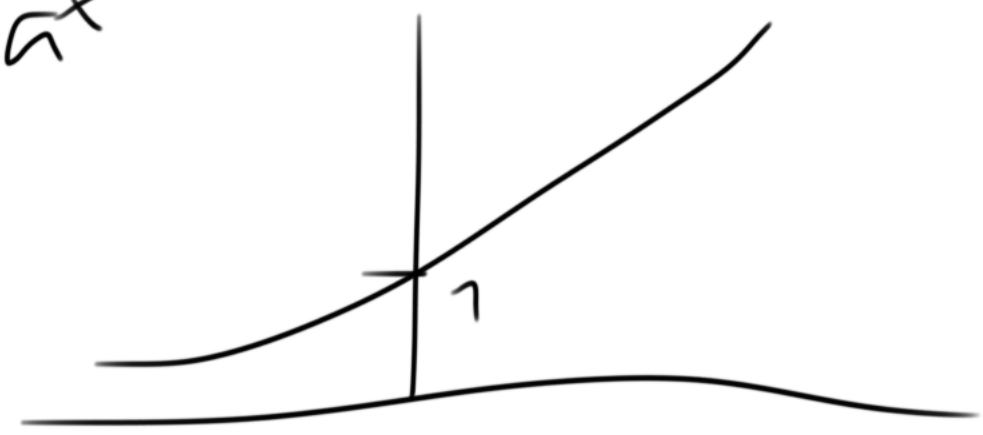
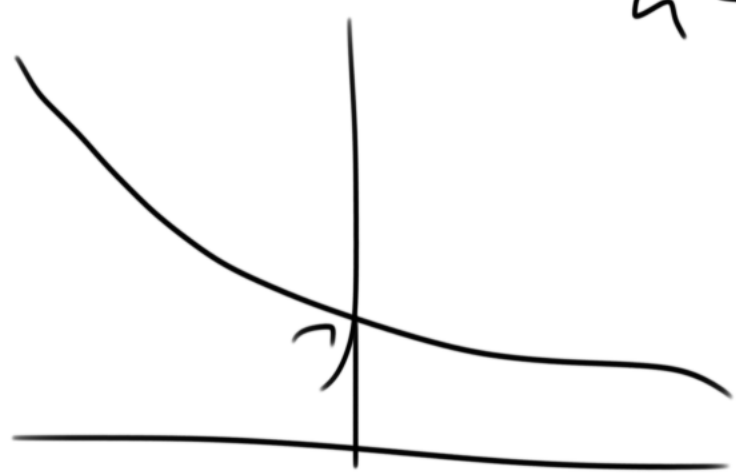
Exponenciální a log. fce

$a > 1$

a^x

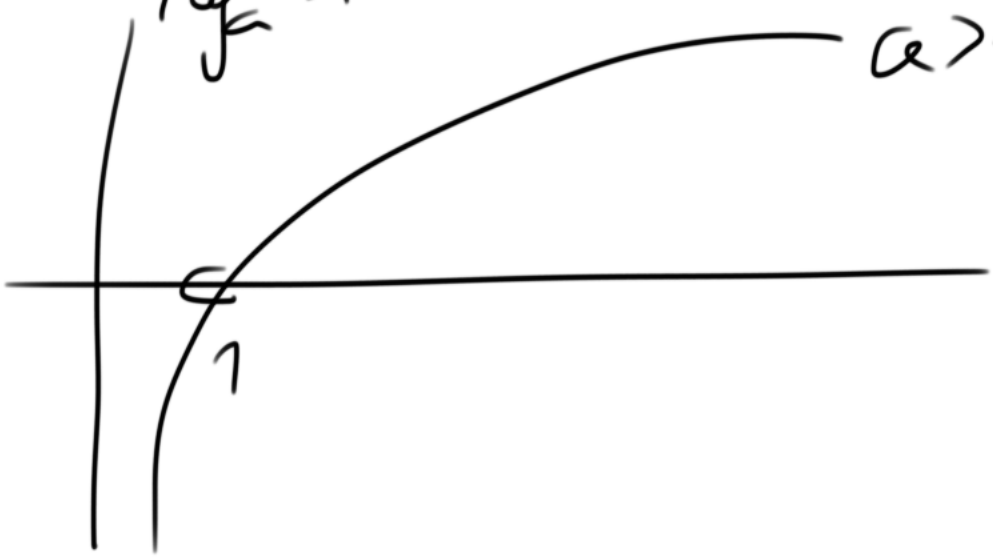


$a < 1$

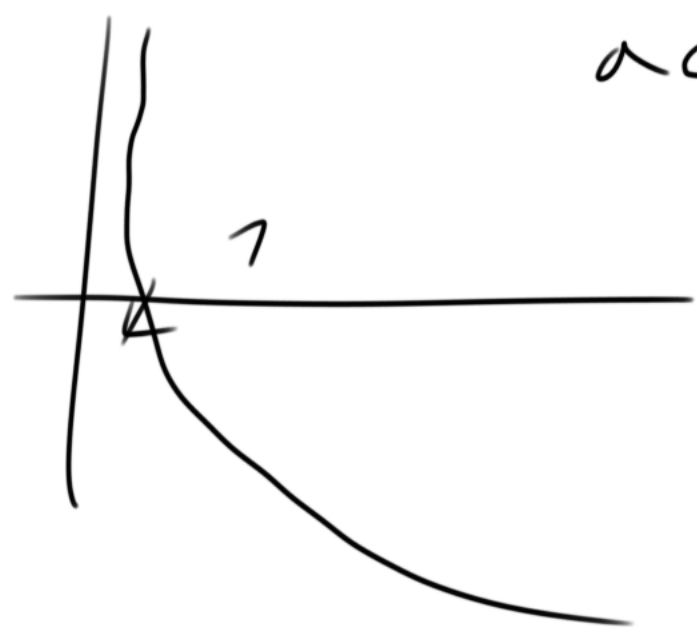


$\log_a x$

$a > 1$



$a < 1$



$a^0 = 1$

$\log_a 1 = 0$

$a^x > 0 \quad \forall x \in \mathbb{R}$

$D_f = (0, \infty)$

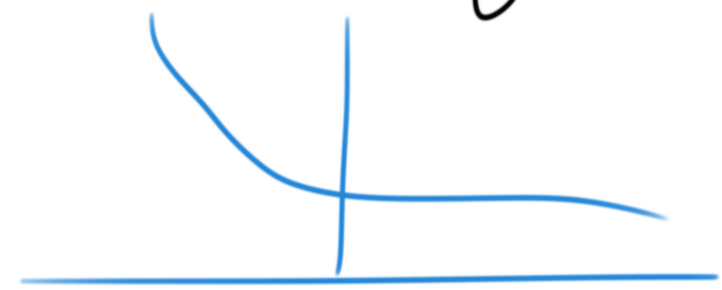
$D_f = \mathbb{R}$

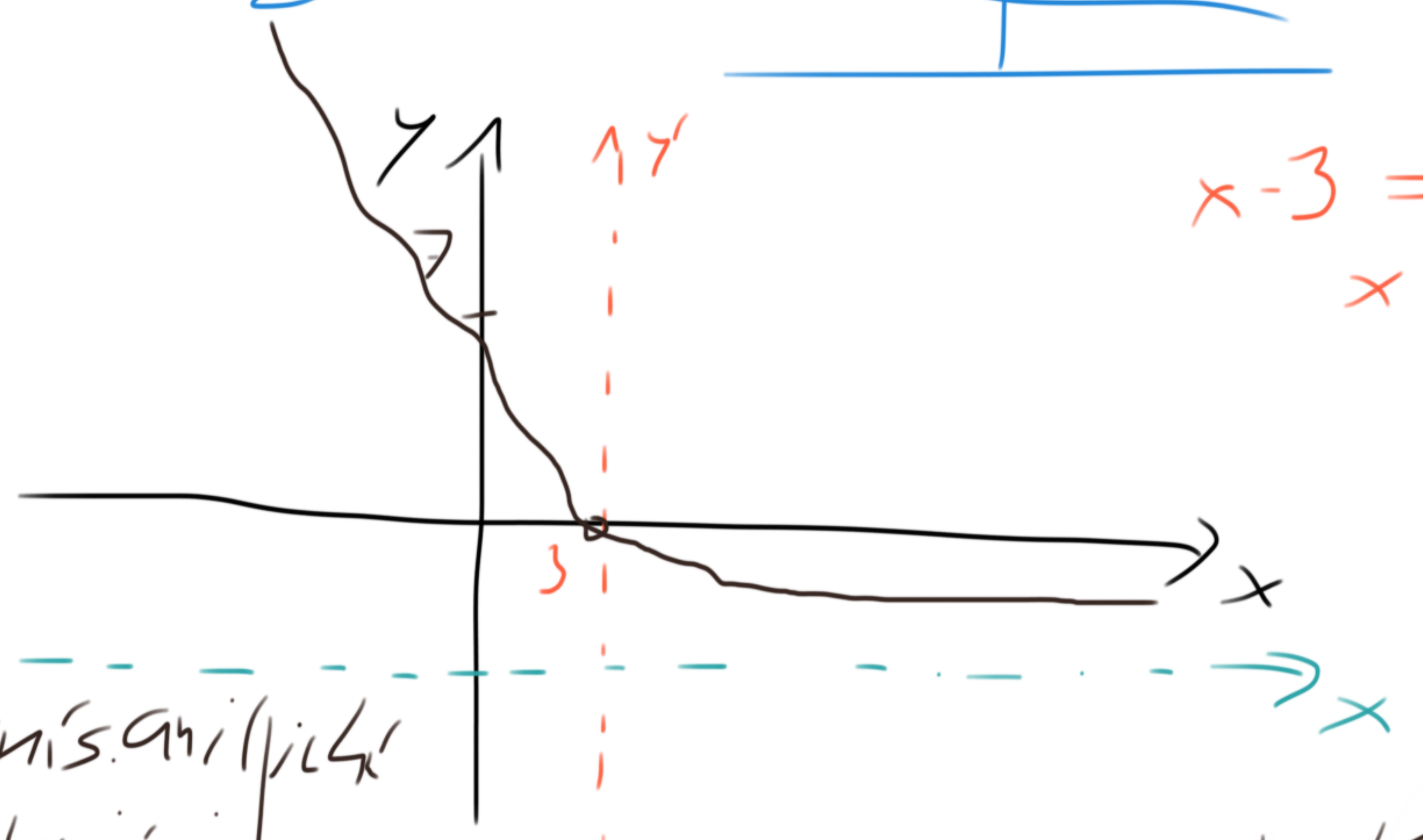
$H_f = \mathbb{R}$

desmos.com

$f: y = \left(\frac{1}{2}\right)^{x-3} - 1$ graf

$a = \frac{1}{2} < 1$





$x - 3 = 0$

$x = 3$

neníšanilický klesající

$y' = \left(\frac{1}{2}\right)^{x'}$

$x' = 0 : y' = 1$

průs. y:

$y = \left(\frac{1}{2}\right)^{0-3} - 1 = 8 - 1 = 7$

$0 = \left(\frac{1}{2}\right)^{x-3} - 1$

$\left(\frac{1}{2}\right)^{x-3} = \left(\frac{1}{2}\right)^{0} \Rightarrow x = 3$

$y = \log_2(x+4) - 1$

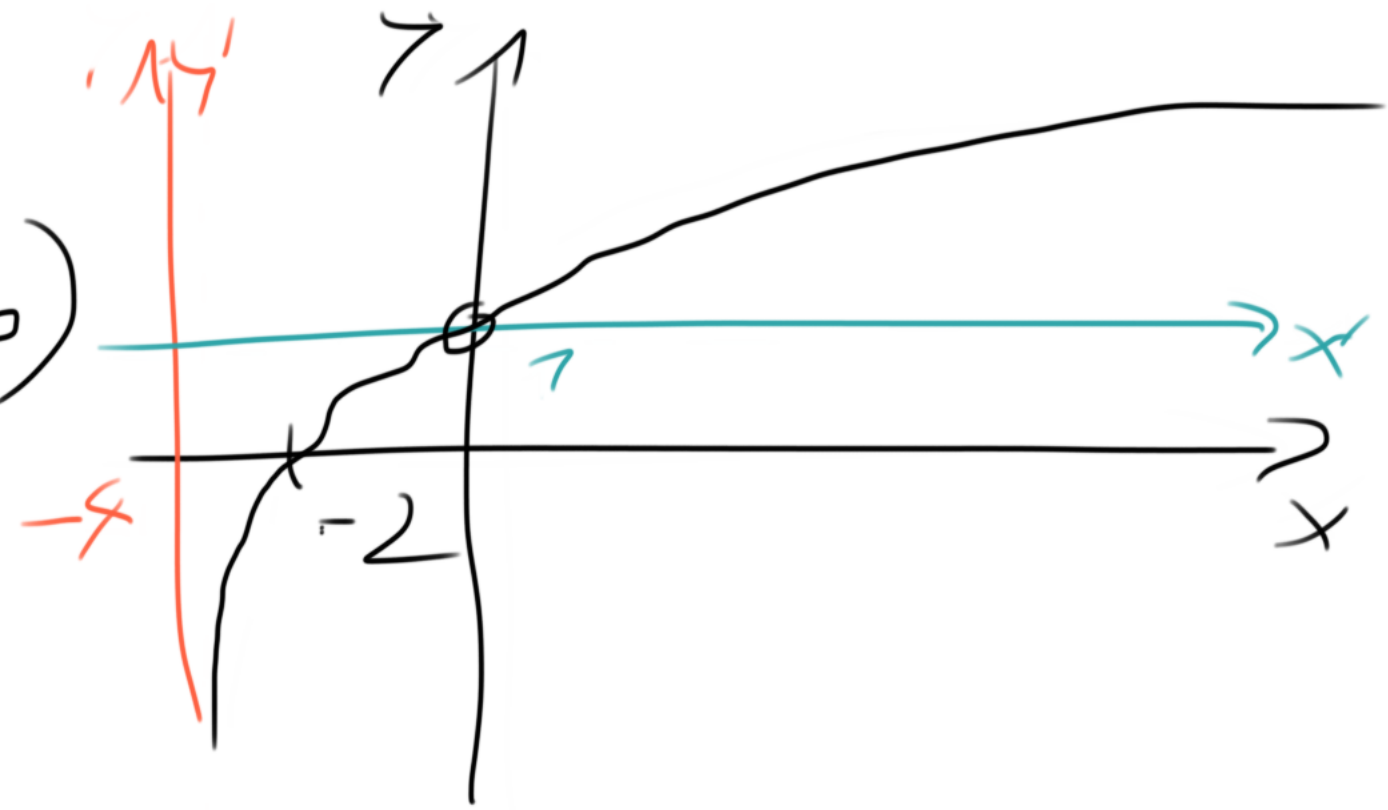
$a = 2 > 1$

$x + 4 > 0$

$x > -4$

$D_f = (-4, \infty)$

$H_f = \mathbb{R}$



priesek y

$x = 0: \quad y = \log_2(0+4) - 1$

$= \log_2 4 - 1$

$= 2 - 1 = 1$

priesek x

$0 = \log_2(x+4) - 1$

$1 = \log_2(x+4)$

$\log_2 2 = \log_2(x+4)$

$2 = x + 4$

$x = -2$

$$2^x \cdot 8^x = 2^x \cdot (2^3)^x = 2^x \cdot 2^{3x} = \underline{2^{4x}}$$

$$\underline{a^r \cdot a^s = a^{r+s}}$$

$$(a^r)^s = a^{r \cdot s}$$

$$\log_a x = s \iff a^s = x$$

$$\log_a (x \cdot y) = \log_a x + \log_a y$$

$$\log_a \left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a x^r = r \cdot \log_a x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Vypočtěte

$$\log_{\frac{1}{3}} 9 = -2$$

$$9 = 3^2 = \left(\frac{1}{3}\right)^{-2}$$

$$\log_{\frac{1}{3}} \left(\frac{1}{3}\right)^{-2} = -2 \cdot \log_{\frac{1}{3}} \frac{1}{3}$$

$$= -2$$

$$= -2$$

$$\log_8 \sqrt{2}$$

$$\sqrt{2} = 2^{1/2} = \left(8^{1/3}\right)^{1/2} = 8^{1/6}$$

$$8 = 2^3 \Rightarrow 2 = \sqrt[3]{8}$$

$$= 8^{1/3}$$

$$\log_8 \sqrt{2} = \log_8 8^{1/6} = \frac{1}{6}$$

$$\log_{\sqrt{5}} 125 = \log_{\sqrt{5}} (5 \cdot 25) = \underbrace{\log_{\sqrt{5}} 5}_{1} + \log_{\sqrt{5}} 25$$

$$= 1 + \log_{\sqrt{5}} 5^2 = 1 + 2 = 3$$

mplete x :

$\log_3 x = 4 \quad \Rightarrow \quad \underline{x = 3^4 = 81}$

$\log_a x = s \iff a^s = x$

$a = 3$

$s = 4 \qquad x = a^s$

$\log_5 x = 0 \Rightarrow x = 5^0 = 1$

$\log_5 x = -\frac{3}{5} \Rightarrow x = \underline{10^{-\frac{3}{5}}}$

$\log = \log_{10}$

$= \sqrt[5]{0{,}001}$ ✗

$x = 10^{-\frac{3}{5}} = \frac{1}{10^{\frac{3}{5}}}$

$= \frac{1}{\sqrt[5]{0{,}001}}$

$\log_{\frac{1}{5}} x = -1 \Rightarrow \underline{x = \frac{1}{5}^{-1} = 5}$

$\log_a 27 = 3$

$a^3 = \underbrace{27}_{3} = 3 \cdot 9 = 3^3$

$a^3 = 3^3$

$a = 3$

$\log_a 4 = \frac{1}{4}$

$a^{\frac{1}{4}} = 4 \quad / \cdot 4$

$a = 4^4$

$a = 256$

$\log_a \sqrt{8} = 3$

$a^3 = \sqrt{8} \quad / \cdot \frac{1}{3}$

$a = \left(\sqrt{8}\right)^{\frac{1}{3}} = \left(\sqrt{2^3}\right)^{\frac{1}{3}}$

$\boxed{a = \sqrt{2}}$

vyjádři pomocí $\log a$, $\log b$, $\log c$ ...

$$\log \frac{a^2 b^3}{100\sqrt{c}} = \log(a^2 \cdot b^3) - \log(100\sqrt{c})$$

$$= \log a^2 + \log b^3 - \log 100 - \log \sqrt{c}$$

$$\log 100 = \log 10^2$$

$$\log \sqrt{c} = \log c^{\frac{1}{2}}$$

$$= 2\log a + 3\log b - 2 \cdot \underbrace{\log 10}_{1} - \frac{1}{2}\log c$$

$$= \underline{2\log a + 3\log b - \frac{1}{2}\log c - 2}$$

Řešte na $\mathbb{R}$

1) $2^{3x-1} \cdot 4 = 8^{x+1} \cdot \left(\frac{1}{2}\right)^x$

$$2^{3x-1} \cdot 2^2 = (2^3)^{x+1} \cdot (2^{-1})^x$$

$$2^{3x+1} = 2^{3x+3} \cdot 2^{-x}$$

$$2^{3x+1} = 2^{2x+3}$$

$3x+1 = 2x+3$

$x = 2$

2) $\frac{3^x}{2 \cdot 3^{\sqrt{3}}} = 4,5 \quad / \cdot 2 \cdot 3^{\sqrt{3}}$

$3^x = 9 \cdot 3^{\sqrt{3}}$

$3^x = 3^2 \cdot 3^{\sqrt{3}}$

$3^x = 3^{\sqrt{3}+2}$

$\boxed{x = \sqrt{3} + 2}$

3) $3^x + 3^{x+1} = 108$

$3^x + 3^{x+1} = 3 \cdot 36$

$3^x + 3^{x+1} = 3^3 \cdot 2^2$

$x + x + 1 = 3 + 2$ ✗

$3^x + 3^x \cdot 3 = 3^3 \cdot 2^2$

$3^x \underbrace{(1+3)}_{4} = 3^3 \cdot 2^2$

$3^x = 3^3$

$\boxed{x = 3}$

$\boxed{a^{f(x)} = b^{g(x)} \Rightarrow a = b}$

4) $3^x + 3^{x+1} = 7 \cdot 4^x - 4^{x+1}$

$3^x(1+3) = 4^x \cdot (7-4)$

$3^x \cdot 4 = 4^x \cdot 3 \quad / : 3^x \; / : 3$

$\frac{4}{3} = \frac{4^x}{3^x}$

$\frac{4}{3} = \left(\frac{4}{3}\right)^x$

$\underline{\underline{x = 1}}$

5) $3^x = 10 \quad / \cdot \log_3(\ldots)$

$\log_3 3^x = \log_3 10$

$x = \log_3 10$

6) $2^x \cdot 3^{x-1} = 6$

$2^x \cdot 3^x \cdot 3^{-1} = 6$

$6^x = 18 \quad / \log_6$

$x = \log_6 18$

$2^x \cdot 3^x = 6$

$6^x = 6$

$x = 1$

7) $4^{2x} - 6 \cdot 4^x + 8 = 0$

$y = 4^x$

$y^2 - 6y + 8 = 0$

$(y-4)(y-2) = 0$

$y_{1,2} = 4, 2$

$4^{x_1} = 4$

$x_1 = 1$

$4^{x_2} = 2 \quad / \log_2$

$\log_2 4^{x_2} = \log_2 2$

$x_2 \cdot 2 = 1$

$x_2 = \frac{1}{2}$

$4^{2x} - 2 \cdot 4^x - 8 = 0$

$y = 4^x$

$y^2 - 2y - 8 = 0$

$(y-4)(y+2) = 0$

$y_{1,2} = 4, -2$

$x_1 = 1$

$4^{x_2} = -2$

$\mathbb{R}$

$a^x > 0 \quad \forall x \in \mathbb{R}$

Logaritmické rce

$\log_2(x+1) = 3$

$\log_2(x+1) = \log_2 8$

$x+1 = 8$

$x = 7$

$\log_2 z = 3$

$2^3 = z$

$8 = z$

Zk.:

$LS = \log_2(7+1)$

$= \log_2 8$

$= 3$

$LS = PS$ ✓

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4. $\log_3(2x-1) = 12$

$\log_3(2x-1) = 3$

$\log_3(2x-1) = \log_3 27$ $\quad 3^3 = 27$

$2x-1 = 27$

$x = 14$

$$\log x = 2\log 5 + \log 4$$

$$\log x = \log 25 + \log 4$$

$$\log x = \log 100 \qquad \log 100 = 2$$

$$x = 100$$

$$\frac{\log_3 x}{1+\log_3 2} = 2 \qquad / (1 + \log_3 2)$$

$$\log_3 x = 2 + 2\log_3 (2)$$

$$\log_3 x = \log_3 9 + \log_3 4$$

$$\log_3 x = \log_3 36$$

$$\boxed{x = 36}$$

$$\log_8 \sqrt{x+30} + \log_8 \sqrt{x} = 1$$

$$\log_8 \sqrt{x} = \log_8 x^{\frac{1}{2}} = \frac{1}{2}\log_8 x$$

$x+30 > 0$

$x > 0$

$x > -30$

$\boxed{x > 0}$

$$\frac{1}{2}\log_8(x+30) + \frac{1}{2}\log_8 x = 1 \quad / \cdot 2$$

$$\log_8[x\cdot(x+30)] = 2$$

$$\log_8[x\cdot(x+30)] = \log_8 64$$

$$\log_8[x^2+30x] = \log_8 64$$

$$x^2+30x-64 = 0$$

$$(x+32)(x-2) = 0$$

$$-2\cdot 32 = -64$$

$$32x - 2x = 30x$$

$\boxed{x_1 = -32}$ $\boxed{x_2 = 2}$

$$\log_2^2 x + 2\log_2 x - 3 = 0$$

$$y = \log_2 x$$

$$y^2 + 2y - 3 = 0$$

$$(y+3)\cdot(y-1) = 0$$

$$y_1 = -3 \qquad y_2 = 1$$

$$\log_2 x_1 = -3 \qquad \log_2 x_2 = 1$$

$$x_1 = 2^{-3}$$

$$\boxed{x_1 = \frac{1}{8}} \qquad \boxed{x_2 = 2}$$

$$\log x^5 - \log x^4 + \log x^3 = 12$$

$$\log \frac{x^5 \cdot x^3}{x^4} = 12$$

$$\log x^4 = 12$$

$$\log x^4 = \log 10^{12}$$

$$x^4 = 10^{12}$$

$$\boxed{x = 10^3}$$