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1) $NSD(78,66) = 2\cdot 3 = 6$ $\quad$ $78 = 2\cdot 39 = 2\cdot 3\cdot 13$

$nsn(78,30) = \underbrace{2\cdot 3\cdot 5}_{30}\cdot 13$ $\quad$ 78 $\quad$ $66 = 2\cdot 33 = 2\cdot 3\cdot 11$

$= 390$ $\quad$ $30 = 2\cdot 15 = 2\cdot 3\cdot 5$

$78 = 6\cdot 13$

$66 = 6\cdot 11$

$$\frac{1}{30} - \frac{7}{78} = \frac{78 - 7\cdot 30}{30\cdot 78} = \cancel{\frac{78-210}{2540}} = \cancel{\frac{-132}{2540}}$$

$$= \frac{13 - 7\cdot 5}{390} = \frac{-22}{390} = -\frac{11}{195}$$

2) Usměrnění zlomku

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}\cdot 1 = \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

3) Zjednodušte výraz a určete podmínky

$$(u^2v^3wx^{-7}) : (5uv^{-3}w^2x^{-2}) = \frac{u^2\cdot v^3\cdot w\cdot x^{-7}}{5\cdot u\cdot v^{-3}\cdot w^2\cdot x^{-2}}$$

$a^r = \frac{1}{a^{-r}}$ $\quad$ $u \neq 0$ $\quad$ $v^{-3} = \frac{1}{v^3}$

$a^r\cdot a^s = a^{r+s}$ $\quad$ $v \neq 0$

$w \neq 0$

$x \neq 0$

$$= \frac{1}{5}\cdot\frac{u^2}{u}\cdot\frac{v^3}{v^{-3}}\cdot\frac{w}{w^2}\cdot\frac{x^{-7}}{x^{-2}}$$

$$= \frac{1}{5}u\cdot v^6\cdot w^{-1}\cdot x^{-5} = \frac{uv^6}{5wx^5}$$

4) $$\left(\frac{1}{a+b} + \frac{1}{a-b}\right)\cdot\left(\frac{1}{a} - \frac{1}{b}\right) =$$

$$\frac{a-b+a+b}{(a+b)\cancel{(a-b)}}\cdot\frac{\cancel{b-a}^{\,-\cancel{(a-b)}\;-1}}{a\cdot b} = \frac{2\cancel{a}}{a+b}\cdot\frac{-1}{\cancel{a}b} = \frac{-2}{(a+b)\cdot b}$$

$a \neq 0$ $\quad$ $a \neq b$

$b \neq 0$ $\quad$ $a \neq -b$

5) $$\frac{xy - y - x^2 + x}{xy + y - x^2 - x} = \frac{y(x-1) - x\cdot(x-1)}{y\cdot(x+1) - x(x+1)}$$

$$= \frac{(x-1)\cancel{(y-x)}}{(x+1)\cancel{(y-x)}} = \frac{x-1}{x+1}$$

$x \neq -1$

$y \neq x$

$$\frac{\cancel{(x-y)}}{1 - \cancel{(x-y)}} = \frac{1}{1-1}$$ ✗

$$= \frac{1\cdot\cancel{(x-y)}}{\left(\frac{1}{x-y} - 1\right)\cdot\cancel{(x-y)}} = \frac{1}{\frac{1}{x-y} - 1}$$

6) $$\frac{\sqrt{x+y}}{\frac{\sqrt{x-y}}{\sqrt{x^2-y^2}}} - x - y = \frac{\sqrt{x+y}}{\sqrt{\frac{\cancel{x-y}}{(x+y)\cdot\cancel{(x-y)}}}} - x - y =$$

$A^2 - B^2 = (A+B)\cdot(A-B)$

$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

$$= \frac{\sqrt{x+y}}{\frac{1}{\sqrt{x+y}}} - x - y = \sqrt{x+y}\cdot\frac{\sqrt{x+y}}{1} - x - y$$

$$= x + y - x - y = 0$$

$x \neq y$ $\quad$ $x \neq -y$ $\quad$ $x + y \geq 0$

$x^2 - y^2 \neq 0$ $\quad$ $x - y \geq 0$

$(x+y)\cdot(x-y) \neq 0$ $\quad$ $x^2 - y^2 \geq 0$

7) $$\sqrt[5]{\left(\frac{c^{1/2}\cdot c^{-1/3}}{c^{-5/6}}\right)^{-3}} = \left(\frac{c^{1/2}\cdot c^{-1/3}}{c^{-5/6}}\right)^{-\frac{3}{5}}$$

$$= \left(\frac{c^{+1/6}}{c^{-5/6}}\right)^{-\frac{3}{5}} = \left(c^{6/6}\right)^{-\frac{3}{5}} = c^{-3/5}$$

$$= \sqrt[5]{\frac{1}{c^3}}$$

$$= \left(\sqrt[5]{c}\right)^{-3}$$

$\sqrt[r]{a} = a^{\frac{1}{r}}$

$\sqrt[r]{a^s} = a^{\frac{s}{r}}$

$a^r\cdot a^s = a^{r+s}$

$a^{-r} = \frac{1}{a^r}$

$c^{1/2} = \sqrt{c} \longrightarrow c \geq 0$

$c \neq 0$

$\boxed{c > 0}$ $\quad$ $c \in \mathbb{R}^+$

8) $$\left[\left(\frac{x}{y}\right)^2 - \frac{x}{y^2}\right] : \left(\frac{x-1}{y}\right)^2 = \frac{x^2 - x}{y^2}\cdot\left(\frac{y}{x-1}\right)^2$$

$y \neq 0$

$x \neq 1$

$$= \frac{x\cancel{(x-1)}}{(x-1)^{\cancel{2}}} = \frac{x}{x-1}$$

9) $$\frac{\left(\sqrt[4]{u} + \sqrt[4]{v}\right)^2 + \left(\sqrt[4]{u} - \sqrt[4]{v}\right)^2}{u - v} : \frac{2}{\sqrt{u} - \sqrt{v}} =$$

$(A+B)^2 = A^2 + \cancel{2A\cdot B} + B^2$

$(A-B)^2 = A^2 - \cancel{2A\cdot B} + B^2$ $\quad$ $\Big\}$ $\oplus$

$(A+B)^2 + (A-B)^2 = 2A^2 + 2B^2$

$$= \frac{2\sqrt{u} + 2\sqrt{v}}{u - v} : \frac{2}{\sqrt{u} - \sqrt{v}}$$

$$= \frac{(\sqrt{u}+\sqrt{v})\cdot(\sqrt{u}-\sqrt{v})}{u - v} = \frac{u-v}{u-v} = 1$$

$u \neq v$ $\quad$ $u, v \geq 0$

Polynomy

$$p_n(x) = \sum_{i=0}^{n} a_i x^i$$

$n = 0$: $\quad$ $p_0(x) = a_0 x^0 = a_0$ $\quad$ konstanta

$n = 1$: $\quad$ $p_1(x) = a_1 x + a_0$ $\quad$ lineární dvojčlen

$n = 2$: $\quad$ $p_2(x) = a_2x^2 + a_1x + a_0$ $\quad$ kvadratický trojčlen

$n = 3$: $\quad$ kubický

$a \in \mathbb{R}$: $\quad$ $p_n(a) = 0$ $\quad$ a … kořen

Každý polynom st. n má n komplexních kořenů

$$p_n(x) = a_n(x - x_1)\cdot(x - x_2)\cdot\ldots\cdot(x - x_n) \qquad x_{1,\ldots n} \text{ kořeny}$$

n=2

$p_2(x) = ax^2 + bx + c$ $\quad$ $x_{1,2} = \frac{-b \pm \sqrt{D}}{2a}$

- $D > 0$: 2 reálné kořeny $\quad$ $D = b^2 - 4\cdot a\cdot c$ $\quad$ "diskriminant"
- $D = 0$: 1 reálný dvojnásobný kořen
- $D < 0$: nemá reálný kořen

$p(x) = x^2 + 5x - 6$ $\quad$ $D = 25 + 24 = 49$

$a = 1$

$b = 5$

$c = -6$

$$x_{1,2} = \frac{-5 \pm \sqrt{49}}{2} = \frac{-5 \pm 7}{2} = \begin{cases} 1 \\ -6 \end{cases}$$

$$p_2(x) = (x-1)\cdot(x+6)$$

Čtverec dvojčlenu: $\quad$ $ax^2 + bx + c \longrightarrow \bar{a}(x + \bar{b})^2 + \bar{c}$

$$x^2 + 5x - 6 = \Big(\underbrace{x^2 + 2\cdot x\cdot\tfrac{5}{2} + \left(\tfrac{5}{2}\right)^2}_{A^2 + 2\cdot A\cdot B + B^2} - \underbrace{\left(\tfrac{5}{2}\right)^2}_{B^2}\Big) - 6 =$$

$A = x$

$2A\cdot B = 5\cdot x$

$2\cdot B = 5$

$B = \frac{5}{2}$

$(A+B)^2 = A^2 + 2\cdot A\cdot B + B^2$

$A = x$

$B = \frac{5}{2}$

$$= \left(x + \tfrac{5}{2}\right)^2 - 6 - \left(\tfrac{5}{2}\right)^2 = \left(x + \tfrac{5}{2}\right)^2 - \frac{49}{4}$$

$$\left(x+\tfrac{5}{2}\right)^2 - \frac{49}{4} = \underbrace{\left(x+\tfrac{5}{2}\right)^2}_{A^2} - \underbrace{\left(\tfrac{7}{2}\right)^2}_{B^2} = \underbrace{\left(x + \tfrac{5}{2} + \tfrac{7}{2}\right)}_{A+B}\cdot\underbrace{\left(x + \tfrac{5}{2} - \tfrac{7}{2}\right)}_{A-B}$$

$$= (x+6)\cdot(x-1)$$

$$x^2 - x - 2 = \left(x - \tfrac{1}{2}\right)^2 - 2 - \left(\tfrac{1}{2}\right)^2$$

$A = x$

$-2\cdot AB$

$-2\cdot xB$

$-2\cdot x\cdot\frac{1}{2}$

$-x$

$$= \left(x - \tfrac{1}{2}\right)^2 - \frac{9}{4}$$

$$= \left(x - \tfrac{1}{2}\right)^2 - \left(\tfrac{3}{2}\right)^2 = \left(x - \tfrac{1}{2} + \tfrac{3}{2}\right)\cdot\left(x - \tfrac{1}{2} - \tfrac{3}{2}\right)$$

$$(x+1)\cdot(x-2)$$

$$x^2 + \underline{2x} + 3 = (x+1)^2 + 3 - 1 = (x+1)^2 + 2$$

$D = 4 - 4\cdot 3 = -8 < 0$

$$= x^2 + 2x + 1 + 2$$

$$= x^2 + 2x + 3$$

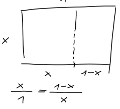
$2x = 2\cdot \overset{x}{A}\cdot B$

$2 = 2B$

$1 = B$

$A^2 - B^2$ $\quad$ $A^2 + B^2 = (A + iB)\cdot(A - iB)$

Zlatý řez $\quad$ golden ratio



$x^2 = 1 - x$

$x^2 + x - 1 = 0$

$$\frac{x}{1} = \frac{1-x}{x}$$

$$x^2 + x - 1 = \left(x + \tfrac{1}{2}\right)^2 - 1 - \left(\tfrac{1}{2}\right)^2$$

$$= \left(x + \tfrac{1}{2}\right)^2 - \frac{5}{4} = \left(x + \tfrac{1}{2}\right)^2 - \left(\frac{\sqrt{5}}{2}\right)^2 = \left(x + \tfrac{1}{2} + \frac{\sqrt{5}}{2}\right)\left(x + \tfrac{1}{2} - \frac{\sqrt{5}}{2}\right)$$

$$x_{1,2} = \frac{-1 \pm \sqrt{5}}{2} = \varphi$$