· Sepište všechny typy rovnice roviny σ dané body A, B, C. Zakreslete ji

$A = [2,1,6]$ $B = [0,-1,-6]$ $C = [-1,2,0]$

$\vec{u} = \vec{AB} = (-2,-2,-12) \longrightarrow \vec{u} = (1,1,6)$

$\vec{v} = \vec{AC} = (-3,1,-6) \longrightarrow \vec{v} = (-3,1,-6)$

$\sigma = \{[2+s-3t, 1+s+t, 6+6s-6t]; s,t \in \mathbb{R}\}$

hledá se normálový vektor: · $\vec{n} = (a,b,c)$

$a + b + 6c = 0$ $\quad \vec{u}\cdot\vec{n} = 0$

$-3a + b - 6c = 0$ $\quad \vec{v}\cdot\vec{n} = 0$

· $\vec{n} = \vec{u} \times \vec{v}$ ✓ $\quad \vec{n}\cdot\vec{u} = 0$, $\vec{n}\cdot\vec{v} = 0$

$\vec{u} = (1, 1, 6)$ $1\ 1\ 6$

$\vec{v} = (-3, 1, -6)$ $-3\ 1\ -6$

$\vec{u}\times\vec{v} = (u_2v_3 - u_3v_2, u_3v_1 - u_1v_3, u_1v_2 - u_2v_1)$

$\vec{n} = \vec{u}\times\vec{v} = (1\cdot(-6) - 6\cdot 1, 6\cdot(-3) - 1\cdot(-6), 1\cdot 1 - 1\cdot(-3))$

$= (-12, -12, 4) \longrightarrow \vec{n} = (3, 3, -1)$

Zkouška: $\vec{n}\cdot\vec{u} = 3 + 3 - 6 = 0$ ✓

$\vec{n}\cdot\vec{v} = -9 + 3 + 6 = 0$ ✓

$\sigma: 3x + 3y - z + d = 0$ $\quad C = [-1,2,0] \in \sigma$:

$-d = \vec{C}\cdot\vec{n}$

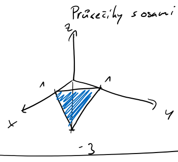
$-d = -3 + 6 - 0 = 3$

$d = -3$

$\boxed{\sigma: 3x + 3y - z - 3 = 0}$

Úsekový tvar: $\boxed{\sigma: x + y - \frac{z}{3} = 1}$

Průsečíky s osami: $P_x = [1,0,0]$, $P_y = [0,1,0]$, $P_z = [0,0,-3]$



Obecná rovnice roviny ζ: $A = [1,1,1]$, $B = [5,1,-3]$, $C = [2,0,2]$

$\vec{u} = \vec{AB} = (4,0,-4) \longrightarrow \vec{u} = (1,0,-1)$ $\ 1\ 0\ -1$

$\vec{v} = \vec{AC} = (1,-1,1)$ $\longrightarrow \vec{v} = (1,-1,1)$ $\ 1\ -1\ 1$

$\vec{n} = \vec{u}\times\vec{v} = (-1,-2,-1)$

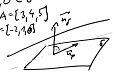
$\zeta: x + 2y + z + d = 0$ $\longrightarrow \vec{n} = (1,2,1) = \vec{v}\times\vec{u}$

$-d = \vec{n}\cdot\vec{A} = 4$

$d = -4$

$\boxed{\zeta: x + 2y + z - 4 = 0}$

Obecnou rci roviny δ: $\delta \parallel \vec{y}$ ← osa y $\quad A, B \in \delta$, $A = [3,4,5]$, $B = [-2,1,0]$



$\vec{y} = (0,1,0)$

$\vec{n}_\delta\cdot\vec{y} = 0$

$\vec{n}_\delta = (a, 0, c)$ $\quad a, c \in \mathbb{R}$

$p \parallel \delta \Rightarrow \vec{n}_\delta \cdot \vec{u}_p = 0$

$\rightarrow \delta: ax + cz + d = 0$

$A \in \delta$: $3a + 5c + d = 0$

$B \in \delta$: $-2a + d = 0$ $\Big\} \ominus$

$5a + 5c = 0 \longrightarrow a = -c \rightarrow \vec{n}_\delta = (a, 0, -a)$

např. $\vec{n}_\delta = (1, 0, -1)$

$\rightarrow -2 + d = 0$

$d = 2$

$\delta: \boxed{x - z + 2 = 0}$

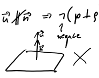
Vzájemná poloha přímky a roviny:

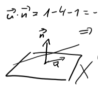
$p = \{[2+t, 3+2t, 1-t], t \in \mathbb{R}\}$ $\quad \rho: x - 2y + z - 5 = 0$

$\vec{u} = (1,2,-1)$ $\quad \vec{n} = (1,-2,1)$

$\vec{u} \not\parallel \vec{n} \Rightarrow \neg(p \perp \rho)$ $\quad \vec{u}\cdot\vec{n} = 1 - 4 - 1 = -4 \neq 0$

negace $\quad \Rightarrow p \not\parallel \rho$, $p \not\subset \rho$





$(2+t) - 2(3+2t) + (1-t) - 5 = 0$

$2 + t - 6 - 4t + 1 - t - 5 = 0$

$-4t - 8 = 0$

$t = -2$ $\quad \nearrow p, t = -2: P = [0,-1,3]$

· $p \cap \rho = \{P\}$

· $P \in p \cap \rho$

$q = \{[2+t, 3t, 1-t], t \in \mathbb{R}\}$ $\quad \vartheta = \{[1+s+2r, 3s+3r, 1-s-3r]; r,s \in \mathbb{R}\}$

$2 + t = 1 + s + 2r$

$3t = 3s + 3r \longrightarrow t = s + r$

$1 - t = 1 - s - 3r$

$2 + \cancel{s} + r = 1 + \cancel{s} + 2r \longrightarrow r = 1$

$1 - s - r = 1 - s - 3r$

$1 - \cancel{s} - 1 = 1 - \cancel{s} - 3$

$-1 = -3 \longrightarrow NR. \Rightarrow \boxed{q \parallel \vartheta}$

$\vec{u}_q = (1,3,-1)$ $\quad \vec{v} = (1,3,-1)$, $\vec{w} = (2,3,-3)$

$\vec{u}_q \parallel \vec{v}$ ⟨ $q \parallel \vartheta$ / $q \subset \vartheta$

Vzájemná poloha rovin:

$\mu = \{[3+t-h, 5+t, -t+2h]; t, h \in \mathbb{R}\}$

$\nu = \{[3+s-4p, 6+2s-3p, 1+5p]; s, p \in \mathbb{R}\}$

$3 + t - h = 3 + s - 4p$

$5 + t = 6 + 2s - 3p$ $\quad \Rightarrow$ moc práce

$-t + 2h = 1 + 5p$

vyplatí se přechod k obecným rovnicím.

μ: $\vec{u} = (1,1,-1)$, $\vec{v} = (-1,0,2)$

$\vec{n} = \vec{u}\times\vec{v} = (2,-1,1)$

$\mu: 2x - y + z + d = 0$

$A = [3,5,0] \in \mu$: $6 - 5 + d = 0$

$\mu: 2x - y + z - 1 = 0$

ν: $\vec{u} = (1,2,0)$, $\vec{v} = (-4,-3,5)$

$\vec{n} = \vec{u}\times\vec{v} = (10,-5,5) \longrightarrow \vec{n} = (2,-1,1)$

$\vec{n}_\mu \parallel \vec{n}_\nu \Rightarrow$ ⟨ $\mu = \nu$ / $\mu \parallel \nu$ $\quad \nu: 2x - y + z + d = 0$

$B = [3,6,1] \in \nu$: $6 - 6 + 1 + d = 0$

$d = -1$

$\nu: 2x - y + z - 1 = 0$

$\Rightarrow \boxed{\mu = \nu}$

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odchylka přímek v prostoru:

$p = \{[2+t, t, 7-2t], t \in \mathbb{R}\}$, $q = \{[4-h, 5, -3+h], h \in \mathbb{R}\}$

$\vec{u} = (1,1,-2)$ $\quad \vec{v} = (-1,0,1)$

$\vec{u}\cdot\vec{v} = -3 \longrightarrow \neg(p \perp q)$

$$\cos\varphi = \frac{|\vec{u}\cdot\vec{v}|}{|\vec{u}|\cdot|\vec{v}|} = \frac{|-3|}{\sqrt{1+1+4}\cdot\sqrt{1+0+1}} = \frac{3}{\sqrt{6}\cdot\sqrt{2}} = \frac{3}{\sqrt{3}\cdot\sqrt{2}\cdot\sqrt{2}} = \frac{\sqrt{3}}{2}$$

$\varphi = \frac{\pi}{6} = 30°$

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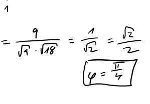
odchylka přímky a roviny:

$p = \{[4-2t, 1-2t, t], t \in \mathbb{R}\}$

$\xi: x + 4y + z - 1 = 0$ $\quad \vec{u} = (-2,-2,1)$, $\vec{n} = (1,4,1)$



$$\cos\alpha = \frac{|\vec{u}\cdot\vec{n}|}{|\vec{u}|\cdot|\vec{n}|}$$

$\alpha + \varphi = \frac{\pi}{2}$

$$\cos\alpha = \cos\left(\frac{\pi}{2} - \varphi\right) = \underbrace{\cos\frac{\pi}{2}}_{0}\cdot\cos\varphi + \underbrace{\sin\frac{\pi}{2}}_{1}\cdot\sin\varphi = \sin\varphi$$

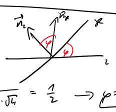
$$\sin\varphi = \frac{|\vec{u}\cdot\vec{n}|}{|\vec{u}|\cdot|\vec{n}|} = \frac{|-2-8+1|}{\sqrt{4+4+1}\cdot\sqrt{1+16+1}} = \frac{9}{\sqrt{9}\cdot\sqrt{18}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$\boxed{\varphi = \frac{\pi}{4}}$

Odchylka dvou rovin:

z: $2x + y - z + 4 = 0$ $\quad \vec{n}_z = (2,1,-1)$

$\varkappa$: $2x + 4y + 2z - 5 = 0$ $\quad \vec{n}_\varkappa = (2,4,2)$



$$\cos\varphi = \frac{|4+4-2|}{\sqrt{4+1+1}\ \sqrt{4+16+4}} = \frac{6}{\sqrt{6}\cdot\sqrt{24}} = \frac{6}{\sqrt{6}\cdot\sqrt{6}\cdot\sqrt{4}} = \frac{1}{2} \rightarrow \varphi = \frac{\pi}{3} = 60°$$