Nonsmooth Newton methods for frictional contact problems in flexible multi-body systems

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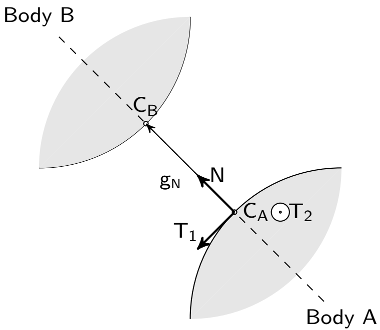
Objectives

Signorini's condition and Coulomb's friction



- gap function $g_{\mathrm{N}} = (C_B - C_A)\mathrm{N}$.
- reaction forces

$$r = r_{\mathrm{N}}\mathrm{N} + r_{\mathrm{T}}, \qquad \text{with } r_{\mathrm{N}} \in \boldsymbol{R} \text{ and } r_{\mathrm{T}} \in \boldsymbol{R}^2.$$

- Signorini condition at position level

$$0 \leqslant g_{\mathrm{N}} \perp r_{\mathrm{N}} \geqslant 0.$$

- relative velocity

$$u = u_{\mathrm{N}}\mathrm{N} + u_{\mathrm{T}}, \qquad \text{with } u_{\mathrm{N}} \in \boldsymbol{R} \text{ and } u_{\mathrm{T}} \in \boldsymbol{R}^2.$$

- Signorini condition at velocity level

$$\begin{cases} 0 \leqslant u_{\mathrm{N}} \perp r_{\mathrm{N}} \geqslant 0 & \text{if } g_{\mathrm{N}} \leqslant 0 \\ r_{\mathrm{N}} = 0 & \text{otherwise.} \end{cases}$$

Signorini's condition and Coulomb's friction

Modeling assumption

Let μ be the coefficient of friction. Let us define the Coulomb friction cone K which is chosen as the isotropic second order cone

$$K = \{r \in \mathbb{R}^3 \mid \|r_{\mathrm{T}}\| \leqslant \mu r_n\}. \tag{1}$$

The Coulomb friction states

- for the sticking case that

$$u_{\mathrm{T}} = 0, \quad r \in K \tag{2}$$

- and for the sliding case that

$$u_{\mathrm{T}} \neq 0, \quad r \in \partial K, \exists\, \alpha > 0, r_{\mathrm{T}} = -\alpha u_{\mathrm{T}}. \tag{3}$$

Disjunctive formulation of the frictional contact behavior

$$\begin{cases} r = 0 & \text{if } g_{\mathrm{N}} > 0 \quad \text{(no contact)} \\ r = 0, u_{\mathrm{N}} \geqslant 0 & \text{if } g_{\mathrm{N}} \leqslant 0 \quad \text{(take–off)} \\ r \in K, u = 0 & \text{if } g_{\mathrm{N}} \leqslant 0 \quad \text{(sticking)} \\ r \in \partial K, u_{\mathrm{N}} = 0, \exists\, \alpha > 0, u_{\mathrm{T}} = -\alpha r_{\mathrm{T}} & \text{if } g_{\mathrm{N}} \leqslant 0 \quad \text{(sliding)} \end{cases} \tag{4}$$

Signorini's condition and Coulomb's friction

Second Order Cone Complementarity (SOCCP) formulation [5]

- Modified relative velocity $\hat{u} \in \boldsymbol{R}^3$ defined by

$$\hat{u} = u + \mu \|u_{\mathrm{T}}\| \mathrm{N}. \tag{5}$$

- Second-Order Cone Complementarity Problem (SOCCP)

$$K^\star \ni \hat{u} \perp r \in K \tag{6}$$

if $g_{\mathrm{N}} \leqslant 0$ and $r = 0$ otherwise. The set $K^\star$ is the dual convex cone to K defined by

$$K^\star = \{u \in \boldsymbol{R}^3 \mid r^\top u \geqslant 0, \quad \text{for all } r \in K\}. \tag{7}$$

Signorini's condition and Coulomb's friction

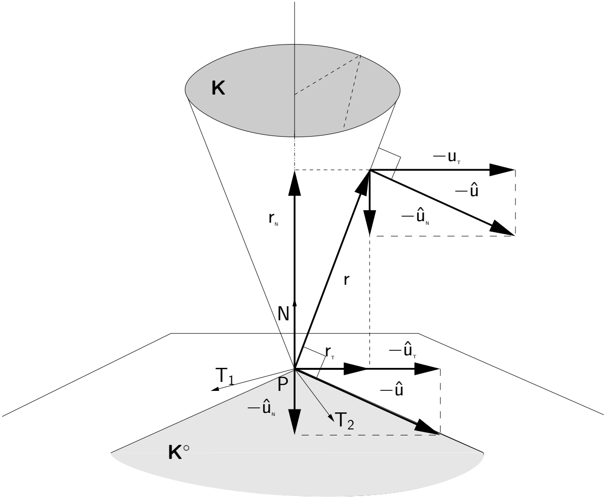


Figure: Coulomb's friction and the modified velocity $\hat{u}$. The sliding case.

3D frictional contact problem

Multiple contact notation

For each contact $\alpha \in \{1, \dots n_c\}$, we have

- the local velocity : $u^\alpha \in \boldsymbol{R}^3$, and

$$u = [[u^\alpha]^\top, \alpha = 1 \dots n_c]^\top$$

- the local reaction vector $r^\alpha \in \boldsymbol{R}^3$

$$r = [[r^\alpha]^\top, \alpha = 1 \dots n_c]^\top$$

- the local Coulomb cone

$$K^\alpha = \{r^\alpha, \|r^\alpha_{\mathrm{T}}\| \leqslant \mu^\alpha |r^\alpha_{\mathrm{N}}|\} \subset \boldsymbol{R}^3$$

and the set K is the cartesian product of Coulomb's friction cone at each contact, that

$$K = \prod_{\alpha=1\dots n_c} K^\alpha \tag{8}$$

and $K^\star$ is dual.

3D frictional contact problems

Problem 1 (General discrete frictional contact problem)

Given

- *a symmetric positive definite matrix* $M \in \mathbb{R}^{n \times n}$,
- *a vector* $f \in \mathbb{R}^n$,
- *a matrix* $H \in \mathbb{R}^{n \times m}$,
- *a vector* $w \in \boldsymbol{R}^m$,
- *a vector of coefficients of friction* $\mu \in \boldsymbol{R}^{n_c}$,

find three vectors $v \in \mathbb{R}^n$, $u \in \boldsymbol{R}^m$ *and* $r \in \boldsymbol{R}^m$, *denoted by* $\mathrm{FC/I}(M, H, f, w, \mu)$ *such that*

$$\begin{cases} Mv = Hr + f \\ u = H^\top v + w \\ \hat{u} = u + g(u) \\ K^\star \ni \hat{u} \perp r \in K \end{cases} \tag{9}$$

with $g(u) = [[\mu^\alpha \|u_T^\alpha\| \mathrm{N}^\alpha]^\top, \alpha = 1 \dots n_c]^\top$. □

3D frictional contact problems

Problem 2 (Reduced discrete frictional contact problem)

Given

- *a symmetric positive semi–definite matrix* $W \in \mathbb{R}^{m\times m}$,
- *a vector* $q \in \mathbb{R}^m$,
- *a vector* $\mu \in \boldsymbol{R}^{n_c}$ *of coefficients of friction,*

find two vectors $u \in \boldsymbol{R}^m$ *and* $r \in \boldsymbol{R}^m$, *denoted by* $\mathrm{FC/II}(W, q, \mu)$ *such that*

$$\begin{cases} u = Wr + q \\ \hat{u} = u + g(u) \\ K^\star \ni \hat{u} \perp r \in K \end{cases} \qquad (10)$$

with $g(u) = [[\mu^\alpha \| u_T^\alpha \| \mathrm{N}^\alpha]^\top, \alpha = 1 \dots n_c]^\top$. □

Relation with the general problem

$W = H^\top M^{-1} H$ and $q = H^\top M^{-1} f + w$.

3D frictional contact problems

Rank of the H matrix and hyperstaticity

The rank of the H matrix (ratio number of contacts unknows/number of d.o.f) plays an important role.

- Rigid multibody systems (high degree of hyperstaticity): Generically : $3n_c \ggg n$. H is NOT full column rank and W is rank deficient.
- Flexible multibody systems. Generically : $3n_c < n$. H may be full column rank and W is full rank.

Effect on convergence of numerical methods

- First order iterative methods solves all the problems but very slowly
- Nonsmooth Newton methods are inefficient.

Nonsmooth Equations based methods

Nonsmooth Newton on $F(r) = 0$

$$r_{k+1} = r_k - \Phi^{-1}(r_k)(F(r_k)), \qquad \Phi(r_k) \in \partial F(r_k)$$

- Alart–Curnier Formulation [1]

$$F_{\mathrm{ac}}(r) := \left[\begin{array}{l} r_{\mathrm{N}} - P_{\boldsymbol{R}_+^{n_c}}(r_{\mathrm{N}} - \rho_{\mathrm{N}}(Wr+q)_{\mathrm{N}}), \\ r_{\mathrm{T}} - P_{D(\mu,(r_{\mathrm{N}} - \rho(Wr+q)_{\mathrm{N}})_+)}(r_{\mathrm{T}} - \rho_{\mathrm{T}}(Wr+q)_{\mathrm{T}}) \end{array}\right], \quad \rho_{\mathrm{N}} > 0, \rho_{\mathrm{T}} > 0, \tag{11}$$

- Jean – Moreau formulation [7, 4]

$$F_{\mathrm{mj}}(r) := \left[\begin{array}{c} r_{\mathrm{N}} - P_{\boldsymbol{R}_+^{n_c}}(r_{\mathrm{N}} - \rho_{\mathrm{N}}(Wr+q)_{\mathrm{N}}) \\ r_{\mathrm{T}} - P_{D(\mu,(r_{\mathrm{N}})_+)}(r_{\mathrm{T}} - \rho_{\mathrm{T}}(Wr+q)_{\mathrm{T}}) \end{array}\right], \quad \rho_{\mathrm{N}} > 0, \rho_{\mathrm{T}} > 0. \tag{12}$$

- Direct natural map reformulation

$$F_{\mathrm{nat}}(r) := \left[\; r - P_K\left(r - \rho(Wr+q+g(Wr+q))\right) \;\right], \quad \rho > 0 \tag{13}$$

MUMPS [3, 2] is used for solving linear systems.

Matrix block-splitting and projection based algorithms [9, 8]

Block splitting algorithm with $W^{\alpha\alpha} \in \boldsymbol{R}^3$ (Gauss-Seidel)

$$\begin{cases} u^{\alpha}_{i+1} - W^{\alpha\alpha} P^{\alpha}_{i+1} = q^{\alpha} + \sum_{\beta<\alpha} W^{\alpha\beta} r^{\beta}_{i+1} + \sum_{\beta>\alpha} W^{\alpha\beta} r^{\beta}_{i} \\ \widehat{u}^{\alpha}_{i+1} = \left[u^{\alpha}_{\mathrm{N},i+1} + \mu^{\alpha}\, \|u^{\alpha}_{\mathrm{T},i+1}\|, u^{\alpha}_{\mathrm{T},i+1}\right]^{T} \\ \mathbf{K}^{\alpha,*} \ni \widehat{u}^{\alpha}_{i+1} \perp r^{\alpha}_{i+1} \in \mathbf{K}^{\alpha} \end{cases} \tag{14}$$

for all $\alpha \in \{1 \dots m\}$.

One contact point problem

- closed form solutions
- Any solver listed before.

Naming convention

NSN-AC-NLS	Nonsmooth Newton Method using (11) without line-search
NSN-JM-NLS	Nonsmooth Newton Method using (12) without line-search
NSN-NM-NLS	Nonsmooth Newton Method using (13) without line-search
NSN-AC-NLS-HYBRID	Method NSN-AC-NLS with preconditioning with 100 iterations of NSGS-AC
NSGS-AC	Gauss–Seidel method with NSN-AC-NLS as local solver
NSGS-FP-VI-UPK	Gauss–Seidel method with fixed point iterations of $F_{\mathrm{nat}}(r) - r$

Table: Naming convention

Error evaluation

$$\frac{\|F_{\mathrm{nat}}(r)\|}{\|q\|} < \epsilon, \tag{15}$$

Siconos/Numerics

Siconos

Open source software for modelling and simulation of nonsmooth systems

Siconos/Numerics

Collection of C routines to solve FC3D problem

- NonSmoothGaussSeidel : VI based projection/splitting algorithm
- TrescaFixedPoint : fixed point algorithm on Tresca fixed point
- LocalAlartCurnier : semi–smooth newton method of Alart–Curnier formulation
- ProximalFixedPoint : proximal point algorithm
- VIFixedPointProjection : VI based fixed-point projection
- VIExtragradient : VI based extra-gradient method
- . . .

http://siconos.gforge.inria.fr

use and contribute ...

Performance profiles [6]

- Given a set of problems $\mathcal{P}$
- Given a set of solvers $\mathcal{S}$
- A performance measure for each problem with a solver $t_{p,s}$ (cpu time, flops, ...)
- Compute the performance ratio

$$\tau_{p,s} = \frac{t_{p,s}}{\min_{s \in \mathcal{S}} t_{p,s}} \geqslant 1 \tag{16}$$

- Compute the performance profile $\rho_s(\tau) : [1, +\infty] \to [0, 1]$ for each solver $s \in \mathcal{S}$

$$\rho_s(\tau) = \frac{1}{|\mathcal{P}|} \big|\{p \in \mathcal{P} \mid \tau_{p,s} \leqslant \tau\}\big| \tag{17}$$

The value of $\rho_s(1)$ is the probability that the solver s will win over the rest of the solvers.

LMGC90 sheared low wall example

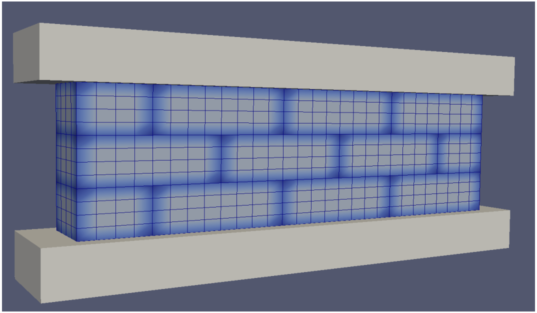
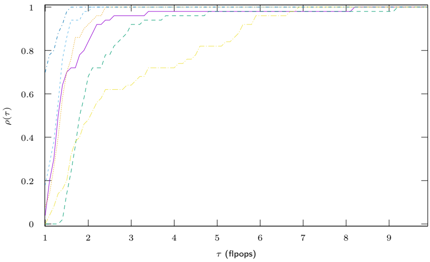


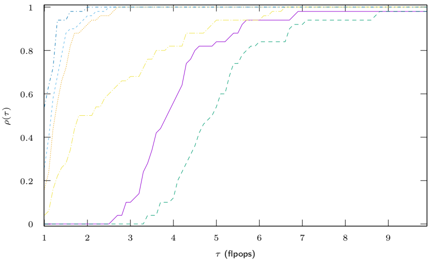
Figure: A low wall meshes with H8

- H8 FE with Linear elastic behavior : $\rho = 2000 \mathrm{kg\,m}^{-3}, E = 2.2 \times 10^9 \mathrm{Pa}, \nu = 0.2$
- $\mu = 0.83$ between block and $\mu = 0.53$ between blocks and supports
- Vertical compression force : 30000N horizontal shear velocity $1 \times 10^{-3} \mathrm{m\,s}^{-1}$.
- Sampling of 50 problems collected in the FCLib with graded difficulty

Results



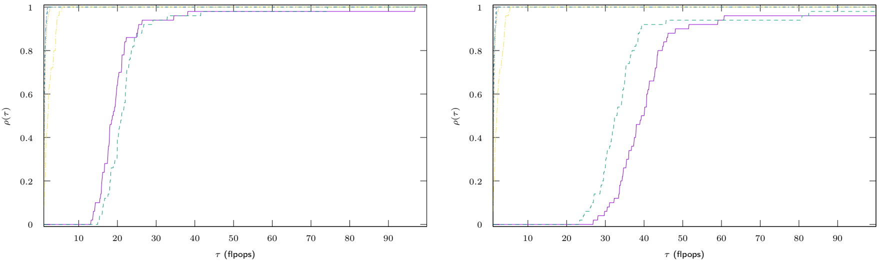
(a) Accuracy $\epsilon = 10^{-2}$



(b) Accuracy $\epsilon = 10^{-3}$

NSGS-AC
NSGS-FP-VI-UPK iter=100
NSN-AC-NLS
NSN-JM-NLS
NSN-NM-NLS
NSN-AC-NLS-HYBRID

Results



(a) Accuracy $\epsilon = 10^{-4}$

(b) Accuracy $\epsilon = 10^{-6}$

NSGS-AC
NSGS-FP-VI-UPK iter=100
NSN-AC-NLS
NSN-JM-NLS
NSN-NM-NLS
NSN-AC-NLS-HYBRID

Conclusions & Perspectives

Conclusions

1. For relatively tight accuracy, nonsmooth Newton methods outperform first order iterative method.
2. NSN-AC-NLS-HYBRID is the most efficient method
3. First order iterative methods are interesting for low accuracy, but are not able to reach tight accuracy,

Perspectives

1. Evaluate the interest to transform rigid model into flexible ones.
2. Study the possibility to take into account the possible nonlinear bulk behavior in the Newton loop
3. HPC and scalability of nonsmooth Newton techniques using MUMPS
4. Continue to set up a collection of benchmarks ➜ FCLIB

FCLIB : a collection of discrete 3D Frictional Contact (FC) problems

Our inspiration: MCPLIB or CUTEst

What is FCLIB ?

- A open source collection of Frictional Contact (FC) problems stored in a specific HDF5 format
- A open source light implementation of Input/Output functions in C Language to read and write problems (Python and Matlab coming soon)

Goals of the project

Provide a standard framework for testing available and new algorithms for solving discrete frictional contact problems share common formulations of problems in order to exchange data

Call for contribution

http://fclib.gforge.inria.fr

Thank you for your attention.

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