Development of discontinuous Galerkin simulation framework for laminar perfect gas hypersonic shock-boundary layer interactions

Vachan Potluri
Mechanical Engineering Department, IIT Bombay

1 Hypersonic flow

The study of hypersonic flows is relevant in design of reentry missiles and hypersonic flights. Loosely, hypersonic flow is said to occur when the free stream Mach number M_∞ exceeds 5. A physical interpretation of this criterion is possible if the definition of Mach number is used. Assuming the fluid to be a calorically perfect gas (c_p and c_v are constants), we have $M = u/a = u/\sqrt{\gamma RT}$, where $\gamma = c_p/c_v$ and R is the specific gas constant. Thus

$$M_\infty \geq 5 \implies M_\infty^2 = \frac{u_\infty^2}{\gamma RT} \geq 25. \tag{1}$$

We now note that the specific thermal energy of the gas $e = c_v T = RT/(\gamma - 1)$ and specific kinetic energy of the gas $q = u^2/2$. Eq. (1) can now be written in a different form:

$$M_\infty^2 = \frac{u_\infty^2}{\gamma RT} = \frac{2}{\gamma(\gamma-1)}\frac{q_\infty}{e_\infty} \geq 25. \tag{2}$$

For a diatomic gas like air ($\gamma = 1.4$), Eq. (2) becomes

$$\frac{q_\infty}{e_\infty} \geq 7$$

which shows that in hypersonic flows, the specific kinetic energy of the gas is atleast about an order higher than the thermal energy.

However, the $M_\infty \geq 5$ criterion only identifies a high velocity flow. It is possible that the free stream is cooled to such a level that this "hypersonic" flow behaves no different from a supersonic flow, eliminating the need for this classification. This is not the case and hypersonic flow is better defined as the flow that has both high velocity and high enthalpy. This combination introduces many complex features to the flow field and some of them are shown in Fig. 1:

- High values of velocity and enthalpy of free stream causes very high temperature in the stagnation region thereby triggering high temperature effects.
- High free stream Mach number along with high temperature effects result in high post-shock density thus compressing the post-shock flow and making the shock and boundary layer sensitive to each other's evolution.
- The high curvature of the nose shock produces a rotational post-shock flow—the entropy layer—which interacts with the boundary layer.

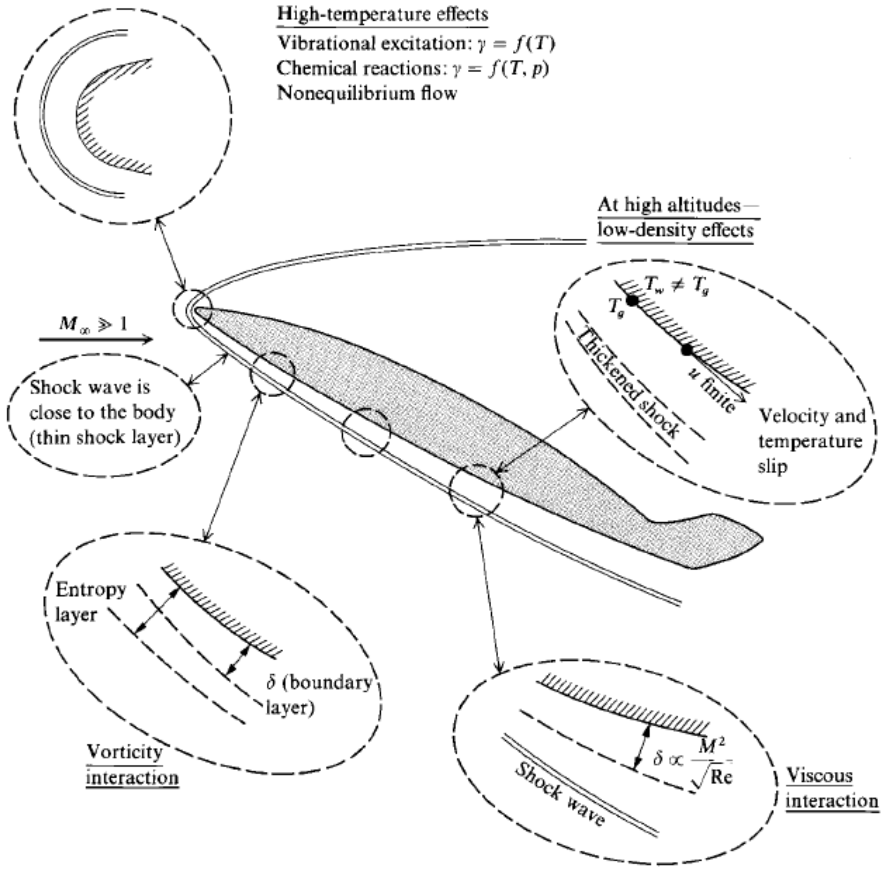

Figure 1: Physical characteristics of hypersonic flow. Adapted from Anderson Jr [1].

2 Hypersonic shock-boundary layer interaction

One specific challenge of hypersonic flow, which we are interested in, is the hypersonic shock-boundary layer interaction (SBLI). To be put simply, SBLI occurs when a shock impinges on a developing boundary layer. In most cases, the shock is strong enough to separate the boundary layer. Fig. 2 shows the simplest form of SBLI: a separated interaction occurring due to an oblique shock impingement on a boundary layer. The flow features can be comprehended as follows.

- The incident shock C_1 pressurises the post-shock boundary layer flow. The subsonic portion of this pressurised boundary layer travels upstream and displaces the incoming boundary layer. The upstream supersonic portion of boundary layer and the inviscid outer flow get compressed due to this displacement and leave the plate at separation point S. This turning of flow produces compression waves which coalesce in the outer inviscid flow region to form the separation shock C_2.
- The separation shock C_2 interacts with the incident shock C_1. Due to interaction with C_2, the incident shock refracts and reaches the separated flow as C_4. Because the separated flow acts like a constant pressure boundary, the refracted incident shock C_4 reflects as expansion wave thus turning the inviscid and supersonic boundary layer flow toward the wall.

- As a result, the inviscid flow and supersonic boundary layer flow reattach at R to form a separation bubble which is distinguishable from unseparated flow by the separation streamline (S). The reattachment imposes further compression by turning the flow parallel to wall, thus leading to the formation of reattachment shock. The reattachment shock combines with refracted separation shock C_3 to form an "apparent" reflected shock.

The interaction of shock induced separation region with the outer supersonic flow produces some interesting features. We note few of them here.

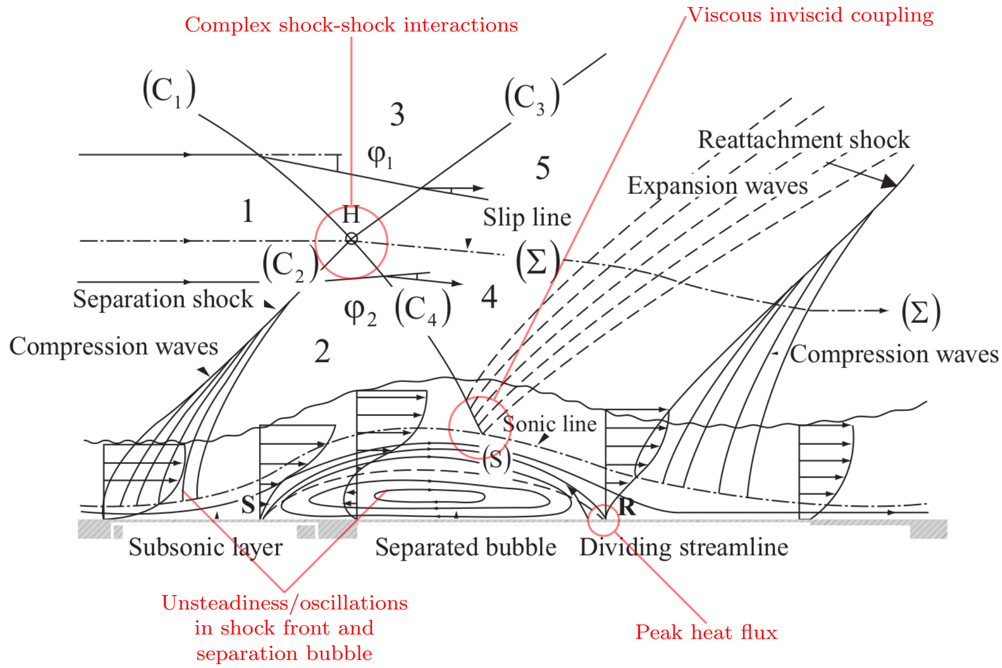

Figure 2: Characteristics of oblique shock-boundary layer interaction. Adapted from Babinsky and Harvey [2].

- The separation streamline (S) develops into a shear layer and produces a stagnation point like flow at the reattachment point R. This leads to localised peaks in surface pressure and heat flux which can affect the design of component on which SBLI occurs. For instance, hypersonic interactions can have 20 times higher peak heat flux compared to undisturbed value in the boundary layer.

- If the incoming boundary layer is laminar, then the separated shear layer may transition before reattachment resulting in a developing transitional/turbulent boundary layer post-reattachment. If the incoming boundary layer is turbulent, the shock is more than just an adverse pressure gradient; it significantly changes turbulence characteristics. The effect of a shock on turbulence characteristics is still an actively pursued area of research.

- Unlike in Fig. 2 where the separation region is relatively small, there can be interactions where separated flow spans a large extent, and such interactions are detrimental to flight. For example, large separation region in supersonic aircraft inlets reduce the performance of engine and may lead to engine stall.

- We can see that the separation bubble significantly alters the outer inviscid flow by introducing additional waves. A strong SBLI (i.e.; separated SBLI) inevitably involves shock-shock interactions. Further, in hypersonic flows where shock layers are thin, the effect of separation bubble is much more prominent in that these shock-shock interactions occur close to the surface and hence, in return, can effect the boundary layer flow immediately downstream. Thus, a strong coupling between viscous and outer inviscid flow is established. These phenomena, especially in internal flows (e.g. supersonic inlets), have the effect of completely changing the flow pattern.

- Incoming free stream and boundary layer disturbances affect the position of shock and thus the extent separation bubble. There can be cases where these disturbances are amplified through feedback to generate sustained large scale oscillations. This occurs generally in transonic SBLI where the downstream is subsonic and hence can affect the position of shock. Supersonic inlet buzz is a manifestation of such sustained large-scale unsteadiness.

The aforementioned list of complexities discussed above is by no means exhaustive. In practical applications, phenomena like three-dimensional separated region, coupling between surface ablation and flow, and thermo-chemical non-equilibrium effects will get added to the list.

SBLI occurs ubiquitously in both internal and external features of a hypersonic vehicle. For instance, prediction and control of SBLI occurring on the body of a vehicle is crucial for design and navigation of the vehicle. Control of SBLI occurring, for example, in supersonic aircraft inlets also plays a critical role in propulsion of the vehicle. All the aforementioned complexities make the numerical simulation of hypersonic SBLI challenging.

3 Numerical simulation of hypersonic perfect gas SBLI

Hypersonic flow simulations, and specifically SBLI computations, are extremely sensitive to the numerical scheme and grid used. For illustration, Fig. 3 shows the results of flow over a blunt capsule simulated by varying the parameter ε of the scheme. Notice how significantly the surface heat flux results (right half of the figures) varies when ε is varied from 0 to 0.3. Fig. 4 shows the variation in the outer shock location of flow over double cone with change in limiter of the numerical scheme. We can notice the significant differences in prediction of separation and reattachment points as the numerical scheme is changed. In fact, one of the solutions does not predict separation at all. The sensitivity to grid resolution and design has also been highlighted in the literature and generation of workable grids can actually take more time compared to the simulation run time [6].

The high sensitivity of numerical simulations is an issue in applications like scramjet inlets where the flow features are sensitive to flight conditions. Small numerical errors in such cases can produce completely erroneous results. In this work, we aim at performing discontinuous Galerkin (DG) simulations of relatively simple laminar perfect gas hypersonic SBLI. We hope that these high order accurate simulations can alleviate the issue of sensitivity. Further, since the DG method is relatively new and still a topic of research, this work will also be relevant in study of DG shock limiters since hypersonic SBLI cases can serve as stringent test cases. Finally, the impetus for this work also comes from the fact that DG methods have high parallel computation scalability which render them usable even in full-scale high resolution simulations of practical problems.

A framework pens2D: Parallel Explicit Navier-Stokes solver for 2D flows, is being developed using deal.II [3, 4, 5] finite element library.

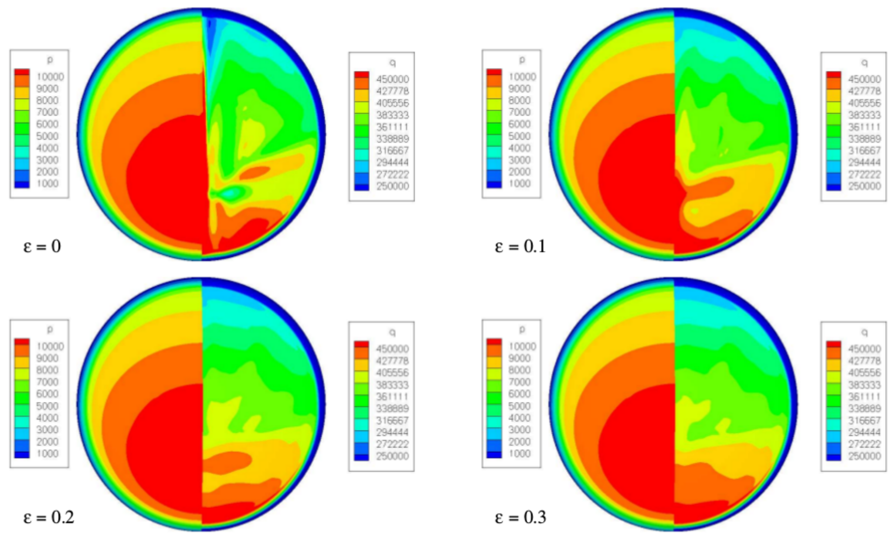

Figure 3: Sensitivity of surface pressure and heat flux in Mach 21 flow over blunt capsule to a parameter ε used in a numerical scheme. Figure shows the projected view along vehicle's axis. Adapted from Candler, Mavriplis, and Trevino [6].

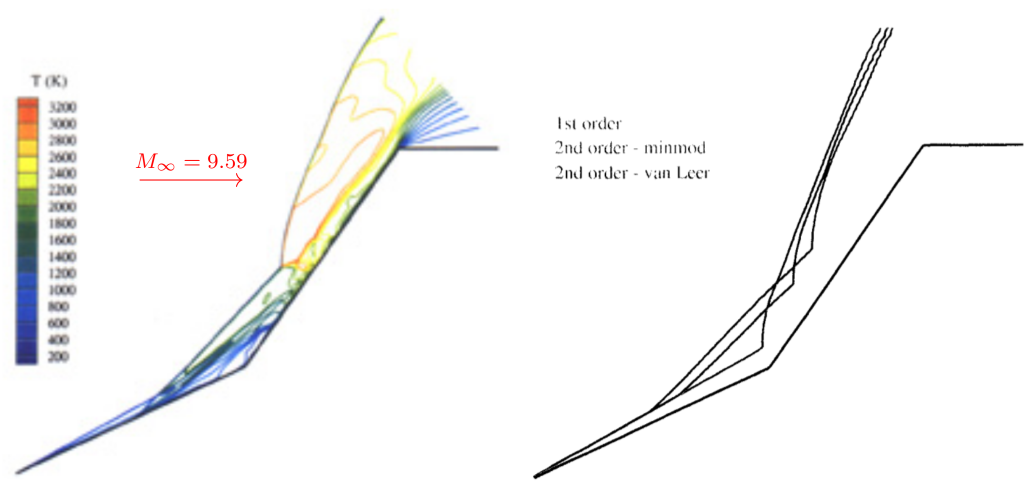

Figure 4: Temperature contour (left) and outer shock location (right) computed with different limiters by corrected Roe's scheme and explicit second order time stepping for run Mach 9.59 flow over a double cone geometry. Adapted from Candler, Nompelis, and Druguet [7].

References

[1] John D Anderson Jr. *Hypersonic and high-temperature gas dynamics*. American Institute of Aeronautics and Astronautics, 2006.

[2] Holger Babinsky and John K Harvey. *Shock Wave-Boundary-Layer Interactions*. Cambridge Aerospace Series. Cambridge University Press, 2011.

[3] W. Bangerth, R. Hartmann, and G. Kanschat. "deal.II – a General Purpose Object Oriented Finite Element Library". In: *ACM Trans. Math. Softw.* 33.4 (2007), pp. 24/1–24/27.

[4] Daniel Arndt et al. "The deal.II finite element library: Design, features, and insights". In: *Computers & Mathematics with Applications* (2020).

[5] Daniel Arndt et al. "The `deal.II` Library, Version 9.2". In: *Journal of Numerical Mathematics* 28.3 (2020), pp. 131–146. DOI: `10.1515/jnma-2020-0043`.

[6] Graham Candler, Dimitri Mavriplis, and Loretta Trevino. "Current status and future prospects for the numerical simulation of hypersonic flows". In: *47th AIAA Aerospace Sciences Meeting Including the New Horizons Forum and Aerospace Exposition*. 2009, p. 153.

[7] Graham Candler, Ioannis Nompelis, and Marie-Claude Druguet. "Navier-Stokes Predictions of Hypersonic Double-Cone and Cylinder-Flare Flow Fields". In: *39th Aerospace Sciences Meeting and Exhibit*. 2001, p. 1024.