Exp a log

Vypočtěte:

$\log_{\frac{1}{3}}\left(\frac{1}{3}\right)^{1} = 1$

$\log_a x$

$\log_8 \sqrt{2} = \log_8 \sqrt{\sqrt[3]{8}}$ $\quad 2^3 = 8$, $2 = \sqrt[3]{8}$

$= \log_8 \left(8^{\frac{1}{3}}\right)^{1/2}$

$= \log_8 8^{\frac{1}{6}}$

$= \frac{1}{6}$

$\log_5 \underset{5^3}{125} = 3$

$\log_2 0{,}125 = \log_2 \frac{1}{8} = \log_2 2^{-3} = -3$ $\quad \frac{1}{8} = 8^{-1}$

$= \log_2 1 - \log_2 8 = 0 - 3 = -3$

Najdete x: $\quad \log_a x = y \iff a^y = x$

$\log_3 x = 4 \implies x = 3^4 = 81$

$\log_5 x = 0 \qquad x = 5^0 = 1$

$\log x = -\frac{3}{5} \qquad x = 10^{-\frac{3}{5}} = \sqrt[5]{\frac{1}{1000}}$

Najdete a:

$\log_a 27 = 3 \qquad a^3 = 27 = \left(\sqrt[3]{27}\right)^3 = 3^3$

$\rightarrow a = 3$

$\log_a 4 = \frac{1}{4} \qquad a^{\frac{1}{4}} = 4 \quad /^4$

$a = 256$

Vyjádřete pomocí $\log a$, $\log b$, $\log c$

$\log\left(\frac{a^2 \cdot b^3}{100 \cdot \sqrt{c}}\right) = \log a^2 + \log b^3 - \log 100 - \log \sqrt{c}$ $\quad \sqrt{c} = c^{\frac{1}{2}}$

$= 2 \cdot \log a + 3 \cdot \log b - 2 - \frac{1}{2} \log c$

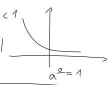
$\log_a x^r = r \cdot \log_a x$

Graf + všechno

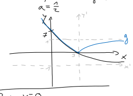
$f: y = \left(\frac{1}{2}\right)^{x-3} - 1$ $\quad a = \frac{1}{2}$

$g: y = |f(x)|$

$a < 1$



$a^0 = 1$



průs. s x:

$\left(\frac{1}{2}\right)^{x-3} - 1 = 0$

$\left(\frac{1}{2}\right)^{x-3} = 1$

$\left(\frac{1}{2}\right)^{x-3} = \left(\frac{1}{2}\right)^0$

$x - 3 = 0$

$x = 3$

$P_x = [3, 0]$

P_y: $x = 0$

$y = \left(\frac{1}{2}\right)^{0-3} - 1 = 2^3 - 1 = 7$

$P_y = [0, 7]$

prostá, klesající v D_f $\quad D_f = \mathbb{R}$, $H_f = (-1, \infty)$

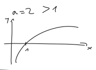
→ zdola omezená

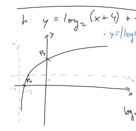
není periodická, sudá ani lichá

$h: y = \log_2 (x+4) + 1$

$i: y = |\log(x+4)| + 1$

$a = 2 > 1$





P_x: $\log_2 (x+4) = -1$ $\quad \log_2 x = -1$

$\log_2 (x+4) = \log_2 \frac{1}{2}$

$x + 4 = \frac{1}{2}$

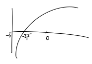
$x = -\frac{7}{2}$

$P_x = [\frac{7}{2}, 0]$

P_y: $y = \log_2 (0+4) + 1$

$= \log_2 4 + 1$

$= 2 + 1 = 3 \longrightarrow P_y [0, 3]$

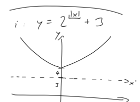


$D_h = (-4, \infty)$ $\quad$ prostá, rostoucí D_h

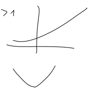
$H_h = \mathbb{R}$ $\quad$ není per, s. ani l.

neomezená

$i: y = 2^{|x|} + 3$



$a > 1$



sudá: $i(-x) = 2^{|-x|} + 3 = 2^{|x|} + 3 = i(x)$

$D_i = \mathbb{R} \quad H_i = \langle 4, \infty)$ $\quad$ zdola omezená

$\searrow$ v $(-\infty, 0)$, $\nearrow$ v $(0, \infty) \implies$ není monotónní $\implies$ není prostá

Průsečík s x: $0 = 2^{|x|} + 3$

$2^{|x|} = -3 \quad N\bar{R}$

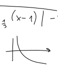
$b: y = |\log_{\frac{1}{3}} (x-1)| - 3$

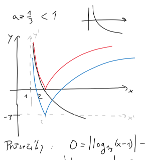
posun v x

abs. h.

posun v y

$a = \frac{1}{3} < 1$





$\log_{\frac{1}{3}} (x-1)$

$|\log_{\frac{1}{3}} (x-1)|$

$|\log_{\frac{1}{3}} (x-1)| - 3 = b(x)$

Průsečíky: $0 = |\log_{1/3} (x-1)| - 3$

$|\log_{1/3} (x-1)| = 3$

$\log_{\frac{1}{3}} (x_1 - 1) = -3$

$\log_{\frac{1}{3}} (x_1 - 1) = \log_{\frac{1}{3}} \left(\frac{1}{3}\right)^{-3}$

$x_1 - 1 = 27$

$x_1 = 28$

$\log_{1/3} (x_2 - 1) = 3$

$x_2 - 1 = \left(\frac{1}{3}\right)^3$

$x_2 = \frac{28}{27}$

$P_1 = [28, 0] \qquad P_2 = [\frac{28}{27}, 0]$

Exp a log rce

Řešte v $\mathbb{R}$

$2^{3x-1} \cdot 4 = 8^{x+1} \cdot \left(\frac{1}{2}\right)^x$ $\qquad a^{f(x)} = a^{g(x)}$

$2^{3x-1} \cdot 2^2 = \left(2^3\right)^{x+1} \cdot \left(2^{-1}\right)^x$ $\qquad \left(a^r\right)^s = a^{r \cdot s}$

$2^{3x-1+2} = 2^{3x+3} \cdot 2^{-x}$ $\qquad \log_a a^{\bigcirc} = \bigcirc$, $\bigcirc = \bigcirc$

$2^{3x+1} = 2^{2x+3} \quad / \log_2$

$\left[\log_2 2^{3x+1} = \log_2 2^{2x+3}\right]$

$3x + 1 = 2x + 3$

$\boxed{x = 2}$

$3^x + 3^{x+1} = 108$

$3^x + 3^x \cdot 3 = 108$

$3^x \cdot (1 + 3) = 108 \quad /:4$

$3^x = 27$

$x = 3$

3
9
27
81
243

$3^x + 3^{x+1} = 27 + 81$

$1 \cdot 27 \quad 3 \cdot 27$

$4 \cdot 27$

$3^x + 3^{x+1} = 7 \cdot 4^x - 4^{x+1}$

$3^x \cdot 4 = (7 - 4) \cdot 4^x$

$3^x \cdot 4 = 3 \cdot 4^x \quad /:3^x \quad :3$

$\frac{4}{3} = \frac{4^x}{3^x}$

$\frac{4}{3} = \left(\frac{4}{3}\right)^x$

$x = 1$

$2^x \cdot 3^{x-1} = 6 \qquad 6 = 2 \cdot 3$

$2^x \cdot 3^x \cdot 3^{-1} = 6$

$6^x \cdot \frac{1}{3} = 6$

$6^x = 18 \quad / \log_6$

$\boxed{x = \log_6 18}$

$4^{2x} - 2 \cdot 4^x - 8 = 0$

$\left(4^x\right)^2 - 2 \cdot 4^x - 8 = 0 \qquad u = 4^x$

$u^2 - 2u - 8 = 0 \qquad u_1 = 4$

$(u - 4) \cdot (u + 2) = 0 \qquad u_2 = -2$

$4^{x_1} = 4 \qquad 4^{x_2} = -2 \qquad a^x > 0 \ \forall x \in \mathbb{R}$

$\boxed{x_1 = 1} \qquad N\bar{R}$

$\log x^5 - \log x^4 + \log x^3 = 12$

$x > 0$

$\log \frac{x^5 \cdot x^3}{x^4} = 12$

$\log \frac{x^8}{x^4} = 12$

$\log x^4 = 12 \quad /:4$

$\log x = 3$

$x = 10^3$

$\log_2^2 x + 2 \log_2 x - 3 = 0$ $\qquad \log_a^2 x = \left(\log_a x\right)^2$

$u = \log_2 x$ $\qquad \log_a x^2 = \log_a \left(x^2\right)$

$u^2 + 2u - 3 = 0 \qquad u_1 = -3 \qquad \log_2 x_1 = -3 \qquad x_1 = 2^{-3}$

$(u + 3) \cdot (u - 1) = 0 \qquad u_2 = 1 \qquad \log_2 x_2 = 1 \qquad x_2 = 2$

$\log_8 \sqrt{x + 30} + \log_8 \sqrt{x} = 1 \quad /\cdot 2$

$\frac{1}{2} \log_8 (x + 30) + \frac{1}{2} \log_8 x -$

$\log_8 (x + 30) + \log_8 x = 2$

$x + 30 \geq 0$

$x \geq -30$

$\boxed{x \geq 0}$

log: $\boxed{x > 0}$

$\log_8 \left[x \cdot (x + 30)\right] = 2 \qquad 2 = \log_8 8^2$

$x \cdot (x + 30) = 64$

$x^2 + 30x - 64 = 0$

$(x + 32) \cdot (x - 2) = 0 \qquad x_1 = -32 \ \times$

$\boxed{x_2 = 2}$

$\frac{\log_3 x}{1 + \log_3 2} = 2$