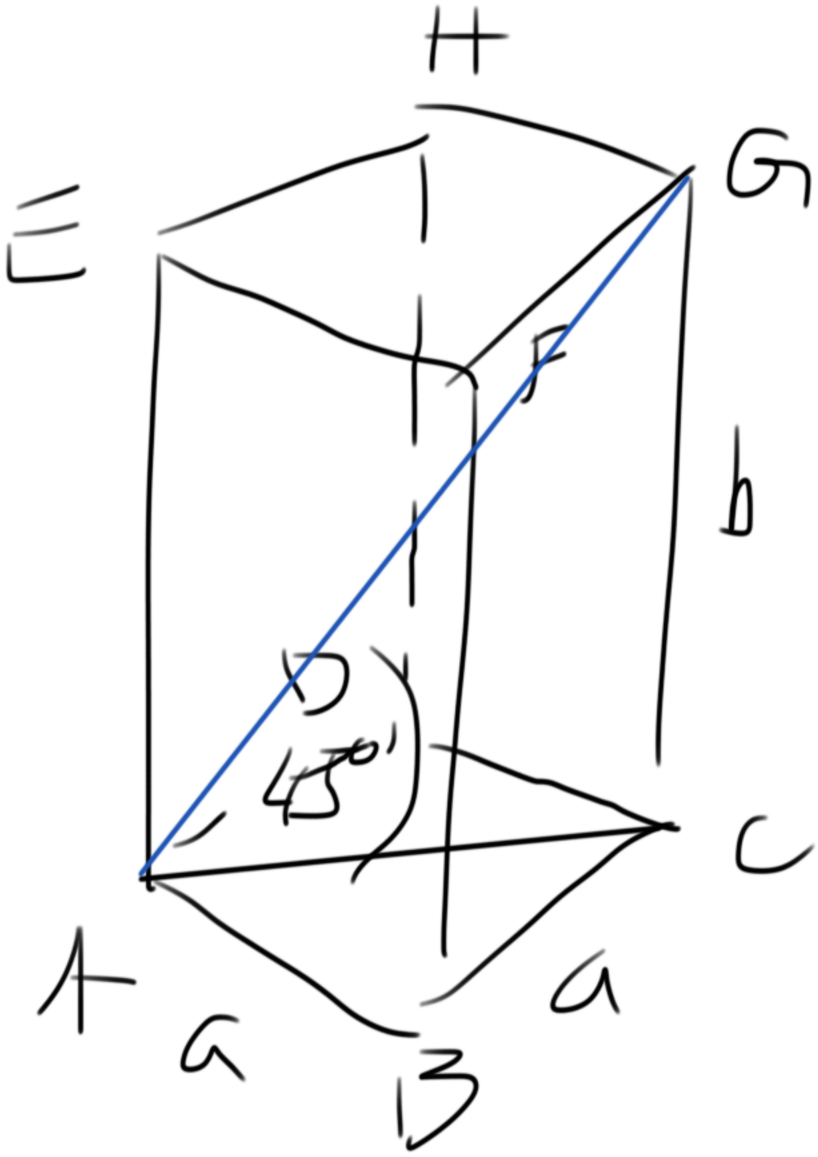


$V = 64\ cm^3$

čtvercová podstava

$b = \sqrt{2}\,a$

$a = 2^{\frac{11}{6}}$

$S = ?$

$$b = \sqrt{2}\,a = 2^{\frac{1}{2}} \cdot 2^{\frac{11}{6}} = 2^{\frac{11+3}{6}} = 2^{\frac{14}{6}}$$

$$S = 2 \cdot a^2 + 4 \cdot a \cdot b$$

$$= 2^{\frac{6}{6}} \cdot 2^{\frac{22}{6}} + \overset{2^2}{4} \cdot 2^{\frac{11}{6}} \cdot 2^{\frac{14}{6}} =$$

$$= 2^{\frac{28}{6}} + 2^{\frac{11+14+12}{6}} = \underline{2^{\frac{28}{6}} + 2^{\frac{37}{6}}}$$

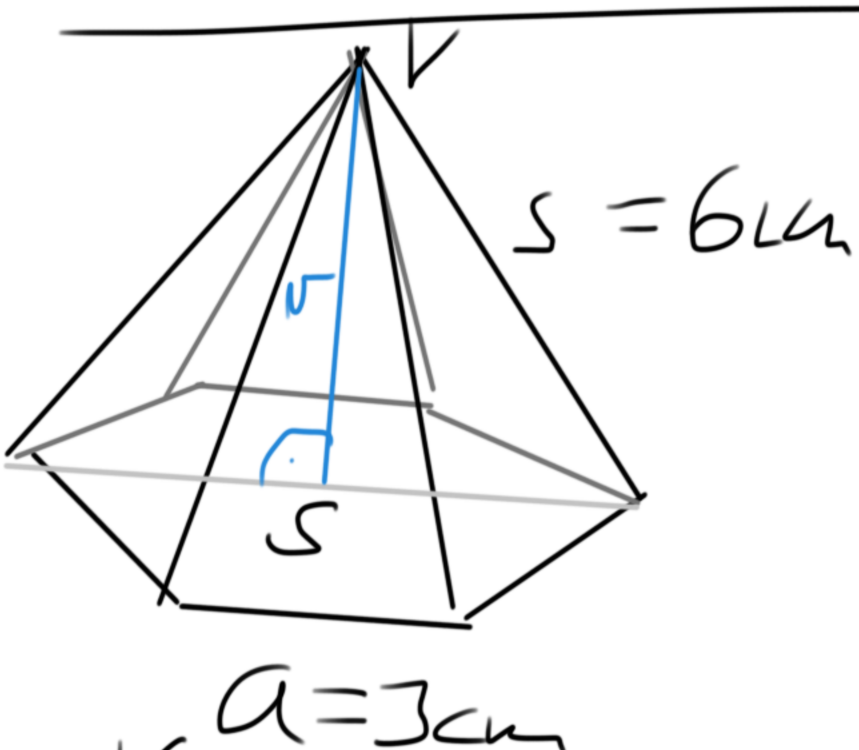
$$= 2^{\frac{28}{6}}\left(1 + 2^{\frac{3}{2}}\right) = 2^4 \cdot 2^{\frac{4}{6}}\left(1 + \sqrt{2} \cdot 2\right)$$

$$= 16 \cdot \sqrt[3]{4}\left(1 + \sqrt{2} \cdot 2\right)$$

2) Vypočítejte V, S pravidelného 6-bokého jehlanu

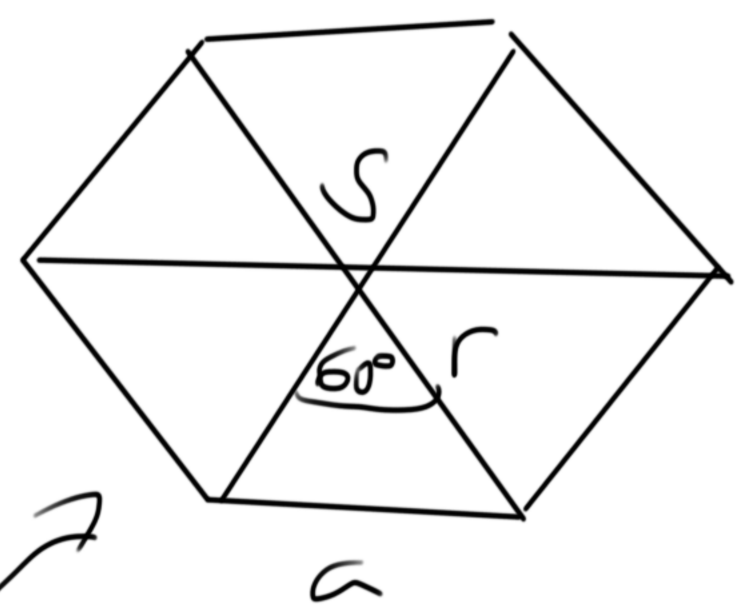
délka podstavní hrany 3cm

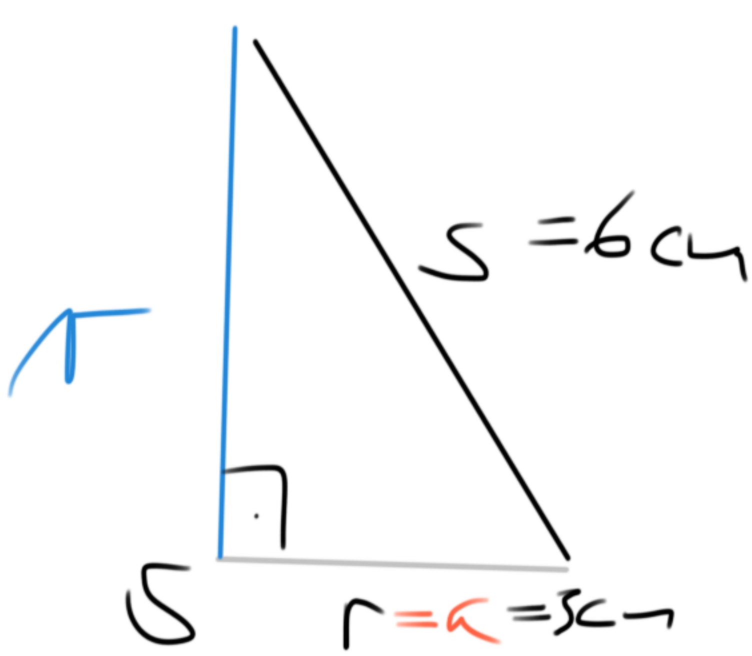
délka boční hrany : 6cm



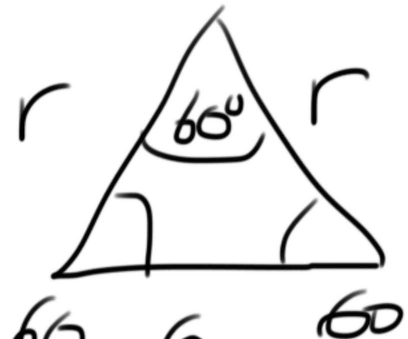
$$V=\frac{1}{3}S_p\cdot v$$

$$S=S_p+S_{pl}$$





všechny Δ jsou rovnoramenné



$\Rightarrow$ rovnostranný

$\underline{r=a}$

$$s^2=v^2+a^2$$

$$v=\sqrt{s^2-a^2}$$

$$\underline{v=\sqrt{36-9}=\sqrt{27}=3\sqrt{3}}$$

$$\sin 30° = \frac{a/2}{r}$$

$$r=\frac{a/2}{1/2}=a$$

$v = 3\cdot\sqrt{3}$ $a = 3\,cm$ $s = 6\,cm$

$S_p = ?$ šestiúhelník

$$S_n = \frac{n}{4}\cdot a^2 \cot\left(\frac{\pi}{n}\right)$$

$$S_p = S_6 = \frac{6}{4} a^2\cdot\sqrt{3} = \frac{3}{2}\sqrt{3}\,a^2 = \underline{\frac{27}{2}\sqrt{3}}\ cm^2$$

$$\cot\left(\frac{\pi}{6}\right) = \frac{\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

$$V = \frac{1}{3}\,S_p\cdot v = \frac{1}{\cancel{3}}\cdot\frac{27}{2}\sqrt{3}\cdot\cancel{3}\sqrt{3} = \frac{81}{2} = \underline{40{,}5}\ cm^3$$

$$S = S_p + S_{pl}$$

plášť ← 6 rovnoramenný $\triangle$

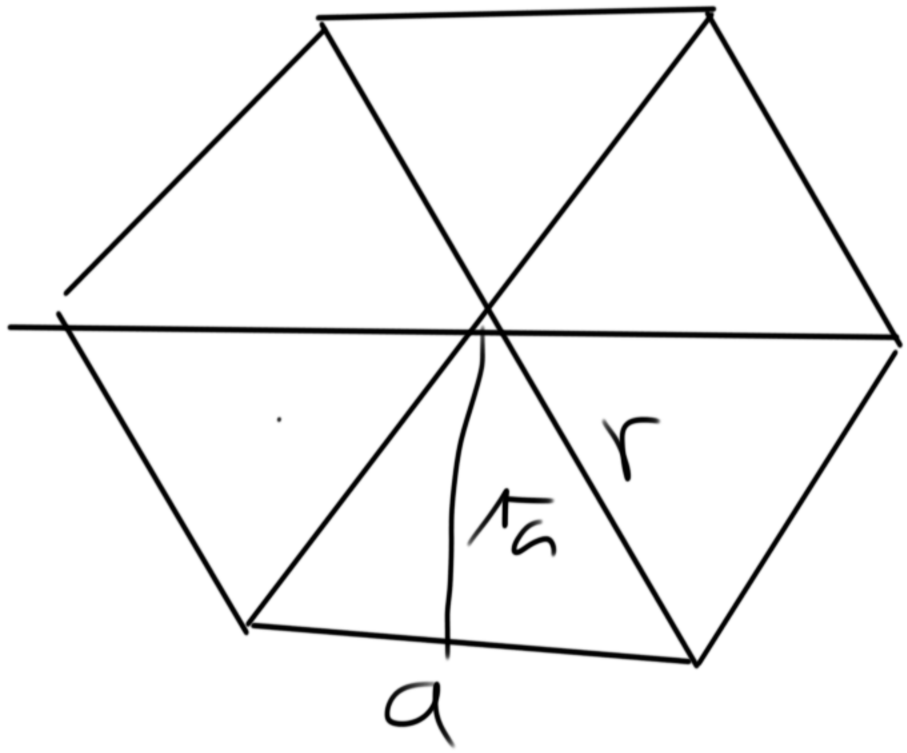
$$S_\triangle = \frac{1}{2}\,a\cdot v_a$$

$$v_a = \sqrt{s^2 - \frac{a^2}{4}} = \sqrt{36 - \frac{9}{4}}$$

$$= \sqrt{\frac{144-9}{4}} = \frac{\sqrt{135}}{2}$$

$$S_\triangle = \frac{1}{2}\,a\,v_a = \frac{3}{2}\cdot\frac{\sqrt{135}}{2} = \frac{3\sqrt{135}}{4}$$

$$S = S_p + 6\cdot S_\triangle = \frac{27}{2}\sqrt{3} + 6\cdot\frac{3}{4}\sqrt{135}$$

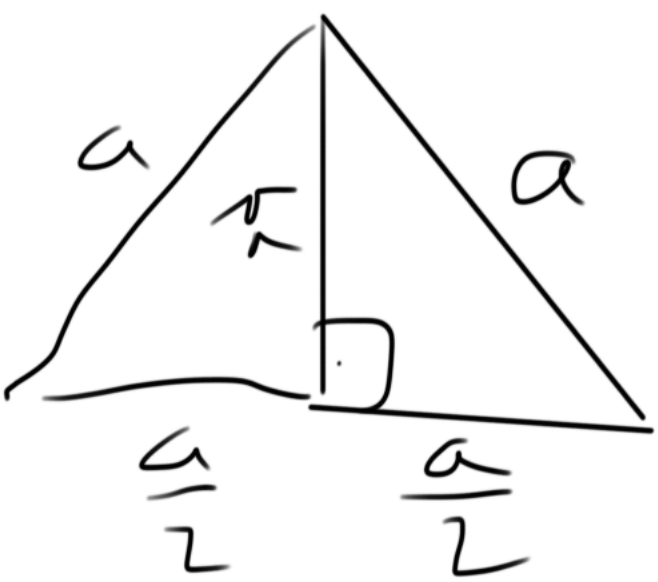


$S = 6 \cdot S_\Delta$

6-úhelník:

$r = a$

$S_\Delta = \frac{1}{2} a \cdot r_a$



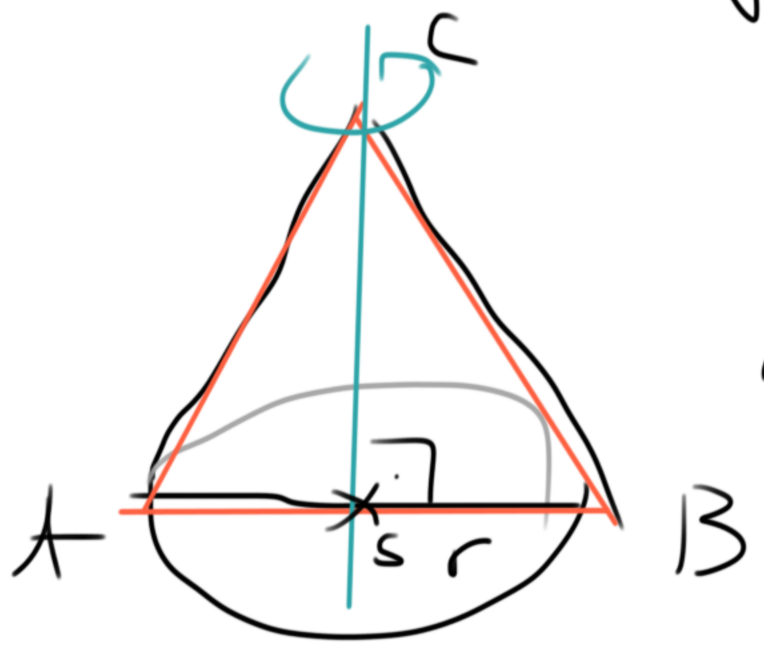
$r_a = \sqrt{a^2 - \frac{a^2}{4}}$

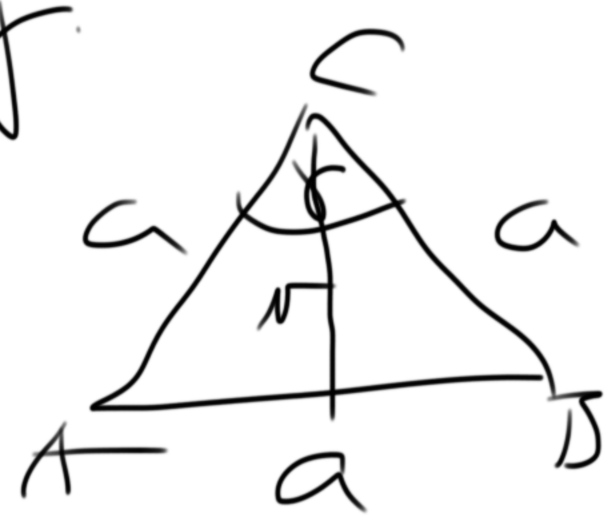
$r_a = \sqrt{\frac{3}{4}a^2} = \frac{\sqrt{3}}{2} a$

$S_\Delta = \frac{1}{2} \cdot a \cdot \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{4} a^2$

$S = 6 \cdot S_\Delta = \frac{3\sqrt{3}}{2} a^2$

3) $V = ?$ $S = ?$ rovnostranného kužele, který vznikl rotací rovnostranného $\triangle ABC$ o straně $a = 4$ cm kolem úhlu γ





$$V = \frac{1}{3} S_p \cdot v$$

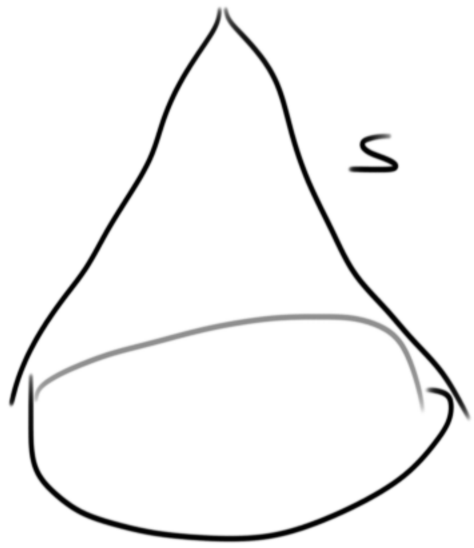
$$S = S_p + S_{pl}$$

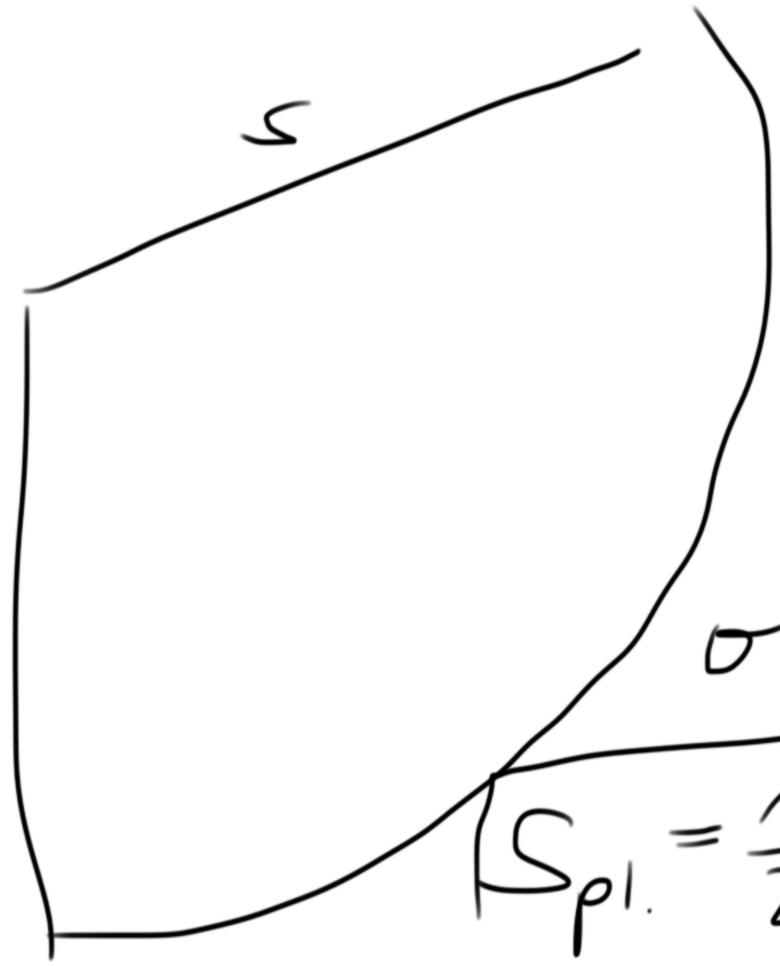
$$v = \sqrt{a^2 - \frac{a^2}{2}} = \frac{\sqrt{3}}{2} a$$

$$S_p = \pi \left(\frac{a}{2}\right)^2 = \pi \frac{a^2}{4}$$

$$V = \frac{1}{3} \pi \frac{a^2}{4} \cdot \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{3 \cdot 8} \pi a^3$$

$a = 4$ cm : $V = \frac{\sqrt{3}}{3 \cdot 8} \pi 64 = \frac{8\sqrt{3}}{3} \pi \text{ cm}^3$





$$\boxed{S_{pl.} = \frac{1}{2} \cdot \sigma \cdot s}$$

obvod podstavy = σ … délka oblouku výseče

délka hrany = s … poloměr výseče

$s = a$ … hrana $\triangle ABC$

$$\sigma = 2\pi\left(\frac{a}{2}\right) = \pi \cdot a$$

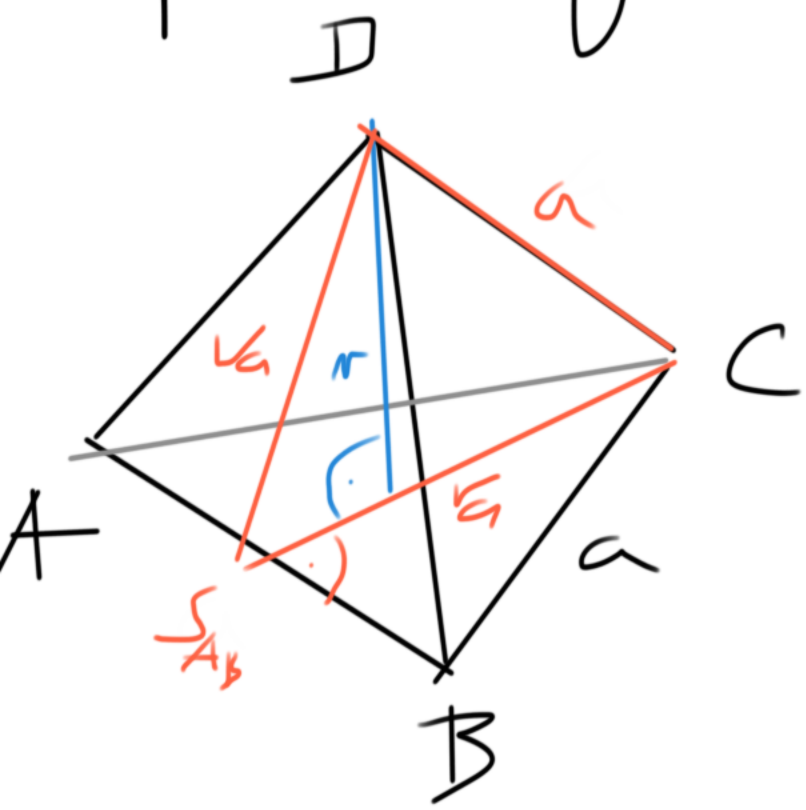
$$S_{pl} = \frac{1}{2} \cdot \pi a \cdot a = \frac{1}{2}\pi a^2 = 8\pi \text{ cm}^2$$

$a = 4$ cm

$$S = S_p + S_{pl} = 4\pi + 8\pi = \underline{12\pi \text{ cm}^2}$$

$$S_p = \pi\left(\frac{a}{2}\right)^2 = 4\pi \text{ cm}^2$$

5) Odvoďte vzorec pro objem čtyřstěnu.

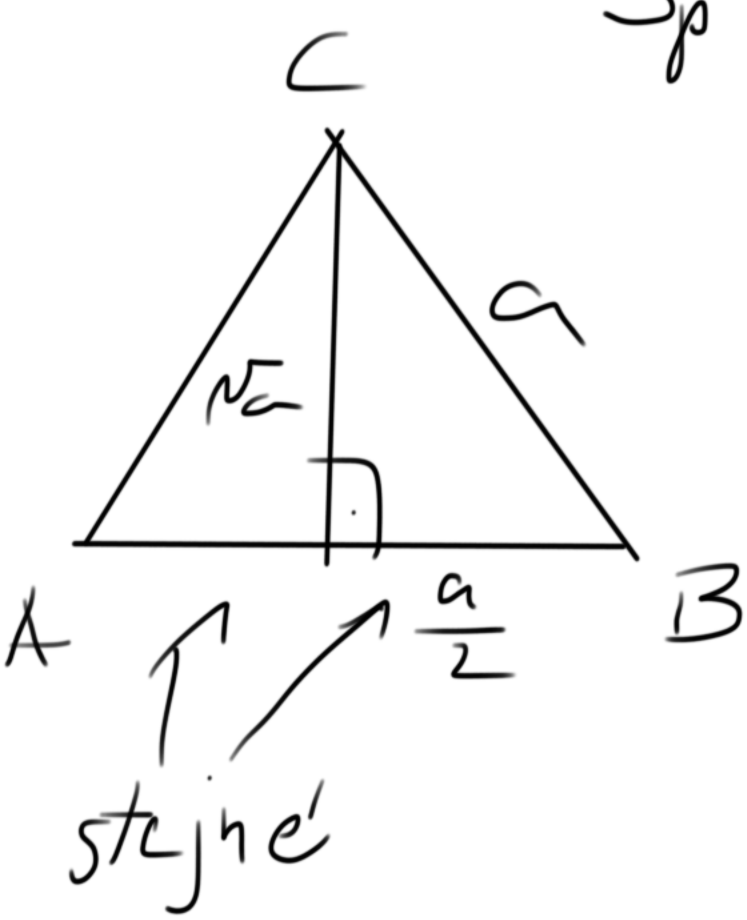


čtyřstěn

prav. 3-boký jehlan

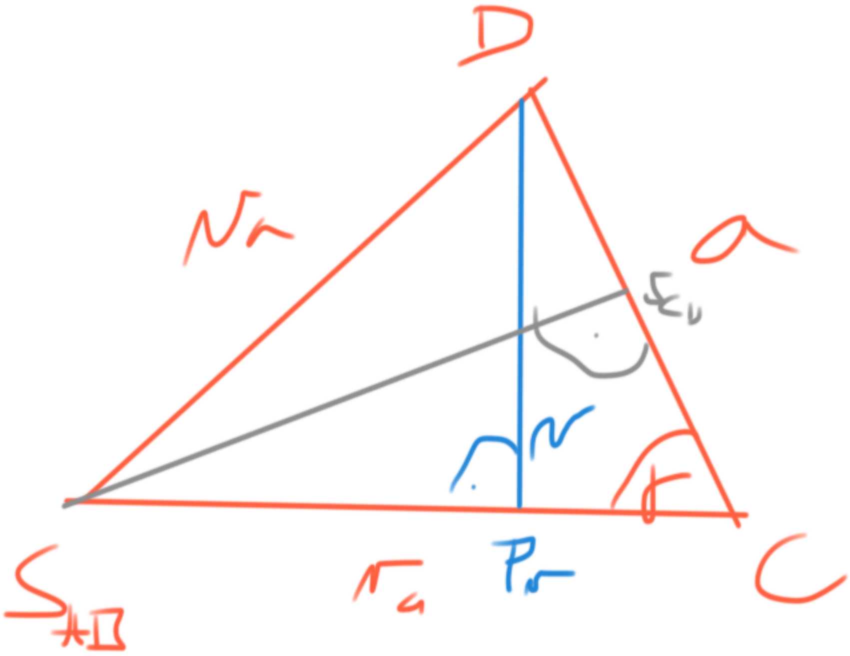
$$V = \frac{1}{3} \cdot S_p \cdot v$$

S_p – rovnostranný △

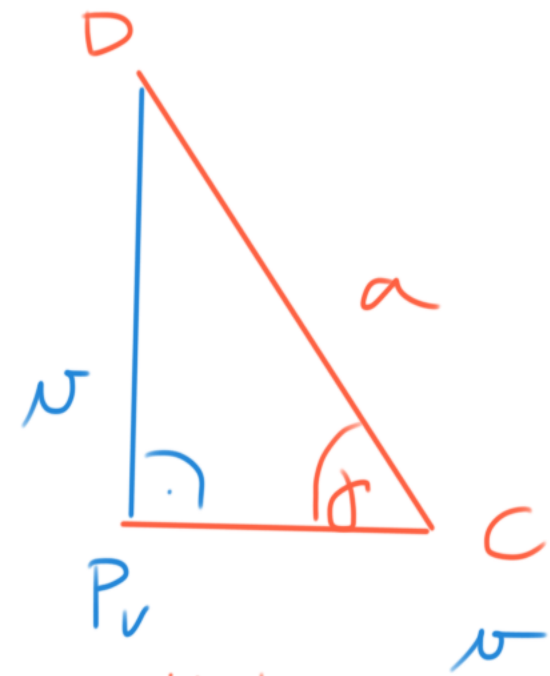


$$v_a = \frac{\sqrt{3}}{2} a$$

$$S_p = \frac{1}{2} \cdot a \cdot \frac{\sqrt{3}}{2} a = \frac{\sqrt{3}}{4} a^2$$



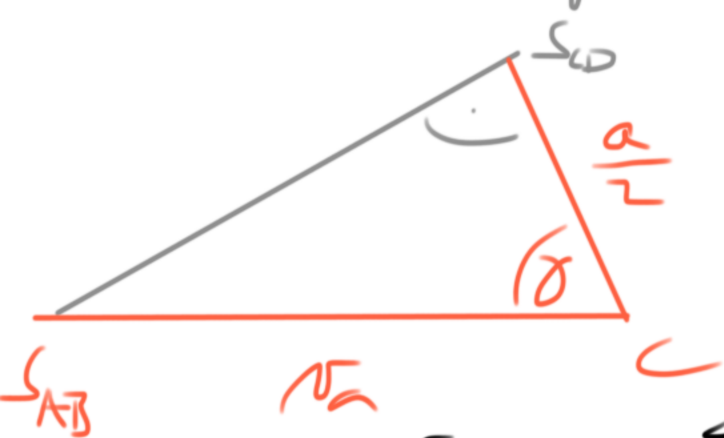
$v_a = \frac{\sqrt{3}}{2}a$



Hledáme $\sin\gamma$

$\sin\gamma = \frac{v}{a}$

$v = a \cdot \sin\gamma$



$$\cos\gamma = \frac{\frac{a}{2}}{v_a} = \frac{a/2}{\frac{\sqrt{3}}{2}a} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$v_a = \frac{\sqrt{3}}{2}a$

$\sin\gamma$?

$$\cos^2\gamma + \sin^2\gamma = 1$$

$$\sin\gamma = \sqrt{1-\cos^2\gamma}$$

$$= \sqrt{1-\frac{3}{9}} = \sqrt{\frac{6}{9}}$$

$$\boxed{\sin\gamma = \frac{\sqrt{6}}{3}}$$

$$\boxed{\begin{aligned}\sin\gamma &= \frac{\sqrt{6}}{3}\\ v &= a\cdot\frac{\sqrt{6}}{3}\end{aligned}}$$

$$V = \frac{1}{3}S_p \cdot v$$

$$S_p = \frac{\sqrt{3}}{4}a^2$$

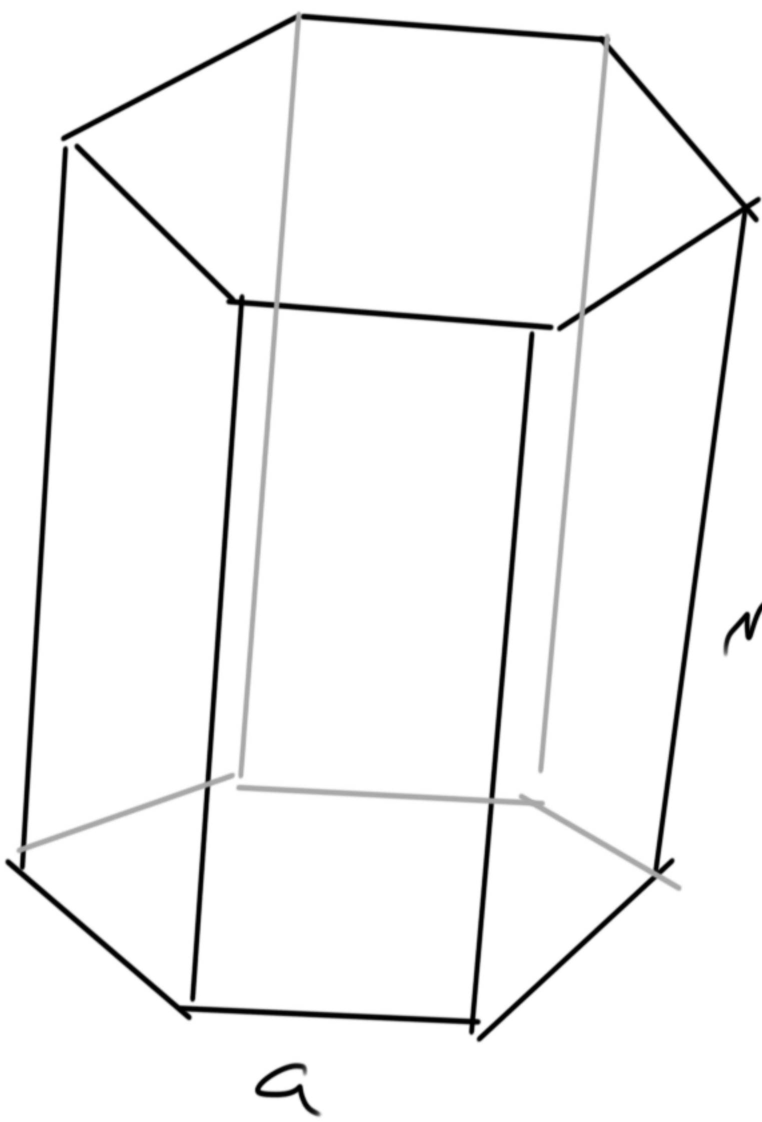
$$v = a\cdot\frac{\sqrt{6}}{3}$$

$$V = \frac{1}{3}\cdot\frac{\sqrt{3}}{4}a^2\cdot a\cdot\frac{\sqrt{6}}{3} = \frac{\cancel{\sqrt{3}}\,\sqrt{\cancel{3}\cdot 2}}{3\cdot\cancel{3}\cdot 4}a^3 = \underline{\frac{\sqrt{2}}{12}a^3}$$

další úloha:

$S, V = ?$ pravidelný 6-boký hranol

$a = 4\,cm \quad v = 6\,cm$



$V = S_p \cdot v$

$S = 2 \cdot S_p + S_{pl}$

$S_p = 6 \cdot S_\Delta$

$= \frac{3}{2}\sqrt{3}\,a^2 = 24\sqrt{3}\,cm^2$

$V = S_p \cdot v = 24\sqrt{3} \cdot 6$

$= 144\sqrt{3}\,cm^3$

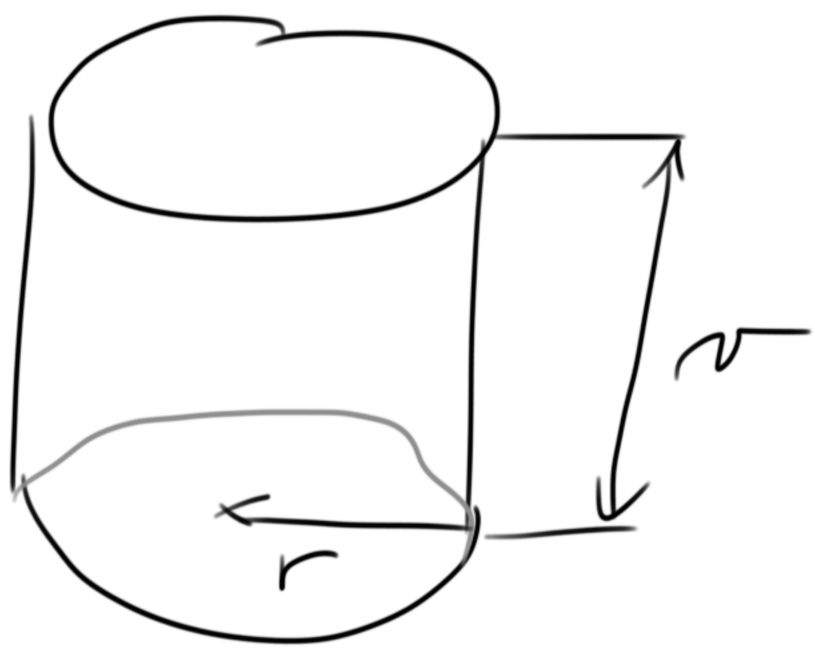
Plášť: $6 \times$ obdélník $a \times v$

$S_{pl} = 6 \cdot a \cdot v = 144$

$S = 2 \cdot S_p + S_{pl} = 48\sqrt{3} + 144$

$= 12(12 + 4\sqrt{3})\,cm^2$

Určete rozměry válcové nádoby o objemu 5 litrů, jestliže výška nádoby se rovná polovině průměru podstavy.



$$V = S_p \cdot v$$

$$S_p = \pi r^2$$

$$V = 5\,l$$

$$v = \frac{d}{2} = r$$

$$V = \pi r^2 \cdot r = \pi r^3$$

$$V = 5\,l = 5\,dm^3 = 5\,(10\,cm)^3 = 5 \cdot 10^3\,cm^3$$

$$1\,dm = 10\,cm \qquad = 5000\,cm^3$$

$$5000 = \pi r^3$$

$$r = \sqrt[3]{\frac{5000}{\pi}} = \sqrt[3]{\frac{5 \cdot 10^3}{\pi}} = \sqrt[3]{\frac{5}{\pi}}\,10$$

$$\boxed{r = \sqrt[3]{\frac{5}{\pi}} \cdot 10\,cm} \doteq 1{,}2 \cdot 10 \sim 12\,cm, \quad v = r = 12\,cm$$

$$11{,}6754$$