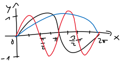
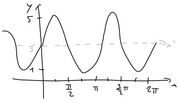
Goniometrie

Graf a všechno.

$f: \ y = 2 \cdot \sin(2x) + 3$





$D_f = \mathbb{R}$

$H_f = \langle 1, 5 \rangle$

$T = \pi$

není prostá, sudá, ani lichá

$2x = \frac{\pi}{2} + k \cdot 2\pi$

$x = \frac{\pi}{4} + k \cdot \pi$

lok. maxima: $x = \left\{ \frac{\pi}{4} + k \cdot \pi \right\}$

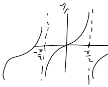
lok. minima: $x = \left\{ \frac{3}{4}\pi + k \cdot \pi \right\}$

klesá v $\left(\frac{\pi}{4} + k \cdot \pi, \frac{3}{4}\pi + k \cdot \pi \right)$ $k \in \mathbb{Z}$

roste v $\left(\frac{3}{4}\pi + k \cdot \pi, \frac{5}{4}\pi + k \cdot \pi \right)$

$g: \ y = \left| -tg\left(2x - \frac{\pi}{3}\right) + 2 \right|$

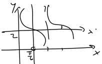
$T = \frac{\pi}{2}$



$\boxed{T = \pi}$

0 : $x = k \cdot \pi$

as. : $x = \frac{\pi}{2} + k \cdot \pi$



$tg\left(2x - \frac{\pi}{3}\right) = 0$

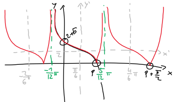
$2x - \frac{\pi}{3} = k \cdot \pi$ $k \in \mathbb{Z}$

$\left(x = \frac{\pi}{6} + k \cdot \frac{\pi}{2} \right.$

$2x - \frac{\pi}{3} = \frac{\pi}{2} + k \cdot \pi$

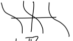
$2x = \frac{5}{6}\pi + k \cdot \pi$

$\left(x = \frac{5}{12}\pi + k \cdot \frac{\pi}{2} \right.$



$g: \ y = \left| -tg\left(2x - \frac{\pi}{3}\right) + 2 \right|$

$-tg\, x$:



$D_g = \mathbb{R} \setminus \left\{ \frac{5}{12}\pi + k \cdot \frac{\pi}{2} \right\}$

$H_g = \mathbb{R}_0^+$

není prostá, sudá, ani lichá

Průsečík s y: $x = 0$ $g(0) = \left| -tg\left(-\frac{\pi}{3}\right) + 2 \right| = \left| tg\left(\frac{\pi}{3}\right) + 2 \right| = \left| \sqrt{3} + 2 \right| = 2 + \sqrt{3}$

s x: $0 = \left| -tg\left(2x - \frac{\pi}{3}\right) + 2 \right|$

$tg\left(2x - \frac{\pi}{3}\right) = 2$ ⟶ nemá hezké řešení

⟶ P

klesá v $\left(\frac{5}{12}\pi + k \cdot \frac{\pi}{2}, \underbrace{P + \frac{\pi}{2}} + k \cdot \frac{\pi}{2} \right)$

roste v $\left(P + k \cdot \frac{\pi}{2}, \frac{5}{12}\pi + k \cdot \frac{\pi}{2} \right)$

Obloukova míra

$\alpha = \frac{15}{6}\pi = 15 \cdot 30° = 450°$

$\frac{\pi}{6} = 30°$

$\alpha = \frac{15}{6}\pi = \underbrace{\frac{12}{6}\pi}_{360°} + \underbrace{\frac{3}{6}\pi}_{90°} = 450°$

$\beta = 20° = \frac{\pi}{9}$

$\downarrow$ 20° .... x $\downarrow$

360° 2π

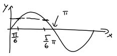
$x = \frac{20}{360} \cdot 2\pi$

$= \frac{20}{180} \cdot \pi = \frac{\pi}{9}$

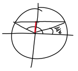
Vyčíslení goniometrických fcí

$\sin\left(\frac{5}{6}\pi\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

I.



II.

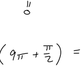


III. $\sin\left(\frac{5}{6}\pi\right) = \sin\left(\pi - \frac{\pi}{6}\right) = \underset{\overset{\shortparallel}{0}}{\sin \pi} \cdot \cos\left(-\frac{\pi}{6}\right) + \underset{\overset{\shortparallel}{-\sin\left(\frac{\pi}{6}\right)}}{\sin\left(-\frac{\pi}{6}\right)} \cdot \underbrace{\cos(\pi)}_{-1}$

$= \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\cos\left(\frac{3}{4}\pi\right) = -\frac{\sqrt{2}}{2}$

$\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$



$\cos\left(\pi - \frac{\pi}{4}\right) = \underset{\overset{\shortparallel}{-1}}{\cos(\pi)} \cdot \underset{\overset{\shortparallel}{\cos\left(\frac{\pi}{4}\right)}}{\cos\left(-\frac{\pi}{4}\right)} - \underset{\overset{\shortparallel}{0}}{\sin(\pi)} \cdot \sin\left(-\frac{\pi}{4}\right)$

$= -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$\cot g\left(\frac{19}{2}\pi\right) = \cot g\left(9\pi + \frac{\pi}{2}\right) = \cot g\left(\frac{\pi}{2}\right)$

$= \frac{\cos\left(\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{2}\right)} = 0$

$9\pi + \frac{\pi}{2} = \frac{\pi}{2}$ ✗ ošklivé

$\cot g\left(9\pi + \frac{\pi}{2}\right) = \cot g(9\pi) + \cot g\left(\frac{\pi}{2}\right)$ ✗

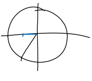
$\cos(240°) = \cos\left(\frac{4}{3}\pi\right) = -\frac{1}{2}$

240° x

360° 2π

$x = \frac{240}{360} \cdot 2\pi$

$= \frac{2}{3} \cdot 2\pi$



	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
s	0	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
c	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

$\sin^2 x + \cos^2 x = 1$

$\cos x = \sqrt{1 - \sin^2 x}$

$\sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

Goniometrické výrazy

Zjednodušte a stanovte podm.

- $(\cos x - \sin x)^2 + (\cos x + \sin x)^2 =$

  $= 2 \cdot (\underbrace{\cos^2 x + \sin^2 x}_{1}) = 2 \qquad \forall x \in \mathbb{R}$

- $\operatorname{cotg} x \cdot \cos x + \sin x = \frac{\cos^2 x}{\sin y} + \sin x = \frac{\cos^2 x + \sin^2 x}{\sin x} = \frac{1}{\sin x}$

  $\operatorname{cotg} x \to \frac{\cos x}{\sin x} \qquad \forall x \in \mathbb{R} \setminus \{k \cdot \pi, k \in \mathbb{Z}\}$

- $2 \operatorname{tg} x \cdot \sqrt{\cos^2 x - \cos^2 x \cdot \sin^2 x} = 2 \cdot \operatorname{tg} x \sqrt{\cos^2 x \cdot (1 - \sin^2 x)}$

  $= 2 \cdot \operatorname{tg} x \sqrt{\cos^4 x}$

  tg: $\boxed{\forall x \in \mathbb{R} \setminus \{\frac{\pi}{2} + k \cdot \pi, k \in \mathbb{Z}\}}$ $= 2 \cdot \frac{\sin x}{\cos x} \cdot \cos^2 x = 2 \cdot \sin x \cdot \cos x$

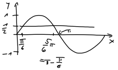
  $= \sin(2x)$

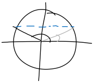
  $\underbrace{\cos^2 x}_{\geq 0} \cdot \underbrace{(1 - \sin^2 x)}_{\geq 0} \geq 0$ ✓

Goniometrické rovnice

Řešte v $\mathbb{R}$

- $\sin x = \frac{1}{2}$

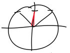




$x_1 = \frac{\pi}{6} + k \cdot 2\pi$

$x_2 = \frac{5}{6}\pi + k \cdot 2\pi \qquad k \in \mathbb{Z}$

- $\sin\left(2x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$



$\sin\left(\frac{5}{6}x\right)$

$\hookrightarrow T = \frac{6}{5} 2\pi$

$u = 2x - \frac{\pi}{4}$

$\to \sin(u) = \frac{\sqrt{2}}{2}$

$u_1 = \frac{\pi}{4} + k \cdot 2\pi$

$u_2 = \frac{3}{4}\pi + k \cdot 2\pi$

$2x_1 - \frac{\pi}{4} = \frac{\pi}{4} + k \cdot 2\pi \qquad 2x_2 - \frac{\pi}{4} = \frac{3}{4}\pi + k \cdot 2\pi$

$\underline{x_1 = \frac{\pi}{4} + k \cdot \pi} \qquad \underline{x_2 = \frac{\pi}{2} + k \cdot \pi}$

- $\sin(3x) - 2\cos^2(3x) = -1$

  $\sin(2x + x) = \sin(2x) \cdot \cos x + \sin(x) \cdot \cos 2x$

  tudy ne

  $\cos^2(3x) = 1 - \sin^2(3x)$

  $\to \sin(3x) - 2 \cdot (1 - \sin^2(3x)) = -1$

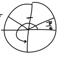
  $2 \cdot \sin^2(3x) + \sin(3x) - 1 = 0 \qquad u = \sin(3x)$

  $2 \cdot u^2 + u - 1 = 0$

  $u_{1,2} = \frac{-1 \pm \sqrt{9}}{4} = \begin{cases} \frac{-1+3}{4} = \frac{1}{2} \\ \frac{-1-3}{4} = -1 \end{cases}$

$\underline{u_1}$

$\sin(3x_1) = \frac{1}{2}$



$3x_{1,1} = \frac{\pi}{6} + k \cdot 2\pi$

$x_{1,1} = \frac{\pi}{18} + k \cdot \frac{2}{3}\pi$

$3x_{1,2} = \frac{5}{6}\pi + k \cdot 2\pi$

$x_{1,2} = \frac{5}{18}\pi + k \cdot \frac{2}{3}\pi$

$u_2 = -1$

$\sin(3x_2) = -1$

$3x_2 = \frac{3}{2}\pi + k \cdot 2\pi$

$x_2 = \frac{\pi}{2} + k \cdot \frac{2}{3}\pi$

Co kdyby $u = 2 \longrightarrow \sin(3x) = 2$

$\Rightarrow N\bar{R}$.

$|\sin(x)| \leq 1 \quad \forall x \in \mathbb{R}$

$\sin(x) = \frac{2}{5} \qquad x = 0, \ldots -$

$\sin(2x) = ?$

$\sin(2x) = 2 \cdot \sin x \cdot \cos x$

$\cos x = \sqrt{1 - \sin^2 x} = \sqrt{1 - \left(\frac{2}{5}\right)^2} = \sqrt{1 - \frac{4}{25}} = \sqrt{\frac{21}{25}} = \frac{\sqrt{21}}{5}$

$\sin(2x) = 2 \cdot \frac{2}{5} \cdot \frac{\sqrt{21}}{5} = \underline{\frac{4\sqrt{21}}{25}}$