AE708 assignment 2

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Notes

Points 2-4 are taken from slide 20 of the file `Rocket_Performance_Parameters_Nozzles.pdf`.

1. Simulations are performed using OpenFOAM-v9.
2. All nozzles use throat radius $R_t = 1$ m and a converging section as a 75° arc of radius $1.5R_t$.
3. All nozzles use $\varepsilon = 4 = (R_e/R_t)^2$.
4. Just after the throat, all nozzles have an arc of angle θ_n and radius $0.4R_t$. For Rao nozzle(s), $\theta_n = 30°$ while $\theta_n = 15°$ for conical nozzle.
5. Conical nozzle uses a half-cone angle of 15°.
6. Two Rao nozzles are considered with the diverging section length 60% ($K = 0.6$) and 80% ($K = 0.8$) of the conical nozzle.
7. All geometries are 3° bodies of revolution. Axisymmetric simulations are done in OpenFOAM.
8. Stagnation conditions: $p_0 = 287\,\mathrm{Pa}$, $T_0 = 1\,\mathrm{K}$ and $\rho_0 = 1\,\mathrm{kg\,m^{-3}}$.
9. Initial conditions: discontinuity at throat with stagnation conditions imposed on left, and design pressure and temperature on right (without velocity).
10. Boundary conditions
    - Inlet: `totalPressure` for p, `totalTemperature` for T, `zeroGradient` for $\mathbf{U}$.
    - Outlet
      - Design condition: `zeroGradient` for p, T and $\mathbf{U}$.
      - Off design: `fixedValue` for p, `zeroGradient` for T and $\mathbf{U}$.
    - Wall: `zeroGradient` for p and T, `slip` for $\mathbf{U}$.
    - Azimuthal faces: `wedge` for p, T and $\mathbf{U}$.

Listing 1: Python code for designing Rao nozzle

```
# See https://moodle.iitb.ac.in/pluginfile.php/336817/mod_assign/introattachment/0/
    Bell_nozzle_design_methodology.pdf?forcedownload=1

import numpy as np
import matplotlib.pyplot as plt

# Parameters
R_t = 1 # throat radius
epsilon = 4 # exit area to throat area ratio
R_e = R_t*(epsilon**0.5) # exit radius
theta_n = 30*np.pi/180 # initial nozzle contour angle after circular segment
Rc_div = 0.4*R_t # curvature radius after divergent section
x_n = Rc_div*np.sin(theta_n) # x coordinate of point N (x axis starts at throat)
y_n = R_t + Rc_div*(1-np.cos(theta_n)) # y coordinate of point N (y axis starts from
    nozzle axis)
```

```
theta_conical = 15*np.pi/180
L_conical = (R_e-R_t)/np.tan(theta_conical) # conical nozzle length with 15 degree cone
    angle
K = 0.8 # length of Rao nozzle relative to conical nozzle with 15 degree cone angle

# Solve for coefficients
coeff_matrix = np.array([
    [2*y_n, 1, 0],
    [R_e**2, R_e, 1],
    [y_n**2, y_n, 1]
])
rhs_vector = np.array([1/np.tan(theta_n), K*L_conical, x_n])
solution = np.linalg.solve(coeff_matrix, rhs_vector)
np.set_printoptions(formatter={'float': '{: 0.3e}'.format})
print("Solution coefficients: {}".format(solution))

# Plot for comparison
L_rao = solution[0]*R_e**2 + solution[1]*R_e + solution[2] # length of Rao nozzle
fig, ax = plt.subplots(1,1)
y = np.linspace(y_n, R_e)
x_rao = solution[0]*y**2 + solution[1]*y + solution[2]
x_conical = (y-y_n)/np.tan(theta_conical) + x_n
ax.plot(x_rao, y, "b-", label="Rao")
ax.plot(x_conical, y, "r--", label="Conical")
ax.grid()
ax.legend()
plt.show()

# Data output (from the point N to the exit)
## pre-throat: circular arc profile
## post-throat: smaller circular arc followed by conical/rao profile
# np.savetxt("rao_profile.dat", np.vstack((x_rao, y)).T)
```

Listing 2: Gmsh code for generating Rao nozzle mesh

```
/*
Point numbering convention:
    0xx: nozzle contour (converging and diverging), except parabolic profile
    1xx: temporary points
    2xx: parabolic profile spline
Line numbering convention:
    0xx: nozzle contours, including parabolic profile spline
Curve loops, surface convention:
    0xx: all
*/

R_t = 1; // throat radius
theta_n = 30*Pi/180; // angle at point N
theta_i = 75*Pi/180; // angle at inlet section
epsilon = 4; // exit area to throat area ratio
R_e = R_t*Sqrt(epsilon); // exit radius
Rc_div = 0.4*R_t; // radius of curvature just after throat in diverging section
Rc_conv = 1.5*R_t; // radius of curvature of converging section
```

```
a = 1.280e+00; // coefficients for parabola
b = -9.650e-01;
c = -2.041e-01;

n_parabola = 100; // number of points to be defined on parabola

x_n = Rc_div*Sin(theta_n); // coordinates of point N
y_n = R_t + Rc_div*(1-Cos(theta_n));
x_i = -Rc_conv*Sin(theta_i); // coordinates of inlet section
y_i = R_t + Rc_conv*(1-Cos(theta_i));
x_e = a*R_e^2 + b*R_e + c; // x coordinate of exit section
revolution_angle = 3*Pi/180; // body's revolution angle

// converging section (circular arc)
Point(100) = {0, R_t+Rc_conv, 0}; // arc center
Point(1) = {x_i, y_i, 0};
Point(2) = {0, R_t, 0};
Point(3) = {x_i, 0, 0};
Point(4) = {0, 0, 0};
Circle(1) = {1, 100, 2};
Line(2) = {3,4};
Line(3) = {3,1};
Line(4) = {4,2};
Curve Loop(1) = {2,4,-1,-3};

// circular arc after throat
Point(101) = {0, R_t+Rc_div, 0}; // arc center
Point(5) = {x_n, y_n, 0};
Point(6) = {x_n, 0, 0};
Circle(5) = {2, 101, 5};
Line(6) = {4,6};
Line(7) = {6,5};
Curve Loop(2) = {6,7,-5,-4};

// parabolic portion
/// define points on the parabolic section
For i In {1:n_parabola}
    y = y_n + (R_e-y_n)*i/n_parabola;
    Point(200+i-1) = {a*y^2 + b*y + c, y, 0};
EndFor
BSpline(8) = {5,200:200+n_parabola-1};
Point(7) = {x_e, 0, 0};
Line(9) = {6,7};
Line(10) = {7,200+n_parabola-1};
Curve Loop(3) = {9,10,-8,-7};



// plane surfaces, transfinite and recombine; extrude and physical entities
Transfinite Curve{1} = 41 Using Bump 0.1;
Transfinite Curve{2} = 41 Using Bump 0.15;
Transfinite Curve{5,6} = 21;
Transfinite Curve{8,9} = 31 Using Progression 1.1;
Transfinite Curve{3,4,7,10} = 31;
```

```
For i In {1:3}
    Plane Surface(i) = {i};
    Transfinite Surface{i};
    Recombine Surface{i};
    out~{i}[] = Extrude{
        {1,0,0},
        {0,0,0},
        revolution_angle
    }{
        Surface{i};
        Layers{1};
        Recombine;
    };
EndFor
// rotate all entities for symmetry about xy plane
Rotate{
    {1,0,0},
    {0,0,0},
    -0.5*revolution_angle
}{
    Point{:}; Curve{:}; Surface{:}; Volume{:};
}
Physical Volume("volume", 1) = {};
Physical Surface("back", 1) = {};
Physical Surface("front", 2) = {};
Physical Surface("inflow", 3) = {};
Physical Surface("outflow", 4) = {};
Physical Surface("wall", 5) = {};
For i In {1:3}
    Physical Volume(1) += {out~{i}[1]};
    Physical Surface(1) += {i};
    Physical Surface(2) += {out~{i}[0]};
    Physical Surface(5) += {out~{i}[3]};
EndFor
Physical Surface(3) += {out_1[4]};
Physical Surface(4) += {out_3[2]};

Mesh 3;
```

Listing 3: Gmsh code for generating Conical nozzle mesh

```
/*
Point numbering convention:
    0xx: nozzle contour (converging and diverging), except parabolic profile
    1xx: temporary points
    2xx: parabolic profile spline
Line numbering convention:
    0xx: nozzle contours, including parabolic profile spline
Curve loops, surface convention:
    0xx: all
*/

R_t = 1; // throat radius
theta_conical = 15*Pi/180; // half cone angle
```

```
theta_n = theta_conical;
theta_i = 75*Pi/180; // angle at inlet section
epsilon = 4; // exit area to throat area ratio
R_e = R_t*Sqrt(epsilon); // exit radius
Rc_div = 0.4*R_t; // radius of curvature just after throat in diverging section
Rc_conv = 1.5*R_t; // radius of curvature of converging section

x_n = Rc_div*Sin(theta_n); // coordinates of point N
y_n = R_t + Rc_div*(1-Cos(theta_n));
x_i = -Rc_conv*Sin(theta_i); // coordinates of inlet section
y_i = R_t + Rc_conv*(1-Cos(theta_i));
x_e = x_n + (R_e-y_n)/Tan(theta_conical); // x coordinate of exit section
revolution_angle = 3*Pi/180; // body's revolution angle

// converging section (circular arc)
Point(100) = {0, R_t+Rc_conv, 0}; // arc center
Point(1) = {x_i, y_i, 0};
Point(2) = {0, R_t, 0};
Point(3) = {x_i, 0, 0};
Point(4) = {0, 0, 0};
Circle(1) = {1, 100, 2};
Line(2) = {3,4};
Line(3) = {3,1};
Line(4) = {4,2};
Curve Loop(1) = {2,4,-1,-3};

// circular arc after throat
Point(101) = {0, R_t+Rc_div, 0}; // arc center
Point(5) = {x_n, y_n, 0};
Point(6) = {x_n, 0, 0};
Circle(5) = {2, 101, 5};
Line(6) = {4,6};
Line(7) = {6,5};
Curve Loop(2) = {6,7,-5,-4};

// conical portion
/// define points on the parabolic section
Point(7) = {x_e, R_e, 0};
Point(8) = {x_e, 0, 0};
Line(8) = {5,7};
Line(9) = {6,8};
Line(10) = {8,7};
Curve Loop(3) = {9,10,-8,-7};



// plane surfaces, transfinite and recombine; extrude and physical entities
Transfinite Curve{1} = 41 Using Bump 0.1;
Transfinite Curve{2} = 41 Using Bump 0.15;
Transfinite Curve{5,6} = 11;
Transfinite Curve{8,9} = 41 Using Progression 1.1;
Transfinite Curve{3,4,7,10} = 31;
For i In {1:3}
    Plane Surface(i) = {i};
```

```
    Transfinite Surface{i};
    Recombine Surface{i};
    out~{i}[] = Extrude{
        {1,0,0},
        {0,0,0},
        revolution_angle
    }{
        Surface{i};
        Layers{1};
        Recombine;
    };
EndFor
// rotate all entities for symmetry about xy plane
Rotate{
    {1,0,0},
    {0,0,0},
    -0.5*revolution_angle
}{
    Point{:}; Curve{:}; Surface{:}; Volume{:};
}
Physical Volume("volume", 1) = {};
Physical Surface("back", 1) = {};
Physical Surface("front", 2) = {};
Physical Surface("inflow", 3) = {};
Physical Surface("outflow", 4) = {};
Physical Surface("wall", 5) = {};
For i In {1:3}
    Physical Volume(1) += {out~{i}[1]};
    Physical Surface(1) += {i};
    Physical Surface(2) += {out~{i}[0]};
    Physical Surface(5) += {out~{i}[3]};
EndFor
Physical Surface(3) += {out_1[4]};
Physical Surface(4) += {out_3[2]};

Mesh 3;
```

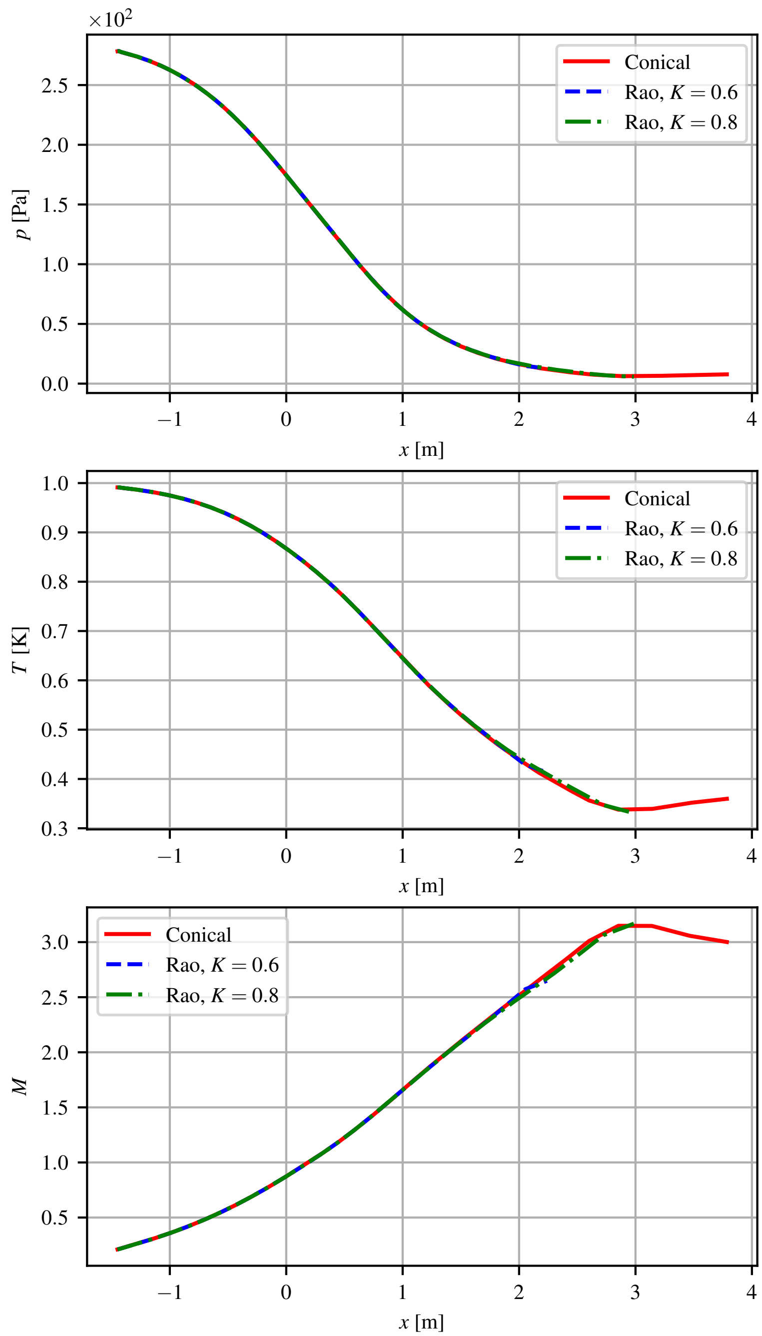


Figure 1: Pressure, Temperature and Mach number variation along the nozzle.

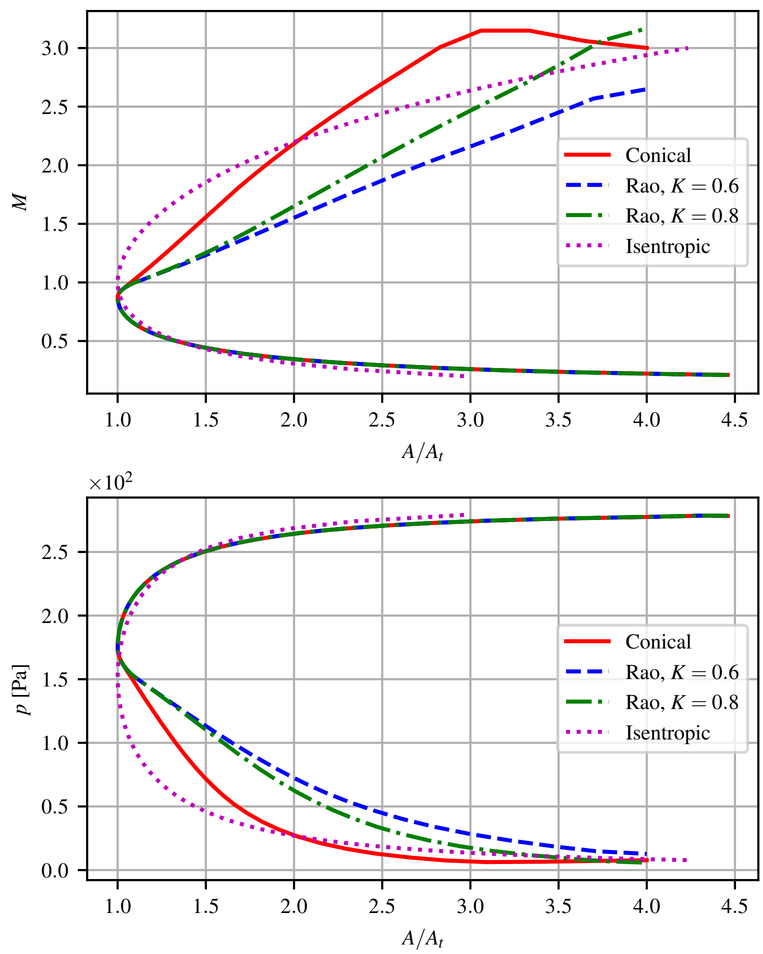
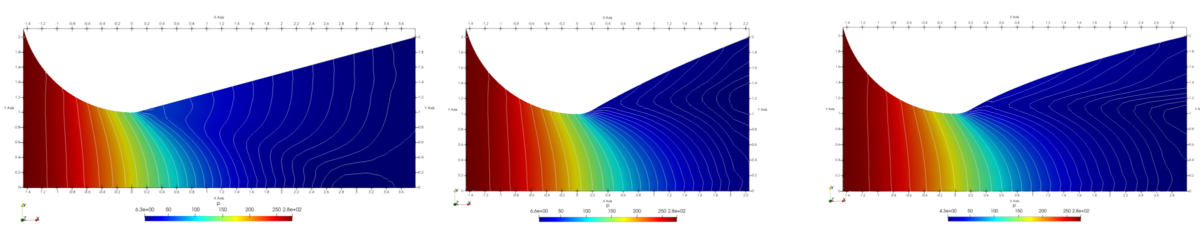


Figure 2: Mach number and pressure variation versus the area ratio.



(a) Conical (b) Rao, $K = 0.6$ (c) Rao, $K = 0.8$

Figure 3: Pressure surface plot with 30 Mach number contours. Clearly, the Mach number contours are not radial and there is significant multidimensional nature to the flow. Quasi-1D analysis will fail here.