6.12.

- goniometrické funkce
- exp. a log. funkce

a rovnice

- úvod do AG

$\sin x, \cos x, tg x, cotg x$:

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$tg\, x$	0	$\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\times$
$cotg\, x$	$\times$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

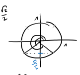
$\boxed{\sin^2 x + \cos^2 x = 1}$

$\sin x = \frac{1}{2} \rightarrow \cos x = \sqrt{1-\sin^2 x} = \sqrt{1-\frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

$tg x = \frac{\sin x}{\cos x}$

$cotg x = \frac{\cos x}{\sin x} = \frac{1}{tg x}$

$\sin x = -\frac{\sqrt{2}}{2}$



Tabulka $\Rightarrow x = l \cdot \frac{\pi}{4}$

$$\boxed{\begin{aligned} x_1 &= \tfrac{5}{4}\pi + k\cdot 2\pi \\ x_2 &= \tfrac{7}{4}\pi + k \cdot 2\pi \end{aligned} \quad k \in \mathbb{Z}}$$

$\sin\left(\frac{13}{2}\pi\right) = \sin\left(\overbrace{6\pi}^{3\cdot 2\pi} + \frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$ ✓

$\frac{13}{2}\pi = \frac{12}{2}\pi + \frac{\pi}{2} = 6\pi + \frac{\pi}{2} + \frac{\pi}{2}$ ✗

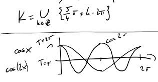
$K = \{\frac{5}{4}\pi + k \cdot 2\pi; k \in \mathbb{Z}\} \cup \{\frac{7}{4}\pi + k\cdot 2\pi; k \in \mathbb{Z}\}$

$\sum_{i=1}^{n} i = 1+2+3+\ldots+n$

A_i … množiny $i = 1, \ldots n$

$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \ldots \cap A_n$

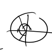
$K = \bigcup_{k\in\mathbb{Z}} \overbrace{\{\frac{5}{4}\pi + k\cdot 2\pi\}}^{A_k}$



$\cos 2x = -\frac{1}{2}$

$u = 2x$

$\cos u = -\frac{1}{2}$



$u_1 = \frac{2}{3}\pi + k\cdot 2\pi$

$u_2 = \frac{4}{3}\pi + k \cdot 2\pi$

$u_1 = \frac{2}{3}\pi$

$2x_1 = \frac{2}{3}\pi$

$x_1 = \frac{\pi}{3} + k 2\pi$ ✗

$2x_1 = \frac{2}{3}\pi + k\cdot 2\pi$

$x_1 = \frac{1}{3}\pi + \underline{k\cdot\pi}$

$x_2 = \frac{2}{3}\pi + \underline{k\cdot\pi}$

→ $\cos(nx)$ … $T = \frac{2\pi}{n}$

$\sin(nx)$ … –

$tg x, cotg x$: $T = \pi$

$f(x) = g(x)$ $\quad f, g$ goniometrické fce $\longrightarrow$ gon. rce.

$\sin^2 x + \cos^2 x = 1 \quad \forall x \in \mathbb{R}$

$\sin(x \pm y) = \sin x \cdot \cos y \pm \sin y \cdot \cos x$

$\cos(x \pm y) = \cos x \cdot \cos y \mp \sin x \cdot \sin y$

$\sin(2x) = 2 \cdot \sin x \cdot \cos x$

$\cos(2x) = \cos^2 x - \sin^2 y$

$\forall x, y \in \mathbb{R}$

$\sin^2 x + \cos^2 y = ?$

Řešte v $\mathbb{R}$: $\quad 2\cdot \sin^2(x) + 3 \cdot \cos(x) = 0$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$2 - 2\cdot\cos^2 x + 3\cdot \cos x = 0$

$u = \cos x$

$2 - 2u^2 + 3u = 0$

$u_{1,2} = \frac{-3 \pm \sqrt{9+16}}{-4} = \left\langle \begin{matrix} -\frac{1}{2} \\ 2 \end{matrix} \right.$

$u_1 = -\frac{1}{2}$

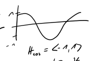
$\cos x_1 = -\frac{1}{2}$

$$\boxed{\begin{aligned} x_{11} &= \tfrac{2}{3}\pi + k\cdot 2\pi \\ x_{12} &= \tfrac{4}{3}\pi + k \cdot 2\pi \end{aligned}}$$

$u_2 = 2$

$\cos x_2 = 2$

NŘ.



$H_{\cos} = \langle -1, 1 \rangle$

$\cos x \neq 2 \quad \forall x \in \mathbb{R}$

$f: y = -tg\left(|x| - \frac{\pi}{2}\right) + 1$

$+1$ … posun v y ↑

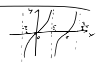
$-\frac{\pi}{2}$ … posun v x →

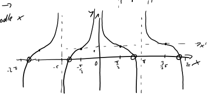
$(-1)\cdot$ … převrácení podle x

$|x|$ … f … sudá

$tg x$

$T = \pi$





Průsečíky sx:

$0 = -tg\left(|x| - \frac{\pi}{2}\right) + 1$

$tg\left(|x| - \frac{\pi}{2}\right) = 1$

$|x| - \frac{\pi}{2} = \frac{\pi}{4} + k\cdot\pi$

$D_f = \mathbb{R} \setminus \{l \cdot \pi\} \quad l \in \mathbb{Z}$

$H_f = \mathbb{R}$

f je sudá

není prostá, není omezená

↗ $(-\pi - k\pi, 0 - k\pi) \quad k \in \mathbb{N}_0$

↘ $(0 + k\pi, \pi + k\pi)$

$T = \pi$

$x \in (-\infty, 0)$

$|x| = -x$

$-x - \frac{\pi}{2} = \frac{\pi}{4} + k\cdot\pi$

$-x = \frac{3}{4}\pi + k\cdot\pi$

$\boxed{x = -\frac{3}{4}\pi - k\cdot\pi}$

pokud $k = -1$

$x = \frac{\pi}{4}$ ✗ $\notin (-\infty, 0)$

$k \in \mathbb{N}_0$

$x \in \langle 0, \infty)$

$|x| = x$

$x - \frac{\pi}{2} = \frac{\pi}{4} + k\cdot\pi$

$\boxed{x = \frac{3}{4}\pi + k\cdot\pi}$

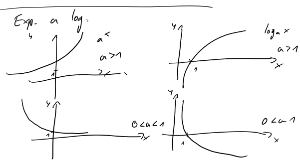
$k \in \mathbb{N}_0$

$P_x = [\frac{3}{4}\pi + k\pi, 0]$

$P_x = [-\frac{3}{4}\pi - k\cdot\pi, 0] \quad k \in \mathbb{N}_0$

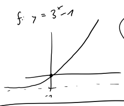
desmos.com $\qquad$ wolframalpha.com

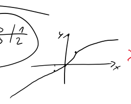
Exp. a log.:



$f: y = 3^x - 1$

x	-1	0	1
y	$-\frac{4}{3}$	0	2





✗

Řešte v $\mathbb{R}$: $\quad \log_6(x+1) + \log_6(x) = 1$

$\log_6(x+1) \rightarrow x > -1$

$\log_6(x) \rightarrow x > 0$

$\log_6[x\cdot(x+1)] = \log_6 6 \quad / 6^{(\ldots)}$

$6^{\log_6 [x\cdot(x+1)]} = 6^{\log_6 6}$

$x\cdot(x+1) = 6$

→ chceme $\log_6(\ldots) = \log_6(\ldots)$

$\log_a(x\cdot y) = \log_a x + \log_a y$

$\log_a a = 1$

$\log_a a^x = a^{\log_a x} = x$

vs: $\quad \log_6(x+1) + \log_6(x) = \log_6 6$

$x+1 \quad + \quad x \quad = 6$ ✗ ŠPATNÉ

$x^2 + x - 6 = 0$

$(x+3)\cdot(x-2) = 0$

$x_1 = -3$ ✗

$\boxed{x_2 = 2}$

$3^{2u+1} + 2^{2u+1} - 5 \cdot 6^u = 0$

$3^{2u+1} + 2^{2u+1} = 5 \cdot 6^u \quad / : 6^u$

$6^u = 2^u \cdot 3^u$

$\frac{3^{2u}\cdot 3}{2^u \cdot 3^u} + \frac{2^{2u} \cdot 2}{2^u \cdot 3^u} = 5$

$6^u > 0 \quad \forall u \in \mathbb{R}$

$\frac{3^{2u}}{3^u} = \frac{3^u \cdot 3^u}{3^u} = 3^u$

$\frac{3^u}{2^u}\cdot 3 + \frac{2^u}{3^u}\cdot 2 = 5$

$\left(\frac{3}{2}\right)^u \cdot 3 + \left(\frac{2}{3}\right)^u \cdot 2 = 5$

$x = \left(\frac{3}{2}\right)^u \longrightarrow x^{-1} = \left(\frac{2}{3}\right)^u \qquad x^{-1} = \frac{1}{x}$

$x \cdot 3 + \frac{1}{x}\cdot 2 = 5 \quad / \cdot x$

$3x^2 - 5x + 2 = 0$

$x_{1,2} = \frac{5 \pm \sqrt{25-24}}{6} = \left\langle \begin{matrix} 1 \\ \frac{2}{3} \end{matrix} \right.$

$x_1 = 1$

$1 = \left(\frac{3}{2}\right)^{u_1}$

$\left(\frac{3}{2}\right)^0 = \left(\frac{3}{2}\right)^{u_1}$

$\boxed{u_1 = 0}$

$x_2 = \frac{2}{3}$

$\frac{2}{3} = \left(\frac{3}{2}\right)^{u_2}$

$\left(\frac{3}{2}\right)^{-1} = \left(\frac{3}{2}\right)^{u_2}$

$\boxed{u_2 = -1}$

$3 \cdot 3^u \cdot (3^u - 2^u) = 2 \cdot 2^u \cdot (3^u - 2^u) \quad / : \boxed{(3^u - 2^u)}$

$3^u \neq 2^u$

$3 \cdot 3^u = 2 \cdot 2^u$

$\frac{3^u}{2^u} = \frac{2}{3}$

→ $\boxed{u = -1}$

$3^u = 2^u$: $\quad 3 \cdot 3^u \cdot 0 = 2 \cdot 2^u \cdot 0$

$0 = 0$ ✓

$\boxed{u = 0}$

$2^x \cdot 3^{x-1} = 6$

$2^x \cdot 3^{x-1} = 2 \cdot 3$

$2^x \cdot \frac{3^x}{3} = 2 \cdot 3 \quad / \cdot 3$

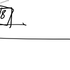
$6^x = 3 \cdot 6$

$6^x = 18 \quad / \log_6(\ldots)$

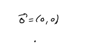
$\log_6 6^x = \log_6 18$

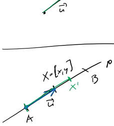
$\log_a a^x = x$

$\boxed{x = \log_6 18}$



$\vec{o} = (0,0)$





$X = A + t\cdot\vec{u}$

$X' = A + \frac{3}{2}\vec{u}$