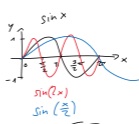
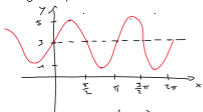


Goniometrie

graf a vřocko

$$f: y = 2 \cdot \sin(2x) + 3$$



periodická s $T = \pi$
kemi'practa', suda' ani' licha

$$D_f = \mathbb{R} \quad H_f = \langle 1, 5 \rangle \text{ otevřená}$$

$$3. v: x = 0 + k \cdot \frac{\pi}{2}, k \in \mathbb{Z}$$

$$\text{lok. maxima: } x = \frac{\pi}{4} + k \cdot \pi, k \in \mathbb{Z}$$

$$\text{lok. minima: } x = \frac{\pi}{4} + \frac{\pi}{2} + k \cdot \pi, k \in \mathbb{Z}$$

$$\text{klesá v } \left(\frac{\pi}{4} + k \cdot \pi, \frac{3\pi}{4} + k \cdot \pi \right)$$

$$\text{roste v } \left(\frac{3\pi}{4} + k \cdot \pi, \frac{5\pi}{4} + k \cdot \pi \right)$$

$$\sin(-3x+2)$$

$$\sin: \text{ minima v } x = \frac{3}{2}\pi + k \cdot 2\pi$$

$$-3x+2 = \frac{3}{2}\pi + k \cdot 2\pi$$

$$-3x = \frac{3}{2}\pi - 2 + k \cdot 2\pi \rightarrow x = -\frac{\pi}{2} + \frac{2}{3} + k \cdot \frac{2}{3}\pi$$

$$g: y = \left| -\tan\left(2x - \frac{\pi}{3}\right) + 2 \right|$$

$$\text{'nulky': } 2x - \frac{\pi}{3} = k \cdot \pi$$

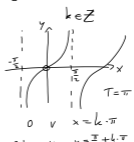
$$2x = k \cdot \pi + \frac{\pi}{3}$$

$$x = \frac{k \cdot \pi}{2} + \frac{\pi}{6}$$

$$\text{asymptoty: } 2x - \frac{\pi}{3} = \frac{\pi}{2} + k \cdot \pi$$

$$2x = \frac{5\pi}{6} + k \cdot \pi$$

$$x = \frac{5\pi}{12} + k \cdot \frac{\pi}{2}$$



$$g: y = \left| -\tan\left(2x - \frac{\pi}{3}\right) + 2 \right|$$

$$y = \left| -\tan(2x) \right|$$

$$\frac{\pi}{2} + 1 \cdot \frac{\pi}{2} = \frac{4}{2}\pi = 2\pi$$

$$\frac{\pi}{2} - 1 \cdot \frac{\pi}{2} = -\frac{1}{2}\pi$$

$$\tan\left(2x - \frac{\pi}{3}\right) = 2$$

$$D_f = \mathbb{R} \setminus \left\{ \frac{5\pi}{12} + k \cdot \frac{\pi}{2} \right\}$$

$$H_f = \mathbb{R}_0^+$$

$$\text{klesající v } \left(\frac{3\pi}{2} + k \cdot \pi, \pi + k \cdot \pi \right)$$

$$\text{rostoucí v } \left(\pi + k \cdot \pi, \frac{3\pi}{2} + k \cdot \pi \right)$$

Oblouková míra

$$\alpha = \frac{15}{6}\pi = 15 \cdot 30^\circ = 450^\circ$$

$$\frac{\pi}{6} = 30^\circ$$

$$\frac{15}{6}\pi = \frac{12}{6}\pi + \frac{3}{6}\pi = 360^\circ + 90^\circ = 450^\circ$$

$$\beta = 20^\circ = \frac{\pi}{9}$$

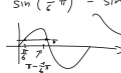
$$\sqrt[20^\circ]{360^\circ} \dots \dots \dots \sqrt[2\pi]{2\pi}$$

$$x = \frac{20}{360} \cdot 2\pi$$

$$x = \frac{1}{9} \cdot \pi$$

Výčíslení goniometrických funkcí

$$\text{Vypočítejte: } \sin\left(\frac{5}{6}\pi\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$



$$\sin\left(\pi - \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) \cos\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{6}\right) \sin\left(\frac{\pi}{6}\right) = \frac{1}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = 0$$



$$\sin\left(\frac{5}{6}\pi\right) = \frac{1}{2}$$

$$\cos\left(\frac{3}{4}\pi\right) = -\frac{\sqrt{2}}{2}$$

$$\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$



$$\cot\left(\frac{1}{2}\pi\right) = \cot\left(9\pi + \frac{\pi}{2}\right) = \cot\left(\frac{\pi}{2}\right) = \frac{\cos\left(\frac{\pi}{2}\right)}{\sin\left(\frac{\pi}{2}\right)} = 0$$

$$T = \pi$$

$$9\pi + \frac{\pi}{2} \neq \frac{\pi}{2} \text{ ošklivé}$$

$$\cot\left(9\pi + \frac{\pi}{2}\right) \neq \cot(9\pi) + \cot\left(\frac{\pi}{2}\right) \text{ ještě ošklivější}$$

$$\cos(240^\circ) = \cos\left(\frac{2}{3} \cdot 2\pi\right) = -\frac{1}{2}$$

$$240^\circ \dots \dots \dots x$$

$$360^\circ \dots \dots \dots 2\pi$$



$$\cos\left(\frac{4}{3}\pi\right) = \cos\left(2\pi - \frac{2}{3}\pi\right)$$

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
s	0	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
c	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
t					
ct					

$$\sin^2 x + \cos^2 x = 1$$

$$\cos x = \sqrt{1 - \sin^2 x}$$

$$x = \frac{\pi}{6}$$

$$\cos \frac{\pi}{2} = \sqrt{1 - \left(\frac{1}{2}\right)^2}$$

$$= \sqrt{1 - \frac{1}{4}}$$

$$= \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

Goniometrické výrazy

Zjednodušte a určete podmínky

$$\bullet (\cos x - \sin x)^2 + (\cos x + \sin x)^2 =$$

$$2 \cdot (\underbrace{\cos^2 x + \sin^2 x}_1) = 2 \quad \forall x \in \mathbb{R}$$

$$\bullet \cot x \cdot \cos x + \sin x = \frac{\cos^2 x}{\sin x} + \sin x = \frac{\cos^2 x + \sin^2 x}{\sin x} = \frac{1}{\sin x}$$

$$\downarrow$$

$$\frac{\cos x}{\sin x}$$

$$x \in \mathbb{R} \setminus \{k \cdot \pi, k \in \mathbb{Z}\}$$

$$\bullet 2 \tan(x) \cdot \sqrt{\cos^2 x - \cos^2 x \cdot \sin^2 x} =$$

$$= 2 \cdot \tan x \cdot \sqrt{\cos^2 x (1 - \sin^2 x)} = 2 \cdot \tan x \cdot \cos^2 x$$

$$= 2 \cdot \sin x \cdot \cos x = \sin(2x)$$

$$\tan: x \in \mathbb{R} \setminus \left\{ \frac{\pi}{2} + k \cdot \pi, k \in \mathbb{Z} \right\}$$

$$\cos^2 x (1 - \sin^2 x) \geq 0$$

$$\begin{matrix} > 0 & > 0 \\ \forall x \in \mathbb{R} \end{matrix}$$

Goniometrické rovnice

Řešte v \mathbb{R}

$$\sin(x) = \frac{1}{2}$$

$$x_1 = \frac{\pi}{6} + k \cdot 2\pi$$

$$x_2 = \frac{5\pi}{6} + k \cdot 2\pi$$

$$\sin\left(2x - \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$u = 2x - \frac{\pi}{4}$$

$$\sin(u) = \frac{\sqrt{2}}{2}$$

$$2x_1 - \frac{\pi}{4} = \frac{\pi}{4} + k \cdot 2\pi$$

$$x_1 = \frac{\pi}{4} + k \cdot \pi$$



$$u_1 = \frac{\pi}{4} + k \cdot 2\pi$$

$$u_2 = \frac{3\pi}{4} + k \cdot 2\pi$$

$$2x_2 - \frac{\pi}{4} = \frac{3\pi}{4} + k \cdot 2\pi$$

$$x_2 = \frac{\pi}{2} + k \cdot \pi$$

$$x_1 = \frac{\pi}{4} + k \cdot \pi$$

$$x_2 = \frac{3\pi}{4} + k \cdot \pi$$

$$\bullet \sin(3x) - 2 \cos^2(3x) = -1$$

$$\cos^2 = 1 - \sin^2$$

$$\sin(3x) - 2 \cdot (1 - \sin^2(3x)) = -1$$

$$2 \cdot \sin^2(3x) + \sin(3x) - 1 = 0 \quad u = \sin(3x)$$

$$2u^2 + u - 1 = 0$$

$$\text{f je lineární } \Leftrightarrow$$

$$f(ax+by) = af(b)+bf(y)$$

$$u_{1,2} = \frac{-1 \pm \sqrt{9}}{4} = \begin{cases} \frac{1}{2} \\ -1 \end{cases}$$

$$\sin(3x_1) = \frac{1}{2}$$

$$3x_{1,1} = \frac{\pi}{6} + k \cdot 2\pi$$

$$x_{1,1} = \frac{\pi}{18} + k \cdot \frac{2}{3}\pi$$

$$3x_{1,2} = \frac{5\pi}{6} + k \cdot 2\pi$$

$$x_{1,2} = \frac{5\pi}{18} + k \cdot \frac{2}{3}\pi$$

$$\sin(3x_2) = -1$$

$$3x_2 = \frac{3\pi}{2} + k \cdot 2\pi$$

$$x_2 = \frac{3\pi}{2} + k \cdot \frac{2}{3}\pi$$

$$x_2 = \frac{3\pi}{2} + k \cdot \frac{2}{3}\pi$$

$$\text{Polár: } u = 3$$

$$\sin(3x) = 3$$

$$|\sin(x)| \leq 1 \quad \forall x \in \mathbb{R}$$

$$\Rightarrow \text{NR}$$