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geometrie ($\triangle$, $\square$, planimetrie, stereometrie)
$\longrightarrow$ ANALYTICKÁ GEOMETRIE!

vaclav-alt.github.io heslo: nemamradlekorici
– ukázka testu

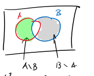
Předběžné termíny průběžného testu:
18.11. 18:00 – 20:00 čtvrtek
27.11. 9:00 – 11:00 sobota

- Množiny
- číselné obory a vlastnosti čísel
- algebraické výrazy
- rovnice a nerovnice

$A \cap B$, $A \cup B$, $\underline{A \setminus B}$, $\bar{A}$

A, B jsou množiny: $A \setminus B = \{x \in A;\ x \notin B\}$

$A = \{1, 2, 3\}$ $B = \{2, 3, 4\}$



$A \setminus B = \{1\}$ $A \setminus B = 1$ ✗
$B \setminus A = \{4\}$

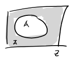
prázdná množina: $\{\}$, $\emptyset$ ale ne $\{\emptyset\}$

$\{\} \neq \{\emptyset\}$
$\emptyset \neq \{\emptyset\}$

- doplněk množiny: $\bar{A}$, A' (v množině Z)

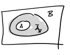
$\bar{A} = \{x \in Z;\ x \notin A\}$

Z .. základní množina



$A = \{n \in \mathbb{N};\ n^2 \leq 16\} = \{1, 2, 3, 4\}$
$B = \{x \in \mathbb{R};\ x^2 \leq 16\} = \langle -4, 4 \rangle$

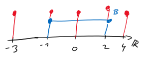
Doplněk A v B
$\bar{A}_B = \{x \in B;\ x \notin A\}$



$\bar{A}_B = B \setminus A$
$\bar{A} = Z \setminus A \implies Z = A \cup \bar{A}$

Určete $A \cup B$, $A \cap B$, $A \setminus B$, $B \setminus A$

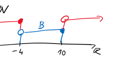
$A = \{-3, -1, 0, 2, 4\}$ $B = \langle -1; 2 \rangle$



$A \cup B = \{-3\} \cup \langle -1, 2 \rangle \cup \{4\}$
$A \cap B = \{-1, 0, 2\}$
$A \setminus B = \{-3, 4\}$
$B \setminus A = (-1, 0) \cup (0, 2)$

$A = \{n \in \mathbb{N};\ n^2 \leq 16\} = \{1, 2, 3, 4\}$
$\bar{A} = \{n \in \mathbb{N};\ n^2 > 16\} = \{5, 6, 7, \ldots\}$

$A \cup \bar{A} = \mathbb{N}$

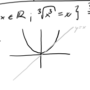


$B = (-4, 10\rangle$
$\bar{B} = (-\infty, -4\rangle \cup (10, \infty)$

$\mathbb{R}^+ = (0, \infty)$ $\overline{\mathbb{R}^+} = (-\infty, 0\rangle = \mathbb{R}_0^-$

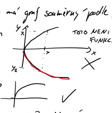
$\{x \in \mathbb{R};\ \sqrt[3]{x^3} = x\} \overset{?}{=} \{x \in \mathbb{R};\ x \geq 0\}$

$f: y = x^2$

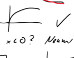


inverzní fce má graf souměrný podle $y = x$

$f^{-1}: y = \sqrt{x}$

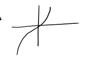


$y = \sqrt{x}$

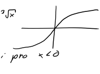


Co když $x < 0$? Neumíme

$f: y = x^3$



$\longrightarrow$ $f^{-1}: y = \sqrt[3]{x}$



umíme i pro $x < 0$

$\{x \in \mathbb{R};\ \underbrace{\sqrt[3]{x^3} = x}\} = \{x \in \mathbb{R};\ x \geq 0\}$

$x = 3$: $\sqrt[3]{3^3} = 3$ ✓
$x = 0$: $\sqrt[3]{0^3} = 0$ ✓
$x = -3$: $\sqrt[3]{(-3)^3} = \sqrt[3]{-(3)^3} = -3$ ✓ (lichá funkce)

$\{x \in \mathbb{R};\ \sqrt[3]{x^3} = x\} = \mathbb{R}$
$\{x \in \mathbb{R};\ x \geq 0\} = \mathbb{R}_0^+$

$\boxed{\mathbb{R} \neq \mathbb{R}_0^+}$

Algebraické výrazy

$(x -)$ nedává mat. smysl $\implies$ není AV

$\frac{\sqrt{x+y}}{x^2}$ je AV

Zjednodušte výraz a stanovte podmínky:

$(A \pm B)^2$
$(A+B) \cdot (A-B)$

$$\left(\frac{1}{a+b} + \frac{1}{a-b}\right) \cdot \left(\frac{1}{a} - \frac{1}{b}\right) = \frac{a-b+a+b}{(a+b) \cdot (a-b)} \cdot \frac{b-a}{a \cdot b}$$

$$= \frac{2a}{(a+b)(a-b)} \cdot \frac{-1 \cdot (a-b)}{a \cdot b} = \frac{-2}{b \cdot (a+b)}$$

$\boxed{a, b \neq 0 \quad a \neq \pm b}$

$b \neq 0$
$a \neq -b$ 2 podm. chybí?

- $$\frac{(\sqrt[4]{u} + \sqrt[4]{v})^2 + (\sqrt[4]{u} - \sqrt[4]{v})^2}{u-v} : \frac{2}{\sqrt{u} - \sqrt{v}} = \frac{2\sqrt{u} + 2\sqrt{v}}{(u-v)} \cdot \frac{\sqrt{u} - \sqrt{v}}{2}$$

$= 1$

$(A+B)^2 = A^2 + 2AB + B^2$
$(A-B)^2 = A^2 - 2AB + B^2$ $\Big\}$ (+)
$\hookrightarrow 2A^2 + 2B^2$

$\sqrt{u}, \sqrt{v} \implies$ $u, v \geq 0$
$u - v \implies$ $u \neq v$

Rovnice a nerovnice

Kvadratický trojčlen: $ax^2 + bx + c$, $a, b, c \in \mathbb{R}$, $a \neq 0$

Doplnění na čtverec dvojčlenu

$ax^2 + bx + c \implies \bar{a} \cdot (x + \bar{b})^2 + \bar{c}$

$\bar{a} = a$ $\bar{b} = \frac{b}{2a}$ $\bar{c} = c - \frac{b^2}{4a}$

$(A+B)^2 = A^2 + 2A \cdot B + B^2$

- $x^2 + 3x + 2 = \left(x + \frac{3}{2}\right)^2 + 2 - \left(\frac{3}{2}\right)^2 = \left(x + \frac{3}{2}\right)^2 - \frac{1}{4}$

$A^2 + 2 \cdot A \cdot B$
$\Downarrow$
$2B = 3$
$B = \frac{3}{2}$

$= \left(x + \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2$ $A^2 - B^2$
$= \left(x + \frac{3}{2} + \frac{1}{2}\right) \cdot \left(x + \frac{3}{2} - \frac{1}{2}\right)$
$= (x + 2) \cdot (x + 1)$

$x_1 = -2$, $x_2 = -1$

Racionální nerovnice

racionální výraz: $\frac{P(x)}{Q(x)}$ P, Q jsou polynomy

Řešte v $\mathbb{R}$:

$\frac{1}{x-2} < \frac{1}{x}$ $/ \cdot x \cdot (x-2)$

$x < x - 2$
$0 < -2$
NŘ

ŠPATNĚ

$2 < 5$ $/ \cdot (-1)$
$-2 < -5$ ŠPATNĚ
$-2 > -5$ ✓

$x \cdot (x-2)$: $- \ - \ + \ - \ + \ +$
$+ \quad 0 \quad - \quad 2 \quad +$ $\mathbb{R}$

I) $x \in (-\infty, 0) \cup (2, \infty)$
$x \cdot (x-2) > 0$

$x < x - 2$
$0 < -2$
NŘ.

II) $x \in (0, 2)$
$x \cdot (x-2) < 0$

$x > x - 2$
$0 > -2$ ✓
$x \in (0, 2)$

$\boxed{x \in (0, 2)}$

alternativně

$\frac{1}{x-2} < \frac{1}{x}$ $/ - \frac{1}{x}$

$\frac{1}{x-2} - \frac{1}{x} < 0$

$\frac{x - (x-2)}{x(x-2)} < 0$

$\frac{2}{x(x-2)} < 0 \implies \boxed{x \in (0, 2)}$

Rovnice s odmocninou

- $\sqrt{x} + x = 2$ $/^2$ $\xrightarrow{/-x}$ $\sqrt{x} = 2 - x$ $/^2$

$x + 2\sqrt{x} \cdot x + x^2 = 4$ $/^2$
$x^4 + \ldots = 16$

$x = 4 - 4x + x^2$
$x^2 - 5x + 4 = 0$
$(x - 4) \cdot (x - 1) = 0$
$x_1 = 1$ $x_2 = 4$

Zk: $x = 1$:
$LS = \sqrt{1} = 1$
$PS = 2 - 1 = 1$
$LS = PS$ ✓

$x = 4$: $LS = \sqrt{4} = 2$
$PS = 2 - 4 = -2$
$LS \neq PS$ ✗

$\boxed{x = 1}$

- $\sqrt{2x+7} + \sqrt{x-5} = \sqrt{3x+2}$ $/^2$

1) $2x + 7 \geq 0 \implies x \geq -\frac{7}{2}$
2) $x - 5 \geq 0 \implies x \geq 5$
3) $3x + 2 \geq 0 \implies x \geq -\frac{2}{3}$

$(\cancel{2x} + \cancel{7}) + 2 \cdot \sqrt{(2x+7) \cdot (x-5)} + (\cancel{x} - \cancel{5}) = \cancel{3x} + \cancel{2}$

$\sqrt{(2x+7) \cdot (x-5)} = 0$

$x_1 = -\frac{7}{2}$ ✗ $x_2 = 5$

Zk: $x = 5$: $LS = \sqrt{17} + \sqrt{0} = \sqrt{17}$
$PS = \sqrt{17}$
$LS = PS$

$\boxed{x = 5}$

Rovnice s absolutní hodnotou

$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$

$|5| = 5$ $|0| = 0$
$|-5| = 5 = -(-5)$

- $|x - 7| = 5$

$|x - a|$ .. vzdálenost x od a

2 — 7 — 12 $\mathbb{R}$

x_1 — a — x_2

$x_1 = 2$
$x_2 = 12$

- $|x - 7| = x - 7$

$x - 7 < 0$ | $x - 7 \geq 0$
$|x-7| = -(x-7)$ 7 $|x-7| = x - 7$

I) $x \in (-\infty, 7)$
$-(x-7) = x - 7$
$-x + 7 = x - 7$
$x = 7$ ✗
NŘ.

II) $x \in \langle 7, \infty)$
$x - 7 = x - 7$
$0 = 0$
$x \in \langle 7, \infty)$

$\boxed{x \in \langle 7, \infty)}$