CV10

Př Najděte kořeny polynomu: $x^3+x^2-4x-4 \overset{?}{=} 0$

„HÁDÁME $\pm 1, \pm 2$"

ZDE $x=-1$ je kořen

dělíme: $x^3+x^2-4x-4 : (x+1) = x^2-4 = (x-2)(x+2)$

$-x^3 \mp x^2 \;|\; \boxed{-4x-4}$

$\Rightarrow \quad x^3+x^2-4x-4 = (x+1)(x+2)(x-1)$

Př Určete D_f + asymptoty

$f(x) = \sqrt{x^2-6x+10}$ $\quad x^2-6x+10 \overset{!}{\geq} 0$

$D = 36-4\cdot 1\cdot 10 < 0$

$\Rightarrow$

$D_f = \mathbb{R}$

a) asymptota v $+\infty$

$$\lim_{x\to+\infty} \sqrt{x^2-6x+10} = \lim_{x\to+\infty} \sqrt{x^2\left(1-\frac{6}{x}+\frac{10}{x^2}\right)} =$$

$$= \lim_{x\to+\infty} |x| \sqrt{1-\frac{6}{x}+\frac{10}{x^2}} = \left(\lim_{x\to+\infty} x\right)\left(\lim_{x\to+\infty}\left(\sqrt{1-\frac{6}{x}+\frac{10}{x^2}}\right)\right) =$$

$$= +\infty \cdot \sqrt{1} = +\infty$$

$$\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\sqrt{x^2-6x+10}}{x} = \lim_{x\to+\infty} \frac{\cancel{x}\sqrt{1-\frac{6}{x}+\frac{10}{x^2}}}{\cancel{x}} =$$

$= \sqrt{1} = \underset{\neq 0}{1} = a$ ... f bude mít asymptotu, dopočtěme b

$$\lim_{x\to+\infty} f(x) - ax = \lim_{x\to+\infty}\left(\sqrt{x^2-6x+10} - x\right) =$$

$$= \lim_{x\to+\infty}\left(\sqrt{x^2-6x+10} - \sqrt{x^2} \cdot \frac{\sqrt{x^2-6x+10}+\sqrt{x^2}}{\sqrt{x^2-6x+10}+\sqrt{x^2}}\right) =$$

$$= \lim_{x\to+\infty} \frac{-6x+10}{\sqrt{x^2-6x+10}+\sqrt{x^2}} = \lim_{x\to+\infty} \frac{-6x+10}{|x|(\sqrt{1-\frac{6}{x}+\frac{10}{x^2}}+|x|} =$$

$$= \lim_{x\to+\infty} \frac{\cancel{x}\left(-6+\frac{10}{x}\right)}{\cancel{x}\left[\sqrt{1-\frac{6}{x}+\frac{10}{x^2}}+1\right]} = \frac{-6+0}{\sqrt{1}+1} = \frac{-6}{2} = -3$$

f má v $+\infty$ asymptotu $y = ax+b = x-3$

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b) asymptota v $-\infty$

$$\lim_{x\to-\infty} \sqrt{x^2-6x+10} = \lim_{x\to-\infty} \sqrt{x^2\left(1-\frac{6}{x}+\frac{10}{x^2}\right)} =$$

$$= \lim_{x\to-\infty} |x|\sqrt{1-\frac{6}{x}+\frac{10}{x^2}} = \left(\lim_{x\to-\infty}(-x)\right)\cdot\left(\lim_{x\to-\infty}\sqrt{1-\frac{6}{x}+\frac{10}{x^2}}\right) =$$

$$= +\infty \cdot (\sqrt{1}) = +\infty$$

$$\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{\sqrt{x^2-6x+10}}{x} = \lim_{x\to-\infty} \frac{(-\cancel{x})\sqrt{1-\frac{6}{x}+\frac{10}{x^2}}}{\cancel{x}} =$$

$= -1\sqrt{1} = -1 \underset{\neq 0}{=} a$ … f bude mít asymptotu, dopočteme b

$$\lim_{x\to-\infty} \sqrt{x^2-6x+10} - \underbrace{(-x)}_{|x|} = \lim_{x\to-\infty} \sqrt{x^2-6x+10} - \sqrt{x^2} =$$

$$= \lim_{x\to-\infty} \sqrt{x^2-6x+10} - \sqrt{x^2} \cdot \frac{\sqrt{x^2-6x+10} + \sqrt{x^2}}{\sqrt{x^2-6x+10} + \sqrt{x^2}} =$$

$$= \lim_{x\to-\infty} \frac{-6x+10}{\sqrt{x^2-6x+10} + \sqrt{x^2}} = \lim_{x\to-\infty} \frac{x\left(-6+\frac{10}{x}\right)}{|x|\sqrt{1-\frac{6}{x}+\frac{10}{x^2}} + |x|} =$$

$$= \lim_{x\to-\infty} \frac{\cancel{x}\left(-6+\frac{10}{x}\right)}{-\cancel{x}\left[\sqrt{1-\frac{6}{x}+\frac{10}{x^2}}+1\right]} = \frac{-6+0}{-1\left[\sqrt{1}+1\right]} = \frac{-6}{-2} = 3$$

. f má v $-\infty$ asymptotu $y = ax+b = -x+3$