

# Grafy funkcí a jejich vlastnosti

Uvažte  $D_f$ ,  $H_f$ , význačné body, graf, monotoničnost, periodičita, prostota, sudej/lučí, omezená?

$$1) \quad y = -3x + 7$$

$$D_f = \mathbb{R}$$

$$H_f = \mathbb{R}$$

neomezená

$$\text{Průs. s } y: x=0 \Rightarrow y=7$$

$$\text{Průs. s } x: 0=-3x+7$$

$$x=\frac{7}{3}$$

není periodická, ani sudej ani lučí, prostá

$$f(-x) = -3(-x) + 7 = -(-3x + 7) \neq f(x) \neq -f(x)$$

klesající v  $D_f$

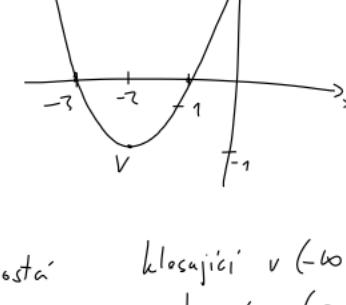
$$2) \quad y = x^2 + 4x + 3$$

$$= (x+2)^2 - 1$$

$$= (x+3)(x+1)$$

$$\Rightarrow x_1 = -3, x_2 = -1$$

$$V = [V_x, V_y] = [-2, -1]$$



$$D_f = \mathbb{R}$$

není prostá

klesající v  $(-\infty, -2)$

$$H_f = (-1, \infty)$$

omezena zedola

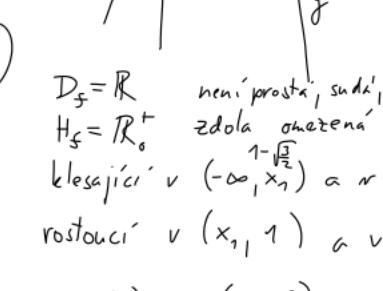
rostoucí v  $(-2, \infty)$

není ani sudej ani lučí.

$$x^2 \begin{cases} \text{sudej pro sudej} \\ \text{lučí a lučí} \end{cases} \quad f: y = x^2 + x \quad f(-x) = x^2 - x \quad \text{ahoj.} \\ g: y = x^2 + 6x \quad \text{ahoj.}$$

$$3) f: y = |-2x^2 + 4x + 1|$$

$$\begin{aligned} g: y &= -2x^2 + 4x + 1 \\ &= -2(x^2 - 2x) + 1 \\ &= -2(x-1)^2 + 1 + 2 \\ &= -2(x-1)^2 + 3 \\ &= -2 \left[ (x-1) - \left( \frac{\sqrt{3}}{2} \right) \right]^2 \\ &= -2 \left( x-1 + \sqrt{\frac{3}{2}} \right) \left( x-1 - \sqrt{\frac{3}{2}} \right) \\ x_1 &= 1 - \sqrt{\frac{3}{2}} < 0 \\ x_2 &= 1 + \sqrt{\frac{3}{2}} \end{aligned}$$

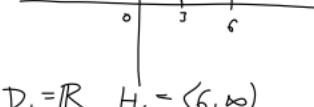


$$D_f = \mathbb{R} \quad \text{není prostá, sudej, ani lučí}$$

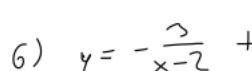
$$H_f = \mathbb{R}_+ \quad \text{zedola omezená}$$

$$\text{klesající v } (-\infty, x_1) \text{ a } (1, x_2) \quad \text{rostoucí v } (x_1, 1) \text{ a v } (x_2, \infty)$$

$$4) \quad y = |x-6| + |x|$$



$$\begin{array}{ccccccc} - & - & + & + & + & + & + \\ \hline I & 0 & II & 6 & III & & \end{array}$$



$$D_f = \mathbb{R}, H_f = (6, \infty)$$

zedola omezená

$$I) \quad x \in (-\infty, 0) \quad y = -x + 6 - x = -2x + 6$$

$$y = -x + 6 + x = 6$$

$$II) \quad x \in (0, 6) \quad y = -x + 6 + x = 6$$

$$y = x - 6 + x = 2x - 6$$

$$x = 3$$

$$6) \quad y = -\frac{2}{x-2} + 6 \quad g: y = |f(x)|$$



$$y' = -\frac{2}{x^2}$$

Průsečík  $y$ :

$$s: y: x=6 \quad y = -\frac{2}{x-2} + 6 = \frac{15}{2}$$

$$s: x: 0 = -\frac{2}{x-2} + 6 \quad 0 = -\frac{2}{x-2} + 6$$

$$\frac{2(x-2)}{2x} = 5 \rightarrow x = \frac{5}{2}$$

prostá

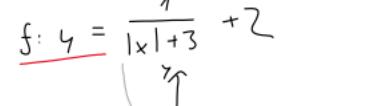
rostoucí v  $(-\infty, 2)$  a v  $(2, \infty)$

$$y = \frac{2x}{x+3} = \frac{2x+6-6}{x+3} = \frac{2(x+3)}{(x+3)} + \frac{-6}{x+3}$$

$$\frac{K}{x+3} = \frac{-6}{x+3} + 2$$

$$D_f \setminus \{x_1, x_2\} \quad y = \left| \frac{1}{x+3} \right| + 2$$

$$\frac{1}{x+3} \quad \left| \frac{1}{x+3} \right| + 2$$



$$f: y = \frac{1}{|x+3|} + 2$$

$f(x)$  sudej

$$H_f = (2, 2 + \frac{2}{3})$$

$$D_f = \mathbb{R}$$