Řešení ukázkového testu

1.1 $K = \langle -3, 3 \rangle$ $L = (-\pi, \pi)$ $M = \langle 0, \pi \rangle$

$K \cup M = \langle -3, \pi \rangle$ $L \cap K = \langle -3, 3 \rangle$ $L \setminus M = (-\pi, 0)$

1.2 $K = \{1, 2, 3, 4, 5\}$ $L = \{1, 3, 5, 15\}$ $M = \{1, 2, 3, 4\}$

$K \cup M = K$ $L \cap K = \{1, 3, 5\}$ $L \setminus M = \{5, 15\}$

2.1

40 l/h ... 9 h

45 l/h ... x

$x = \frac{40}{45} 9 = 8\,h.$

2.2 $\left(\frac{5-2}{4} + \frac{6}{8}\right) : \frac{4-7}{4} = \frac{12}{8} \cdot \frac{4}{-3} =$

$= -2$

2.3 $165 = 3 \cdot 5 \cdot 11$ $110 = 2 \cdot 5 \cdot 11$ $66 = 2 \cdot 3 \cdot 11$

$NSD(165, 110) = 5 \cdot 11 = 55$ $nsn(110, 66) = 2 \cdot 3 \cdot 5 \cdot 11 = 330$

3.1 $5 \cdot (7 + x) = x(2x + 9) - 35$

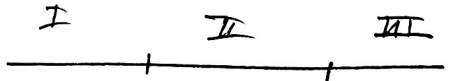
$35 + 5x = 2x^2 + 9x - 35$

$0 = 2x^2 + 4x - 70 \quad / : 2$

$0 = x^2 + 2x - 35$

$0 = (x + 7)(x - 5)$ $\longrightarrow$ $\boxed{x_1 = 5 \quad x_2 = -7}$

3.2 $|x - 2| + |x| = 3$



$|x-2| = -x+2$ $|x| = -x$

I) $x \in (-\infty, 0)$

$-x + 2 - x = 3$

$-2x = 1$

$x = -\frac{1}{2}$

$|x-2| = -x+2$ $|x| = x$

II) $x \in \langle 0, 2)$

$-x + 2 + x = 3$

$2 = 3$

NŘ.

$|x-2| = x-2$ $|x| = x$

III) $x \in \langle 2, \infty)$

$x - 2 + x = 3$

$2x = 5$

$x = \frac{5}{2}$

$K = \{-\frac{1}{2}, \frac{5}{2}\}$

3.3 $\sqrt{x-2} - (x-2) = -2$

$\sqrt{x-2} = x-4 \quad /^2$ → důsledková úprava

$x-2 = x^2 - 8x + 16$ → nutná zkouška

$0 = x^2 - 9x + 18$

$0 = (x-3)\cdot(x-6)$

$x_1 = 3 \qquad x_2 = 6$

Zkouška

$x=3$: $LS = \sqrt{3-2} = 1$, $PS = -1$ ✗

$x=6$: $LS = \sqrt{6-2} = 2$, $PS = 6-4 = 2$ ✓

$\boxed{x = 6}$

4. $2x + 3y = -9$

$-3x - \frac{9}{2}y = \frac{3}{2} \quad /\cdot\frac{2}{3}$ } ⊕

$0 = -8$ → NŘ.

5.1 $x^2 - 2(x-1) > x(x-3) + 2$

$x^2 - 2x + 2 > x^2 - 3x + 2$

$x > 0$

$\boxed{x \in \mathbb{R}^+}$

5.2 $\frac{x+3}{x-2} < \frac{1}{x} \quad /-\frac{1}{x}$

$\frac{x^2+3x-x+2}{x(x-2)} < 0$

$\frac{x^2+2x+2}{x(x-2)} < 0$

$x^2+2x+2 = (x+1)^2 + 1 > 0 \quad \forall x \in \mathbb{R}$

	0		2	
−	∄	+	∄	+
−	0	+	2	+
+	0	−	2	+

~~$x \in \mathbb{R}$~~ $\quad x \in (0, 2)$

6.1 $\left(\frac{1}{y} - \frac{1}{x}\right) \cdot \frac{x^2}{x-y} = \frac{x-y}{xy} \cdot \frac{x^2}{x-y} = \frac{x}{y}$

$x \neq y$

$x, y \neq 0$

6.2 $\frac{x^3+x^2+x+1}{x^4-1} = \frac{x^2(x+1)+(x+1)}{(x^2+1)(x+1)\cdot(x-1)} = \frac{(x^2+1)(x+1)}{(x^2+1)(x+1)\cdot(x-1)} = \frac{1}{x-1}$

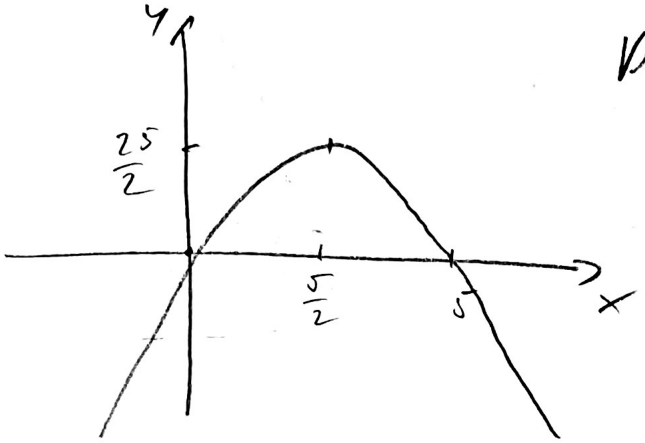
$x \neq \pm 1$

Z. 1. $f: y = -2x^2 + 7x - 6 + 3(x+2)$

$= -2x^2 + 10x = \underline{-2x \cdot (x-5)}$

Průsečíky s osou x: $x_1 = 0 \quad x_2 = 5$

s osou y: $y = 0$



Vrchol: $V = \left[-\frac{b}{2a}, c - \frac{b^2}{4a}\right]$

$= \left[\frac{5}{2}, \frac{25}{2}\right]$

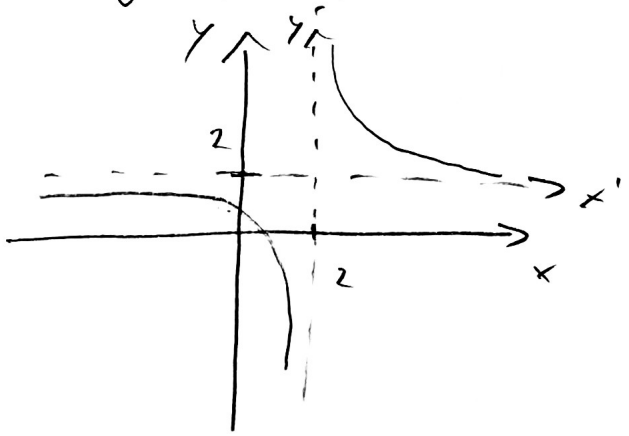
není periodická, sudá ani lichá

$D_f = \mathbb{R}, \quad H_f = (-\infty, \frac{25}{2}\rangle$

omezená shora

$\nearrow$ na $(-\infty, \frac{5}{2})$, $\searrow$ na $(\frac{5}{2}, \infty)$, není prostá

Z. 2 $g: y = \frac{3}{3x-6} + 2 = \frac{1}{x-2} + 2$



Průsečíky:

P_x: $0 = \frac{1}{x-2} + 2$

$-2x + 4 = 1$

$x = \frac{3}{2}$

P_y: $y = -\frac{1}{2} + 2 = \frac{3}{2}$

$P_x = [\frac{3}{2}, 0] \quad P_y = [0, \frac{3}{2}]$

prostá; není periodická; sudá ani lichá;

shora neomezená; klesající na $(-\infty, 2)$ a na $(2, \infty)$

$D_g = \mathbb{R} \setminus \{2\} \quad H_g = \mathbb{R} \setminus \{2\}$

7.3 $h: y = |x-2| + |5-x|$

$|x-2| = -x+2$ $\quad |x-2| = x-2$ $\quad |x-2| = x-2$

$|5-x| = 5-x$ $\quad |5-x| = 5-x$ $\quad |5-x| = x-5$

I) $x \in (-\infty, 2)$

$h(x) = -x+2+5-x$

$= -2x+7$

II) $x \in \langle 2, 5)$

$h(x) = x-2+5-x$

$= 3$

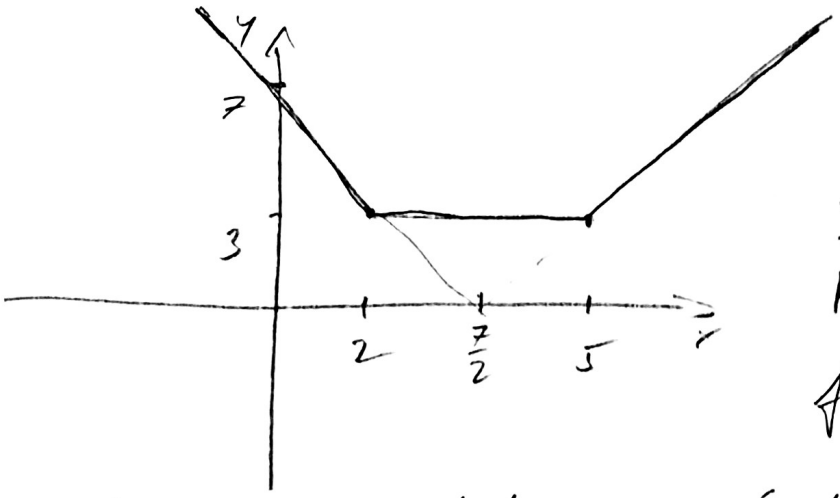
III) $x \in \langle 5, \infty)$

$h(x) = x-2+x-5$

$= 2x-7$

$2x-7 = 0$

$\Leftrightarrow x = \frac{7}{2}$



$D_h = \mathbb{R}$

$H_h = \langle 3, \infty)$

funkce je zdola omezená

není sudá ani lichá, není periodická.

$\nearrow$ na $(5, \infty)$, $\searrow$ na $(-\infty, 2)$, konst. na $(2, 5)$

není prostá.

8.1 Ne. Je pouze omezená shora.

8.2 $\mathbb{R} \setminus \mathbb{Q}$ Čísla, která nelze zapsat jako zlomek dvou celých čísel.

8.3 $A \cap B = \{x ; x \in A \wedge x \in B\}$