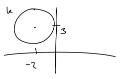
Kuželosečky

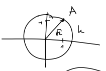
$k: (x+2)^2 + (y-3)^2 = 2 \qquad S=[-2,3] \quad r=\sqrt{2}$

$(x-m)^2 + (y-n)^2 = r^2$



Napište rovnici kružnice k

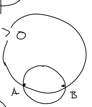
$S=[0,0]$ a $A \in k$, $A=[1,1]$



$k: (x-0)^2 + (y-0)^2 = r^2$

$A \in k: \quad 1^2 + 1^2 = r^2 \rightarrow 2 = r^2$

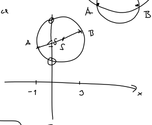
$r=\sqrt{2}$ $\qquad r=\pm\sqrt{2}$, ale $r>0$



$k: ?$ $\qquad A, B \in k$

AB je průměr kružnice

$A[-1,5] \qquad B=[3,7]$



$(x-m)^2 + (y-n)^2 = r^2$

$S_k = S_{AB} = \left[\frac{-1+3}{2}, \frac{5+7}{2}\right] = [1, 6]$

$r = |SB| = \sqrt{(3-1)^2 + (7-6)^2} = \sqrt{4+1} = \sqrt{5}$

$k: (x-1)^2 + (y-6)^2 = 5$

Průsečík s osou y: $\qquad o_y: x=0$

$k: (x-1)^2 + (y-6)^2 = 5$

$1 + y^2 - 12y + 36 = 5$

$y^2 - 12y + 32 = 0$

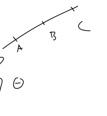
$D = 144 - 128 = 16$

$P_{y1} = [0,4] \leftarrow y_{1,2} = \frac{12 \pm 4}{2} = \begin{cases} 8 \\ 4 \end{cases}$

$P_{y2} = [0,8]$

Napište rovnici kružnice k opsané $\triangle ABC$

$A=[-5,0] \quad B=[2,-1] \quad C=[1,2]$



$k: (x-m)^2 + (y-n)^2 = r^2$

$A: \quad (-5-m)^2 + (0-n)^2 = r^2$

$B: \quad (2-m)^2 + (-1-n)^2 = r^2$

$\quad (1-m)^2 + (2-n)^2 = r^2$

① $25 + 10m + m^2 + n^2 = r^2$

② $4 - 4m + m^2 + 1 + 2n + n^2 = r^2$ $\Big\}\ominus$

③ $1 - 2m + m^2 + 4 - 4n + n^2 = r^2$ $\Big\}\ominus$

①−②: $21 + 14m + 0 \cdot m^2 - 1 - 2n + 0 \cdot n^2 = 0 \cdot r^2$

①−③: $24 + 12m + 0 \cdot m^2 - 4 + 4n + 0 \cdot n^2 = 0 \cdot r^2$

$$\left.\begin{aligned} 14m - 2n &= -20 \quad /\cdot 2 \\ 12m + 4n &= -20 \end{aligned}\right\} \oplus$$

$40m = -60$

$\boxed{m = -\frac{3}{2}}$

$-14 \cdot \frac{3}{2} - 2n = -20$

$-2n = 1 \longrightarrow \boxed{n = -\frac{1}{2}}$

$k: \left(x+\frac{3}{2}\right)^2 + \left(y+\frac{1}{2}\right)^2 = r^2$

A: $[-5,0]$ $\quad \left(-5+\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = r^2$

$\left(-\frac{7}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = r^2$

$\frac{49}{4} + \frac{1}{4} = r^2 \longrightarrow \boxed{k: \left(x+\frac{3}{2}\right)^2 + \left(y+\frac{1}{2}\right)^2 = \frac{25}{2}}$

$\frac{50}{4} = r^2$

$k: ?$ $\qquad$ dotýká přímek $p_1: y=2 \qquad p_2: y=0$

a $M = \left[-\frac{5}{2}, 1-\frac{\sqrt{3}}{2}\right] \in k$

Z obrázku: $S=[m,1]$

$r=1$

$k: (x-m)^2 + (y-1)^2 = 1$

$M \in k: \left(-\frac{5}{2}-m\right)^2 + \left(1-1+\frac{\sqrt{3}}{2}\right)^2 = 1$

$\frac{25}{4} + 5m + m^2 + \frac{3}{4} = 1$

$4m^2 + 20m + 24 = 0$

$m^2 + 5m + 6 = 0$

$(m+3)(m+2) = 0 \longrightarrow m_1 = -3 \qquad m_2 = -2$

$k_1: (x+3)^2 + (y-1)^2 = 1$

$k_2: (x+2)^2 + (y-1)^2 = 1$

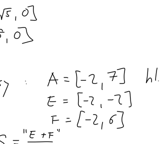
Načrtněte elipsu $e: 4x^2 + 9y^2 = 36 \quad /:36$

kružnice $\quad 9x^2 + 9y^2 = 36 \quad /:9$

$x^2 + y^2 = 4$

$e: \frac{x^2}{9} + \frac{y^2}{4} = 1$

$\frac{x^2}{3^2} + \frac{y^2}{2^2} = 1$



$\rightarrow a=3$

$b=2$

$S=[0,0]$

$a^2 = e^2 + b^2 \qquad E=[-\sqrt{5}, 0]$

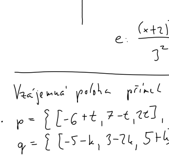
$e = \sqrt{9-4} \qquad F=[\sqrt{5}, 0]$

$= \sqrt{5}$

rovnice + obrázek elipsy : $\quad A=[-2,7]$ hl. vrchol

$E=[-2,-2]$

$F=[-2,6]$



$S = \text{"}\frac{E+F}{2}\text{"}$

$S = S_{EF} = [-2, 2]$

$$\left.\begin{aligned} e &= |SE| = 4 \\ a &= |SA| = 5 \end{aligned}\right\} \begin{aligned} b^2 &= a^2 - e^2 \\ b &= 3 \end{aligned}$$

$e: \frac{(x+2)^2}{3^2} + \frac{(y-2)^2}{5^2} = 1$

Vzájemná poloha přímek v $\mathbb{R}^3$

$p = \{[-6+t, 7-t, 2t], t \in \mathbb{R}\} \qquad \vec{u}_p = (1,-1,2)$

$q = \{[-5-k, 3-2k, 5+k], k \in \mathbb{R}\} \qquad \vec{u}_q = (-1,-2,1)$

$\vec{u}_p \cdot \vec{u}_q = -1 + 2 + 2 = 3 \neq 0$

$x=x \qquad -6+t = -5-k$

$y=y \qquad 7-t = 3-2k$ $\Big\}\oplus$

$z=z \qquad 2t = 5+k$

$1 + 0 = -2 - 3k \longrightarrow k=-1$

$-6+t = -5+1$

$\boxed{t=2}$

Zk: R3: $LS = 4$

$PS = 4$ ✓

$p: t=2: \quad x=-4 \qquad q_{k=-1}: x=-4$

$y=5 \qquad y=5$

$z=4 \qquad z=4$

$p \cap q = \{P=[-4,5,4]\} \implies$ p a q jsou různoběžné.

Rovina

$\rho = \{[1+t+k, 2+3t-k, 5t+k], t,k \in \mathbb{R}\}$

→ obecná rovnice?

→ průsečíky s osami:

$\rho: \quad x = 1+t+k$

$y = 2+3t-k$ $\Big\}\oplus$

$z = 5t+k$ $\Big\}\oplus$

$x+y = 3+4t \quad /\cdot(-2)$

$y+z = 2+8t$ $\Big\}\oplus$

$(-2x-2y) + (y+z) = -6-8t+2+8t$

$-2x - y + z = -4$

$\rho: -2x - y + z + 4 = 0 \qquad \vec{n}_\rho = (-2,-1,1)$

úsekový tvar: $\frac{x}{2} + \frac{y}{4} - \frac{z}{4} = 1$

$\longrightarrow P_x = [2,0,0]$

$P_y = [0,4,0]$

$P_z = [0,0,-4]$