
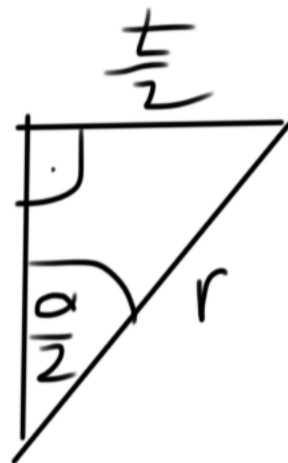
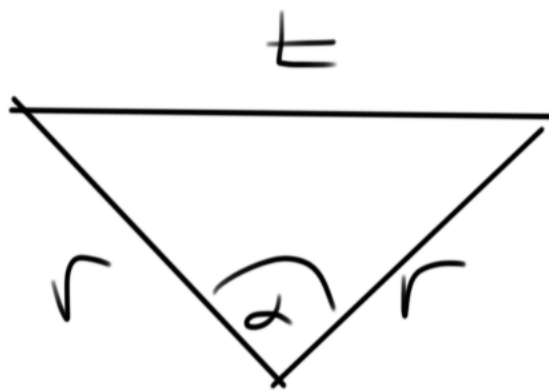
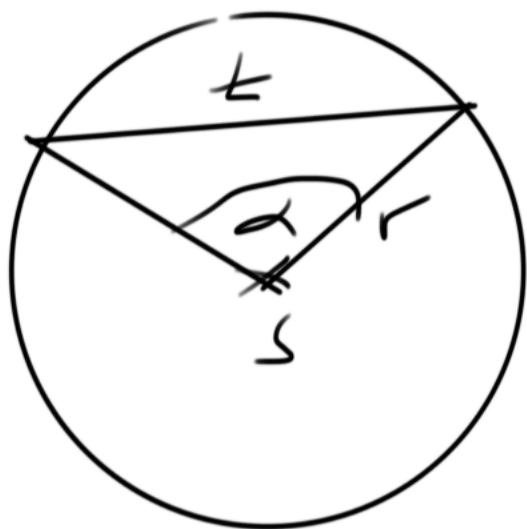


1) Vypočítejte  $O, S$    
 tetiva  $t = 4\text{cm}$  a příslušný  
 středový  $\angle \alpha = 60^\circ$



$$\sin\left(\frac{\alpha}{2}\right) = \frac{\frac{t}{2}}{r}$$

$$r = \frac{t}{2 \cdot \sin\left(\frac{\alpha}{2}\right)}$$

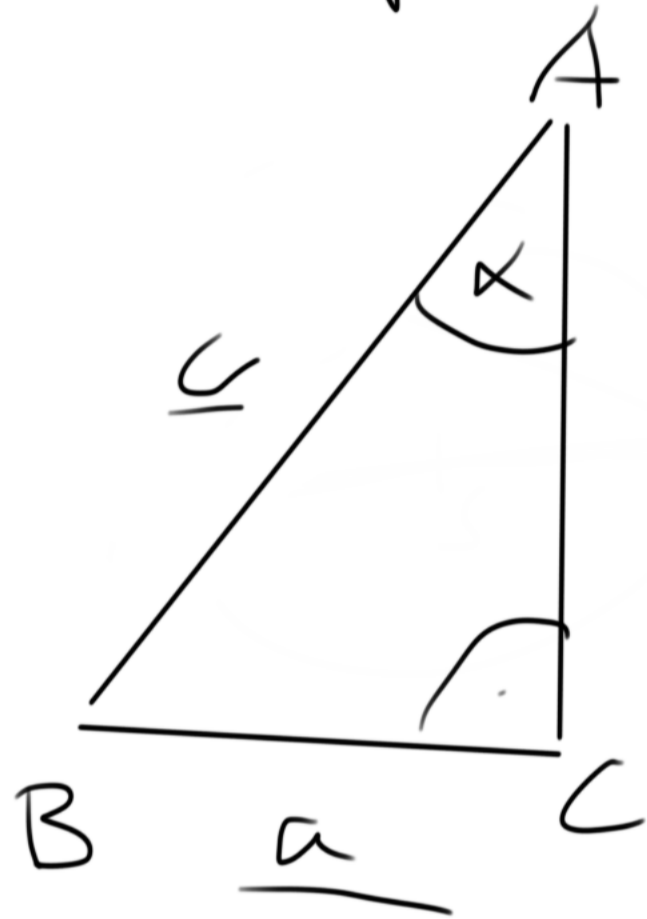
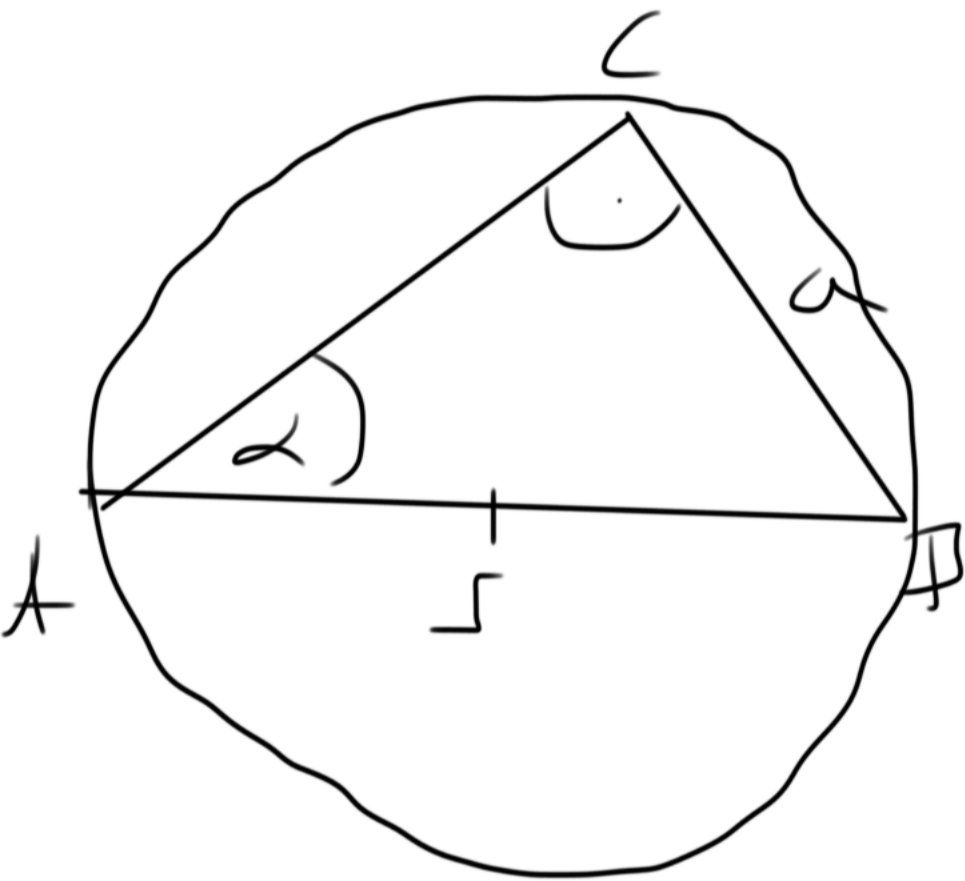
$$\alpha = 60^\circ \rightarrow \frac{\alpha}{2} = 30^\circ \quad \sin 30^\circ = \frac{1}{2}$$

$$r = \frac{t}{2 \cdot \frac{1}{2}} = t = 4\text{cm}$$

$$O = 2\pi r = 8\pi\text{cm}$$

$$S = \pi r^2 = 16\pi\text{cm}^2$$

2) Spočítejte  $O, S$   $\odot$  opsané  
 pravouhlému  $\triangle$  :  $\alpha = 15^\circ$   
 $a = \frac{1}{\sqrt{2+\sqrt{3}}} \text{ cm}$



$$\sin \alpha = \frac{a}{c}$$

$$c = \frac{a}{\sin \alpha}$$

$$a = \frac{1}{\sqrt{2+\sqrt{3}}}$$

$$\alpha = 15^\circ$$

$$\sin 15^\circ = ?$$

$$\sin 30, \cos 30$$

$$\left| \sin \frac{\phi}{2} \right| = \sqrt{\frac{1 - \cos \phi}{2}} \quad \phi \in \left( -\frac{\pi}{2}, \frac{\pi}{2} \right)$$

$$\sin 15^\circ > 0$$

$$\sin 15^\circ = \sqrt{\frac{1 - \cos 30^\circ}{2}}$$

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

$$\sin 15^\circ = \sqrt{\frac{1 - \frac{\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{\frac{2 - \sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{2 - \sqrt{3}}{4}} = \frac{\sqrt{2 - \sqrt{3}}}{2}$$

$$a = \frac{1}{\sqrt{2 + \sqrt{3}}}$$

$$c = \frac{a}{\sin 15^\circ}$$

$$c = \frac{\frac{1}{\sqrt{2 + \sqrt{3}}}}{\frac{\sqrt{2 - \sqrt{3}}}{2}} = \frac{2}{\sqrt{2 + \sqrt{3}} \cdot \sqrt{2 - \sqrt{3}}} = \frac{2}{\sqrt{(2 + \sqrt{3})(2 - \sqrt{3})}}$$

$$(A + B)(A - B) = A^2 - B^2$$

$$= \frac{2}{\sqrt{4 - 3}} = 2$$



$$c = 2 \text{ cm}$$

$$\Rightarrow r = \frac{c}{2} = 1 \text{ cm}$$

$$O = 2\pi r = 2\pi \text{ cm} \quad S = \pi r^2 = \pi \text{ cm}^2$$

3) Odvodte vzorec pro obsah pravidelného  $n$ -úhelníku

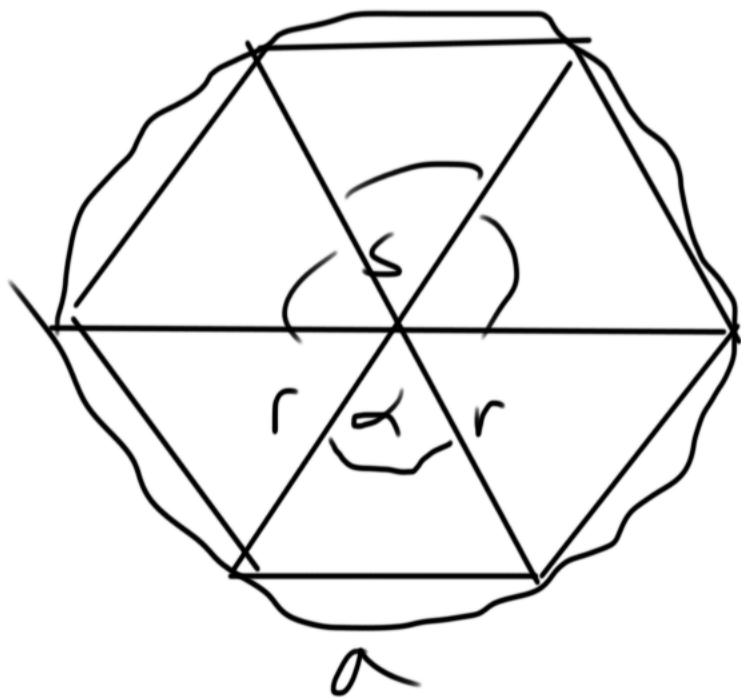
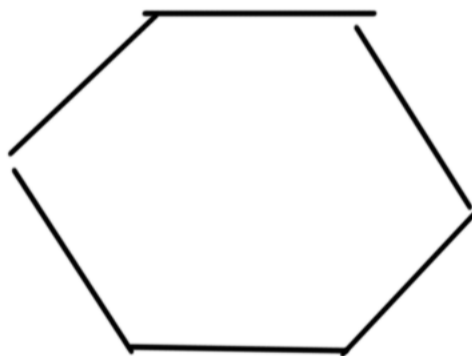
$n$ -úhelník –  $n$  vnitřních úhlů

nepravidelný  $n=5$ :  
 pravidelný



→ všechny strany stejné délky

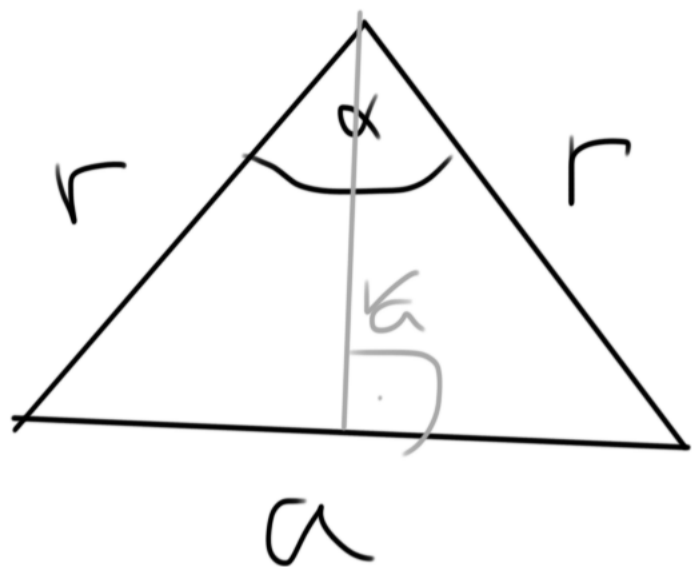
⇒ všechny vnitřní úhly jsou stejné velikosti



$$S_n = n \cdot S_{\Delta}$$

$$\alpha = \frac{2\pi}{n}$$

$$\frac{360}{n}$$

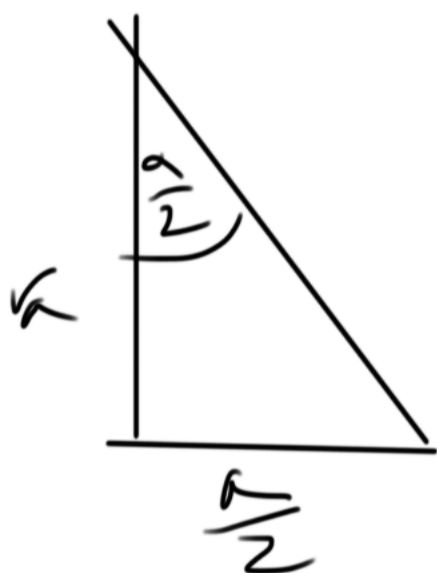


a. - strann

$$\alpha = \frac{2\pi}{n}$$

$$S_n = n \cdot S_{\Delta}$$

$$S_{\Delta} = \frac{1}{2} \cdot a \cdot r_a$$



$$\cot\left(\frac{\alpha}{2}\right) = \frac{r_a}{\frac{a}{2}}$$

$$r_a = \frac{a}{2} \cdot \cot\left(\frac{\alpha}{2}\right)$$

$$S_{\Delta} = \frac{1}{2} \cdot a \cdot r_a = \frac{1}{2} \cdot a \cdot \frac{a}{2} \cot\left(\frac{\alpha}{2}\right)$$

$$S_{\Delta} = \frac{1}{4} a^2 \cot\left(\frac{\alpha}{2}\right)$$

$$S_n = n \cdot S_{\Delta}$$

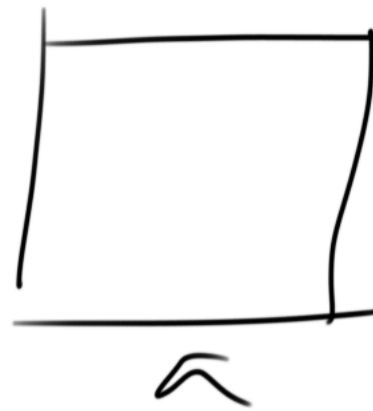
$$\alpha = \frac{2\pi}{n}$$

$$\frac{\alpha}{2} = \frac{\pi}{n}$$

$$S_n = \frac{n}{4} a^2 \cot\left(\frac{\pi}{n}\right)$$

$$S_n = \frac{n}{4} a^2 \cotg\left(\frac{\pi}{n}\right)$$

$n=4$ : čtverec

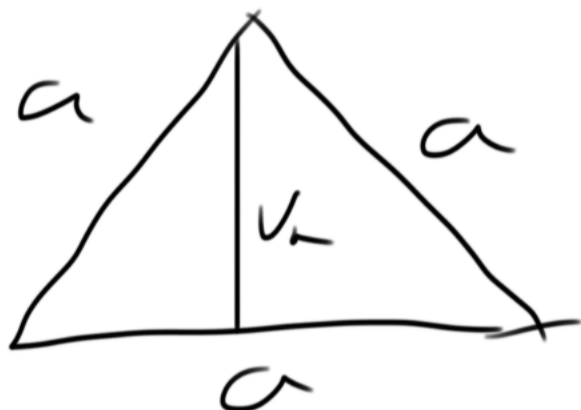


$$S = a^2$$

$$\cotg\left(\frac{\pi}{4}\right) = 1$$

$$S_4 = \frac{4}{4} a^2 \cdot 1 = a^2 \quad \checkmark$$

$n=3$ :



rohovstranný  $\Delta$

$$\cotg\left(\frac{\pi}{3}\right) = \frac{\cos \frac{\pi}{3}}{\sin \frac{\pi}{3}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$$

$$S_3 = \frac{3}{4} a^2 \cdot \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{4} a^2$$

$n=6$ :

$$\cotg\left(\frac{\pi}{6}\right) = \frac{\cos \frac{\pi}{6}}{\sin \frac{\pi}{6}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$$S_6 = \frac{6}{4} a^2 \cdot \sqrt{3} = \frac{3}{2} \sqrt{3} a^2$$

# Vzájemná poloha kružnic

chyba na přednášce

iii)  $r_1 + r_2 > r > r_1$  ~~špatně~~  
 $r_1 + r_2 > r > r_1 - r_2$  správně.

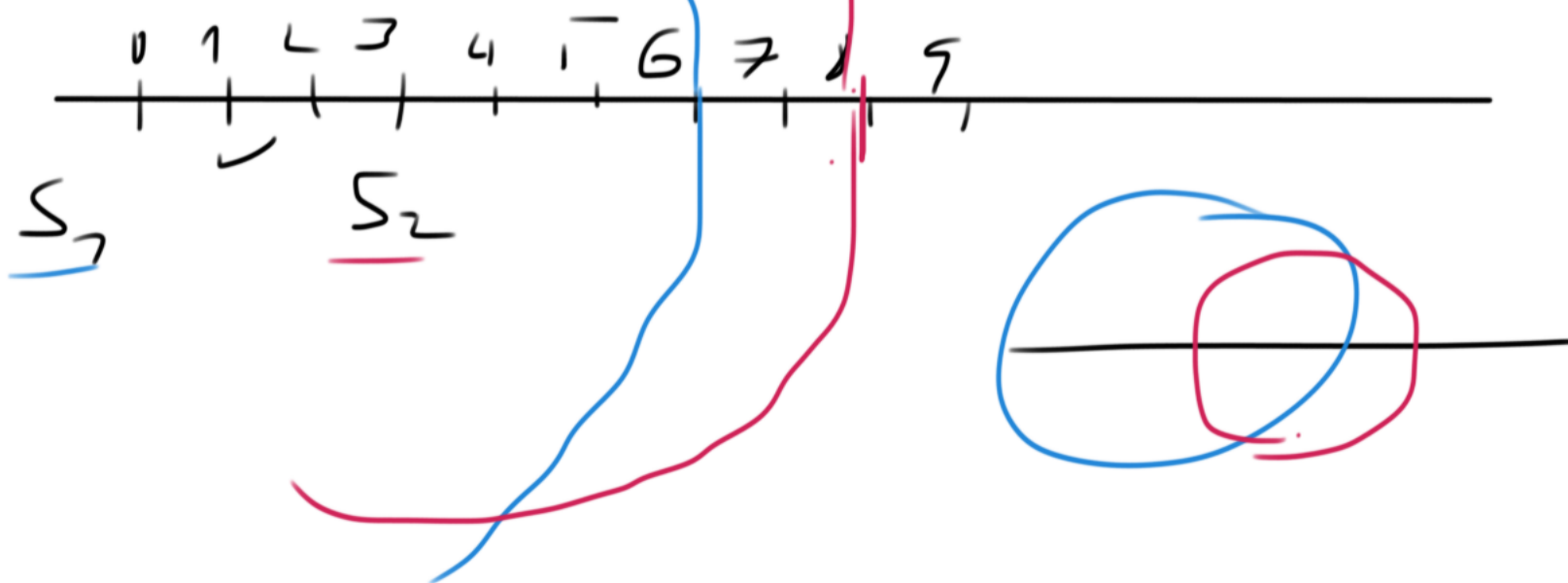
$r_1 = 6\text{ cm}$   $r_2 = 5\text{ cm}$ ,  $r = 3\text{ cm}$

$r_1 + r_2 = 11\text{ cm}$

$r_1 - r_2 = 1\text{ cm}$

$r_1 + r_2 > r > r_1 - r_2$

$\Rightarrow$  Kružnice se protínají  
 $\rightarrow$  2 sp. body

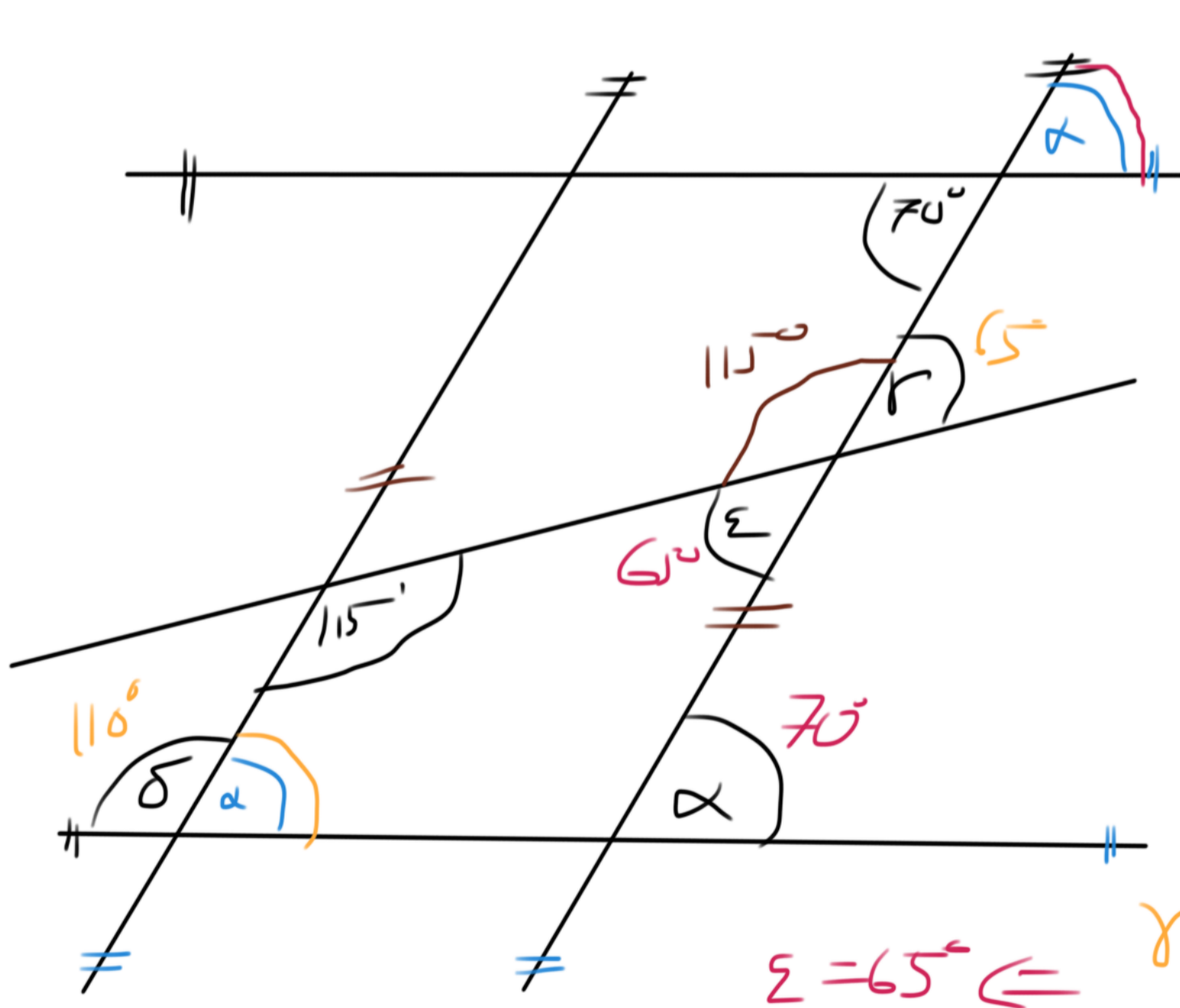
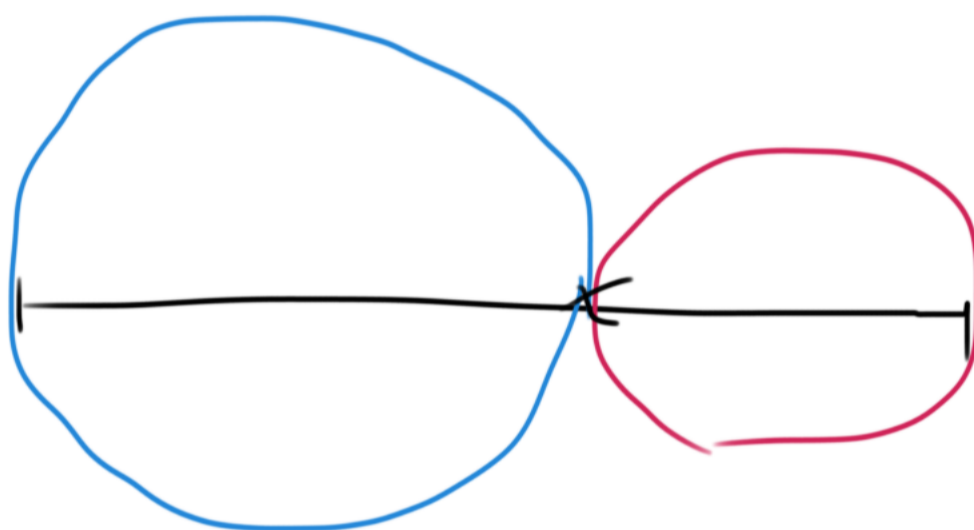


$$r_1 = 32 \text{ cm}, r_2 = 15 \text{ cm}, v = 47 \text{ cm}$$

$$r_1 + r_2 = 47 \text{ cm}$$

$$r_1 + r_2 = v$$

→ vnější dotyk → 1 sp. bod



souhlasí  
vrcholové  
 $\alpha = 70^\circ$

vedlejší  
 $\alpha + \delta = 180^\circ$   
 $\delta = 110^\circ$

střídané

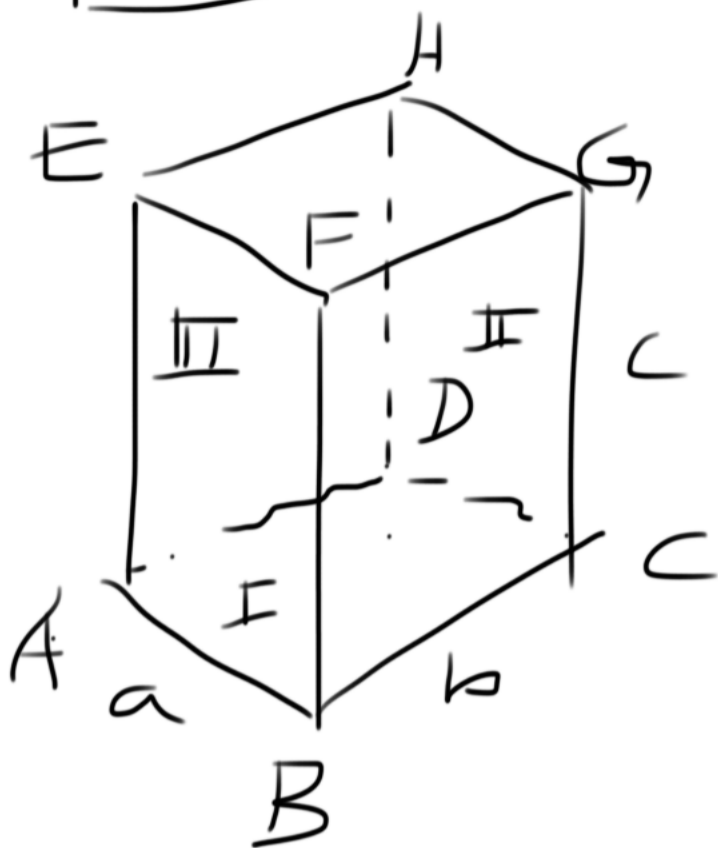
$$\gamma = \epsilon$$

$$\gamma + 115 = 180$$

$$\gamma = 65^\circ$$

$$\epsilon = 65^\circ$$

Kvádr



$$V = a \cdot b \cdot c$$

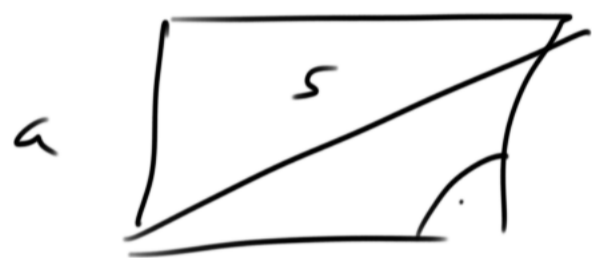
$$S = 2 \cdot (S_I + S_{II} + S_{III})$$

$$S_I = a \cdot b$$

$$S_{III} = a \cdot c$$

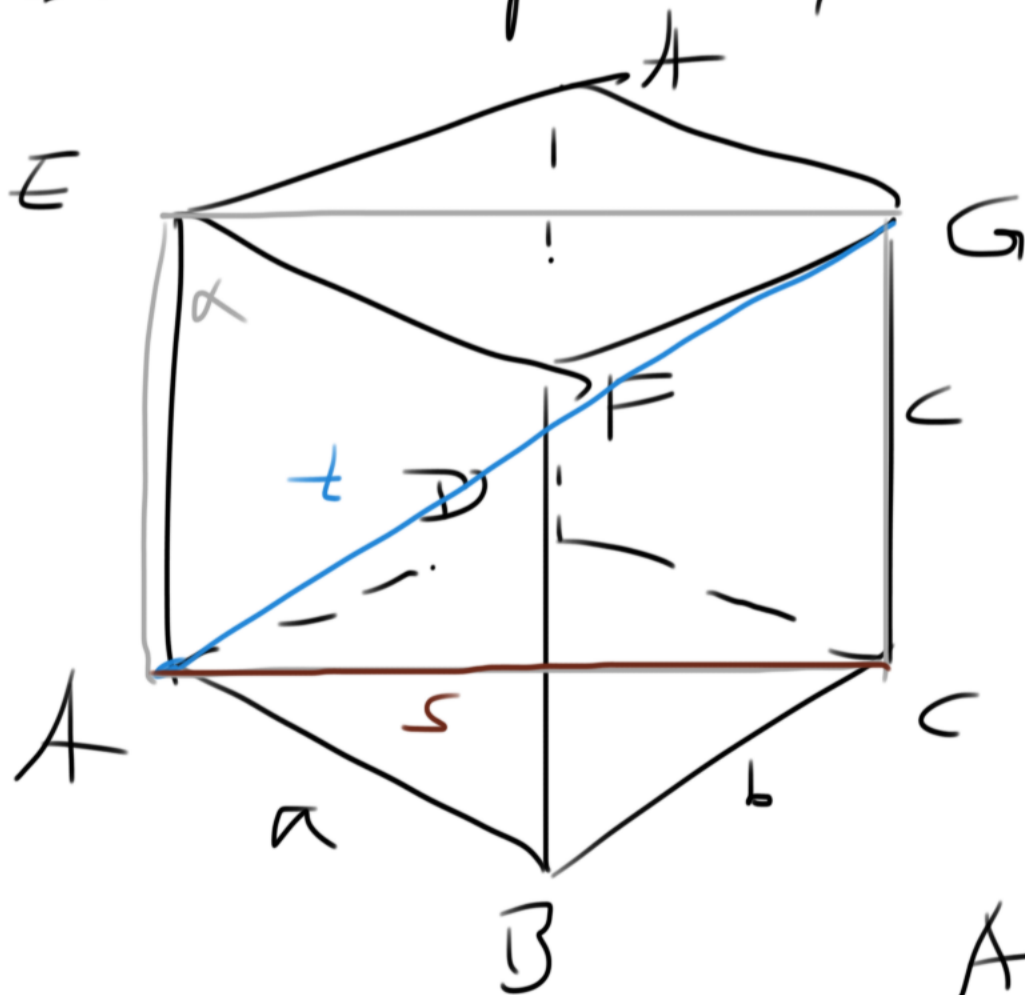
$$S_{II} = b \cdot c$$

stěnové úhlopříčky



$$s = \sqrt{a^2 + b^2}$$

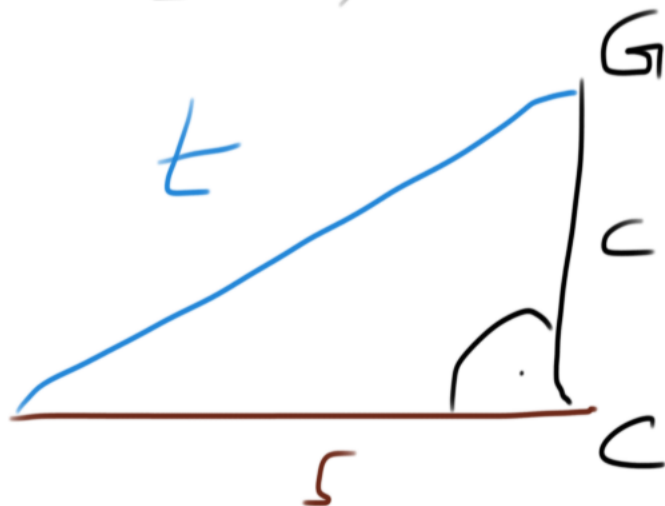
tělesové úhlopříčky



$$t = \overleftrightarrow{AG}$$

$$\alpha \cdot ACG$$

$$t \in \alpha \wedge \alpha \perp AC$$



$$t = \sqrt{s^2 + c^2} = \sqrt{a^2 + b^2 + c^2}$$

b) Délka tělesové úhř. krychle  
je  $3\sqrt{6}$  cm.

a)  $a = ?$  b)  $V = ?$  c)  $S$

krychle:  $a = b = c$

$$V = a^3 \quad S = 6 \cdot S_{\square} = 6 \cdot a^2$$

$$s = \sqrt{a^2 + a^2} = \sqrt{2} a$$

$$t = \sqrt{a^2 + a^2 + a^2} = \sqrt{3} a$$

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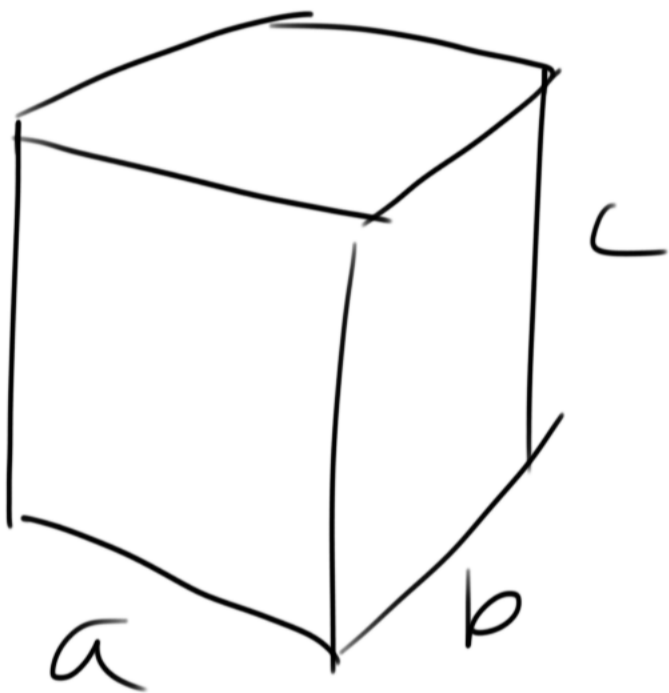
$$\begin{array}{ll} \text{a)} \quad \sqrt{3} a = 3 \cdot \sqrt{6} & \text{b)} \quad V = a^3 \\ a = 3 \cdot \sqrt{2} \text{ cm} & V = 27 \cdot 2^{\frac{3}{2}} \text{ cm}^3 \end{array}$$

$$\begin{array}{l} \text{c)} \quad S = 6 \cdot a^2 \\ S = 6 \cdot (3\sqrt{2})^2 = 6 \cdot 18 = 108 \text{ cm}^2 \end{array}$$

7) krychle:  $a \rightarrow a' = 2a$   
 $V' = ?$   $S' = ?$

$$V = a^3 \quad V' = a'^3 = (2a)^3 = 8a^3 = 8 \cdot V$$

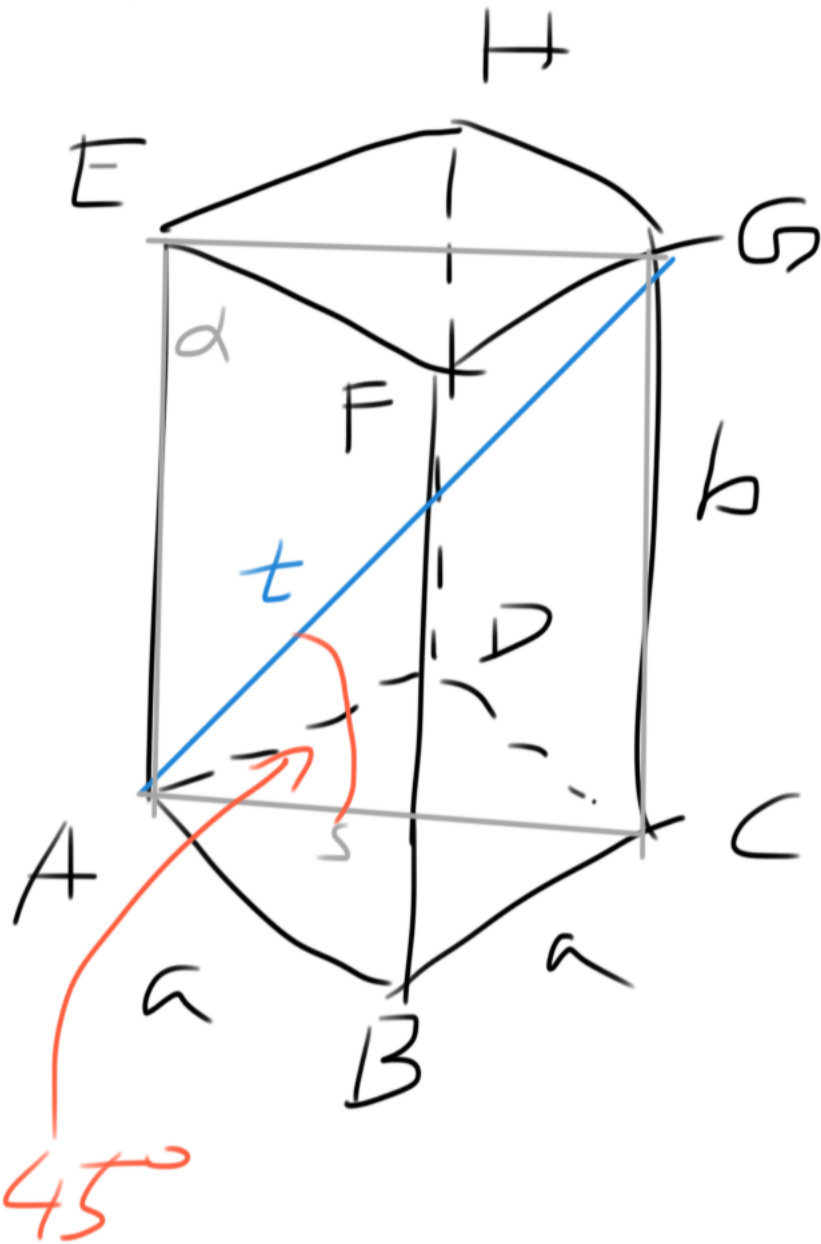
$$S' = 6a'^2 = 6 \cdot (2a)^2 = 4 \cdot 6a^2 = 4 \cdot S$$



$$\begin{aligned} a' &= 2a \\ b' &= b \\ c' &= c \end{aligned}$$

$$\begin{aligned} V' &= a' \cdot b' \cdot c' \\ &= 2a \cdot b \cdot c = 2 \cdot V \end{aligned}$$

8) kvádr se čtvercovou podstavou



$$V = 64 \text{ cm}^3$$

odchylka  $t = \overleftrightarrow{AG}$   
od roviny  $ABC$   
je  $45^\circ$ .

$$S = ?$$

$$t \in \alpha$$

$$\alpha \perp AG$$

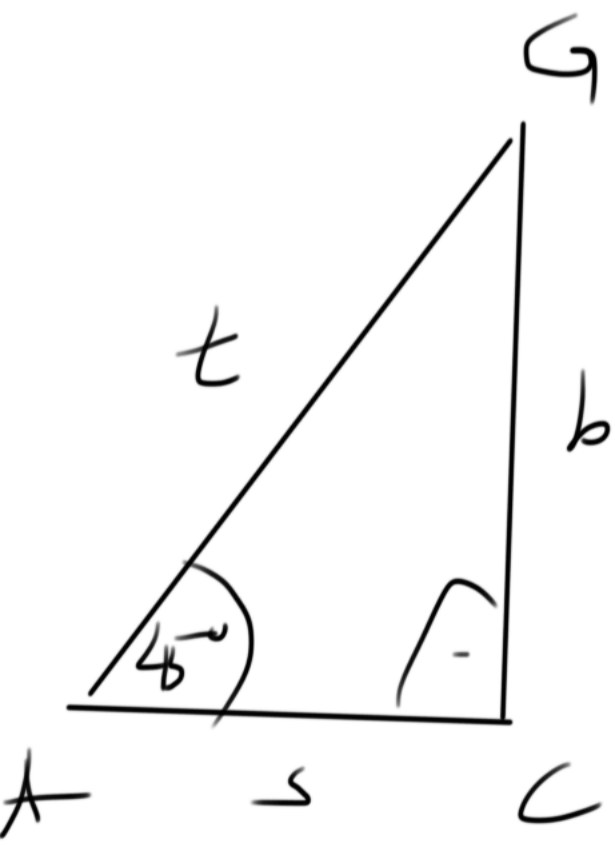
$$\alpha \cap ABC = S$$

1. čtverec:  $s = \sqrt{2} \cdot a$

2.  $45^\circ$   $\tan 45^\circ = \frac{b}{s}$

$$b = \tan 45^\circ \cdot s$$

$$b = \sqrt{2} a$$



$$a \sim \sqrt{2}a$$

$$V = 64 \text{ cm}^3$$

$$V = \sqrt{2} a^3$$

$$\sqrt{2} a^3 = 64$$

$$2^{1/2} a^3 = 2^6$$

$$a^3 = 2^{11/2}$$

$$a = 2^{\frac{11}{6}}$$