Exp. a log

Vypočtěte $\qquad \log_a x = y \iff a^y = x$

$\log_{\frac{1}{3}} \frac{1}{3} = 1 \qquad \left(\frac{1}{3}\right)^1 = \frac{1}{3}$

$\log_8 \sqrt{2} = \log_8 \sqrt{8^{1/3}} = \log_8 8^{1/6} = \frac{1}{6}$

$\left(8^{1/3}\right)^{1/2}$

$2^3 = 8$

$2 = \sqrt[3]{8} = 8^{1/3}$

$\log_5 125 = 3$

$5 \cdot 25$

Najděte x $\qquad \log_a x = y \iff a^y = x$

$\log_3 x = 4 \longrightarrow x = 3^4 = 81$

$\log_5 x = 0 \longrightarrow x = 1$

$\log_{\frac{1}{5}} x = -1 \longrightarrow x = \left(\frac{1}{5}\right)^{-1} = 5 \qquad a^{-1} = \frac{1}{a}$

Najděte a:

$\log_a 27 = 3 \iff 27 = a^3 \quad / \sqrt[3]{\ }$

$a = \sqrt[3]{27} = 3$

$\log_a 4 = \frac{1}{4} \iff a^{\frac{1}{4}} = 4 \quad /^4$

$a = 256$

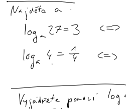
Vyjádřete pomocí $\log a$, $\log b$ a $\log c$

$\log\left(\frac{a^2 \cdot b^3}{100 \cdot \sqrt{c}}\right) = \log a^2 + \log b^3 - \log 100 - \log \sqrt{c}$ $\qquad \sqrt{c} = c^{1/2}$

$\log_a x^r = r \cdot \log_a x \qquad = 2 \cdot \log a + 3 \cdot \log b - 2 - \frac{1}{2} \cdot \log c$

Grafy a všechno

$f: y = \left(\frac{1}{2}\right)^{x-3} - 1 \qquad a = \frac{1}{2} < 1$



průsečík s x:

$\left(\frac{1}{2}\right)^{x-3} - 1 = 0$

$\left(\frac{1}{2}\right)^{x-3} = 1$

$\left(\frac{1}{2}\right)^{x-3} = \left(\frac{1}{2}\right)^0$

$x - 3 = 0$

$x = 3$

$P_x = [3, 0]$

P_y: $x = 0$

$y = \left(\frac{1}{2}\right)^{0-3} - 1 = 7$

$P_y = [0, 7]$

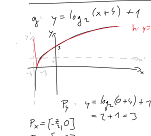
$D_f = \mathbb{R} \qquad H_f = (-1, \infty) \longrightarrow$ zdola omezená

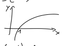
klesající v $D_f \longrightarrow$ monotónní $\longrightarrow$ prostá

ani s. ani l.

$g: y = \log_2(x+4) + 1 \qquad a = 2 > 1$

$h: y = |g(x)|$





P_x: $\log_2(x+4) + 1 = 0$

$\log_2(x+4) = -1$

$\log_2(x+4) = \log_2 \frac{1}{2}$

$x + 4 = \frac{1}{2}$

$x = -\frac{7}{2}$

P_y: $y = \log_2(0+4) + 1$

$= 2 + 1 = 3$

$P_x = \left[-\frac{7}{2}, 0\right]$

$P_y = [0, 3]$

D_g: $x + 4 > 0$

$x > -4$

$D_g = (-4, \infty) \qquad H_g = \mathbb{R}$

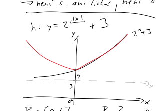
rostoucí v D_g

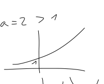
$\to$ monotónní

$\to$ prostá

$\to$ není s. ani lichá, není omezená

$h: y = 2^{|x|} + 3 \qquad a = 2 > 1$





$P_y = [0, 4] \qquad P_x: ? \qquad 0 = 2^{|x|} + 3$

$2^{|x|} = -3$ NŘ $\qquad a^x > 0 \ \forall x \in \mathbb{R}$

$\searrow$ v $(-\infty, 0)$, $\nearrow$ v $(0, \infty)$ $\qquad D_h = \mathbb{R}$, $H_h = \langle 4, \infty)$

sudá: $h(-x) = 2^{|-x|} + 3 = 2^{|x|} + 3 = h(x)$

není prostá, zdola omezená

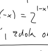
$i: y = \left|\log_{\frac{1}{3}}(x-1)\right| - 3$

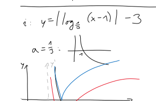
posun v x do 1

abs. hodnota

posun v y do -3

$a = \frac{1}{3}$:





$\log_{\frac{1}{3}}(x-1)$

$\left|\log_{\frac{1}{3}}(x-1)\right|$

$\left|\log_{1/3}(x-1)\right| - 3$

P_x: $\left|\log_{1/3}(x-1)\right| - 3 = 0$

$\left|\log_{1/3}(x-1)\right| = 3$

$\log_{1/3}(x_1 - 1) = -3 \qquad \log_{1/3}(x_2 - 1) = 3$

$\log_{1/3}(x_1 - 1) = \log_{1/3}\left(\frac{1}{3}\right)^{-3} \qquad x_2 - 1 = \left(\frac{1}{3}\right)^3$

$x_1 - 1 = 27 \qquad \boxed{x_2 = \frac{28}{27}}$

$\boxed{x_1 = 28}$

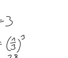
$P_1 = [28, 0]$

$P_2 = \left[\frac{28}{27}, 0\right]$

$\searrow$ v $(1, 2)$

$\nearrow$ v $(1, \infty)$

$\to$ není prostá



zdola omezená $\qquad D_i = (1, \infty)$

není s. ani l. $\qquad H_i = \langle -3, \infty)$

Exp. a log. rce

Řešte v $\mathbb{R}$:

$2^{3x-1} \cdot 4 = 8^{x+1} \cdot \left(\frac{1}{2}\right)^x$

$2^{3x-1} \cdot 2^2 = \left(2^3\right)^{x+1} \cdot \left(2^{-1}\right)^x \qquad a^r \cdot a^s = a^{r+s}$

$\left(a^r\right)^s = a^{r \cdot s}$

$2^{3x-1+2} = 2^{3x+3} \cdot 2^{-x}$

$2^{3x+1} = 2^{2x+3} \quad / \log_2$

$\left[\log_2 2^{3x+1} = \log_2 2^{2x+3}\right] \qquad \log_a a^0 = 0$

$0 = 0$

$3x + 1 = 2x + 3$

$\boxed{x = 2}$

$3^x + 3^{x+1} = 108 \qquad 3$

$3^x + 3^x \cdot 3 = 108 \qquad 9$

$3^x \cdot (1 + 3) = 108 \quad / : 4 \qquad 27$

$3^x = 27 \qquad 81$

$\boxed{x = 3} \qquad 243$

$3^x + 3^{x+1} = 7 \cdot 4^x - 4^{x+1}$

$3^x \cdot (1 + 3) = 4^x \cdot (7 - 4)$

$3^x \cdot 4 = 4^x \cdot 3 \quad / : 4^x \ : 4$

$\frac{3^x}{4^x} = \frac{3}{4}$

$\left(\frac{3}{4}\right)^x = \frac{3}{4} \longrightarrow \underline{x = 1}$

$4^{2x} - 2 \cdot 4^x - 8 = 0$

$u = 4^x$

$u^2 - 2u - 8 = 0 \qquad u_1 = 4 \longrightarrow 4^{x_1} = 4 \quad \underline{x_1 = 1}$

$(u - 4) \cdot (u + 2) = 0 \qquad u_2 = -2 \longrightarrow 4^{x_2} = -2$ NŘ.

$a^{2x} \qquad a^x$

$3^x = 10 \quad / \log_3$

$x = \underline{\log_3 10}$

$\frac{3^x}{2 \cdot 3^{\sqrt{2}}} = 4{,}5 \quad / \cdot 2 \cdot 3^{\sqrt{2}}$

$3^x = 9 \cdot 3^{\sqrt{2}}$

$3^x = 3^{2+\sqrt{2}}$

$x = 2 + \sqrt{2}$

Zk: $LS = \frac{3^{2+\sqrt{2}}}{2 \cdot 3^{\sqrt{2}}} = \frac{9}{2} = 4{,}5$

$\log x^5 - \log x^4 + \log x^3 = 12$

$\log\left(\frac{x^5 \cdot x^3}{x^4}\right) = 12$

$\log x^4 = 12$

$\log x = 3 \quad / 10^{\square}$

$x = 10^3$

$10^{\log x} = 10^3$

$x = 10^3$

$\log_2^2 x + 2 \log_2 x - 3 = 0 \qquad \log_a^2 x = \left(\log_a x\right)^2$

$u = \log_2 x \qquad \log_a x^2 = \log_a\left(x^2\right)$

$= 2 \cdot \log_a x$

$u^2 + 2u - 3 = 0$

$(u + 3) \cdot (u - 1) = 0 \qquad u_1 = -3 \longrightarrow \log_2 x_1 = -3$

$\boxed{x_1 = 2^{-3}}$

$u_2 = 1 \longrightarrow \log_2 x_2 = 1 \longrightarrow \boxed{x_2 = 2}$

$\log_8 \sqrt{x + 30} + \log_8 \sqrt{x} = 1 \quad / \cdot 2$

$\log_8 (x + 30) + \log_8 x = 2 \qquad \log (x+30)^{1/2}$

$\frac{1}{2} \cdot \log (x + 30)$

$\sqrt{x+30} > 0$

$x > -30$

$\sqrt{x} > 0$

$\boxed{x > 0}$

$\log_8\left[(x + 30) \cdot x\right] = \log_8 64$

$(x + 30) \cdot x = 64$

$x^2 + 30x - 64 = 0$

$(x + 32) \cdot (x - 2) = 0$

$x_1 = -32 \qquad \boxed{x_2 = 2}$

$x_{1,2} = \frac{-30 \pm \sqrt{900 + 256}}{2}$

$= \frac{-30 \pm \sqrt{1156}}{2}$

$= \frac{-30 \pm 34}{2} = \begin{cases} 2 \\ -32 \end{cases}$

$\frac{\log_3 x}{1 + \log_3 2} = 2 \quad / \cdot (1 + \log_3 2)$

$\log_3 x = 2 + 2 \cdot \log_3 2$

$\log_3 x = \log_3 9 + \log_3 4$

$\log_3 x = \log_3 36$

$\boxed{x = 36}$