ME699 Stage-1 report

P. Vachan Deep
Roll # 184104002

September 13, 2019

---

1 Introduction

Blood flow is an important functioning of human body which significantly effects how it functions. Since blood transfers many entities from all parts of the body to specific centres, characteristics of blood flow very much decide the state of a living creature. It is of much more interest to visualise blood flow, in some way, in patients, especially in those with heart-related diseases for the purpose of diagnosis.

The visualisation techniques used for blood flow have come a long way from using probes in animals, to using X-rays and now to using MRI. In the latter ones, essentially, the patient is injected with contrast—a chemical which can respond to either X-rays or magnetic field—before exposing him/her to the visualisation equipment. These are hence considered (mechanically) non-invasive.

However, there cannot be *any* measurement without perturbation and these processes are no exception. Of interest for this study is the effect of magnetic field on the flow of blood which is conducting fluid. In particular, blood flow in the aorta, the largest artery of the body, would be considered.

2 Objectives

A simplified geometric model for aorta would be used to identify how a strong magnetic field of strength ~ 1 T would change the characteristics of the flow. The change in the following characteristics would be monitored.

1. Flow rate
2. Helicity, which also accounts for flow separation

Blood flow rate is the most important quantity because it directly effects the functioning of other organs. Helicity is a measure of how much the streamlines and vorticity lines are aligned together. Mathematically,

$$H = \int_V \mathbf{u} \cdot \boldsymbol{\omega} \, \mathrm{d}V = \int_V \mathbf{u} \cdot (\nabla \times \mathbf{u}) \, \mathrm{d}V. \tag{1}$$

It is known that aortic blood flow has helical nature through experiments (for example, see [1]). There have been propositions that this helical nature is to prevent flow separation during late-systole and diastole phase when blood flow rate decreases. Experimental works have proven this to be true. For example, Morbiducci *et al.* [3] argue that one among many functions of flow helicity is to supply fluid from core to the walls when a separation is due. The helicity of blood flow in aorta is therefore an important indicator of cardiac soundness.

The impact of magnetic field on these two characteristics of the flow would be sought to assess whether MRI based flow visualisation is truly a non-invasive technique, i.e.; whether it doesn't alter the flow characteristics by much.

3 Methods

The blood flow will be considered incompressible and Newtonian. Although blood has corpuscles which produce non-Newtonian characteristics, for simplicity, these effects will be ignored (at lease, in the initial phase). The arterial walls will be considered rigid which is a serious crime. However, since the interest is to only see how the magnetic field effects the flow, it is believed that even with this assumption, useful insights will be gained. The MHD (low frequency) approximation would be considered valid.

mhdFoam, the solver for MHD in OpenFOAM [5] will be used for this project. It solves (incompressible) MHD equations Eqs. (2) to (4). It uses a PISO algorithm for solving Eqs. (2) and (3) by treating $\mathbf{B}$ explicitly. Then, a prediction for $\mathbf{B}$ is made using Eq. (4) by treating all terms except the stretching term (first on RHS) implicitly. Then, a correction is done to $\mathbf{B}$ to ensure its solenoidal nature.

$$\nabla \cdot \mathbf{u} = 0 \tag{2}$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} = -\frac{\nabla p}{\rho} + \nu \nabla^2 \mathbf{u} + \frac{1}{\rho \mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} \tag{3}$$

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{B} = (\mathbf{B} \cdot \nabla)\mathbf{u} + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B} \tag{4}$$

Note that Eq. (3) is devoid of the current density since the Ampere's law has been used and, Eq. (4) is a result of combining Ohm's law, Faraday's law and Ampere's along with divergence free natures of $\mathbf{u}$ and $\mathbf{B}$.

The solver has been validated for Hartmann flow with imposed velocity (Fig. 1). This case is available in the tutorials of OpenFOAM. The results of validation are shown in Fig. 2. The finer grid simulation is to ensure that no numerical instabilities arise when the grid is refined. Otherwise, numerical results agree well with exact solution even for the 200×40 grid. Although analytical expression for velocity ratio u/u_0 is available, an expression for u_0 is not, in the few references consulted. Therefore, for the purpose of Fig. 2, u_0 is taken as the maximum velocity of the solution obtained with respective grid.

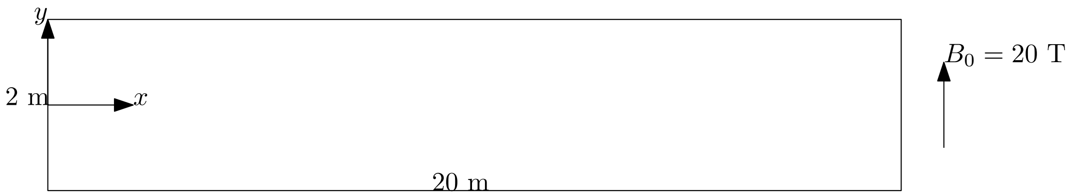

Figure 1: Geometry for Hartmann flow. At the inlet (left side), a velocity of 1 m/s in x direction is specified and all other variables are given zero normal gradient boundary condition. At the outlet (right side), pressure is specified as 0 Pa, velocity is given pressureInletOutletVelocity and magnetic field is given zero normal gradient boundary condition. The top and bottom walls are assumed insulating so that magnetic field takes the value of the external field. No slip for velocity and zero normal gradient for pressure is imposed at these walls.

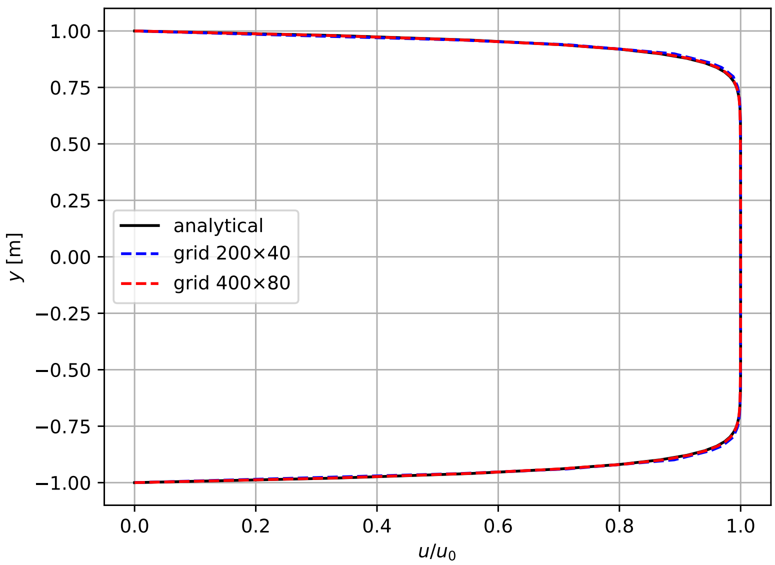

Figure 2: Velocity profile at $x = 18$ m compared with analytical result.

4 Preliminary results

The preliminary results are aimed at examining the effect of magnetic field on a pulsatile flow. It is well known that the velocity profiles are far from parabolic in such flows. For this purpose, a pipe flow case is setup with appropriate fluid properties and inlet conditions. A more complex geometric model would be considered in the future for simulation.

4.1 Properties of blood

The average density of blood is reported to be 1060 kg/m^3 [4]. The kinematic viscosity is found to be about 3×10^{-6} m^2/s [8]. The conductivity of blood as a function of the hematocrit—the volume percentage of red blood cells in blood—is presented by HIRSCH *et al.* [2]. The average hematocrit is reported to be between 40 and 50 percent [6] based on which, the conductivity can be deduced to be about 0.5 mho/m [2].

4.2 Aortic geometry and flow rate

The diameter of cardiac aorta is about 2–3 cm. Although the aortic wall contains taper, a simplistic pipe model will be considered for now, and a realistic model will be used in the future. The blood velocity at the inlet has a maximum of 40 cm/s [7] and oscillates about 60 to 100 times a minute (the regular heart rate). The maximum Reynold's number thus is about 4000. The beginning of cardiac aorta is the only place where the blood flow can enter turbulent transitional regime. However, the simulation will be done assuming laminar flow.

4.3 Case setup

A pipe flow case is setup with a length of 10 cm and a diameter of 3.2 cm. For the "plain" flow (without magnetic field), pisoFoam solver is used to obtain a time accurate solution. All boundary conditions are similar to those in Fig. 1 except at the inlet, where a uniform x velocity $u(t) = 0.25 + 0.15\sin(2\pi ft)$ is imposed, with the frequency f taken as 1 Hz (60 beats per minute). Again, insulating artery (pipe) walls are assumed.

4.4 Results

Firstly, note that the length of pipe chosen is not sufficient for the flow to be fully developed. Comparison between plain and MHD flows is shown in Figs. 3 and 4. It is easy to see that the centreline velocity would be lower for MHD because of the flattening effect that the Lorenz force will have on velocity profile. Both Figs. 3 and 4 confirm this. Further, it also logically follows that that the centreline velocity for MHD case almost equals the instantaneous inlet velocity, as can be confirmed from Fig. 3.

The unexpected result, however, is that the inlet pressure for MHD case is lower than that of plain flow case (recall that the outlet pressure is set to 0 Pa). This is non-intuitive for two reasons.

1. For the same flow rate, an extra (Lorenz) force acts on MHD flow. Hence, it might not be illogical to expect that the pressure difference should be higher in this case.
2. The MHD flow profile is steeper (see Fig. 4) at the wall which causes a higher wall shear stress for this case. This factor should also support higher inlet pressure for MHD flow.

The explanation for lower pressure lies in the unsteadiness of the flow. A total pipe control volume analysis would involve an unsteady term which can counter the two factors stated above. The significance of unsteadiness can be perceived from the fact that the average inlet pressure becomes lower that outlet pressure (see Fig. 3) for significant portion of the time. Further, the comparison of time variation of inlet pressure shows few interesting trends.

- Pressure oscillation lags velocity oscillation by about quarter time period.
- There also exists a small lag between pressure oscillation of plain and MHD flows.
- The inlet average pressure difference almost vanishes when the pressure is rising.

A detailed explanation as to why the MHD flow pressure is lower might involve all these factors and definitely depends on the complete spatial and temporal variation of velocity throughout the pipe. For now, it suffices to note these distinct results and bear them in mind when analysing blood flow in a more complex geometry which is more closer to the shape of aorta. A more detailed analysis of the results presented here can be done when the results obtained in a curved pipe-like flow show similar characteristics.

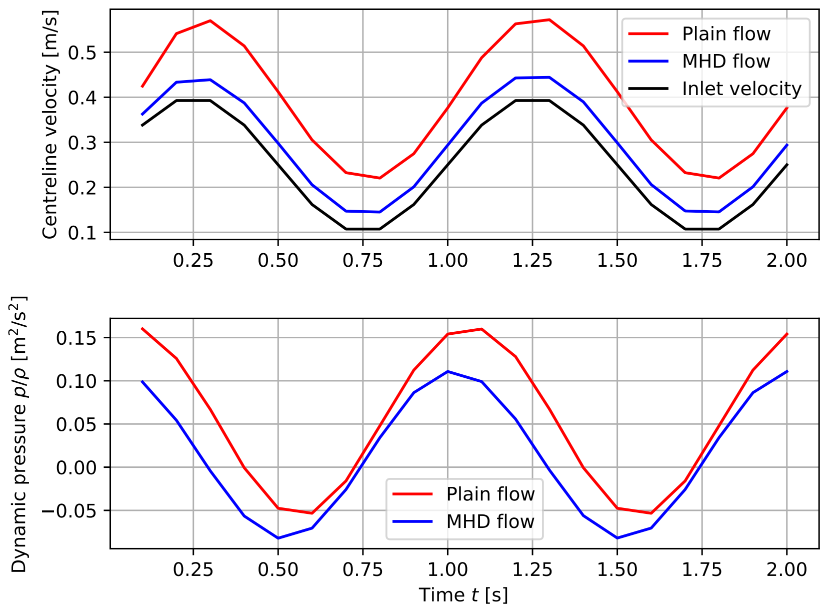

Figure 3: Comparison of time variation of average inlet pressure and centreline velocity at the outlet between plain and MHD flows.

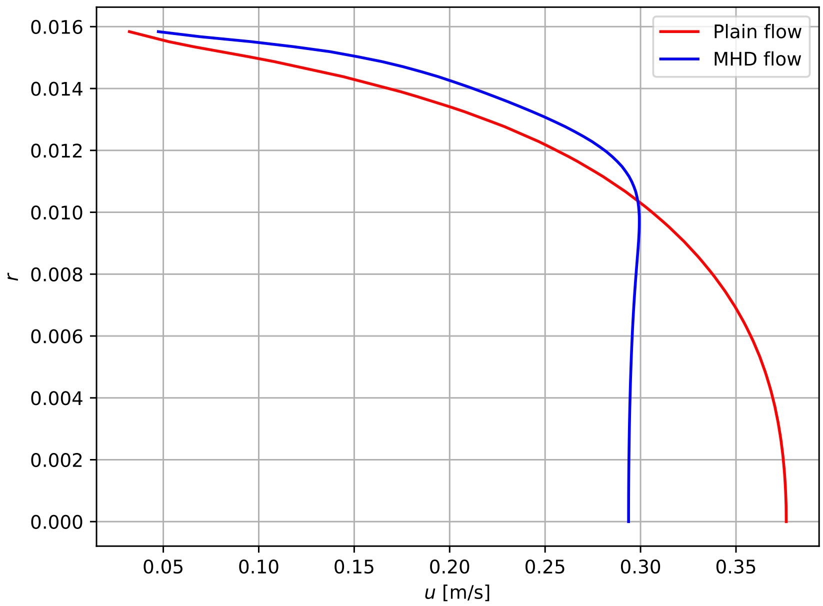

Figure 4: Comparison of velocity profiles at outlet between plain and MHD flows.

5 Future work plan

First, a suitable model for aorta has to be made. Since a 3D simulation would be require to comprehend quantities like helicity (Eq. (1)), the model would be made as simple as possible (for example, simulating only ascending aorta). Then, a pulsatile flow simulation like the one presented above would be done; again for both plain and MHD cases. Efforts would be made to try and understand these results in some more detail. Lastly, if time permits, non-Newtonian effects of blood flow will be captured using an appropriate viscosity model.

References

1. Bogren, H. G. & Buonocore, M. H. 4D magnetic resonance velocity mapping of blood flow patterns in the aorta in young vs. elderly normal subjects. *Journal of Magnetic Resonance Imaging: An Official Journal of the International Society for Magnetic Resonance in Medicine* **10,** 861–869 (1999).
2. HIRSCH, F. G., TEXTER, E. C., WOOD, L. A., BALLARD, W. C., HORAN, F. E., WRIGHT, I. S., FREY, C. & STARR, D. THE ELECTRICAL CONDUCTIVITY OF BLOOD. *Blood* **5,** 1017–1035. ISSN: 0006-4971. eprint: `http://www.bloodjournal.org/content/5/11/1017.full.pdf`. `http://www.bloodjournal.org/content/5/11/1017` (1950).
3. Morbiducci, U., Ponzini, R., Rizzo, G., Cadioli, M., Esposito, A., Montevecchi, F. M. & Redaelli, A. Mechanistic insight into the physiological relevance of helical blood flow in the human aorta: an in vivo study. *Biomechanics and modeling in mechanobiology* **10,** 339–355 (2011).
4. Shmukler, M. *Density Of Blood* [Online; accessed 12-September-2019]. 2004. `https://hypertextbook.com/facts/2004/MichaelShmukler.shtml`.
5. The OpenFOAM Foundation. *OpenFOAM* 2019. `https://openfoam.org/`.
6. Wikipedia contributors. *Hematocrit — Wikipedia, The Free Encyclopedia* [Online; accessed 12-September-2019]. 2019. `https://en.wikipedia.org/w/index.php?title=Hematocrit&oldid=912296822`.
7. Wikipedia contributors. *Hemodynamics — Wikipedia, The Free Encyclopedia* `https://en.wikipedia.org/w/index.php?title=Hemodynamics&oldid=912997579`. [Online; accessed 12-September-2019]. 2019.
8. Wikipedia contributors. *Hemorheology — Wikipedia, The Free Encyclopedia* [Online; accessed 12-September-2019]. 2019. `https://en.wikipedia.org/w/index.php?title=Hemorheology&oldid=915252588`.