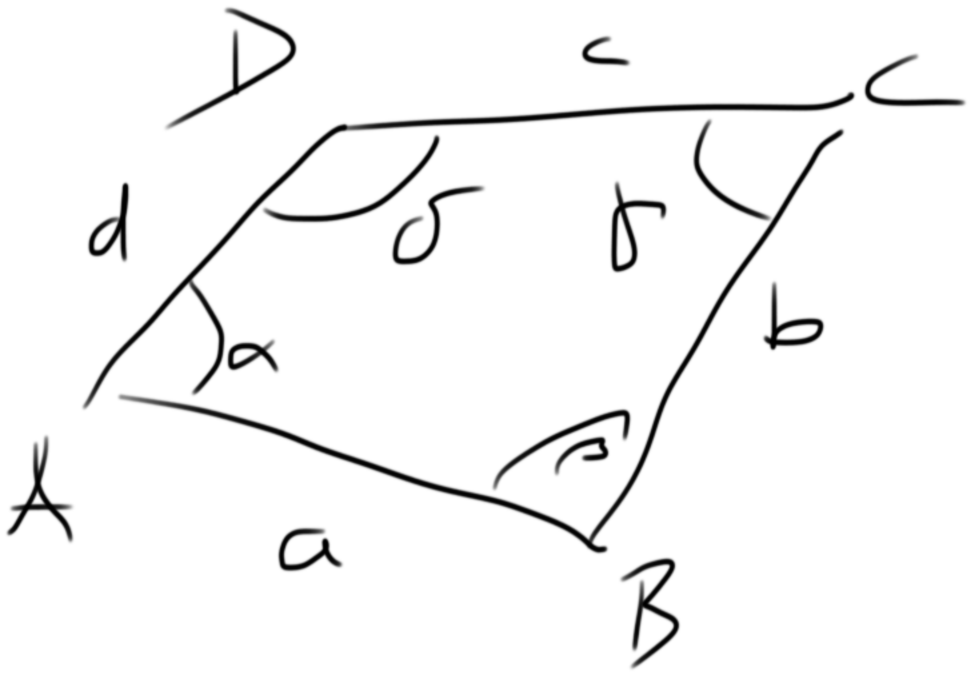
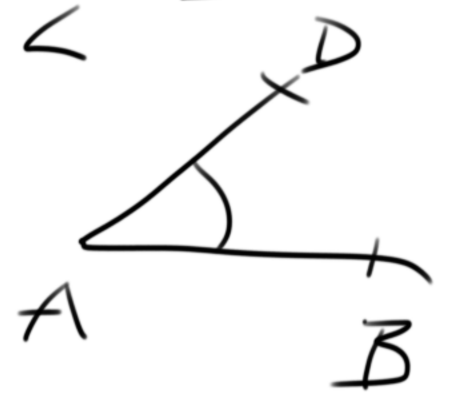
9. ČTYŘÚHELNÍK



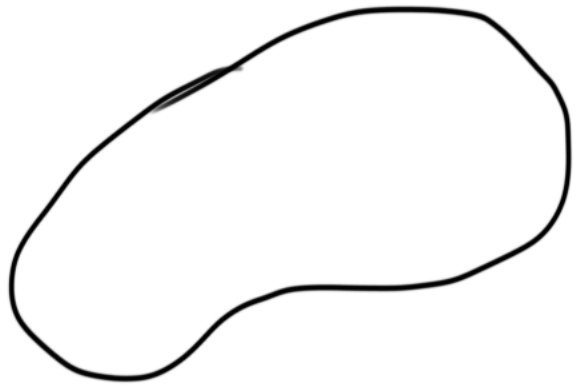
$\sphericalangle DAB = \alpha$

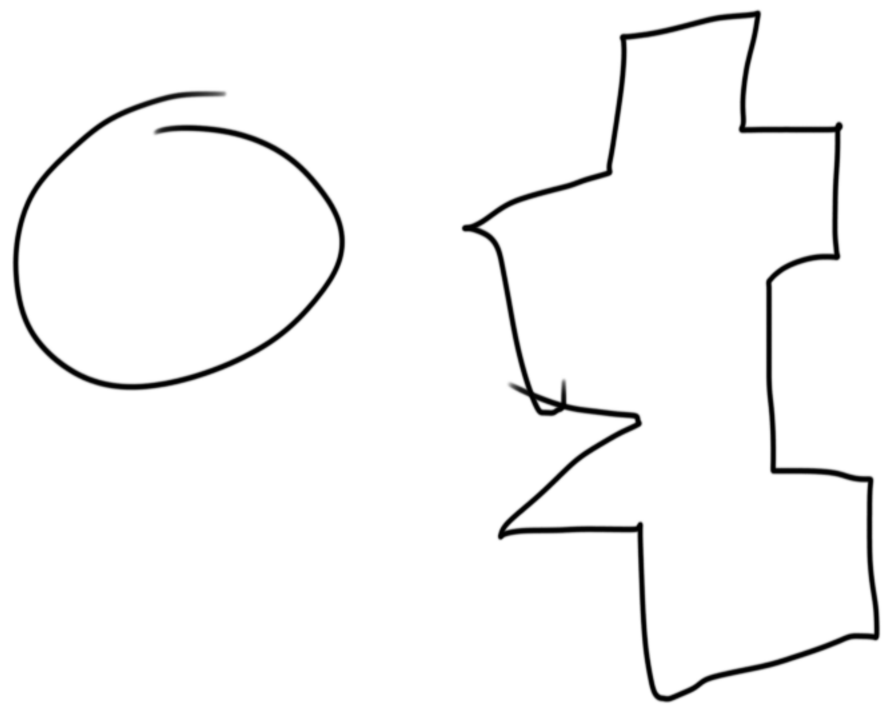


a, b, c, d ... strany
A, B, C, D ... vrcholy
$\alpha, \beta, \gamma, \delta$ ... vnitřní úhly

1. Vlastnost: součet vnitřních úhlů : $360°$

$\triangle$: $180°$ $\square$: $360°$

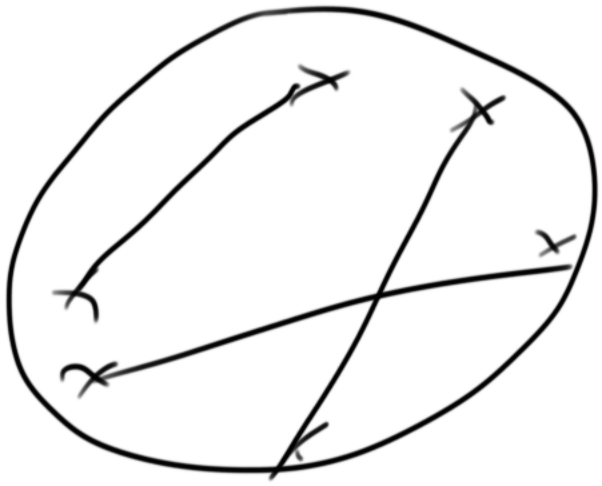




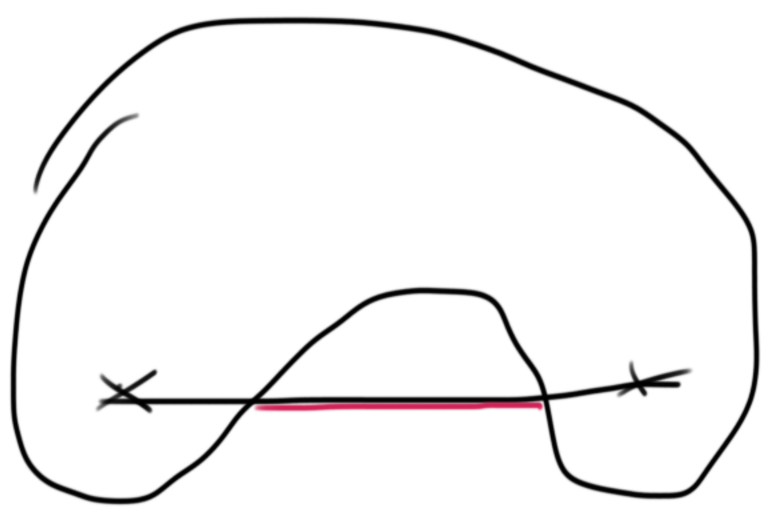
konvexní

nekonvexní

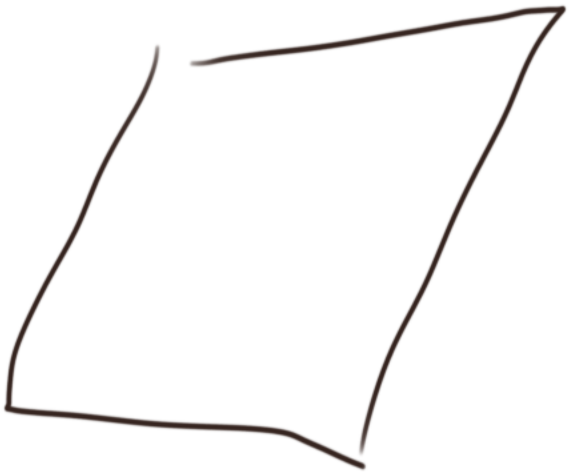
Definice: Rovinný útvar "konvexní" pokud spojnice libovolných 2 vnitř. bodů leží uvnitř útvaru.



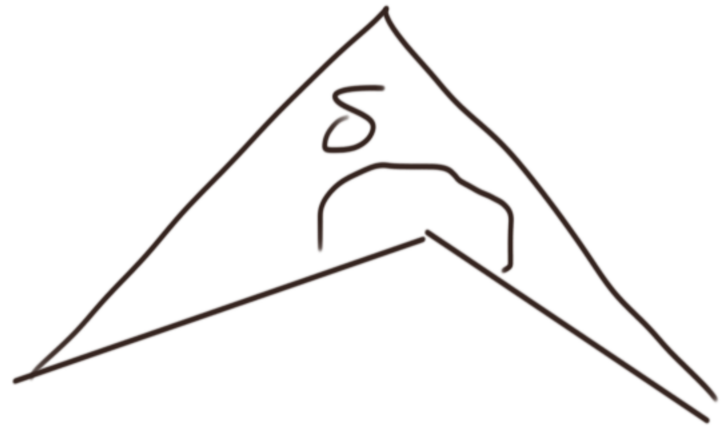
konvexní



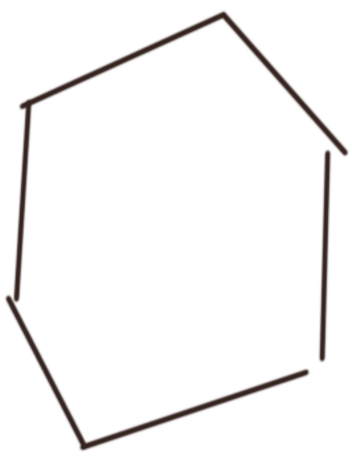
nekonvexní

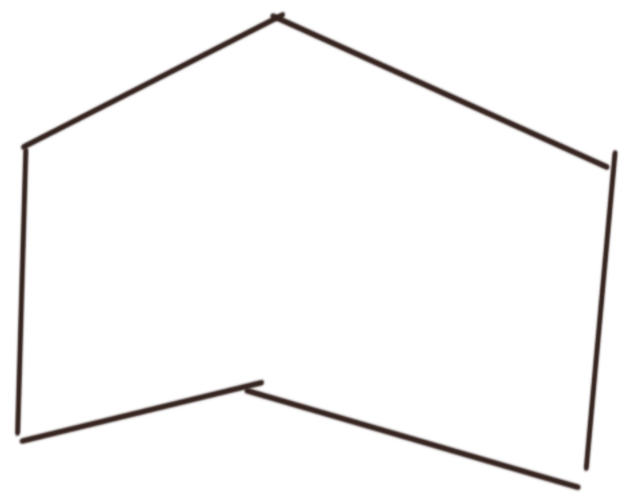


konvexní



nekonvexní



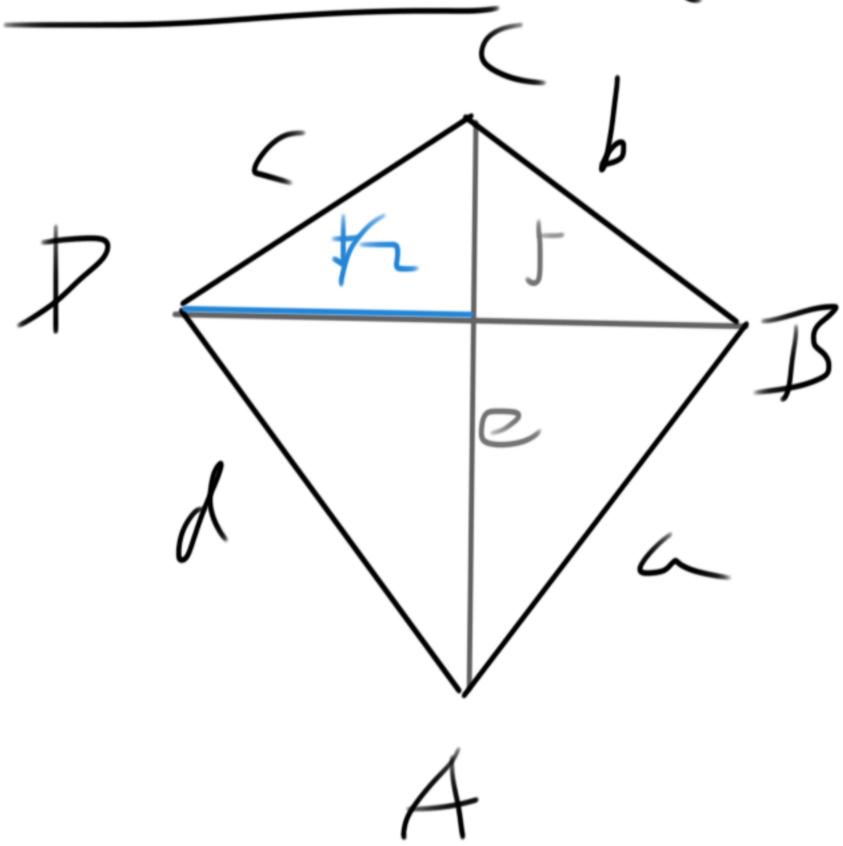


V nekonvexních n-úhelnících:
alespoň jeden úhel $> 180°$

Deltoid

Detoid

Deltoid (drak)



čtyřúhelník

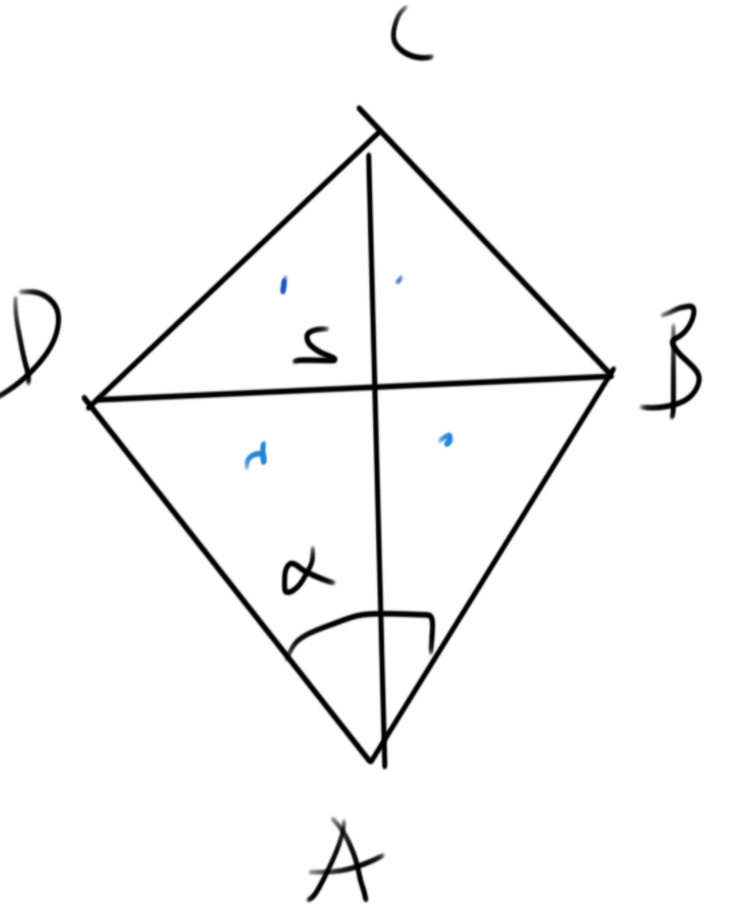
$a = d$

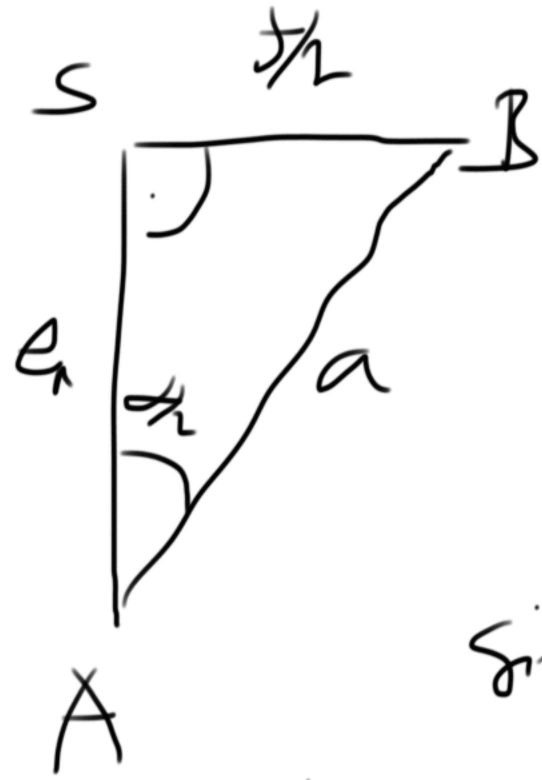
$b = c$

Postřehy

- osově souměrný podle hlavní úhlopř. e
- úhlopříčky jsou na sebe kolmé

hl. úhl. e půlí tu vedlejší





e_a výška u A

$$\sin\left(\frac{\alpha}{2}\right) = \frac{\frac{f}{2}}{a}$$

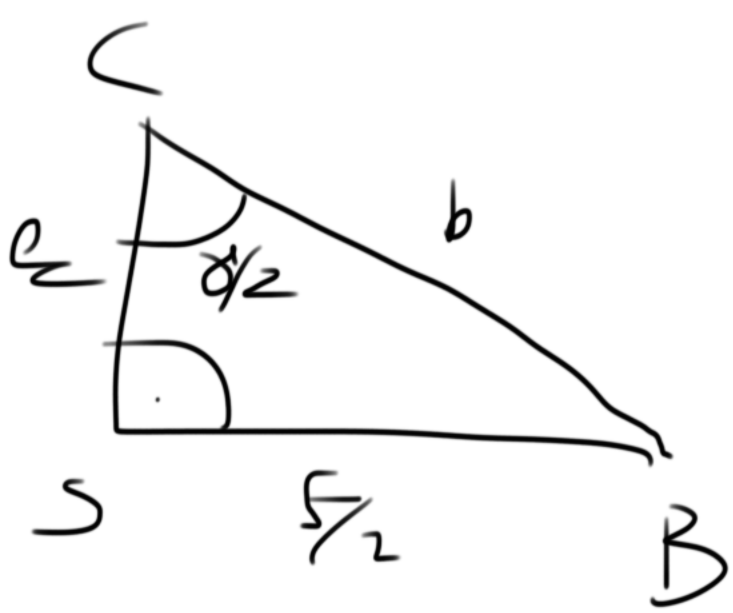
$$\boxed{f = 2 \cdot a \cdot \sin\left(\frac{\alpha}{2}\right)}$$

$$\sin \alpha = \frac{\text{protilehlá}}{\text{přepona}}$$

$$\cos \alpha = \frac{\text{přilehlá}}{\text{přep.}}$$

$$\cos\left(\frac{\alpha}{2}\right) = \frac{e_a}{a}$$

$$\boxed{e_a = a \cdot \cos\left(\frac{\alpha}{2}\right)}$$



$$\boxed{\begin{aligned} f &= 2 \cdot b \cdot \sin\left(\frac{\gamma}{2}\right) \\ e_c &= b \cdot \cos\left(\frac{\gamma}{2}\right) \end{aligned}}$$

$$e_a + e_c = e$$

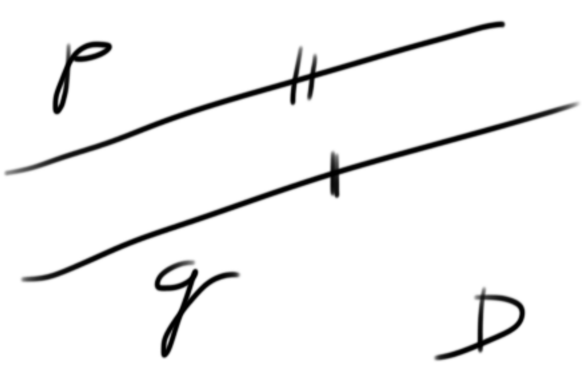
$$S = 2 \cdot S_{\triangleright} + 2 \cdot S_{\triangle} = \cancel{2} \cdot \frac{f}{\cancel{2}} \cdot e_a \cdot \frac{1}{2} + \cancel{2} \cdot \frac{f}{\cancel{2}} \cdot e_c \cdot \frac{1}{2} =$$

$$= \frac{f}{2}\left(e_a + e_c\right) = \boxed{\frac{1}{2} e \cdot f = S}$$

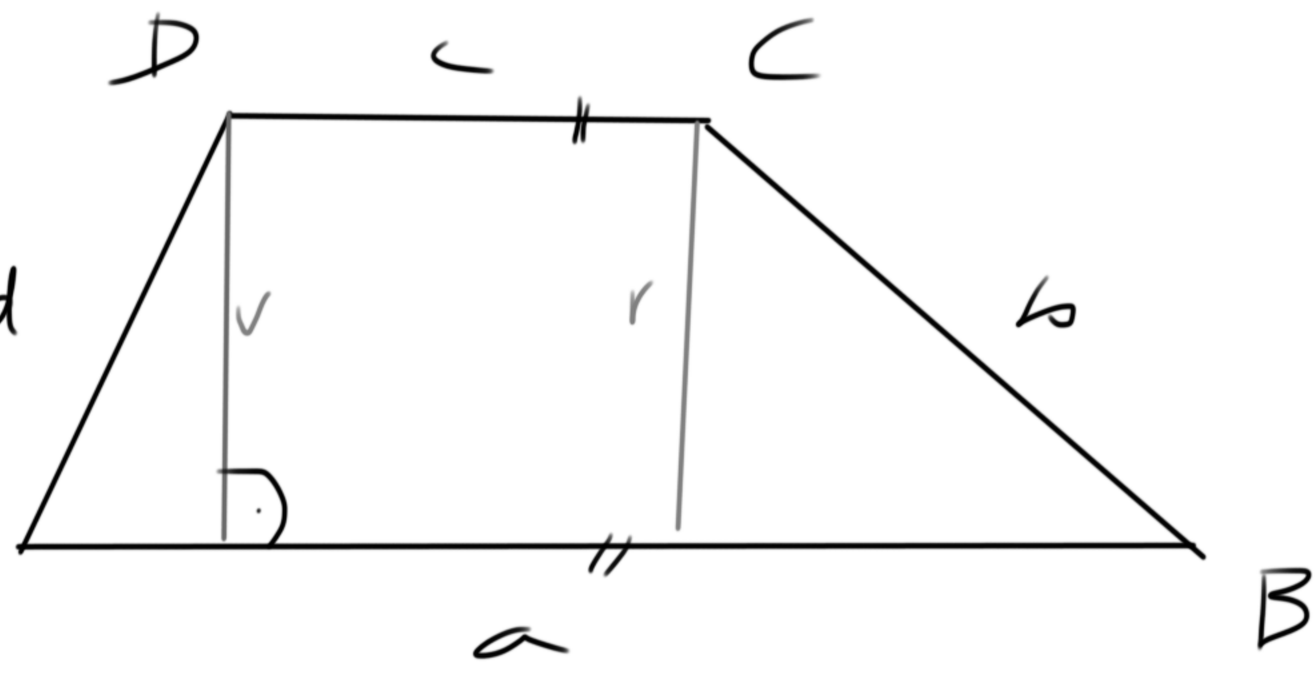
Pro všechny ◻: $O = a+b+c+d$

Lichoběžník

- má 1 dvojici rovnoběžných stran

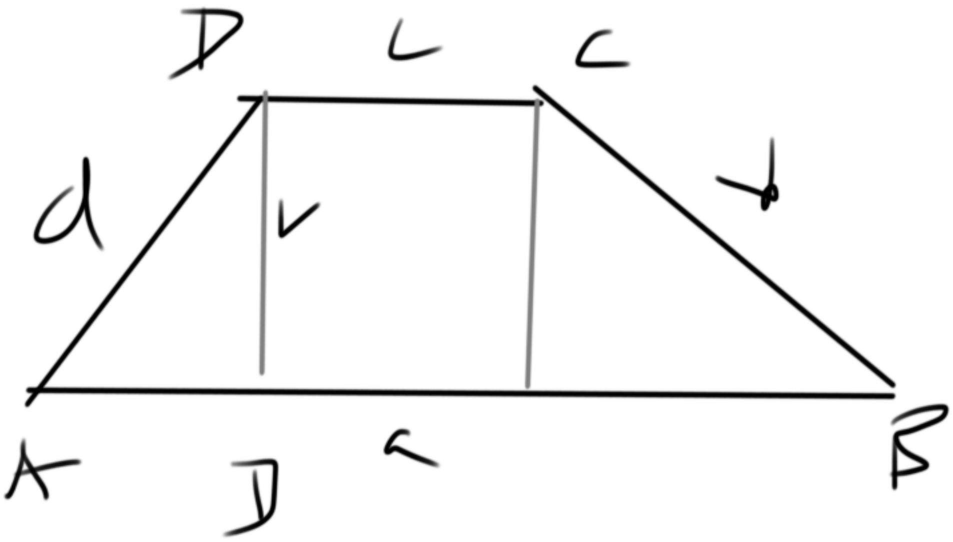


rovnoběžné $p \parallel q$



a, c ... základny b, d ... ramena

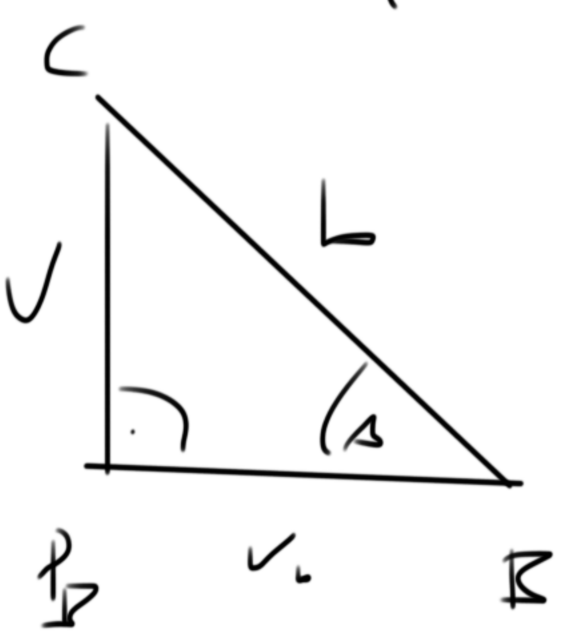
$a \parallel c$



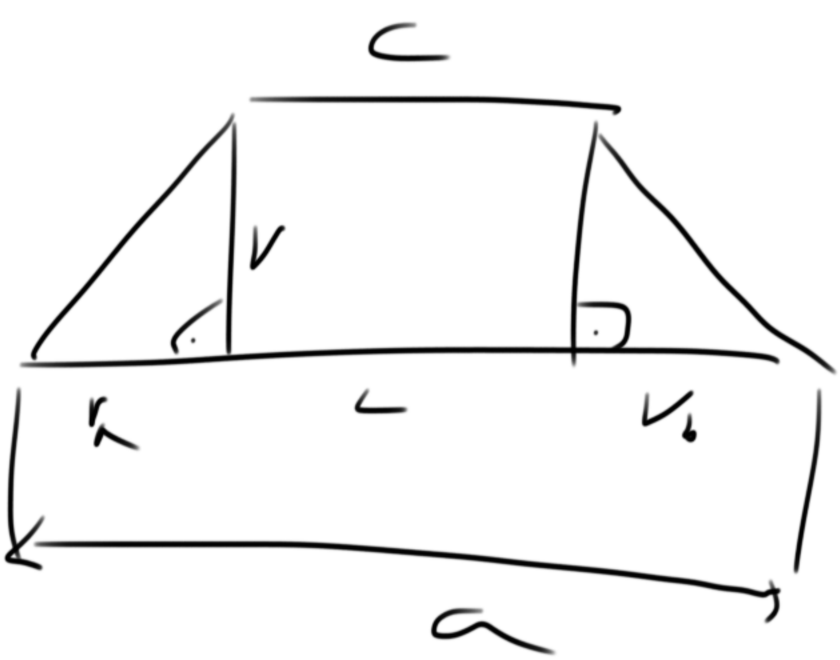
$$\sin\alpha = \frac{v}{d}$$

$$\cos\alpha = \frac{v_a}{d}$$

$$\boxed{\begin{aligned} v &= d\cdot\sin\alpha \\ v_a &= d\cdot\cos\alpha \end{aligned}}$$



$$\boxed{\begin{aligned} v &= b\cdot\sin\beta \\ v_b &= b\cdot\cos\beta \end{aligned}}$$



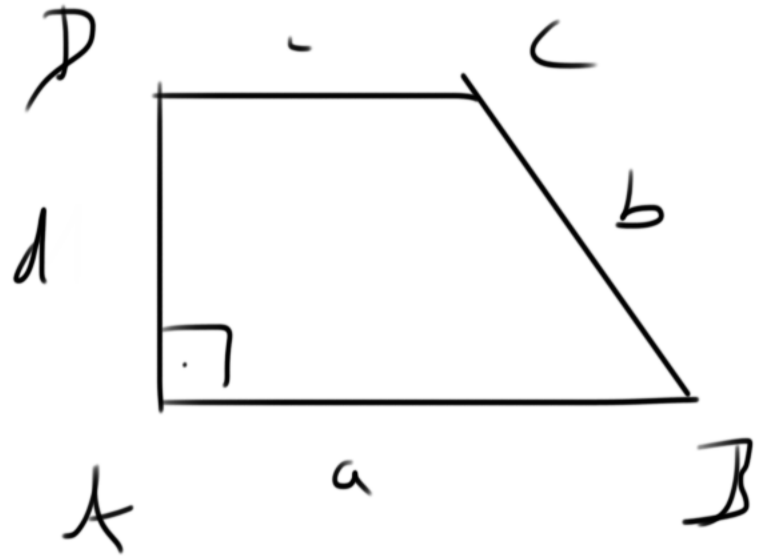
$$\underline{a = c + v_a + v_b}$$

$$S = S_{\triangle} + S_{\square} + S_{\triangle}$$

$$\begin{aligned} S &= \frac{1}{2}v\cdot v_a + c\,v + \frac{1}{2}v\cdot v_b \\ &= \frac{1}{2}v\left(v_a + 2c + v_b\right) \\ &= \frac{1}{2}v\cdot\left(c + v_a + v_b + c\right) \end{aligned}$$

$$\boxed{S = \frac{1}{2}v\cdot(a+c)}$$

· pravoúhlý lichoběžník

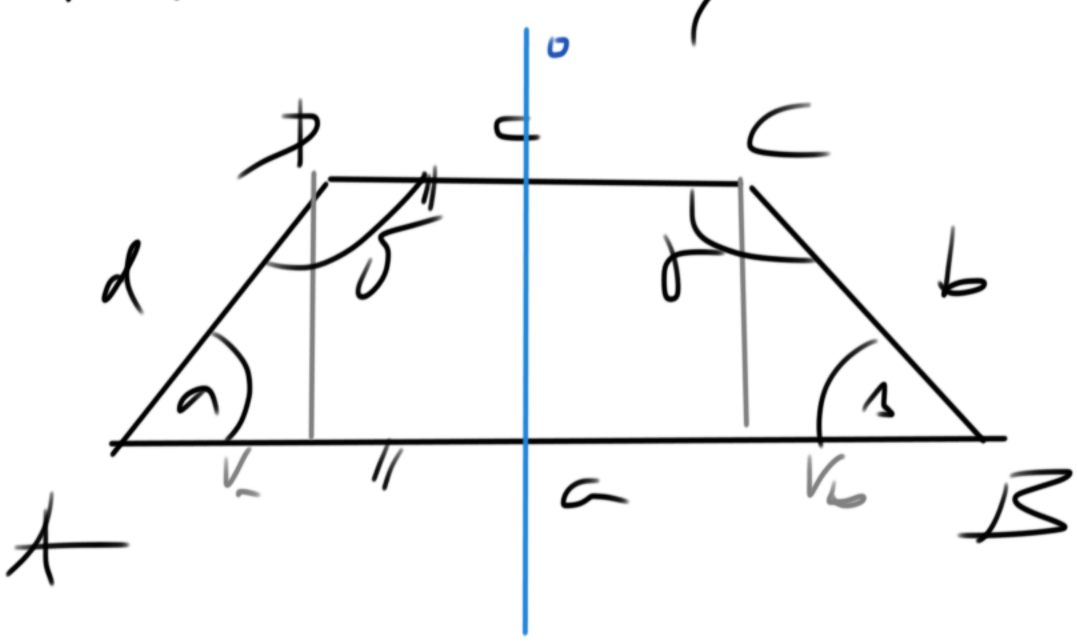


jedno rameno kolmé na základu,

$$S = \frac{1}{2} v \cdot (a+c)$$

$v = d$

· rovnoramenný lichoběžník



$b = d$

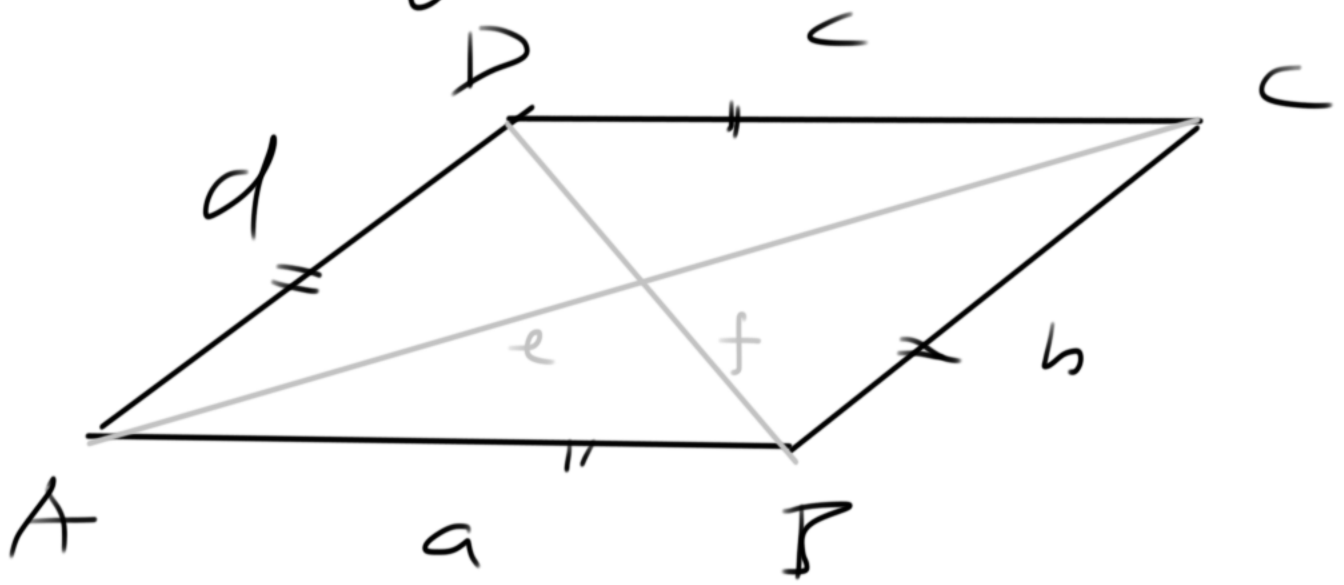
$\Rightarrow v_a = v_b$

osově souměrný $\Rightarrow \alpha = \beta, \gamma = \delta$

Rovnoběžník (kosodélník)

romboid
rhomboid
rhombus

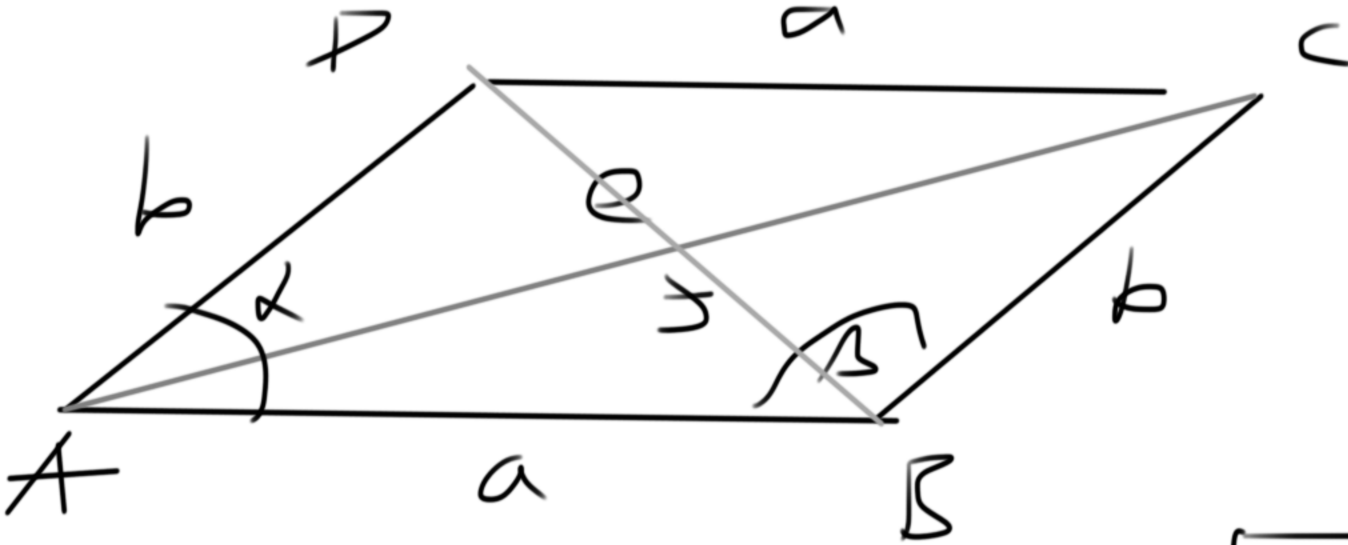
- 2 dvojice rovnoběžných stran



$a \parallel c$
$b \parallel d$

Postřehy

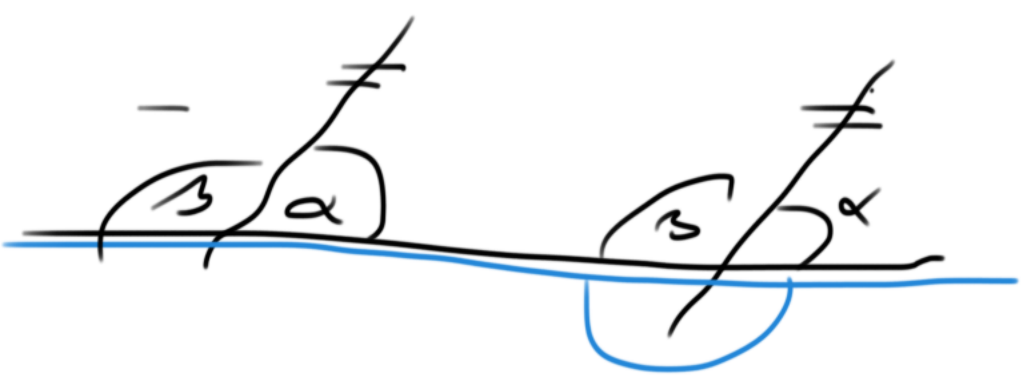
- rovnoběžnost $\Rightarrow$ $a = c$, $b = d$
- $\Rightarrow$ $\alpha = \gamma$, $\beta = \delta$
- <u>nejsou stejně dlouhé</u>, půlí se



cosinová věta

$$e = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cos \beta}$$

$$f = \sqrt{a^2 + b^2 - 2 \cdot a \cdot b \cdot \cos \alpha}$$



$$\alpha + \beta = 180^\circ$$

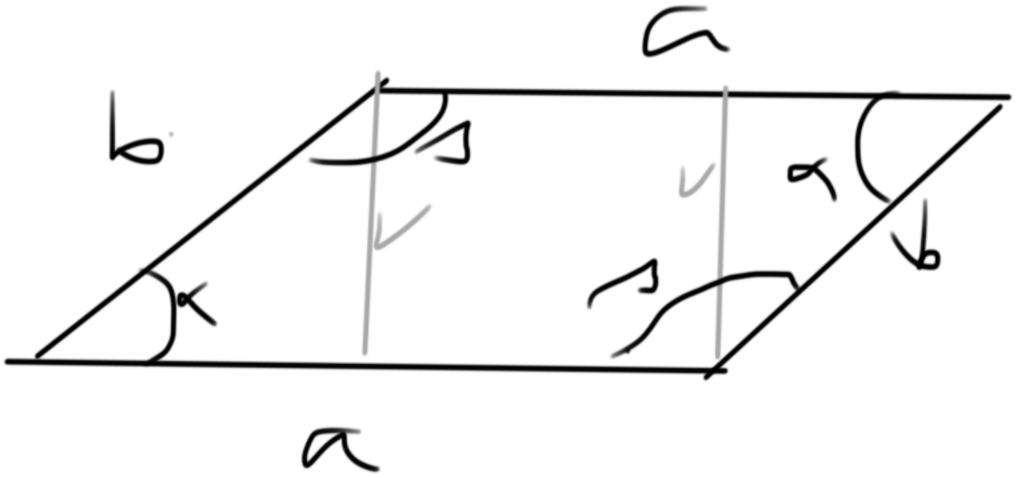
$$\alpha + \beta = \pi$$

$$\beta = \pi - \alpha$$

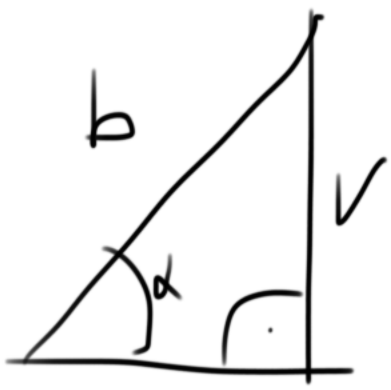
$$\cos \beta = \cos(\pi - \alpha) = \underbrace{\cos \pi}_{-1} \cos \alpha + \underbrace{\sin \pi}_{0} \sin \alpha$$

$$= -\cos \alpha$$

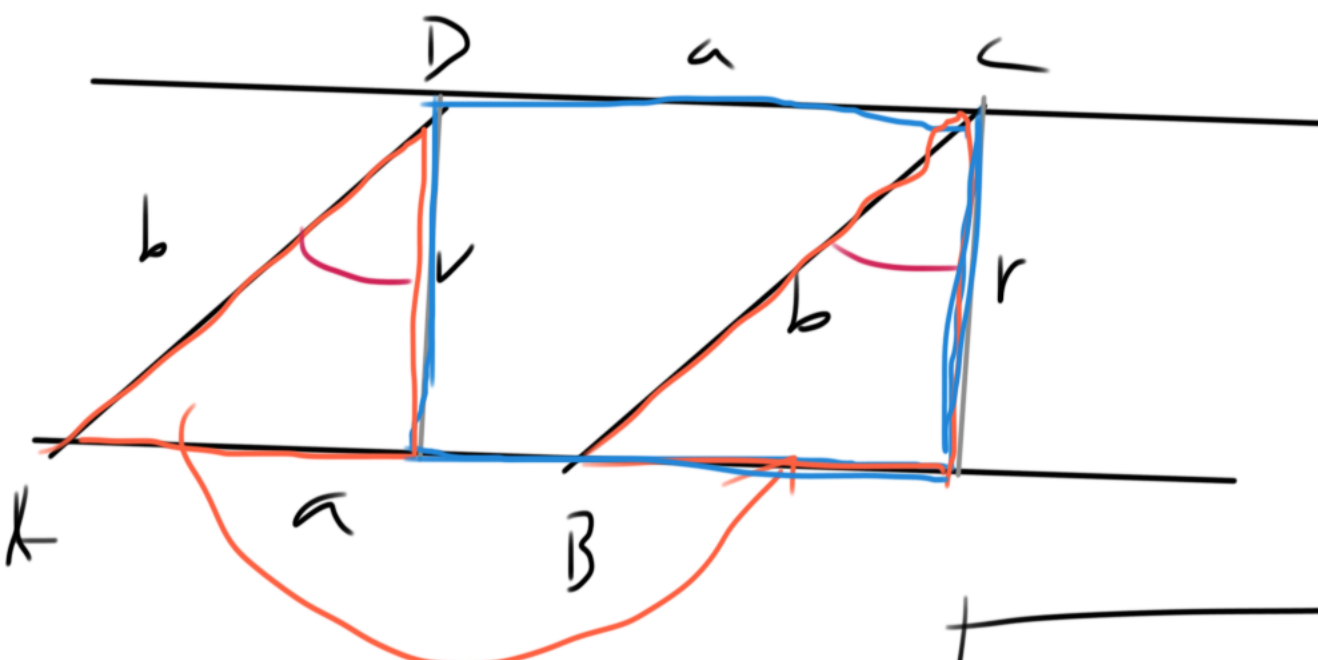
$$\Rightarrow \quad e = \sqrt{a^2 + b^2 + 2 \cdot a \cdot b \cos \alpha}$$



$$S = S_{\triangle} + S_{\square} + S_{\triangledown}$$



$$\underline{v = b \sin\alpha}$$

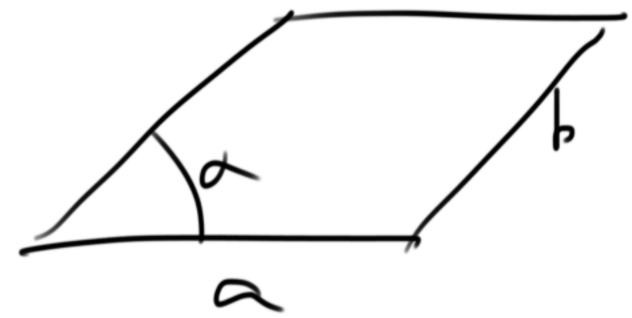


$$S = S_{\square}$$

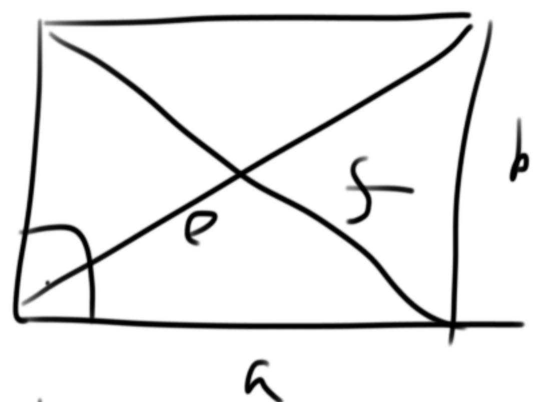
$$\boxed{S = v \cdot a}$$

$$\boxed{S = a \cdot b \cdot \sin\alpha}$$

Speciální případy



- $\alpha = \frac{\pi}{2}$, 90°



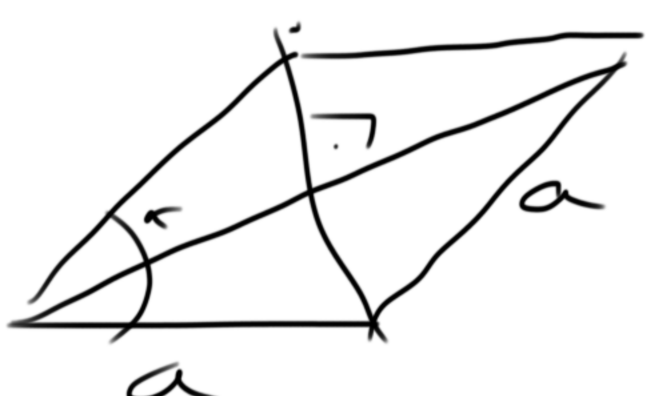
obdélník

- úhlopříčky jsou stejně dlouhé

$e = f$

$S = a \cdot b \sin\frac{\pi}{2} = a \cdot b$

- $a = b$



kosočtverec

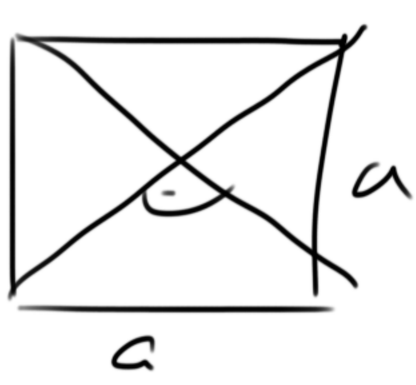
- úhlopříčky jsou na sebe kolmé

$e \perp f$

$S = a \cdot b \sin\alpha \longrightarrow a^2 \sin\alpha$

- $\alpha = \frac{\pi}{2}$, $a = b$

čtverec



$e \perp f \wedge e = f$

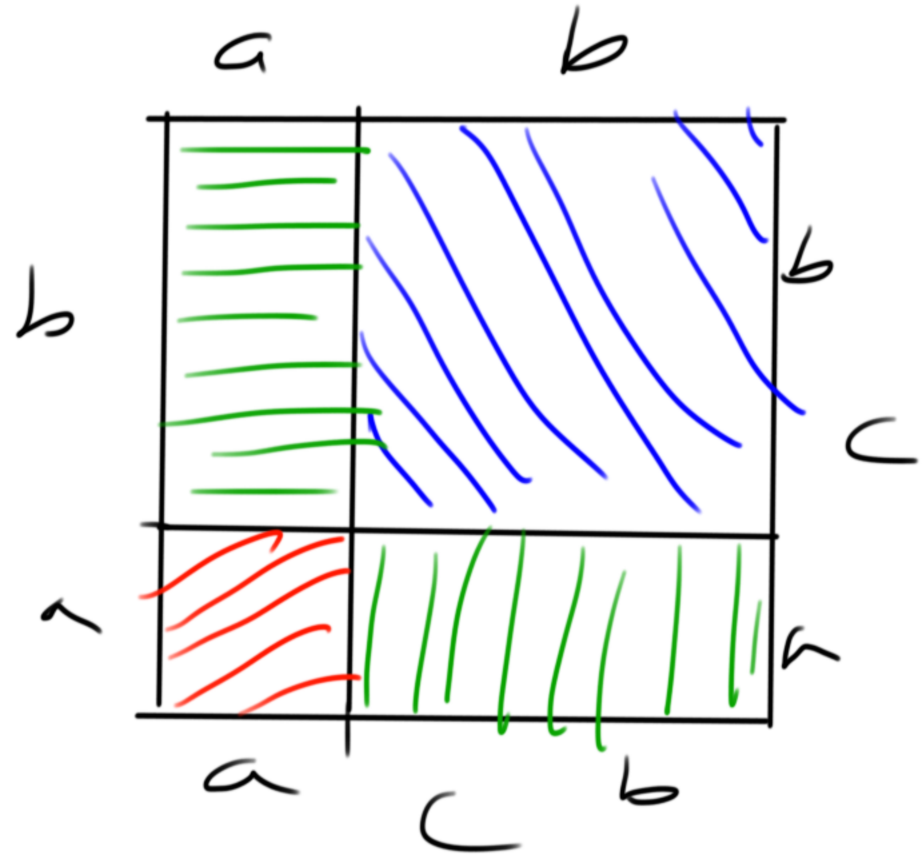
$\boxed{S = a^2}$

Geometrická ilustrace algebraických identit

$$(a+b)^2 = a^2 + 2\cdot a\cdot b + b^2$$

$$(a+b)\cdot(a-b) = a^2 - b^2$$

$a+b = c$ $\qquad c^2$



$$S = c^2$$
$$= (a+b)^2$$

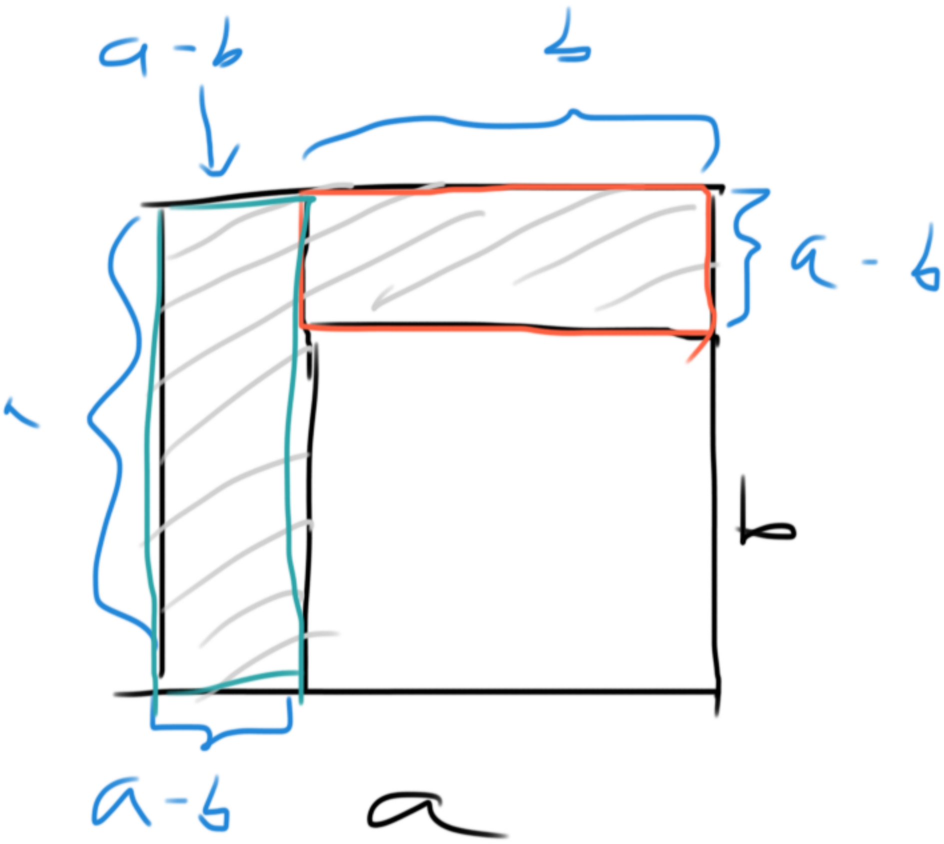
$$S = S_{\square} + S_{\square} + 2\cdot S_{\square}$$

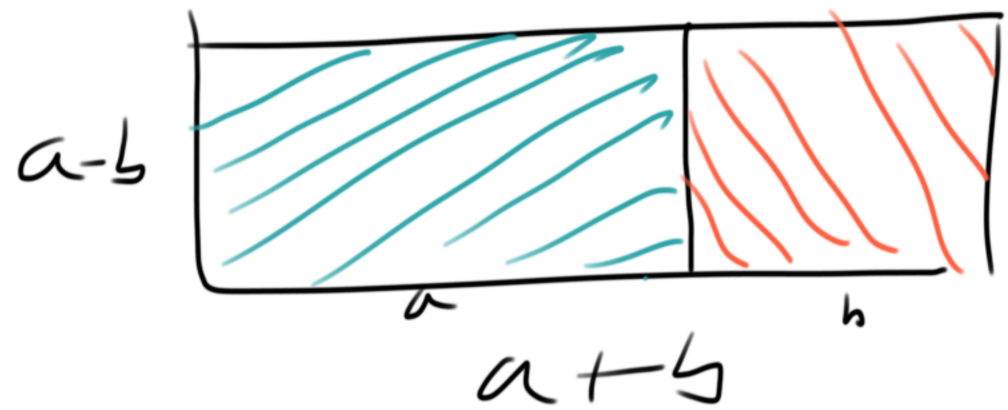
$$S = \underbrace{a^2} + \underbrace{2a\cdot b} + \underbrace{b^2}$$

$$(a+b)^2 = a^2 + 2a\cdot b + c^2$$

$$a^2 - b^2 = (a+b)(a-b)$$

$a > b$





$a\cdot(a-b)$ $\quad$ $b\cdot(a-b)$

Pythagorova věta

$$c^2 = a^2 + b^2$$

$S = \frac{a\cdot b}{2}$

$S = 4\cdot\frac{1}{2}a\cdot b + c^2$

$S = a^2 + b^2 + 2ab$