Goniometrie

úhel
velikost

goniometrické funkce: $\sin x,\ \cos x,\ \operatorname{tg} x,\ \operatorname{cotg} x$

$\tan x$ $\tan^{-1} x$

cyklometrické: $\arcsin x$, $\arccos x$, $\operatorname{arctg} x$, $\operatorname{arccotg} x$ — $\tan^{-1}$ převrácený

Sin x

$f: y = \sin(x)$

$D_f = \mathbb{R}$

$H_f = \langle -1, 1 \rangle$

$\Rightarrow$ omezená

Periodická s periodou $T = 2\pi$

spojitá na D_f

lichá

Průsečík s osou x: $x = k \cdot \pi,\ k \in \mathbb{Z}$

lokální maxima: $x = \frac{\pi}{2} + k \cdot 2\pi,\ k \in \mathbb{Z}$

lokální minima: $x = \frac{3}{2}\pi + k \cdot 2\pi,\ k \in \mathbb{Z}$

Cos x

$f: y = \cos(x)$

$D_f = \mathbb{R}$

$H_f = \langle -1, 1 \rangle$

$\Rightarrow$ omezená

sudá

Periodická s $T = 2\pi$

průs. s x: $x = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$

lok. max: $x = k \cdot 2\pi$

lok. min: $x = \pi + k \cdot 2\pi$

Tangens x

$f: y = \operatorname{tg}(x) = \frac{\sin(x)}{\cos(x)}$

$D_f = \mathbb{R} \setminus \{\frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}\}$

$H_f = \mathbb{R}$

Periodická s $T = \pi$

není omezená (ani s ani b)

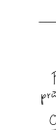
lichá

průs. s x: $x = k \cdot \pi,\ k \in \mathbb{Z}$

asymptoty: $x = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$

Cotangens

$f: y = \operatorname{cotg}(x) = \frac{\cos(x)}{\sin(x)}$



$\operatorname{tg} x = \frac{1}{\operatorname{cotg} x}$

$D_f = \mathbb{R} \setminus \{k\pi,\ k \in \mathbb{Z}\}$

$H_f = \mathbb{R}$

neomezená

lichá: $\operatorname{cotg}(-x) = \frac{\cos(-x)}{\sin(-x)} \overset{?}{=} \frac{\cos(x)}{-\sin(x)}$, $\frac{\cos x}{-\sin x} = -\operatorname{cotg} x$

Periodická s $T = \pi$

průs. s x: $x = \frac{\pi}{2} + k\pi,\ k \in \mathbb{Z}$

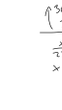
Obloukova míra

úhel

$\alpha = 30°$

1 rad

V obloukové míře úhel o velikosti 1 rad vymezuje na jednotkové kružnici oblouk délky 1



$\alpha = 1$ rad

Plný úhel: 2π

$x = 30°$ ... ? rad

$360° \ldots 2\pi$

$30° \ldots x$

$\frac{x}{2\pi} = \frac{30}{360}$

$x = \frac{2\pi}{12} = \frac{\pi}{6}$

π .. Ludolfovo číslo

iracionální $\rightarrow$ nekonečný, neperiodický desetinný rozvoj

$\pi = 3{,}14159265\ldots$

$\pi \neq 3{,}14$

Užitečné hodnoty

$360° = 2\pi$ $\quad 90° = \frac{\pi}{2}$

$180° = \pi$ $\quad 45° = \frac{\pi}{4}$

$60° = \frac{\pi}{3}$ $\quad 30° = \frac{\pi}{6}$ $\quad 15° = \frac{\pi}{12}$

$135° = 90° + 45° = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3}{4}\pi$

Hodnoty goniometrických funkcí

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin x	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos x	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tg x	0	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	X
cotg x	X	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0

$A = [1, 0]$

$\alpha = 0 \quad x = \cos 0 = 1$

$y = \sin 0 = 0$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$

$\sin\left(\frac{\pi}{6}\pi\right)$

$\frac{7}{6}\pi = \frac{6}{6}\pi + \frac{1}{6}\pi =$

$= 210°$

$\sin\left(\frac{7}{6}\pi\right) = -\frac{1}{2}$

$\cos\left(\frac{7}{6}\pi\right) = \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\cos\left(\frac{7}{6}\pi\right) = \frac{\sqrt{3}}{2}$

Goniometrické identity

$\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$

$\cos(x) = \sin\left(x + \frac{\pi}{2}\right)$

$\operatorname{tg} x = \frac{\sin x}{\cos x}$

$\operatorname{cotg} x = \frac{\cos x}{\sin x}$

$\operatorname{cotg} x = \frac{1}{\operatorname{tg} x}$

$\sin(-x) = -\sin(x)$ lichá f.

$\cos(-x) = \cos(x)$ sudá f.

$\operatorname{tg}(-x) = -\operatorname{tg}(x)$

$\operatorname{cotg}(-x) = -\operatorname{cotg}(x)$

$\sin^2(x) + \cos^2(x) = 1$

$\cos^2 x = (\cos x)^2$

$\cos^2 x \neq \cos(x^2)$

$\sin(2x) = 2\sin(x) \cdot \cos(x)$

$\cos(2x) = \cos^2(x) - \sin^2(x)$

$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$

$\sin(x + k \cdot 2\pi) = \sin x$

$\cos(x + k \cdot 2\pi) = \cos x$

$\operatorname{tg}(x + k\pi) = \operatorname{tg} x$

$\operatorname{cotg}(x + k\pi) = \operatorname{cotg} x$

$k \in \mathbb{Z}$

Př. $\cos\left(\frac{11}{6}\pi\right) = \cos\left(\frac{12}{6}\pi - 2\pi\right) = \cos\left(-\frac{\pi}{6}\right) \overset{\text{sudá}}{=} \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

$\sin\left(\frac{5}{4}\pi\right) = \sin\left(\pi + \frac{\pi}{4}\right) = \sin\pi \cos\frac{\pi}{4} + \sin\frac{\pi}{4} \cos\pi = -\frac{\sqrt{2}}{2}$

Goniometrické rovnice

$f(x) = 0$

obecný postup: 1. vyjednodušení: ekvivalentní úpravy + gon. id.

2. vyjádření podm. rovnat.

3. závěrečné úpravy

$\sin(3x) = 1 - \sin(3x) \quad / + \sin(3x)$

$2\sin(3x) = 1 \quad / 2$

$\sin(3x) = \frac{1}{2}$

$z = 3x \quad \sin(z) = \frac{1}{2} \longrightarrow z_1 = \frac{\pi}{6} + k \cdot 2\pi$

$z_2 = \frac{5}{6}\pi + k \cdot 2\pi$

$3x_1 = \frac{\pi}{6} + k \cdot 2\pi \quad / 3$

$x_1 = \frac{\pi}{18} + k \cdot \frac{2}{3}\pi$

$3x_2 = \frac{5}{6}\pi + k \cdot 2\pi$

$x_2 = \frac{5}{18}\pi + k \cdot \frac{2}{3}\pi$

$k \in \mathbb{Z}$

$\cos^2 x + \cos x = 0$

$y = \cos x$

$y^2 + y = 0$

$y(y + 1) = 0$

$y_1 = 0$

$\cos x = 0$

$x = \frac{\pi}{2} + k\pi$

$y_2 = -1$

$\cos x_2 = -1$

$x_2 = \pi + k \cdot 2\pi$

$y_1 = -1 \quad y_2 = 0 \longrightarrow \cos x = 2$

NŘ, protože $\cos x \leq 1 \quad \forall x \in \mathbb{R}$

Goniometrické fce pro ostré úhly v Δ

$\sin\alpha = \frac{a}{c}$ $\quad \sin\beta = \frac{b}{c}$

$\cos\alpha = \frac{b}{c}$ $\quad \cos\beta = \frac{a}{c}$

$\operatorname{tg}\alpha = \frac{a}{b}$ $\quad \operatorname{tg}\beta = \frac{b}{a}$

$\operatorname{cotg}\alpha = \frac{b}{a}$ $\quad \operatorname{cotg}\beta = \frac{a}{b}$

$\longrightarrow a = c \cdot \sin\alpha$

$c = 1 \longrightarrow a = \sin\alpha$

$b = c \cdot \cos\alpha$

$\Delta \longrightarrow a^2 + b^2 = c^2$

$(c \cdot \sin\alpha)^2 + (c \cdot \cos\alpha)^2 = c^2(\underbrace{\sin^2\alpha + \cos^2\alpha}_{=1})$