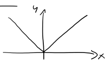
5. 12.

- goniometrické funkce
- exp. a log. funkce

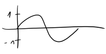
$y = |x|$



omezená zdola

omezená $\Leftrightarrow$ omezená shora $\wedge$ zdola

$y = \sin x$



omezená

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$tg\, x$	0	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	X
$cotg\, x$	X	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

$\sin^2 x + \cos^2 x = 1 \quad \forall x \in \mathbb{R}$

$tg\, x = \frac{\sin x}{\cos x}$

$cotg\, x = \frac{\cos x}{\sin x} = \frac{1}{tg\, x}$

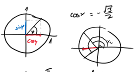
$tg\, x = \tan x$

$cotg\, x = cotan\, x$

Rovnice: $f(x) = g(x)$

$\vdots$

$\cos x = -\frac{\sqrt{3}}{2}$



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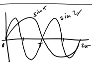
sin x

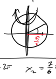
sin(2x)



Kandidátnax: $\frac{\pi}{6}$

$\rightarrow x_1 = \pi - \frac{\pi}{6}$

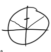
$x_2 = \pi + \frac{\pi}{6}$



$x_1 = \frac{5}{6}\pi + k \cdot 2\pi \qquad x_2 = \frac{7}{6}\pi + k \cdot 2\pi$

$k \in \mathbb{Z}$

$\sin(2x) = \frac{1}{2}$



$u = 2x$

$\sin u = \frac{1}{2}$

$u_1 = \frac{\pi}{6} + k \cdot 2\pi$

$u_2 = \frac{5}{6}\pi + k \cdot 2\pi$

$2x_2 = \frac{5}{6}\pi + k \cdot 2\pi \qquad 2x_1 = \frac{\pi}{6} + k \cdot 2\pi$

$x_2 = \frac{5}{12}\pi + k \cdot \pi \qquad x_1 = \frac{\pi}{12} + k \cdot \pi$

$\sin x \ \ldots \ T = 2\pi$

$\sin(2x) \ \ldots \ T = \pi$

$\sin(nx) \ \ldots \ T = \frac{2\pi}{n} \quad n \in \mathbb{N}$

$\sin x, \cos x \quad T = 2\pi \qquad tg\, x, cotg\, x \quad T = \pi$

$\sin^2 x + \cos^2 x = 1 \quad \forall x \in \mathbb{R} \longrightarrow \sin^2(f(x)) + \cos^2(f(x)) = 1$

$\sin^2(x^2) + \cos^2(x^2) = 1$

$\sin(2x) = 2 \cdot \sin x \cdot \cos x$

$\cos(2x) = \cos^2 x - \sin^2 x$

$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \cdot \sin y$

---

f: $y = -\tan\left(|x| - \frac{\pi}{2}\right) + 1$

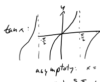
$\rightarrow +1 \ldots$ posun v y

$\rightarrow -\frac{\pi}{2} \ldots$ posun v x do $\frac{\pi}{2}$

$\rightarrow (-1) \ldots$ převrácení podle x

$\rightarrow |x| \rightarrow f \ldots$ sudá

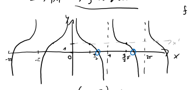
tan x:



asymptoty: $x = \frac{\pi}{2} + k \cdot \pi \quad k \in \mathbb{Z}$

$D = \mathbb{R} \setminus \{\frac{\pi}{2} + k \cdot \pi\}$

$H = \mathbb{R}$



f není definována v

$|x| - \frac{\pi}{2} \neq \frac{\pi}{2} + k \cdot \pi$

$|x| \neq \pi + k \cdot \pi = \pi(k+1) = \pi \cdot l \quad l \in \mathbb{Z}$

$|x| \neq \pi \cdot l$

$x \neq \pi \cdot l$

$D_f = \mathbb{R} \setminus \{\pi \cdot l\} \quad l \in \mathbb{Z}$

$\tan x \rightarrow -\tan x$

$0 = -\tan\left(|x| - \frac{\pi}{2}\right) + 1$

$(-\infty, 0) \quad \langle 0, \infty)$

$|x| = -x \quad |x| = x$

$x \in \langle 0, \infty)$:

$\tan\left(x - \frac{\pi}{2}\right) = 1$

$x - \frac{\pi}{2} = \frac{\pi}{4} + k \cdot \pi$

$x = \frac{3}{4}\pi + k \cdot \pi$

$k \in \mathbb{N}_0$

f sudá $\Rightarrow$ $P_{x1}\left[\frac{3}{4}\pi + k \cdot \pi, 0\right]$, $P_{x2}\left[-\frac{3}{4}\pi - k \cdot \pi, 0\right]$ $\quad k \in \mathbb{N}_0$

$x \in (-\infty, 0)$:

$\tan\left(-x - \frac{\pi}{2}\right) = 1$

$-x - \frac{\pi}{2} = \frac{\pi}{4} + k \cdot \pi$

$-x = \frac{3}{4}\pi + k \cdot \pi$

$x = -\frac{3}{4}\pi - k \cdot \pi \quad k \in \mathbb{N}_0$

$H = \mathbb{R}$

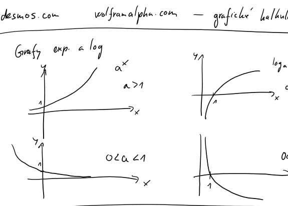
$\nearrow (-\pi - k\pi, 0 - k\pi) \quad k \in \mathbb{N}_0$

$\searrow (0 + k\pi, \pi + k\pi) \quad k \in \mathbb{N}_0$

neomezená, neprostá, sudá, nemá lokální minima ani maxima.

desmos.com wolframalpha.com – grafické kalkulačky

Grafy exp. a log



Řešte v $\mathbb{R}$

$2\sin^2(x) + 3\cos x = 0$

$\rightsquigarrow \sin u = u \quad \sin x = \frac{\sqrt{3}}{2}$

$2 - 2\cos^2 x + 3\cos x = 0$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x \quad \forall x \in \mathbb{R}$

$u = \cos x$

$2 - 2u^2 + 3u = 0$

$2u^2 - 3u - 2 = 0$

$u_{1,2} = \frac{3 \pm \sqrt{9 + 16}}{4} = \frac{3 \pm 5}{4} = \begin{cases} 2 \\ -\frac{1}{2} \end{cases}$

$\cos x_1 = 2$

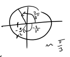
NŘ: $\cos x \leq 1 \quad \forall x \in \mathbb{R}$

$\cos x_2 = -\frac{1}{2}$

$x_2 = \frac{2}{3}\pi + k \cdot 2\pi$

$x_3 = -\frac{2}{3}\pi + k \cdot 2\pi$

$= \frac{4}{3}\pi + (k-1)2\pi \quad k \in \mathbb{Z}$



$\rightsquigarrow \frac{\pi}{3}$

Periodicita: $\cos(x + 2\pi) = \cos x$

$\cos\left(\frac{13}{6}\pi\right) = \cos\left(\frac{\pi}{6} + 2\pi\right) = \cos\left(\frac{\pi}{6}\right)$ ✓

$\frac{13}{6}\pi = \frac{12}{6}\pi + \frac{1}{6}\pi = \frac{\pi}{6} + 2\pi \neq \frac{\pi}{6}$

Řešte v $\mathbb{R}$

$\log_6(x+1) + \log_6(x) = 1$ chceme tvar $\log_6(\ldots) = \log_6(\_)$

$\log_6[(x+1) \cdot x] = \log_6 6 \qquad \log_a(x \cdot y) = \log_a x + \log_a y$

$/ 6^{(\ldots)} \qquad \log_a a = 1$

$\left[6^{\log_6[x(x+1)]} = 6^{\log_6 6}\right]$

$\log_a x, \ a^x$: navzájem inverzní prosté

$x \cdot (x+1) = 6 \leftarrow a^{\log_a x} = \log_a a^x = x$

vs: $\log_6(x+1) + \log_6(x) = \log_6 6$

$x + 1 + x = 6$ ŠPATNĚ.

$x^2 + x - 6 = 0$

$(x+3) \cdot (x-2) = 0 \rightarrow x_1 = -3 \times \ D_{log} = \mathbb{R}^+$, $x_2 = 2$

$9 \cdot 3^x + 3^{-x} = 10 \qquad 3^{-x} = \frac{1}{3^x}$

$9 \cdot 3^x + \frac{1}{3^x} = 10 \quad / \cdot 3^x$

$9 \cdot 3^{2x} + 1 = 10 \cdot 3^x$

$u = 3^x$

$9 \cdot u^2 + 1 = 10u$

$9 \cdot u^2 - 10u + 1 = 0$

$u_{1,2} = \frac{10 \pm \sqrt{100 - 36}}{18} = \begin{cases} 1 \\ \frac{2}{18} = \frac{1}{9} \end{cases}$

$3 + 4^x = 7 \quad / -3$

$4^x = 4^1 \quad / \log_4(\ldots)$

$[\log_4 4^x = \log_4 4^1]$

$x = 1$

$3^{x_1} = 1$

$x_1 = 0$

$3^{x_2} = \frac{1}{9}$

$3^{x_2} = 3^{-2}$

$x_2 = -2$

pokud např. $u = -3$

$3^x = -3 \rightarrow$ NŘ.

$3^x > 0 \quad \forall x \in \mathbb{R}$

$3^x + 3^{x+1} = 7 \cdot 4^x - 4^{x+1} \rightsquigarrow 3^x \times 4^x$

$3^x + 3^x \cdot 3 = 7 \cdot 4^x - 4^x \cdot 4$

$3^x(1 + 3) = 4^x(7 - 4)$

$3^x \cdot 4 = 4^x \cdot 3$

$\frac{3^x}{4^x} = \frac{3}{4}$

$\left(\frac{3}{4}\right)^x = \left(\frac{3}{4}\right)^1$

$\rightarrow x = 1$

$2^x \cdot 3^{x-1} = 6$

$2^x \cdot 3^{x-1} = 2 \cdot 3$

$\frac{2^x \cdot 3^x}{3} = 2 \cdot 3$

$2^x 3^x = 2 \cdot 3^2$

$6^x = 18 \quad / \log_6(\ldots)$

$[\log_6 6^x = \log_6 18]$

$x = \log_6 18$

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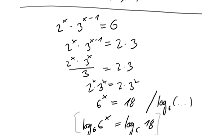
přímka:



p: $X = A + t\vec{u} \qquad t \in \mathbb{R}$ parametr

p: $x = A_x + t u_x$

$y = A_y + t \cdot u_y$

$z = A_z + t u_z$

Vyšetřete vzájemnou polohu přímé p a q $\qquad ax + by + c = 0$

p: $1 \cdot x + 2y + 3 = 0$

q: $3x + 2y + 1 = 0$

$x + 2y = -3$

$3x + 2y = -1$ $\Big\} \ominus$

$2x = 2$

$x = 1$

$\vdots$