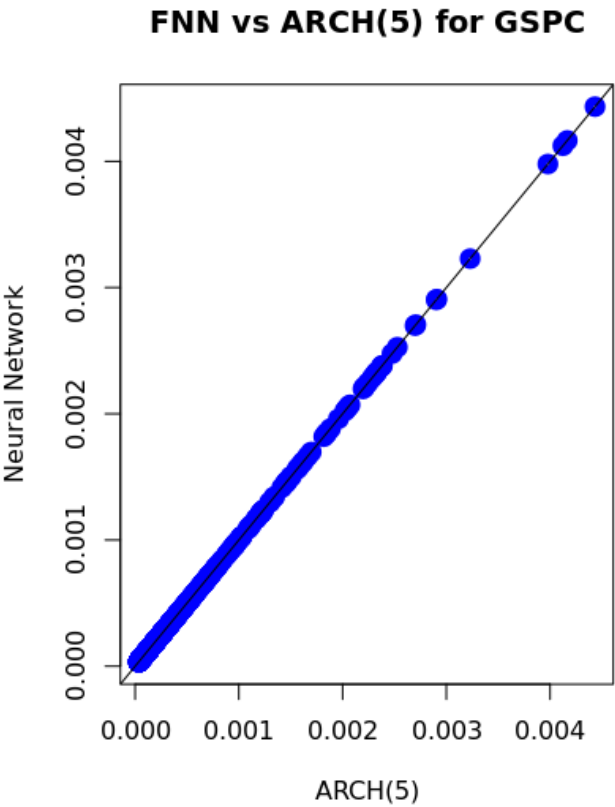


GSPC conditional vol



5. The estimate of the unconditional volatility of the returns for the GARCH(1,1) model is **0.01178**. This is lower than both the ARCH and sample SD values. Thus, it suggests that the GARCH(1,1) model is strongly representative of the underlying random variable/process.

1. The models fit exceptionally well; as shown in part B, the models have settled on identical parameters.



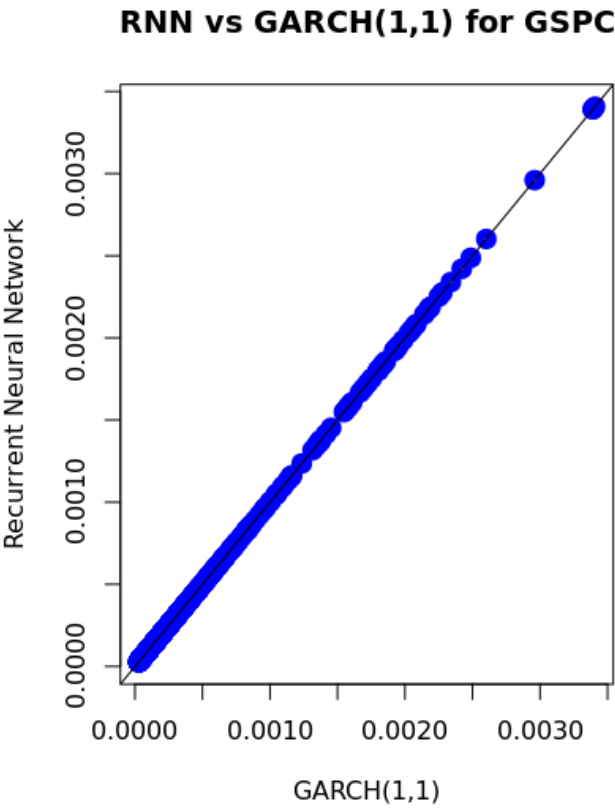
2. Estimated parameters:

Model	Param 1	Param 2	Param 3	Param 4	Param 5	Param 6
ARCH(5)	3.353e-05	6.219e-02	2.275e-01	2.118e-01	1.853e-01	1.525e-01
FNN	3.353e-05	6.219e-02	2.275e-01	2.118e-01	1.853e-01	1.525e-01

Both models have settled on identical parameters.

Q3

1.

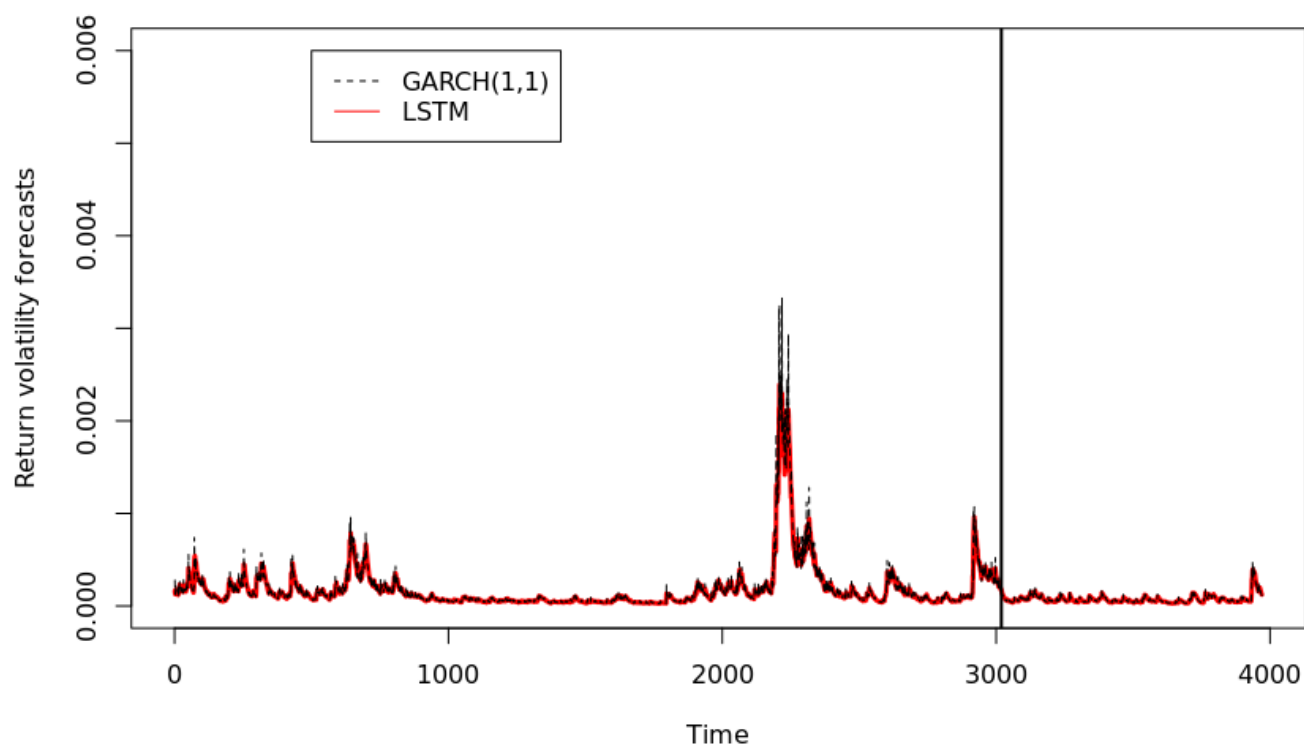


2.

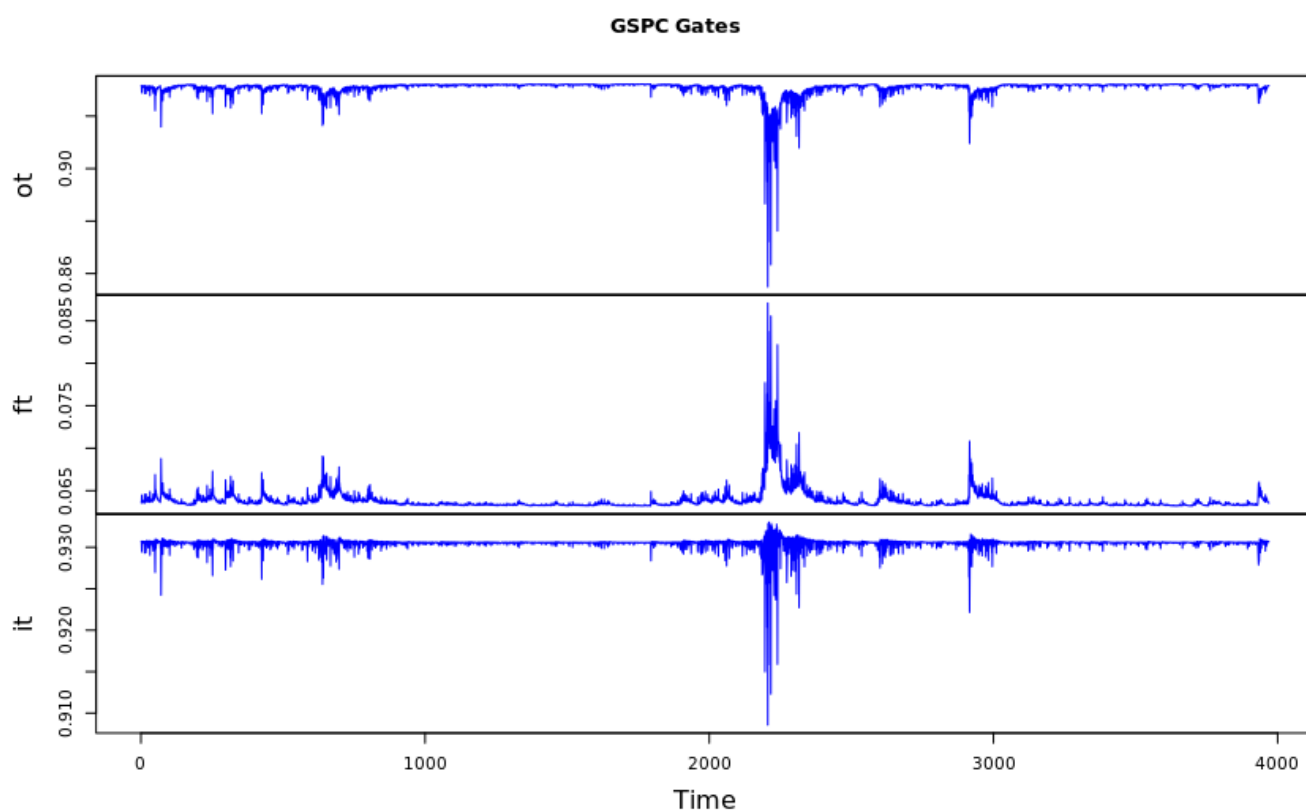
Model	Param 1	Param 2	Param 3	Param 2 + Param 3
GARCH(1,1)	1.741e-06	9.397e-02	8.936e-01	0.9875
RNN	1.741e-06	9.397e-02	8.936e-01	0.9875

Q4

- 1. Done by provided code.
- 2. Done by provided code.
- 3. No such question?
- 4. The performance of the two models is very, very similar. It looks like the LSTM doesn't have volatility peaks as high as GARCH, and LSTM appears to be a little slower to make adjustments than GARCH. Otherwise, the models perform identically.



5. The first gate, o_t , seems to be a score function that tracks whether or not the model made the correct prediction in the next timestep, with larger mistakes resulting in larger drops in score. It may also be a confidence metric. The second gate, f_t , appears to be tracking stock price. The third gate, i_t , looks like it might be measuring some sort of sentiment estimate like volume, as it also has some peaks during the 2008/2009 financial crisis time.



6.

Model	Sum of Absolute Deviation	Sum of Squared Errors	R ²	Akaike Information Criterion
GARCH	0.05885	1.023e-05	0.3434	-7425
LSTM	0.06379	1.219e-05	0.2174	-7403

Based on the sum of squared errors, it looks like GARCH performed a little better than the LSTM. It also had a lower sum of absolute deviation, but they both had quite low R² values. Additionally, their AIC scores are very similar, but by the definition of AIC, the GARCH model is significantly better. This aligns with the other metrics we've seen.

Q5

Based on this homework assignment, we've seen that LSTMs and RNNs are effective, if not perfect, models for forecasting volatility of financial assets. GARCH is designed for estimating volatility of time series, and both the LSTM and RNN had close (but slightly worse) performance. However, the LSTM and RNN might be able to use additional data points more effectively than GARCH. Lastly, the markets are not a closed system - as "hype" and other factors can play as big a role as fundamentals in terms of pricing - therefore, some amount of error is to be expected.