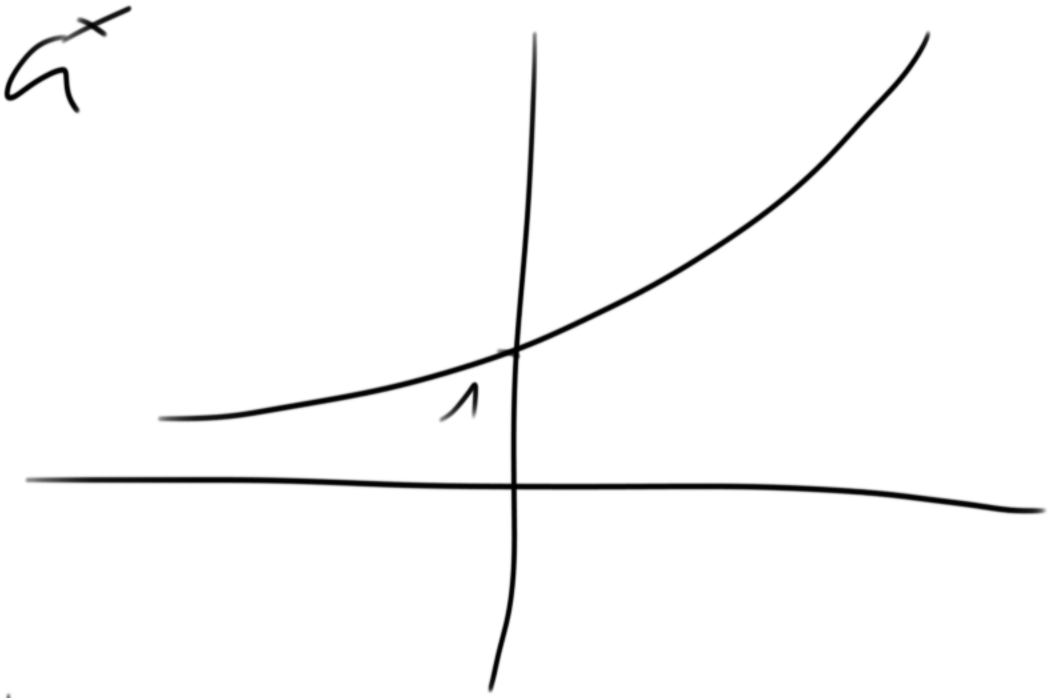
Log. a exp. funkce

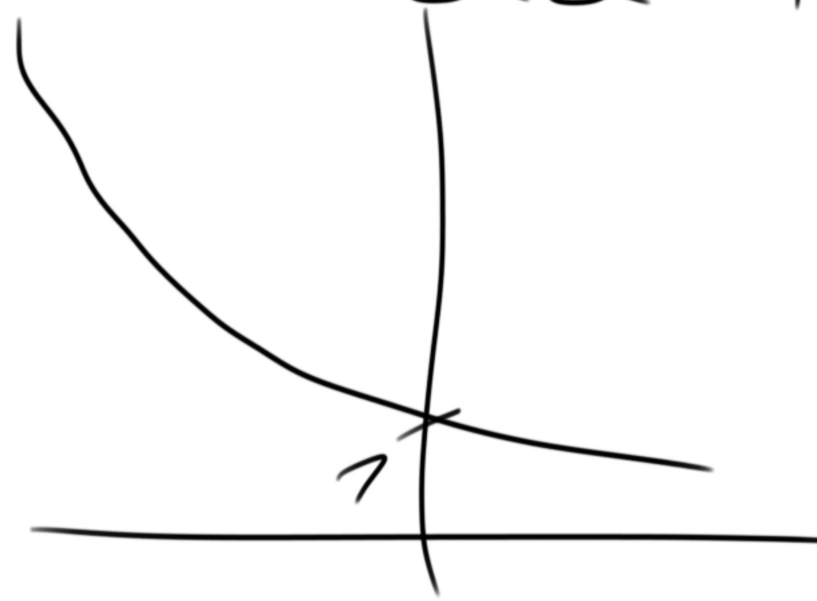
Grafy

a^x

$a>1$

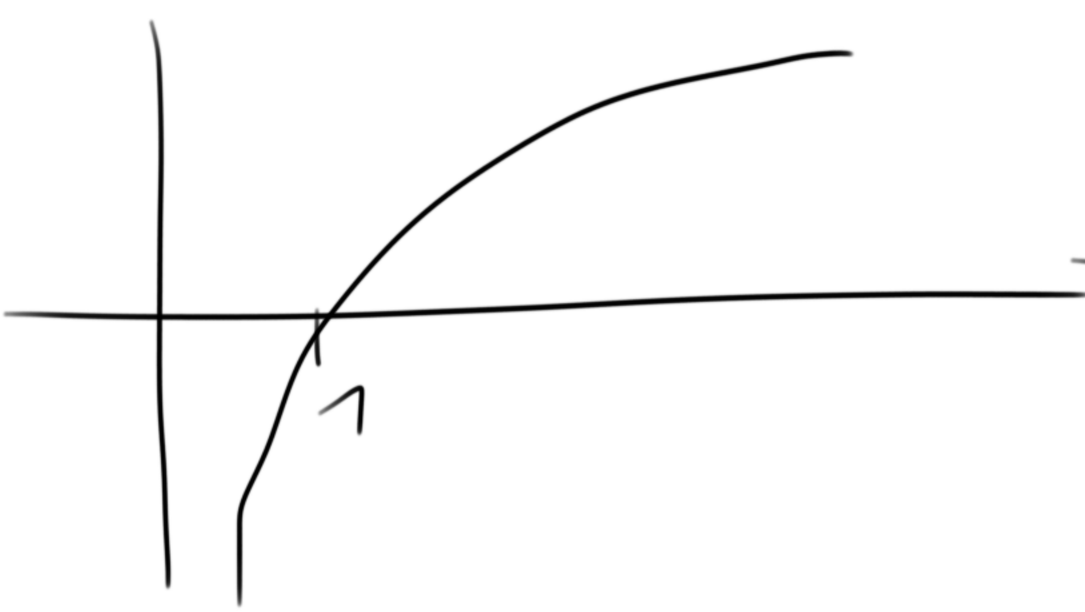


$0<a<1$

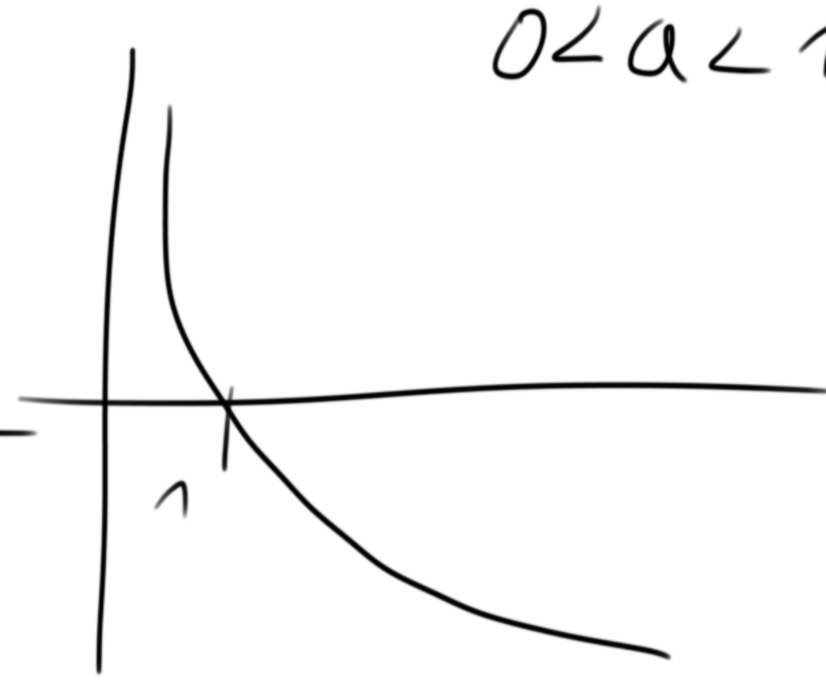


$\log_a x$

$a>1$



$0<a<1$

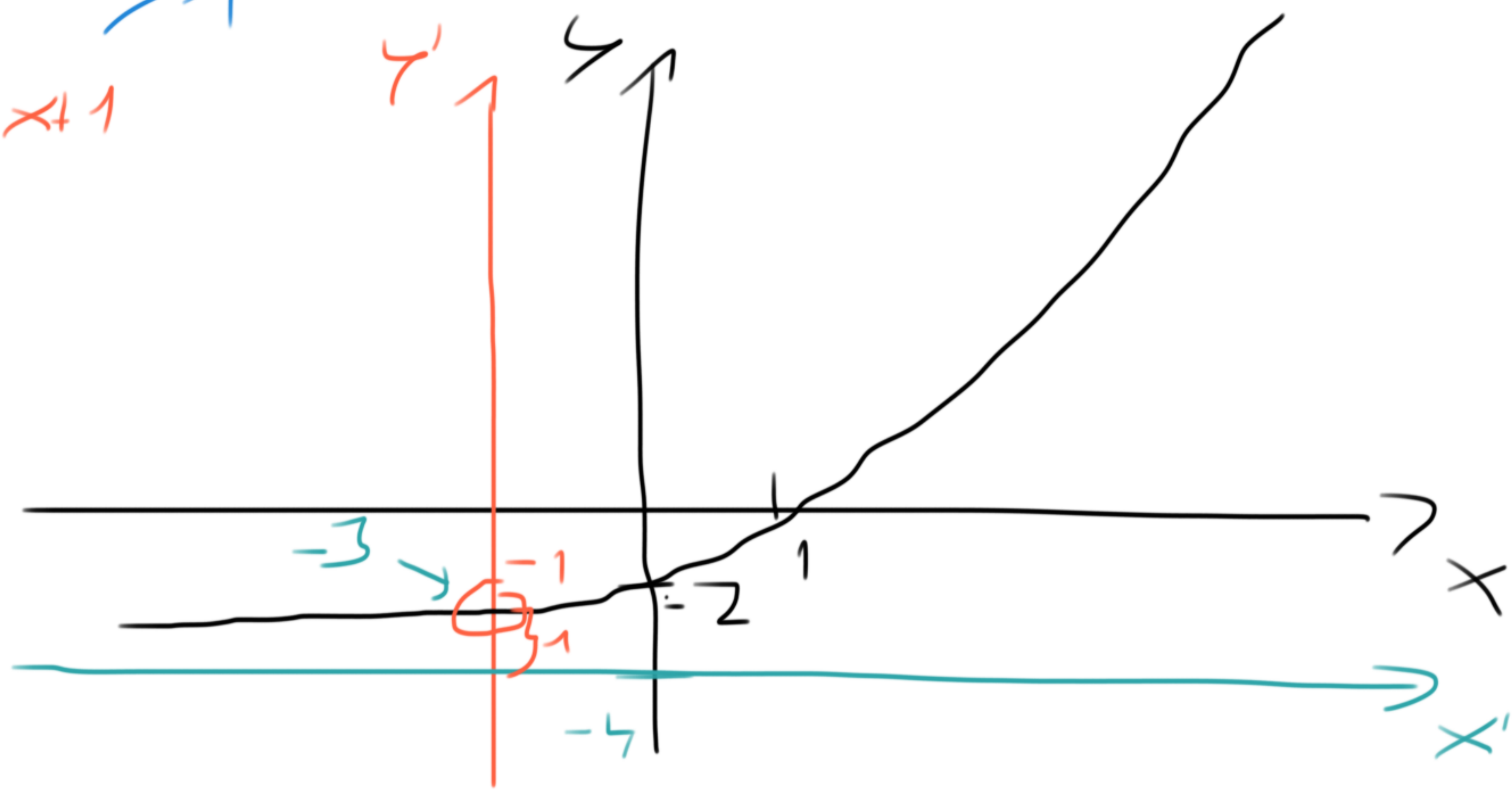


graf, vlastnosti, body

$y = 2^{x+1} - 4$

> 1

nb x+1



průs. s y

$x = 0$

$y = 2^{0+1} - 4$

$= 2 - 4$

$= -2$

průs. s x

$y = 0$

$0 = 2^{x+1} - 4$

$4 = 2^{x+1}$

$2^2 = 2^{x+1}$

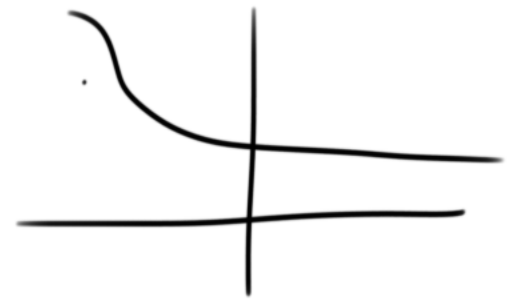
$2 = x + 1$

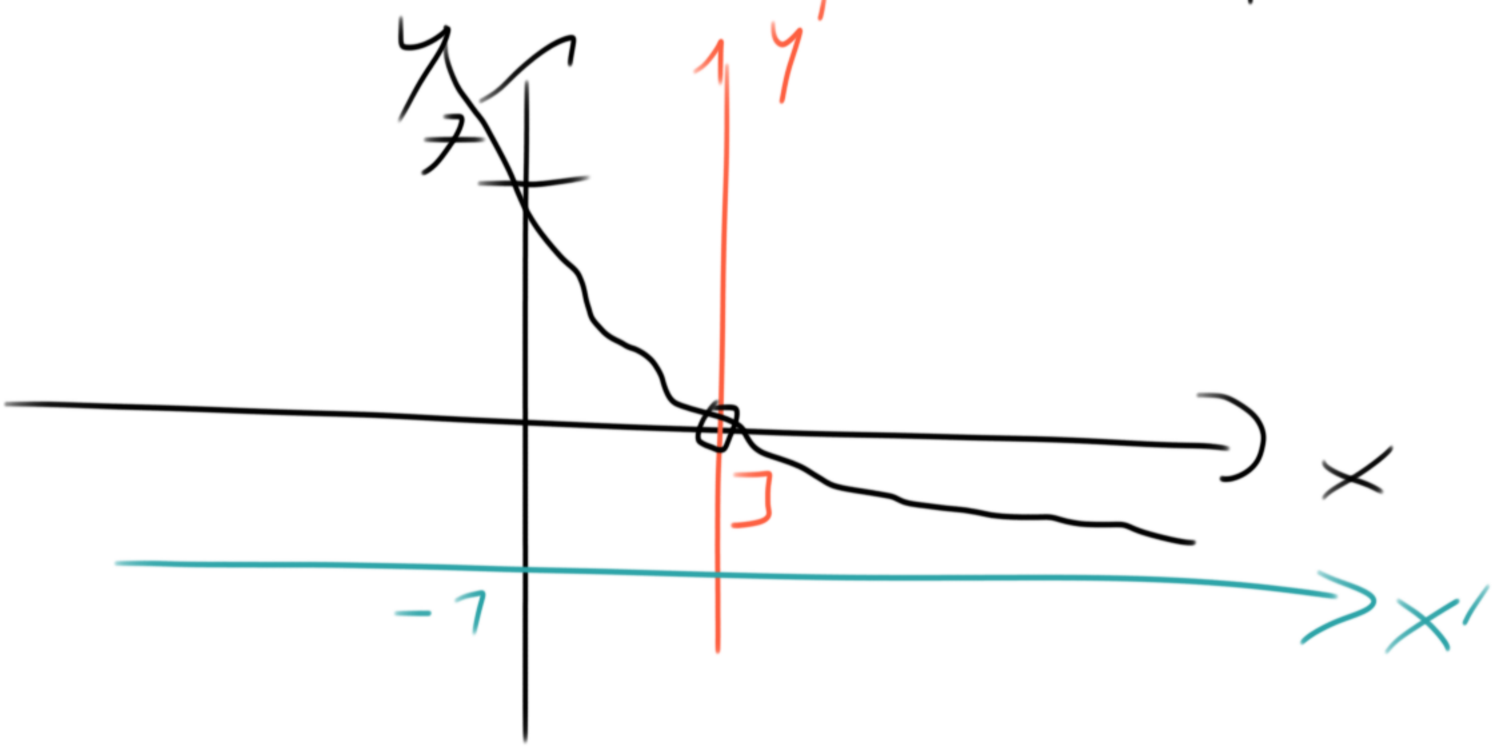
$x = 1$

$y' = 2^{x'} \quad x' = 0 \Rightarrow y' = 1$

$y = \left(\frac{1}{2}\right)^{x-3} - 1$

$a = \frac{1}{2} < 1 \longrightarrow$





$x = 0$:

$y = \left(\frac{1}{2}\right)^{-3} - 1$

$= 8 - 1 = 7$

$y = 0$:

$0 = \left(\frac{1}{2}\right)^{x-3} - 1$

$1 = \left(\frac{1}{2}\right)^{x-3}$

$\left(\frac{1}{2}\right)^{0} = \left(\frac{1}{2}\right)^{x-3}$

$x = 3$

klesající na D_f

$a < 1$

$H_f = (-1, \infty) \Rightarrow$ omezená

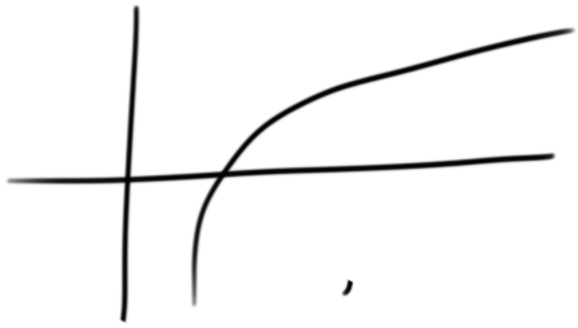
$D_f = \mathbb{R}$

ani sudá ani lichá

prostá

$y = \log_2(x+4)+1$

$a = 2 > 1$



$x+4>0$

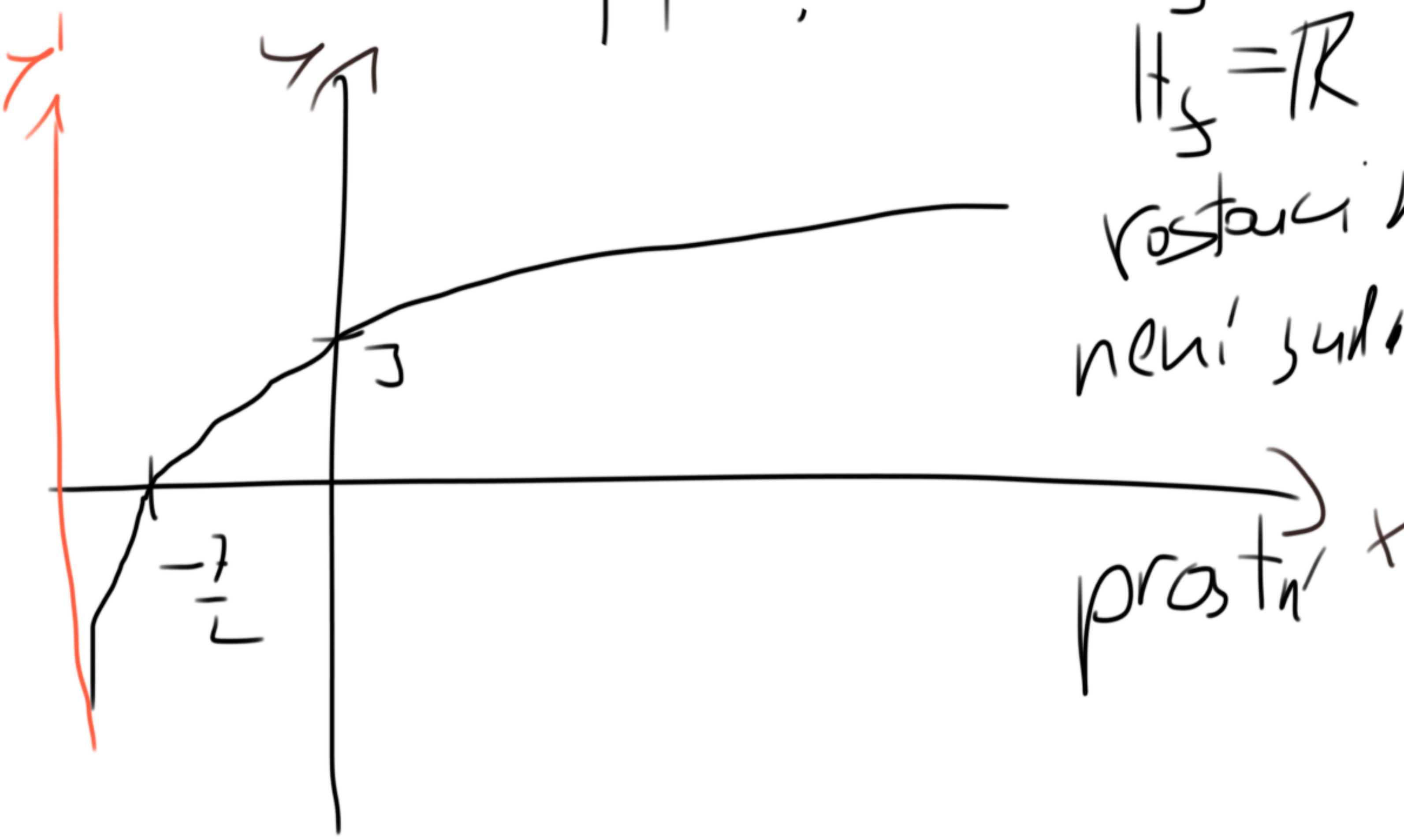
$x>-4$

$D_f = (-4, \infty)$

$H_f = \mathbb{R}$

rostoucí na D_f

není sudá ani lichá



prostá

$x=0$:

$y = \log_2(0+4)+1$

$= \log_2 4 + 1$

$= 2+1 = \underline{3}$

$y=0$

$0 = \log_2(x+4)+1$

$-1 = \log_2(x+4)$

$\log_2 \frac{1}{2} = \log_2(x+4)$

$x = \frac{1}{2} - 4$

$x = -\frac{7}{2}$

desmos.com

$$2^x \cdot 4^{3x} = 2^x \cdot (2^2)^{3x} = 2^x \cdot 2^{6x} = 2^{7x}$$

$$a^r \cdot a^s = a^{r+s}$$

$$(a^r)^s = a^{r \cdot s}$$

$$\log_a x = s \iff a^s = x$$

$$\log_a(x \cdot y) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a x^r = r \cdot \log_a x$$

$$\log_b x = \frac{\log_a x}{\log_a b}$$

$$\log_a 1 = 0 \qquad \log_a a = 1$$

Vypočítejte

$$\log_{\frac{1}{3}} 9 = -2$$

$$9 = 3^2 = \left(\frac{1}{3}\right)^{-2}$$

$$\log_{\frac{1}{3}} \left(\frac{1}{3}\right)^{-2} = -2 \cdot \log_{\frac{1}{3}} \frac{1}{3} = -2$$

$$= -2$$

$$\log_7 \sqrt{7} = \log_7 7^{1/2} = \frac{1}{2}$$

$$\log_5 125 = \log_5 (5 \cdot 25) = \log_5 5 + \log_5 25$$

$$125 = 5 \cdot 25 \qquad = 1 + 2 = 3$$

$$\log_{0,2} 0{,}04 = \log_{0,2} (0{,}2)^2 = 2$$

Reste v $\mathbb{R}$

$\log_3 x = 4 \qquad x = 3^4 = 81$

$\log_a x = s \quad : \quad a^s = x$

$\log_2 x = -\frac{1}{3} \qquad x = 2^{-\frac{1}{3}}$

$$\boxed{x = \frac{1}{2^{1/3}} = \sqrt[3]{\frac{1}{2}}}$$

$\log x = -\frac{3}{5} \qquad x = 10^{-3/5}$

$$\boxed{x = \frac{1}{\sqrt[5]{0,001}}}$$

$$x = 10^{-\frac{3}{5}} = \sqrt[5]{10^{-3}} = \sqrt[5]{\frac{1}{10^3}} = \sqrt[5]{0,001}$$

$\log_{17} x = 1 = \log_{17} 17$

$\log_{17} x = \log_{17} 17$

$$\boxed{x = 17}$$

$\log_a 27 = 3 \iff a^3 = 27 \quad / \sqrt[3]{\ }$

$\underline{a = 3}$

$\log_a \frac{1}{16} = 4 \qquad a^4 = \frac{1}{16} \quad / \sqrt[4]{\ }$

$a = \frac{1}{2}$

$\log_a 5 = -1$

$a^{-1} = 5$

$\frac{1}{a} = 5$

$\boxed{a = \frac{1}{5}}$

Řešte v R

$$2^{3x-1}\cdot 4 = 8^{x+1}\cdot\left(\frac{1}{2}\right)^{x}$$

$$2^{3x-1}\cdot 2^{2} = \left(2^{3}\right)^{x+1}\left(2^{-1}\right)^{x}$$

$$2^{3x+1} = 2^{3x+3}\cdot 2^{-x}$$

$$2^{3x+1} = 2^{2x+3}$$

$$3x+1 = 2x+3$$

$$\underline{x = 2}$$

$$\frac{3^{x}}{2\cdot 3^{\sqrt{3}}} = 4{,}5 \quad /\cdot 2\cdot 3^{\sqrt{3}}$$

$$3^{x} = 9\cdot 3^{\sqrt{3}}$$

$$3^{x} = 3^{2}\cdot 3^{\sqrt{3}}$$

$$3^{x} = 3^{2+\sqrt{3}}$$

$$3^x + 3^{x+1} = 108 \qquad 108 = 3 \cdot 36 = 2^2 \cdot 3^3$$

$$\underline{3^x + 3^{x+1}} = 2^2 \cdot 3^3$$

$$x + x + 1 = 2 \cdot 3 \qquad a^{f(x)} = a^{g(x)}$$

$$3^x \cdot \underbrace{\cancel{(1+3)}}_{4} = \cancel{2^2} \cdot 3^3$$

$$3^x = 3^3$$

$$\boxed{x = 3}$$

$$4^x + 6^x = 2 \cdot 9^x$$

$$2^{2x} + 2^x \cdot 3^x = 2 \cdot 3^{2x} \quad / : 2^x \cdot 3^x$$

$$\underline{\frac{2^x}{3^x} + 1 = 2 \cdot \frac{3^x}{2^x}}$$

PAUZA

$3^x = 10 \quad / \log_3(\ldots)$

$\underbrace{\log_3 3^x} = \log_3 10$

$\boxed{x = \log_3 10} = 2{,}\ldots$

$5^{x+1} = 4 \quad / \log_5(\ldots)$

$\log_5 5^{x+1} = \log_5 4$

$x+1 = \log_5 4$

$\boxed{x = \log_5 4 - 1}$

$$7\cdot 4^{-x+2} = 3\cdot 4^{-x+3} - 5$$

$$7\cdot 4^{-x+2} - 3\cdot 4^{-x+3} = -5$$

$$4^{-x}\cdot(7\cdot 4^{2} - 3\cdot 4^{3}) = -5$$

$$4^{-x}\cdot 4^{2}\underbrace{(7-12)}_{-5} = -5$$

$$4^{-x+2} = 1$$

$$4^{-x+2} = 4^{0}$$

$$-x+2=0$$

$$\boxed{x=2}$$

$$4^{2x} - 6\cdot 4^{x} + 8 = 0 \qquad y_{1,2} = 4, 2$$

$$y = 4^{x} \qquad 4^{x_1} = 4 \quad \boxed{x_1 = 1}$$

$$y^{2} - 6y + 8 = 0 \qquad 4^{x_2} = 2 \quad \boxed{x_2 = \frac{1}{2}}$$

$$(y-4)\cdot(x-2) = 0$$

$$4^{2x} - 2 \cdot 4^x - 8 = 0$$

$$y = 4^x$$

$$y^2 - 2y - 8 = 0$$

$$(y-4)(x+2) = 0$$

$$y_{1,2} = 4, -2$$

$$4^{x_1} = 4$$

$$\boxed{x_1 = 1}$$

$$4^{x_2} = -2$$

NR

$$a^x > 0 \quad \forall x \in R$$

$$4^x + 6^x = 2 \cdot 9^x$$

$$\frac{2^x}{3^x} + 1 = 2 \cdot \frac{3^x}{2^x}$$

$$y = \left(\frac{3}{2}\right)^x$$

$$y^{-1} + 1 = 2 \cdot y \quad / \cdot y$$

$$1 + y = 2y^2$$

$$2y^2 - y - 1 = 0$$

$$y_{1,2} = \frac{1 \pm \sqrt{1+8}}{4} = \begin{cases} 1 \\ -\frac{1}{2} \end{cases}$$

$$\left(\frac{3}{2}\right)^{x_1} = 1$$

$$\boxed{x_1 = 0}$$

$$\left(\frac{3}{2}\right)^{x_2} = -1/2$$

NR

Log. rce

$$\log_2(x+1) = 3 \qquad\qquad \log_2 z = 3$$

$$z = 2^3 = 8$$

$$\log_2 \underline{(x+1)} = \log_2 \underline{8}$$

$$x+1 = 8$$

$$\underline{x = 7}$$

$$4\log_3(2x-1) = 12 \quad / :4$$

$$\log_3(2x-1) = 3 \cdot \log_3 3$$

$$\log_3(2x-1) = \log_3(3^3)$$

$$2x-1 = 27$$

$$\boxed{x = 14}$$

$$\frac{\log_3 x}{1+\log_3 2} = 2 \quad \Big| \cdot (1+\log_3 2)$$

$$\log_3 x = \underbrace{2}_{\log_3 9} + 2\cdot \log_3 2$$

$$\log_3 x = \log_3 9 + \log_3 4$$

$$\log_3 x = \log_3 36$$

$$\boxed{x = 36}$$

$$\log_8 \sqrt{x+30} + \log_8 \sqrt{x} = 1$$

$\log x \quad D_f = \mathbb{R}^+$

$\Rightarrow \quad x+30 > 0 \qquad x > -30$

$x > 0 \qquad \boxed{x > 0}$

$x = -5 \longrightarrow \times$

$$\log_8 \sqrt{x} = \log_8 x^{\frac{1}{2}} = \frac{1}{2}\log_8 x$$

$$\frac{1}{2}\log_8 (x+30) + \frac{1}{2}\log_8 x = 1 \quad / \cdot 2$$

$$\log_8 [x \cdot (x+30)] = 2$$

$$\log_8 [x(x+30)] = \log_8 64$$

$$x^2 + 30x - 64 = 0$$

$$(x+32) \cdot (x-2) = 0$$

$x > 0 \qquad \boxed{\cancel{x_1 = -32} \quad x_2 = 2}$

$$3^x + 3^{x+1} = 7 \cdot 4^x - 4^{x+1}$$

$$3^x(1+3) = 4^x(7-4) \qquad / : 3^x$$

$$3^x \cdot 4 = 4^x \cdot 3 \qquad / : 3$$

$$\frac{4}{3} = \frac{4^x}{3^x}$$

$$\frac{4}{3} = \left(\frac{4}{3}\right)^x$$

$$\boxed{x = 1}$$