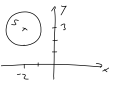
Kuželosečky

Načrtněte kružnici $k: (x+2)^2+(y-3)^2=2$

$(x-m)^2+(y-n)^2=r^2 \qquad S=[m,n]$

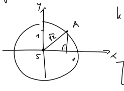
$\rightarrow S=[-2,3]$

$\underline{r=\sqrt{2}}$



$r^2=2 \rightarrow r=\pm\sqrt{2}$, ale $r>0$

Napište rovnici kružnice k : $S=[0,0]$ a $A=[1,1] \in k$



$k: x^2+y^2=r^2$

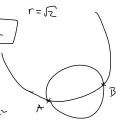
$A \in k: \quad 1^2+1^2=r^2$

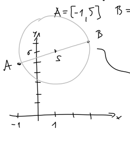
$r=\sqrt{2}$

$\boxed{k: x^2+y^2=2}$

$k: ?$ AB je průměrem k

$A=[-1,5] \quad B=[3,7]$





$(x-m)^2+(y-n)^2=r^2$

$S_k=S_{AB}=\frac{A+B}{2}=\left[\frac{-1+3}{2},\frac{5+7}{2}\right]=[1,6]$

Poloměr?

- $A \in k: \quad (\overset{x-m}{-1-1})^2+(\overset{y-n}{5-6})^2=r^2$

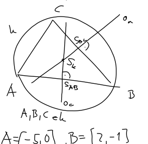
$4 + 1 = r^2$

$r=\sqrt{5}$

- $r=|SB|=\sqrt{(3-1)^2+(7-6)^2}=\sqrt{4+1}=\sqrt{5}$

$\boxed{k: (x-1)^2+(y-6)^2=5}$

$k: ?$ k opsaná $\triangle ABC$



$k: (x-m)^2+(y-n)^2=r^2$

$A \in k: (-5-m)^2+(0-n)^2=r^2$

$B \in k: (2-m)^2+(-1-n)^2=r^2$

$C \in k: (1-m)^2+(2-n)^2=r^2$

$A=[-5,0], B=[2,-1]$

$C=[1,2]$

① $25+10m+m^2 \qquad +n^2=r^2$

② $4-4m+m^2+1+2n+n^2=r^2$

③ $1-2m+m^2+4-4n+n^2=r^2$

①−②: $21+14m+0\cdot m^2-1-2n+0\cdot n^2=0\cdot r^2$

①−③: $24+12m+0\cdot m^2-4+4n+0\cdot n^2=0\cdot r^2$

$14m-2n=-20 \quad /\cdot 2$

$12m+4n=-20$

$40m=-60 \longrightarrow \boxed{m=-\frac{3}{2}}$

$-14\cdot\frac{3}{2}-2n=-20$

$\boxed{n=-\frac{1}{2}}$

$k: \left(x+\frac{3}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=r^2$

- $\underset{[-5,0]}{A} \in k: \left(-5+\frac{3}{2}\right)^2+\left(0+\frac{1}{2}\right)^2=r^2$

$\frac{49}{4}+\frac{1}{4}=r^2$

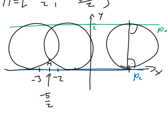
$r^2=\frac{25}{2} \longrightarrow r=\frac{5}{\sqrt{2}}=\frac{5\sqrt{2}}{2}$

$\boxed{k: \left(x+\frac{3}{2}\right)^2+\left(y+\frac{1}{2}\right)^2=\frac{25}{2}}$

- $r=|SA|$

$k: ?$ dotýká se přímek $p_1: y=2$, $p_2: y=0$

a prochází bodem $M=\left[-\frac{5}{2}, 1-\frac{\sqrt{3}}{2}\right]$



Z obrázku: $S=[m,1]$

$r=1$

$k: (x-m)^2+(y-1)^2=1$

$M \in k: \left(-\frac{5}{2}-m\right)^2+\left(1-\frac{\sqrt{3}}{2}-1\right)^2=1$

$\frac{25}{4}+5m+m^2+\frac{3}{4}=1 \quad /\cdot 4$

$4m^2+20m+24=0 \quad /:4$

$m^2+5m+6=0$

$(m+3)\cdot(m+2)=0 \longrightarrow m_1=-3$

$m_2=-2$

$$\boxed{\begin{aligned} k_1&: (x+3)^2+(y-1)^2=1 \\ k_2&: (x+2)^2+(y-1)^2=1 \end{aligned}}$$

Elipsa

Načrtněte: $e: 4x^2+9y^2=36 \quad /:36 \qquad 9x^2+9y^2=36 \quad /:9$

$\frac{x^2}{9}+\frac{y^2}{4}=1 \qquad\qquad x^2+y^2=4$

elipsa $\qquad\qquad$ kružnice

$\rightarrow S=[0,0]$

$\rightarrow a=3$

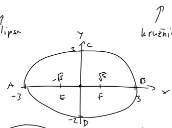
$b=2$

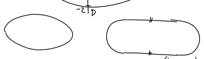
$a^2=e^2+b^2$

$e=\sqrt{9-4}=\sqrt{5}$

$E=[-\sqrt{5},0]$

$F=[\sqrt{5},0]$

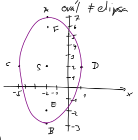




ovál ≠ elipsa

Elipsa: $E=[-2,-2]$, $F=[-2,6]$

$A=[-2,7]$ hl. vrchol



$S_e=S_{EF}=[-2,2]$

$e=|SF|=4 \qquad b=\sqrt{a^2-e^2}=3$

$a=|SA|=5$

$$\boxed{e: \frac{(x+2)^2}{3^2}+\frac{(y-2)^2}{5^2}=1}$$

Vzájemná poloha přímek v $\mathbb{R}^3$

$p=\{[-6+t, 7-t, 2t], t\in\mathbb{R}\}$

$q=\{[-5-k, 3-2k, \underset{4+k}{5+k}], k\in\mathbb{R}\}$

$\vec{u}_p=(1,-1,2)$

$\vec{u}_q=(-1,-2,1)$

$\vec{u}_p \neq \alpha\cdot\vec{u}_q \quad \forall\alpha\in\mathbb{R}$

$\rightarrow p \nparallel q$ a $p \neq q$

$\vec{u}_p\cdot\vec{u}_q=-1+2+2=3\neq 0$

$\rightarrow \neg(p\perp q)$

$x=x$

$y=y$

$z=z$

$-6+t=-5-k$

$7-t=3-2k$

$2t=5+k \qquad 4+k$

$1=-2-3k \longrightarrow k=-1$

$-6+t=-5+1 \longrightarrow t=2$

Zk.: rovnice 3: $LS=4$, $PS=4$ ✓

$p: t=2$

$\rightarrow x=-4$

$y=5$

$z=4$

$q: k=-1$

$x=-4$

$y=5$

$z=4 \qquad z=3$

$P_p=[-4,5,4]$

$P_q=[-4,5,3]$

$\rightarrow p\cap q=\emptyset$

p a q jsou mimoběžky

$p\cap q=\{P=[-4,5,4]\} \Rightarrow p \not\parallel q$

Rovina: $\rho=\{[1+t+k, 2+3t-k, 5t+k], t,k\in\mathbb{R}\}$

→ obecná rovnice

→ průsečíky

$x=1+t+k$

$y=2+3t-k$

$z=5t+k$

$x+y=3+4t \quad /\cdot(-2)$

$y+z=2+8t$

$-2x-2y+y+z=-6-8t+2+8t$

$-2x-y+z=-4$

$\boxed{\rho: -2x-y+z+4=0}$

$\vec{u}=(1,3,5)$

$\vec{v}=(1,-1,1)$

$\vec{n}=(-2,-1,1)$

$\vec{u}\cdot\vec{n}=-2-3+5=0$

$\vec{v}\cdot\vec{n}=-2+1+1=0$

úsekový tvar:

$\rho: \frac{x}{2}+\frac{y}{4}-\frac{z}{4}=1$

$P_x=[2,0,0] \quad P_y=[0,4,0] \quad P_z=[0,0,-4]$