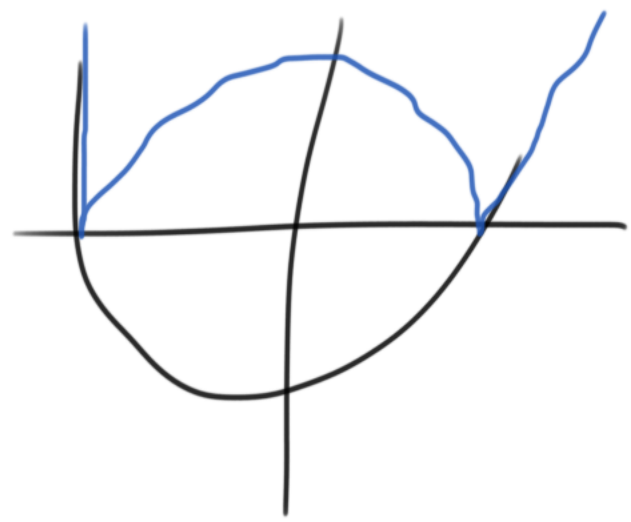
Absolutní hodnota

$$y = |x| \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

$f(x)$

$|f(x)|$



$D_f = \mathbb{R}$

$H_f = \mathbb{R}_0^+$

Zadání: graf, D_f, H_f, význačné body, vlastnosti (monotonie, per. parita, prostá?, omezená)

1) $y = \underline{-3}x + 7$

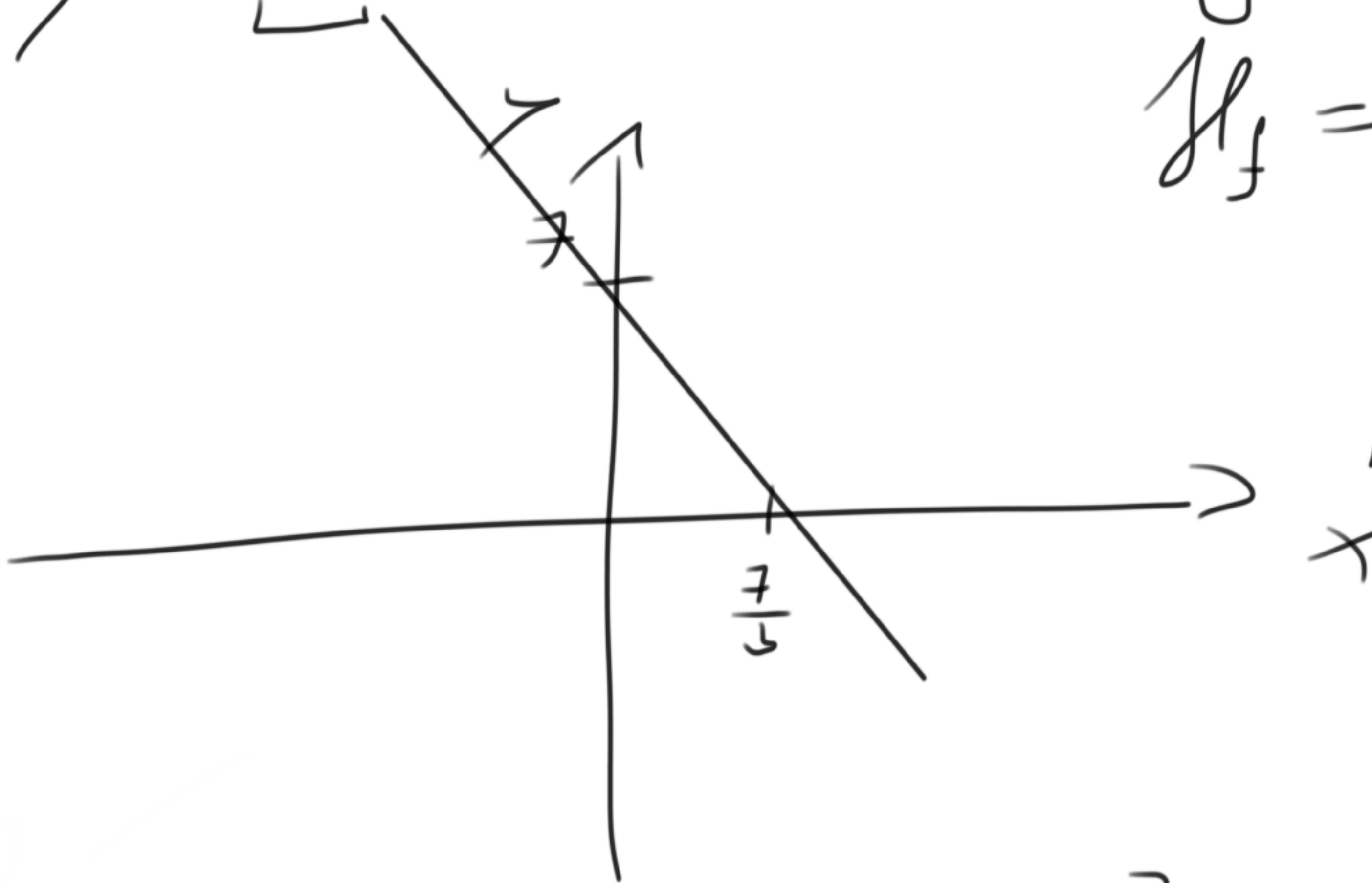
$D_f = \mathbb{R}$

$H_f = \mathbb{R}$

kles. na D_f

neomez.

prostá



prus. s y

$x = 0$

$y = -3 \cdot 0 + 7 = 7$

prus. s x

$y = 0$

$0 = -3x + 7$

$x = \frac{7}{3}$ ✓

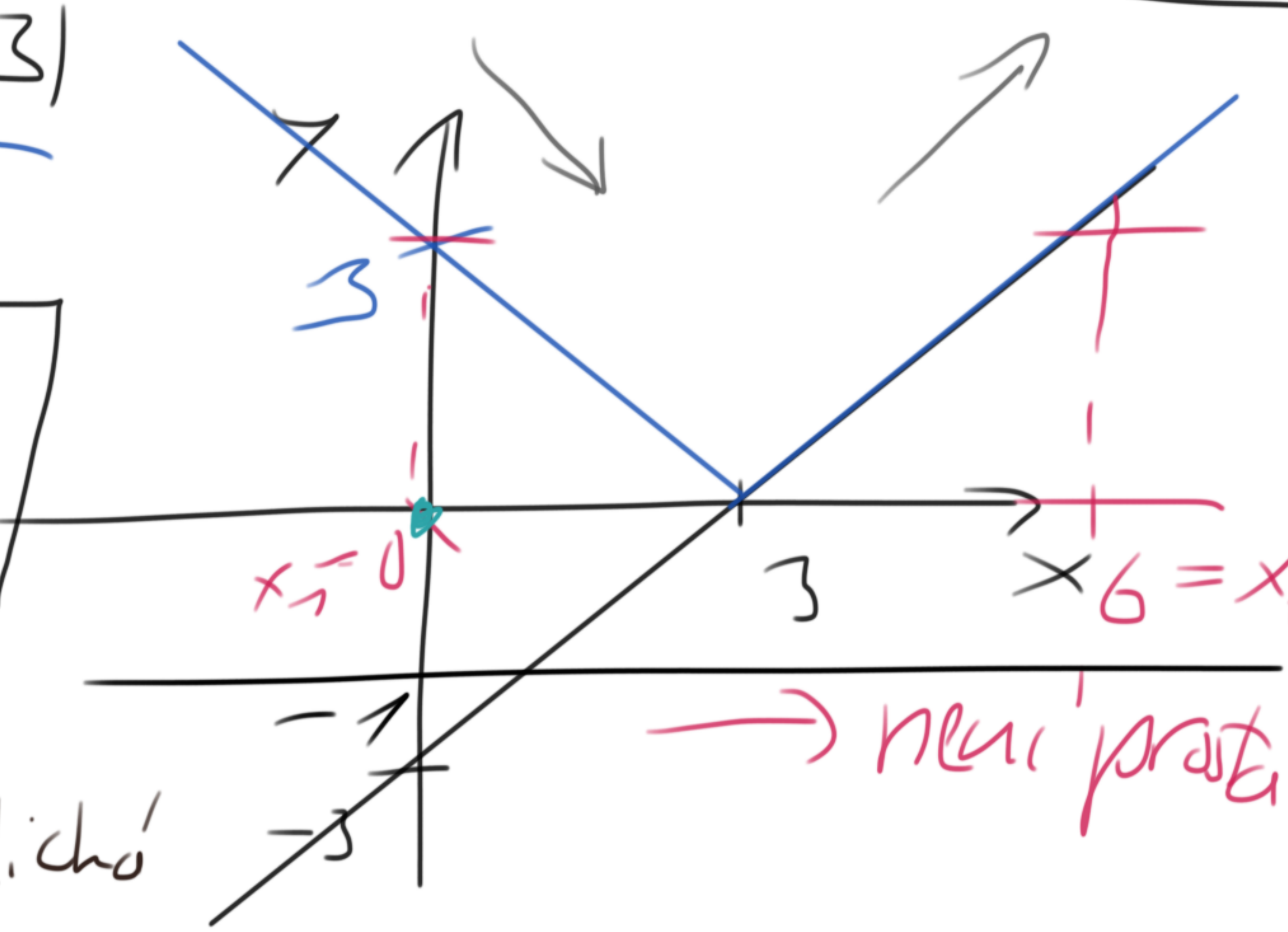
---

2) $y = |x - 3|$

$y = x - 3$

kles. v $(-\infty, 3)$

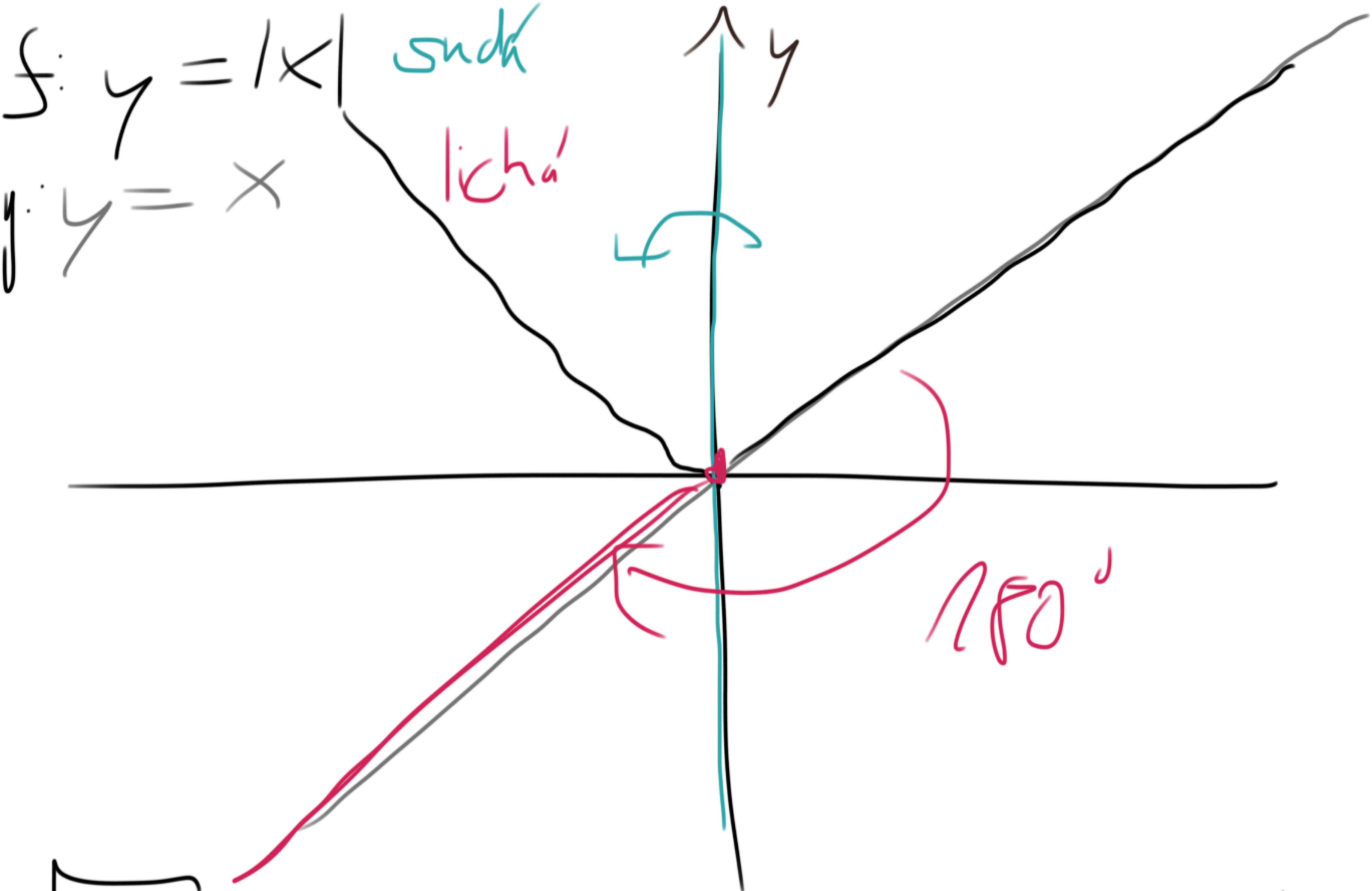
rost. v $(3, \infty)$



→ není prostá

omezená

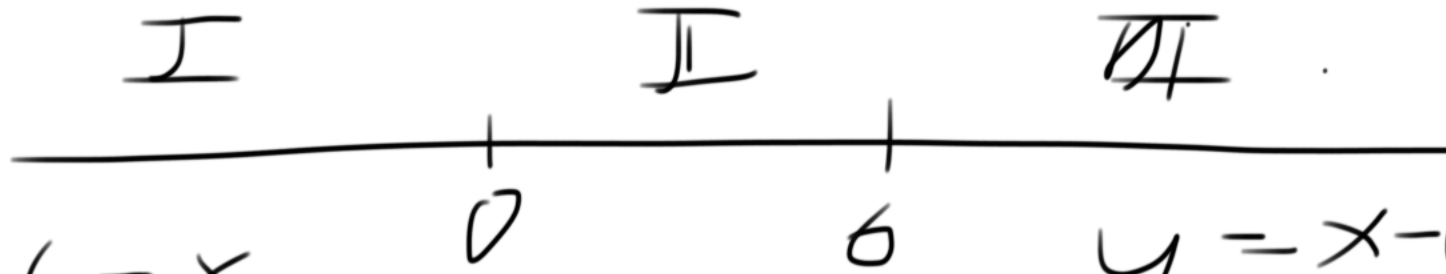
ani sudá ani lichá



$f(-x) = |-x| = |x| = f(x)$ sudá

$g(-x) = -x = -g(x)$ lichá

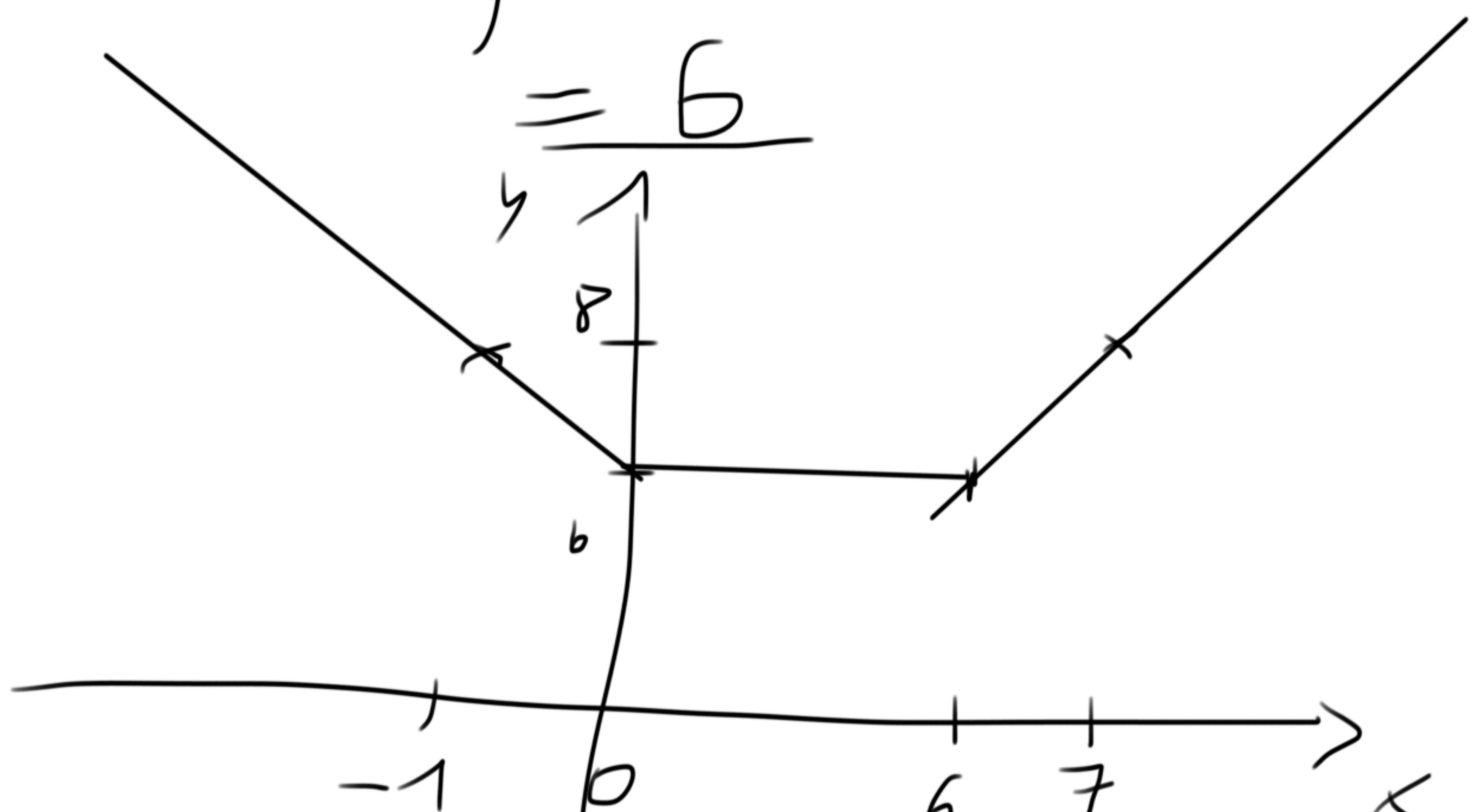
4) $y = |x-6| + |x|$ $\quad D_f = \mathbb{R}$



I: $y = -x + 6 - x = \underline{-2x + 6}$

II: $y = -x + 6 + x = \underline{6}$

III: $y = x - 6 + x = 2x - 6$



$f(0) = 6$ $\quad f(6) = 6$

$f(-1) = 8$ $\quad f(7) = 8$

Parita	H_f	Omezenost	Monotonie (prostá?)
<u>nic</u>	$(6, \infty)$	zdola	$\searrow (-\infty, 0)$ $\nearrow (6, \infty)$ konst. $(0, 6)$

5) $y = x^2 + 4x + 3$

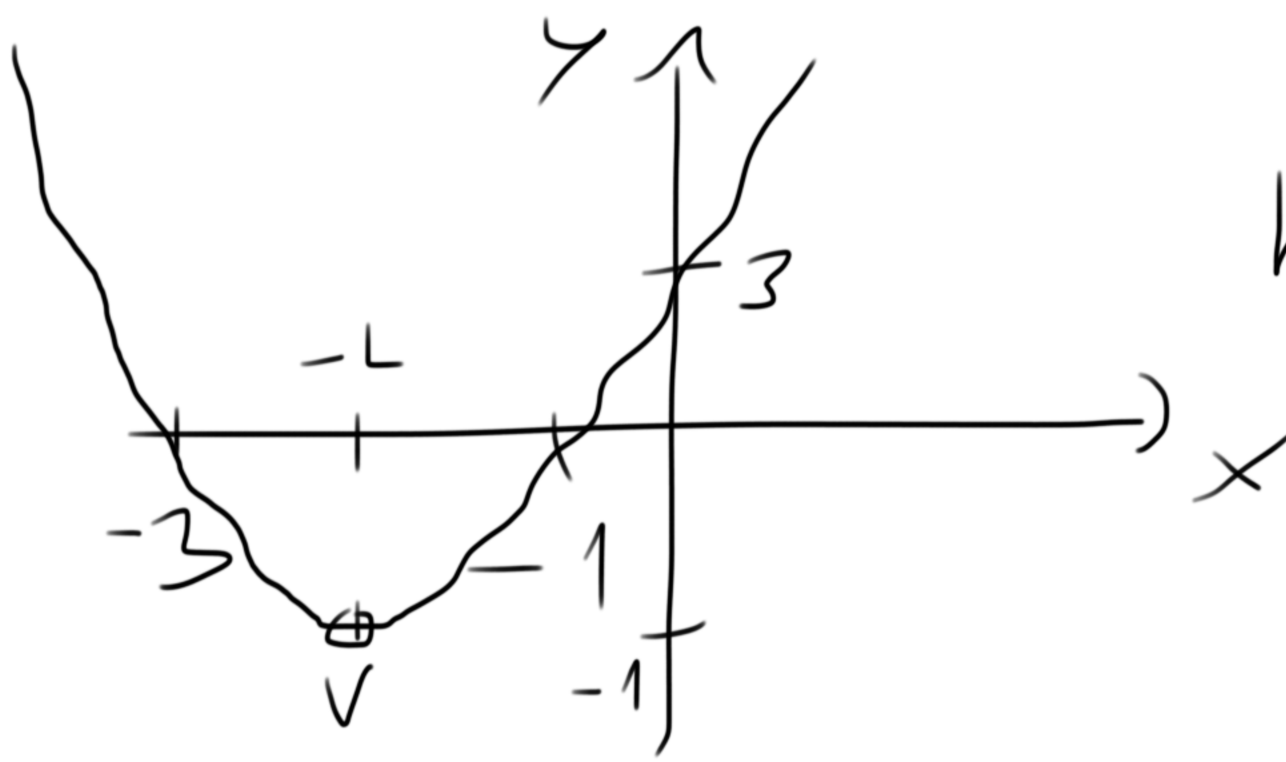
$a = 1 > 0$

průs. s y: $x = 0$ $y = \underset{c}{3}$

průs. s x: $y = 0$

$$0 = x^2 + 4x + 3$$

$$x_{1,2} = \frac{-4 \pm \sqrt{16 - 4 \cdot 3}}{2} = \left\langle \begin{matrix} -1 \\ -3 \end{matrix} \right.$$



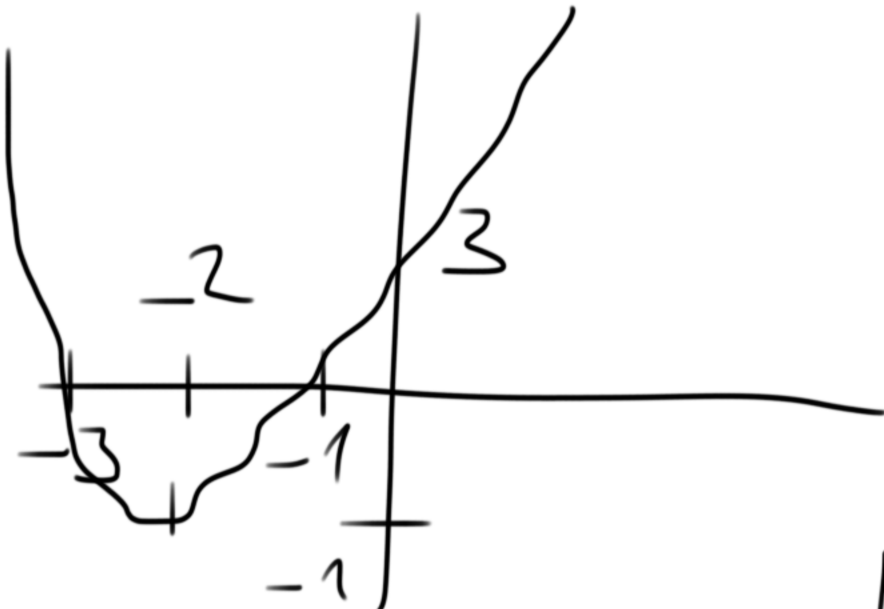
$V = [V_x, V_y]$

$$\boxed{\begin{aligned} V_x &= -\frac{b}{2a} \\ V_y &= c - \frac{b^2}{4a} \end{aligned}}$$

$x^2 + 4x + 3$

$$V = \left[-2, 3 - \frac{16}{4}\right] = [-2, -1]$$

$V_x = -2$ $\quad V_y = f(V_x) = f(-2) = 4 - 8 + 3 = \underline{-1}$



Parita H_f Omez.

X $\langle -1, \infty)$ zdola

Monotonie ↘ v $(-\infty, -2\rangle$

↗ v $\langle -2, \infty)$

---

6) $y = x^2 - 6x + 9$ $\quad A^2 - 2A \cdot B + B^2$

$= (x-3)^2 + 9 - 3^2 = 1 \cdot (x-3)^2 + 0$

$$y = a \cdot (x - B)^2 + C$$

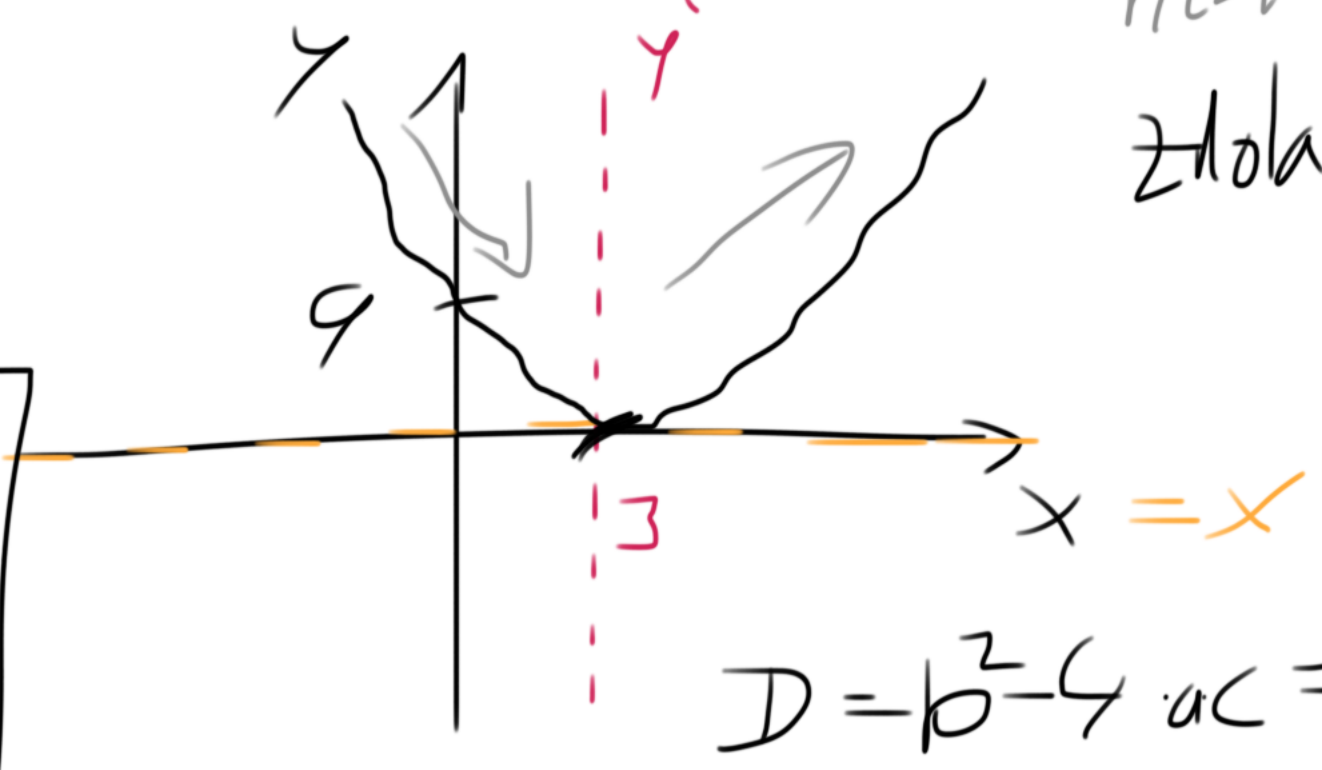
kontrakce
dilatace
v y

posun
v x

posun v y

není prostá

zdola omezená



rozhledem k novým osám

$y' = (x')^2$

$D = b^2 - 4ac = 0$

$y = x^2 + 4x + 3$

$f(-x) = (-x)^2 - 4x + 3 = x^2 - 4x + 3$

$\neq f(x)$

$\longrightarrow$ není sudá

$\neq -f(x)$

$-f(x) = -x^2 - 4x - 3$

x^{2n} . — sudá

x^{2n+1} . . lichá

$y = \boxed{x^2 + 6}$ sudá

$y = 2x$ lichá

14) $y = |x^2 + 2x - 3|$

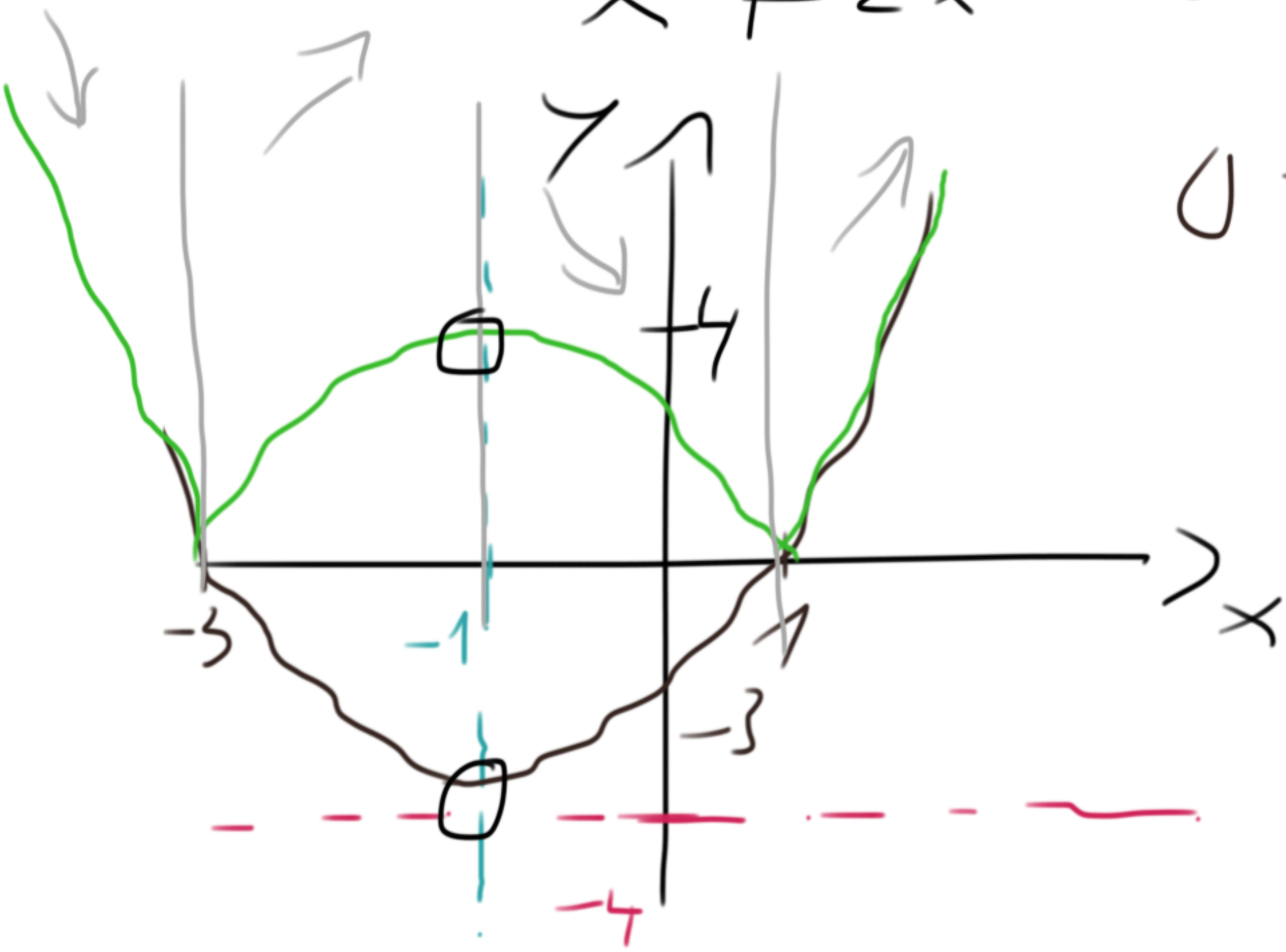
$y = |f(x)|$

$y = x^2 + 2x - 3$

$= (x+1)^2 - 3 - 1^2 = (x+1)^2 - 4$

Zk: $x^2 + 2x \cdot 1 + 1^2 - 4$

$= x^2 + 2x - 3$ ✓



$0 = (x+1)^2 - 4$

$A^2 - B^2$

$= (x+1+2)(x+1-2)$

$A + B \quad A - B$

$= (x+3) \cdot (x-1)$

$x_{1,2} = +1, -3$

20) $y = -\dfrac{3}{x-2} + 6$

$\dfrac{1}{x}$:

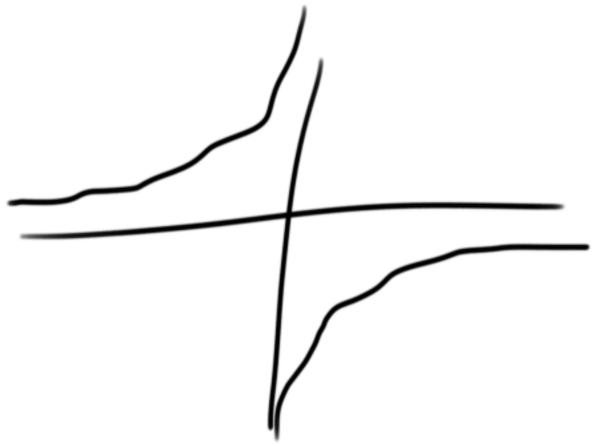
lichá

klesající

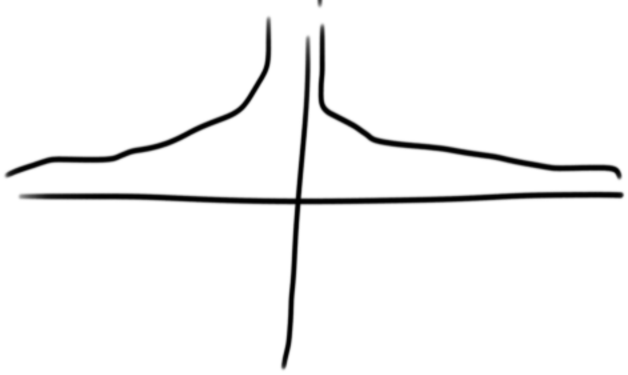
$D_f = \mathbb{R} \setminus \{0\}$

$H_f = \mathbb{R} \setminus \{0\}$

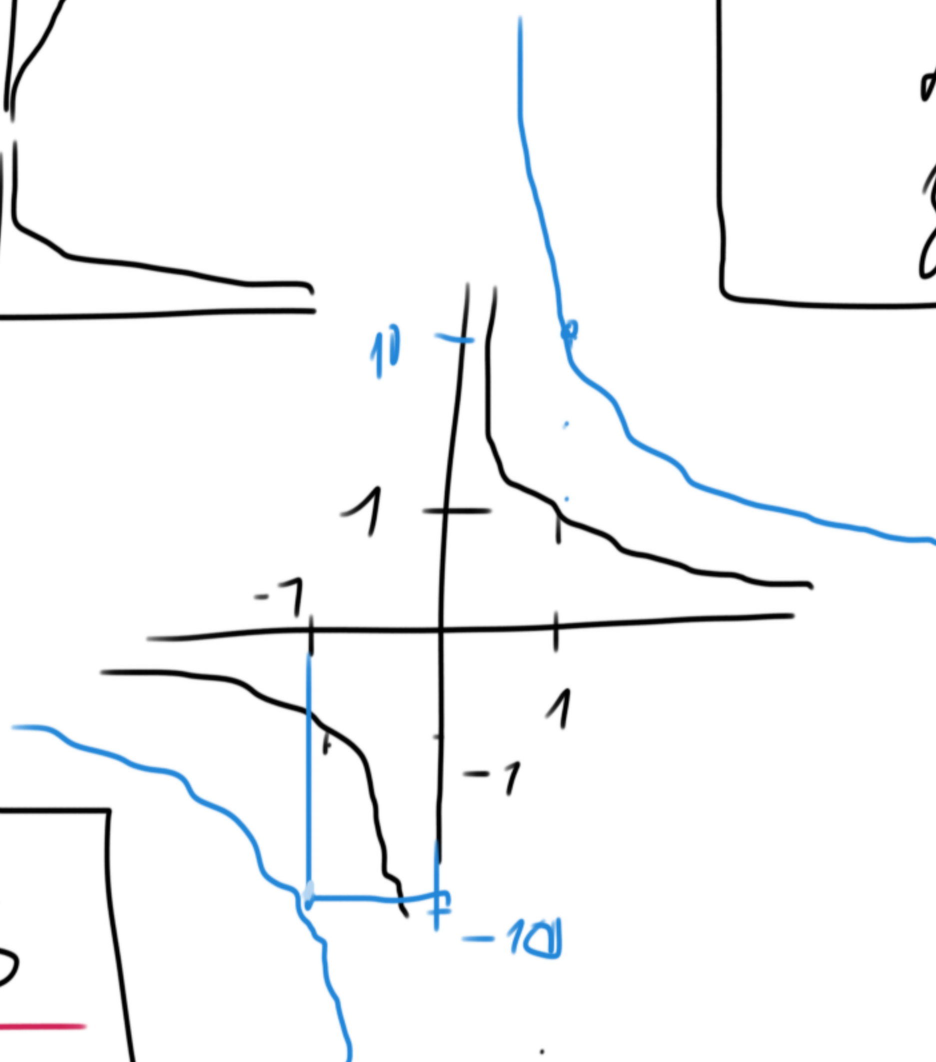
$-\dfrac{1}{x}$:



$\left|\dfrac{1}{x}\right|$:



$\dfrac{1}{x}, \dfrac{10}{x}$



$y = \dfrac{-3}{x-2} + 6$

x	0	1	3
y			

$x = 0 \quad y = +\dfrac{3}{2} + 6 = \dfrac{15}{2}$

$y = 0 \quad 0 = \dfrac{-3}{x-2} + 6$

$3 = 6(x-2)$

$x = \dfrac{1}{2} + 2 = \dfrac{5}{2}$

$D_f = \mathbb{R} \setminus \{2\}$

$H_f = \mathbb{R} \setminus \{6\}$

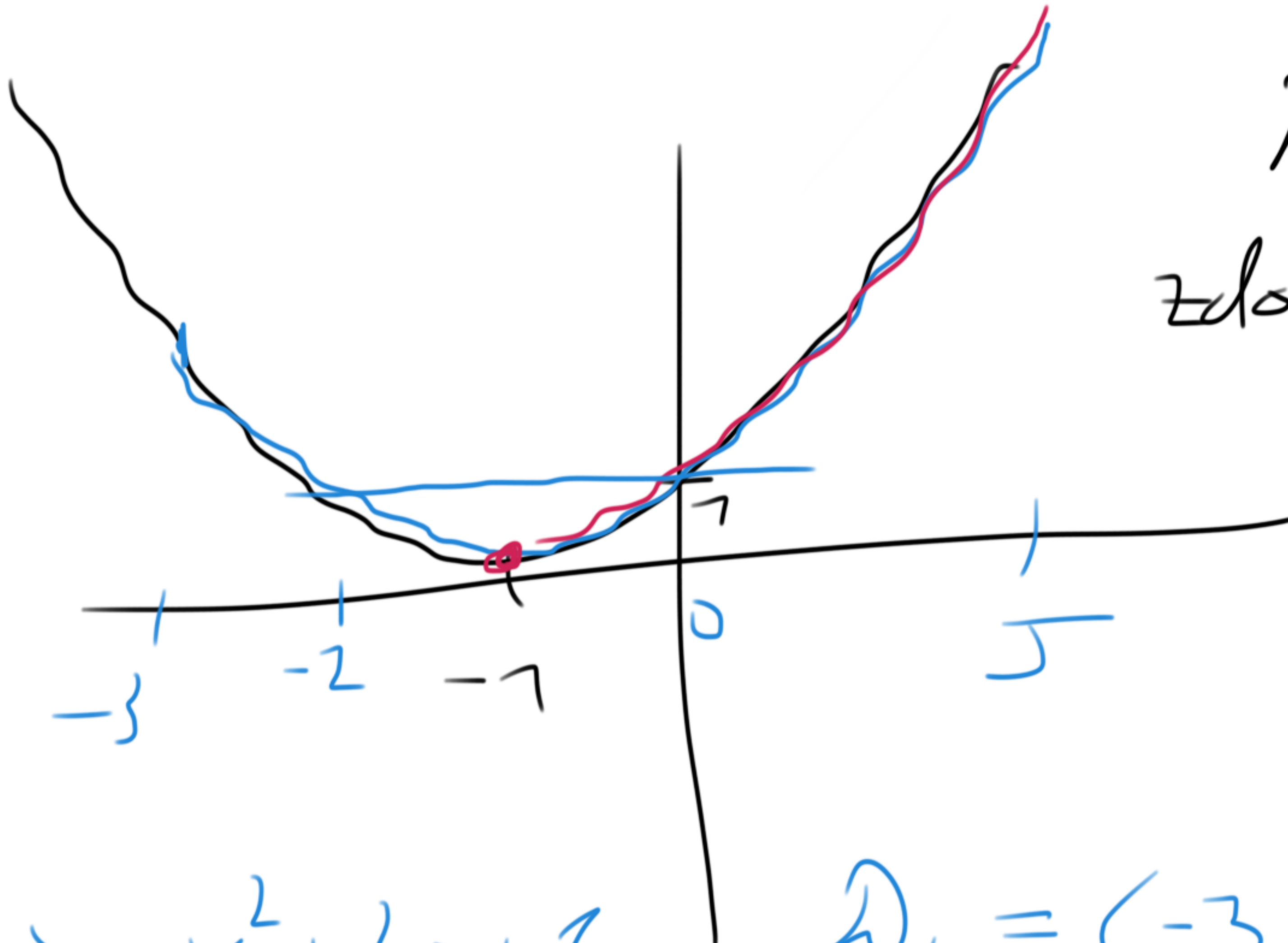
Parita: X

klesající D_f

neomezená

prostá

$y = x^2 + 2x + 1 = (x+1)^2$



$D_f = \mathbb{R}$

zdola omezená

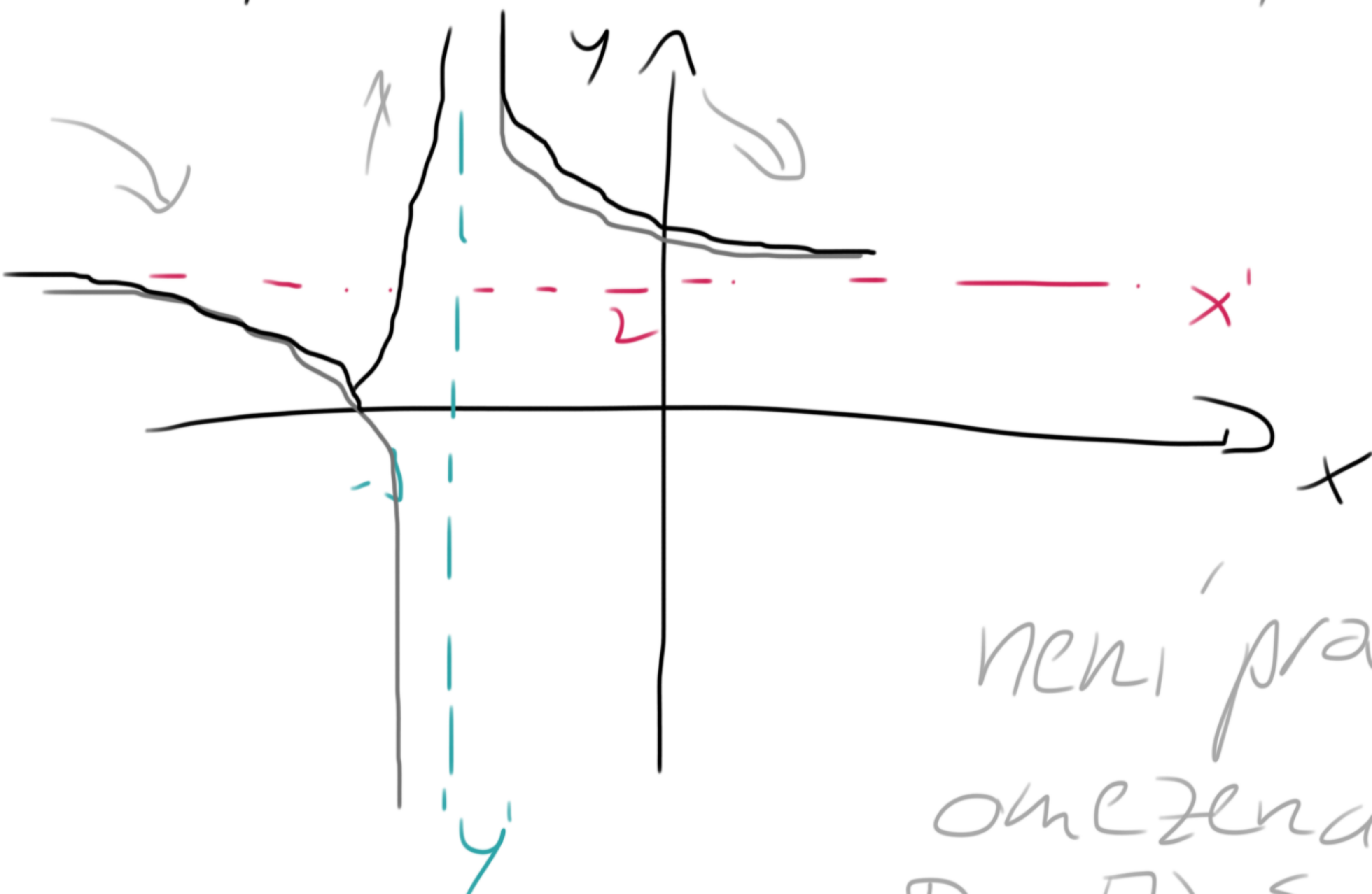
$y = x^2 + 2x + 1$ $\quad D_f = \langle -3, 5 \rangle$

omezená na D_f

$y = x^2 + 2x + 1$ $\quad D_f = \langle -1, 5 \rangle$

rostoucí na $D_f \Rightarrow$ prostá

23) $y = \left|\dfrac{1}{x+3} + 2\right|$ $\quad y = |f(x)|$



není prostá

omezená

$D_f = \mathbb{R} \setminus \{-3\}$

$H_f = \langle 0, \infty)$

24) $\boxed{y = \left|\dfrac{1}{x+3}\right| + 2}$