# Goniom. fce

## Grafy

$f: y = \underbrace{2}\cdot\sin(2x) + \underline{3}$

graf

význačné body

dilatace v y

kontrakce v x

posun v y

$H_{sin} = \langle -1, 1 \rangle$

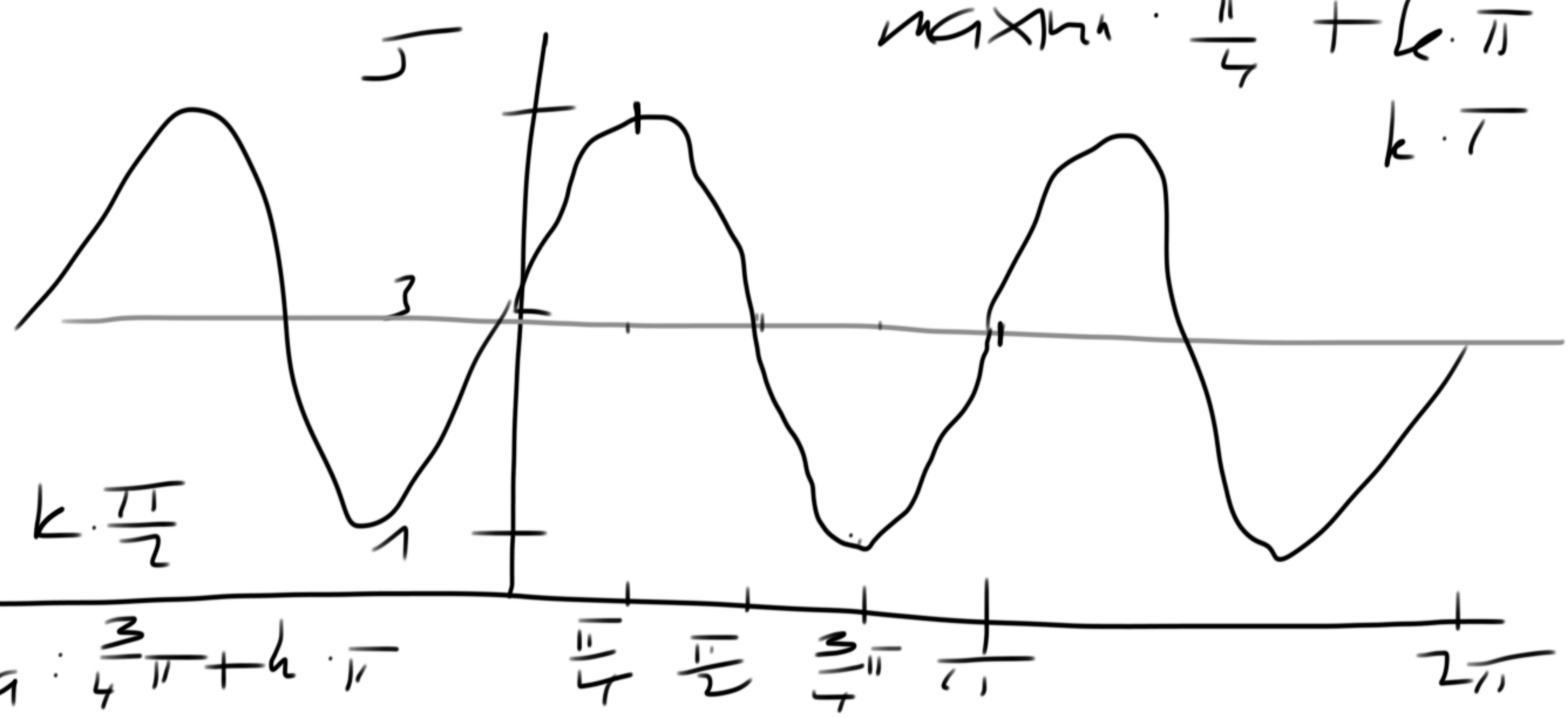
$H_f = \langle 2\cdot(-1)+3, 2\cdot 1+3 \rangle = \underline{\langle 1, 5 \rangle}$

$\sin(x\cdot f)$ $f = 1$ $T = 2\pi$

$\sin(2x)$ $f = 2$ $T = \pi$

maxima: $\frac{\pi}{4} + k\cdot\pi$

$k\cdot T$




(nuly) trojky: $k\cdot\frac{\pi}{2}$

minima: $\frac{3}{4}\pi + k\cdot\pi$

Vyjádřete v obloukové míře

$\alpha = 20^\circ$

$$\begin{array}{ccc} & 2\pi \ \ldots\ldots & 360^\circ \\ \uparrow & x \ \ldots\ldots & 20^\circ \uparrow \end{array}$$

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$$\frac{x}{2\pi} = \frac{20}{360}$$

$$x = 2\pi \frac{1}{18} = \frac{\pi}{9}$$ ✓

$60^\circ \sim \frac{\pi}{3}$ $\quad 60^\circ = 3 \cdot 20^\circ$

$$20^\circ = \frac{1}{3}\frac{\pi}{3} = \frac{\pi}{9}$$ ✓

$\alpha = \frac{11}{12}\pi$

$$\begin{array}{ccc} & \pi \ \ldots\ldots & 180^\circ \\ \nearrow & \frac{11}{12}\pi \ \ldots\ldots & x \nearrow \end{array}$$

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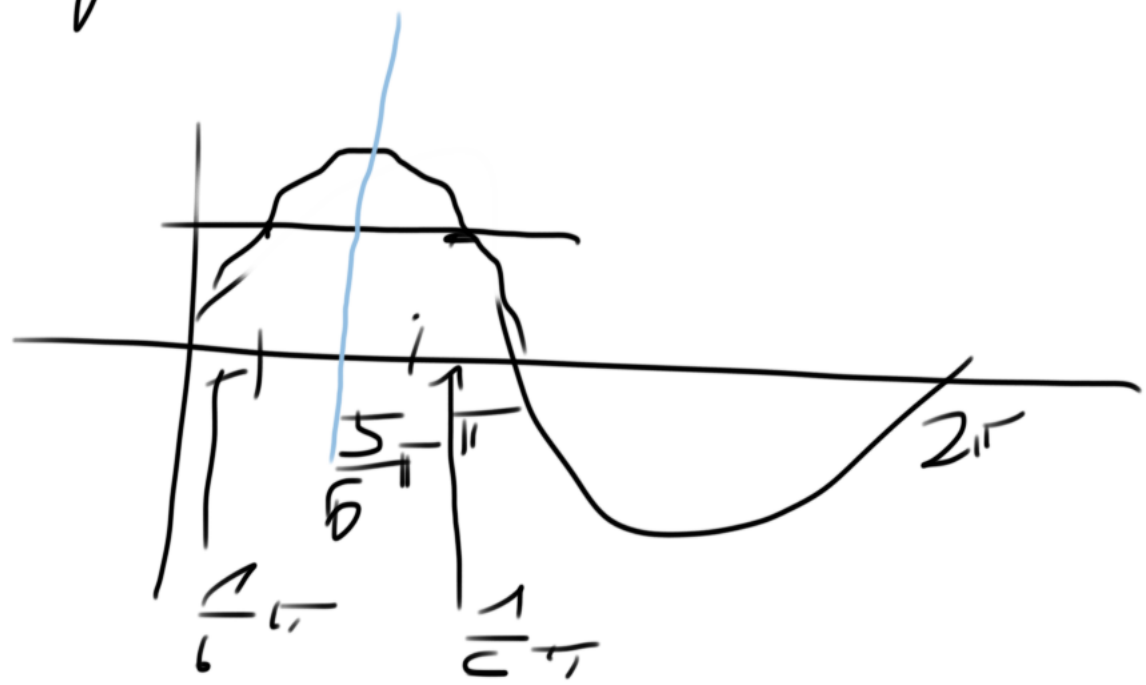
$$\frac{x}{180} = \frac{11}{12}$$

$$x = \frac{11}{12} \cdot 180$$

$$x = 11 \cdot 15 = 165^\circ$$ ✓

# Vyčíslete / spočtete

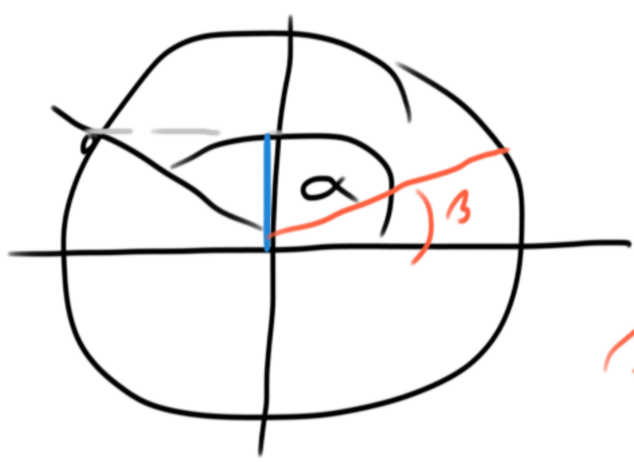
$\sin \frac{5}{6}\pi =$




$\sim \frac{\pi}{6}$

$30°$

$$\sin\left(\frac{5}{6}\pi\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$




sin

$\beta = \frac{\pi}{6}$

$$\sin(\alpha) = \sin(\beta)$$
$$= \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

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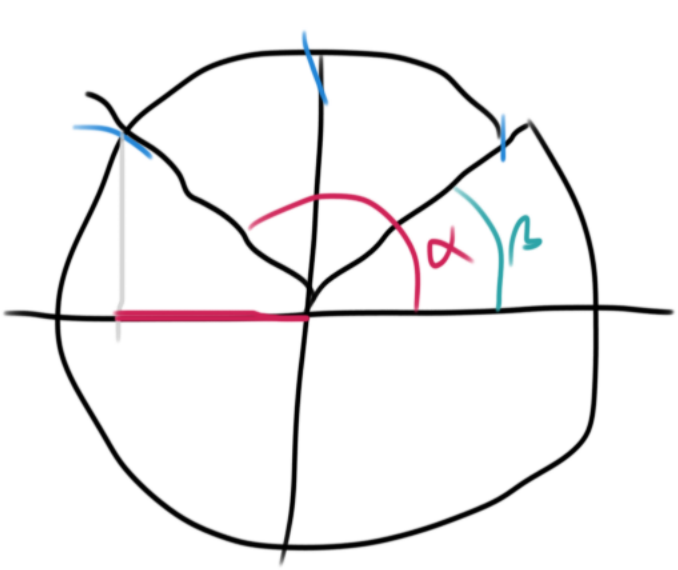
$$\cos \frac{3}{4}\pi = -\underbrace{\cos \frac{\pi}{4}}_{\frac{\sqrt{2}}{2}} = -\frac{\sqrt{2}}{2} \quad \checkmark$$

$\frac{\pi}{4} \sim 45°$

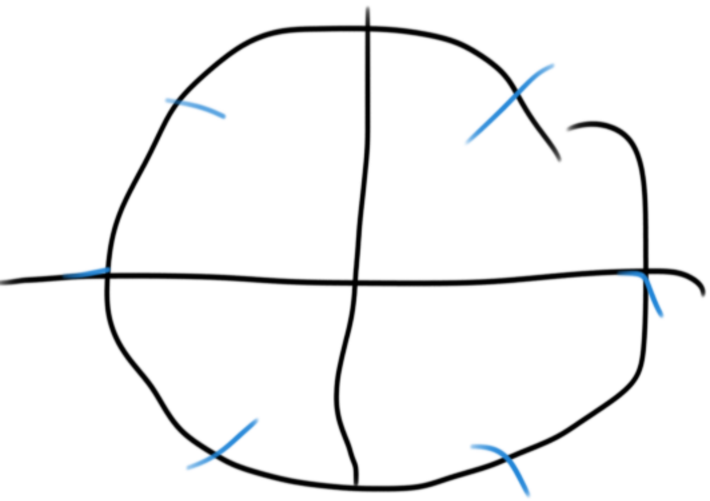



$\cos\alpha = -\cos\beta$

$\beta = \frac{\pi}{4}$

$\sin\left(\frac{15}{3}\pi\right)$

$T = 2\pi$



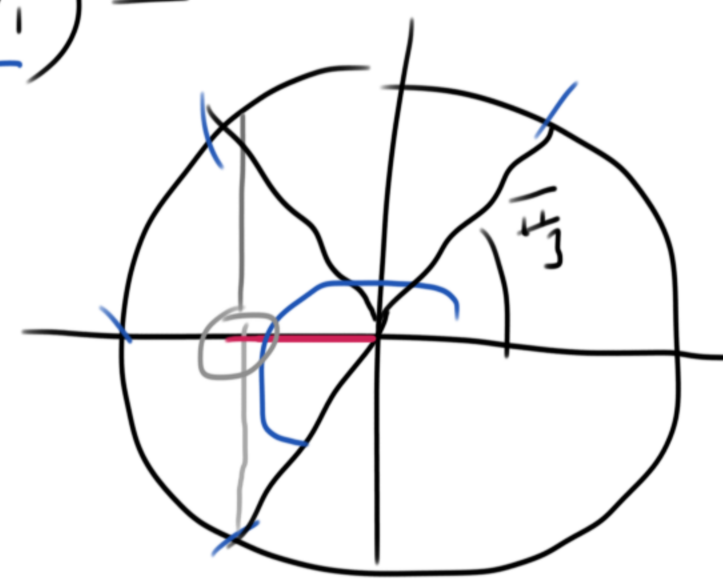
$\sin\left(\frac{15}{3}\pi\right) = \sin(5\pi)$

$= \sin(\pi + 2 \cdot 2\pi) =$

$= \sin(\pi) = 0$

$\cos(240°) = \cos\left(\underbrace{\frac{4}{3}\pi}\right) =$

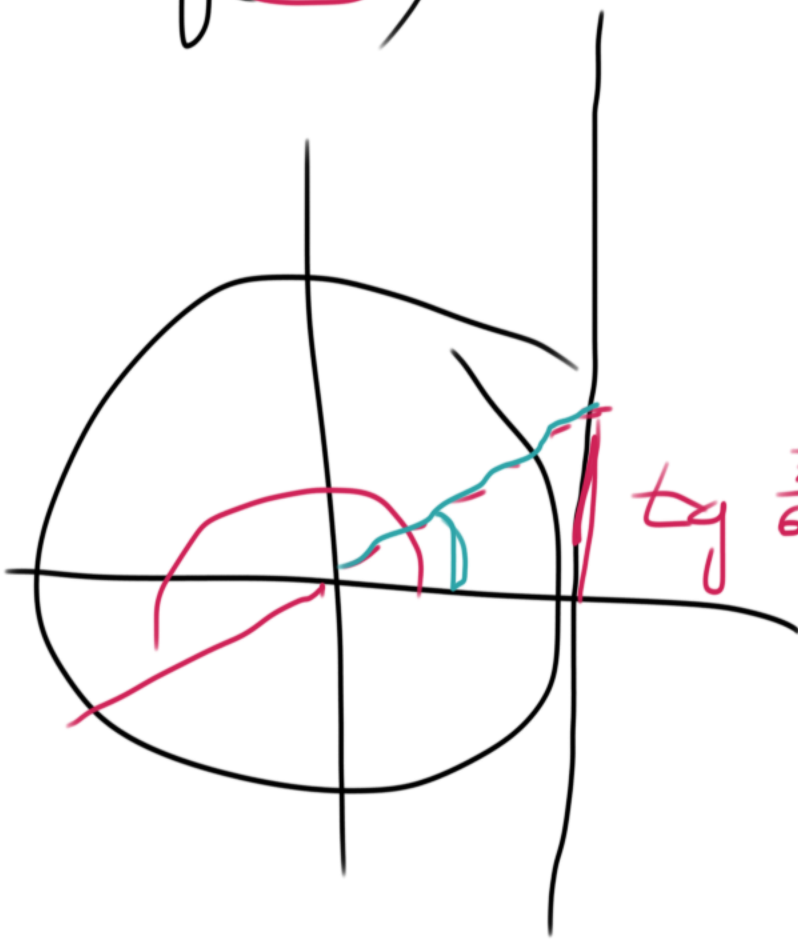
$240° = 4 \cdot 60° \sim 4 \cdot \frac{\pi}{3}$

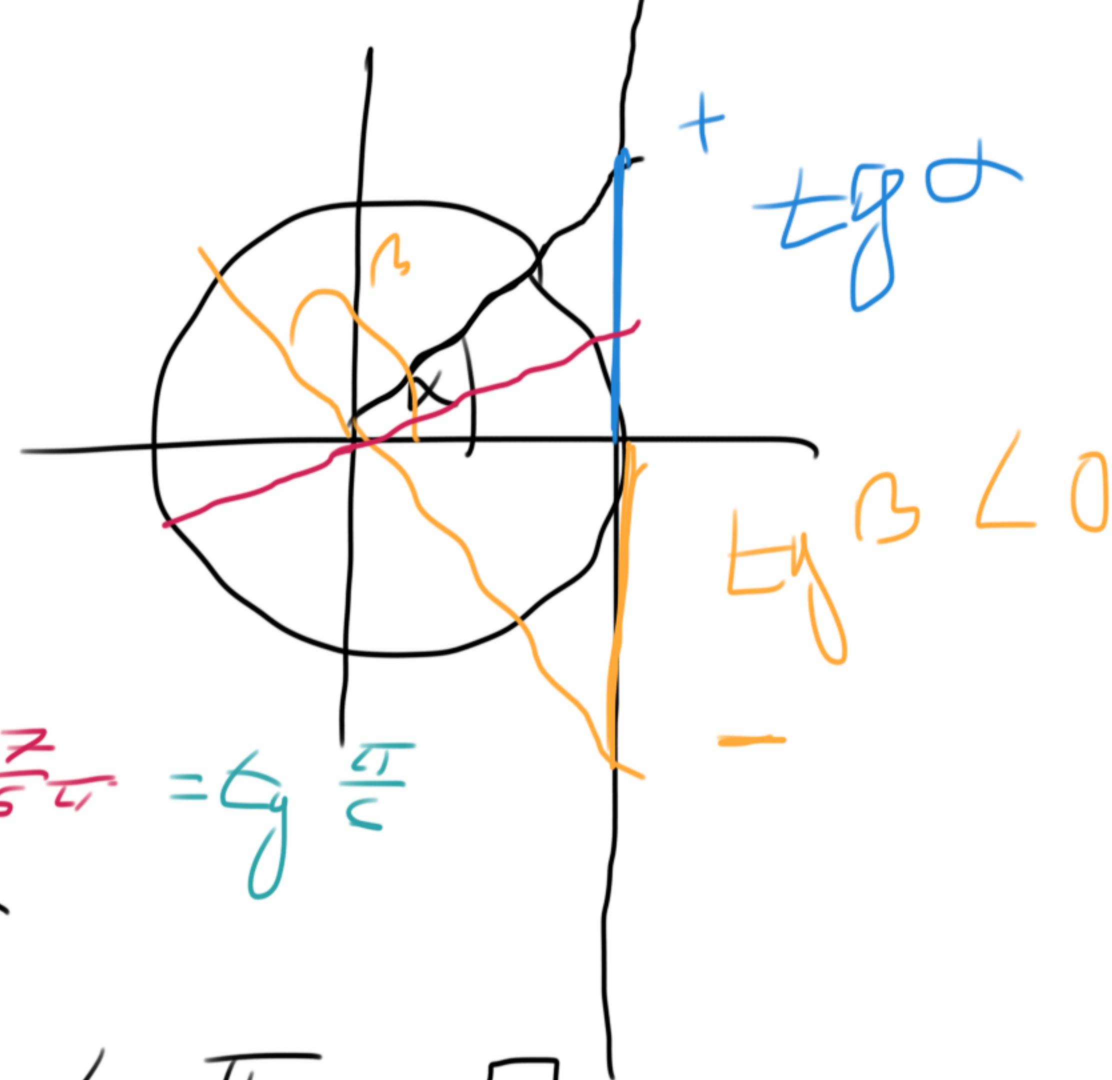



$= \cos\left(\frac{2}{3}\pi\right) = -\cos\left(\frac{\pi}{3}\right)$

$= -\frac{1}{2}$

$$\operatorname{tg}\left(\frac{7}{6}\pi\right) > 0$$




$$\operatorname{tg}\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

$$\operatorname{tg}\frac{\pi}{6} = \frac{\sin\frac{\pi}{6}}{\cos\frac{\pi}{6}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \underline{\frac{\sqrt{3}}{3}}$$

$$\operatorname{tg}\frac{7}{6}\pi = \text{——}\,\|\,\text{——} = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$$

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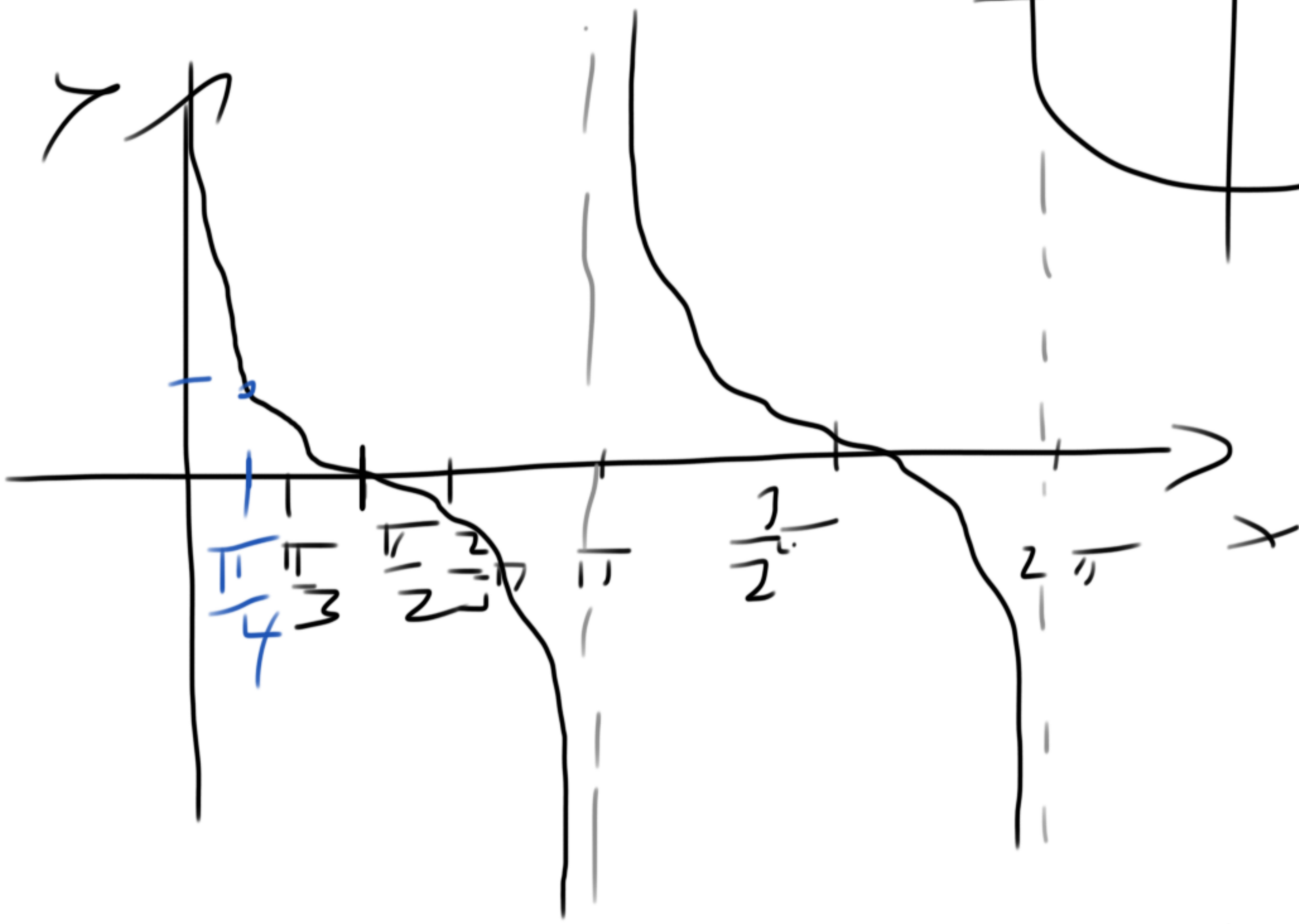
$$\operatorname{tg}\frac{7}{6}\pi = \operatorname{tg}\left(\frac{\pi}{6}+\pi\right) \underset{\substack{\uparrow \\ T=\pi}}{=} \operatorname{tg}\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{3}$$

$\mathrm{cotg}\ \frac{2}{3}\pi$


$$\mathrm{cotg}\ \frac{2}{3}\pi < 0$$

$$\mathrm{cotg}\ \frac{\pi}{3} = \frac{\sqrt{3}}{3}$$

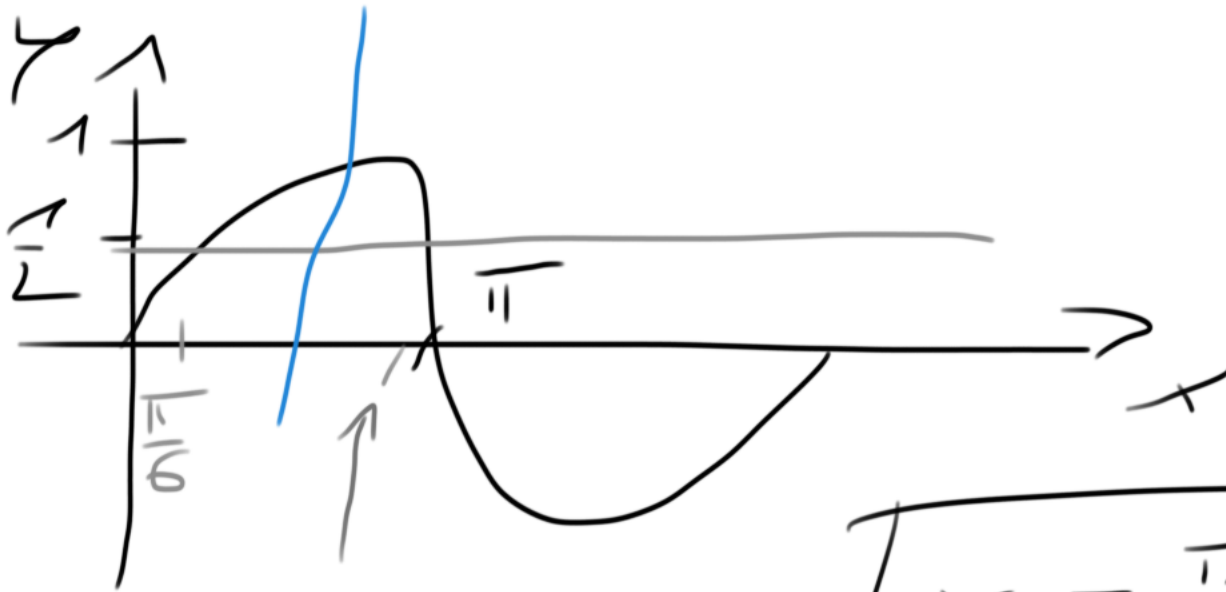
$$\frac{\cos\frac{\pi}{3}}{\sin\frac{\pi}{3}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$\mathrm{cotg}\ \frac{2\pi}{3} = -\frac{\sqrt{3}}{3}$$

$$\mathrm{cotg}\left(\frac{2\pi}{3}\right) = \mathrm{cotg}\left(\frac{2\pi}{3} - \pi\right) = \mathrm{cotg}\left(-\frac{\pi}{3}\right)$$

$$= -\mathrm{cotg}\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{3}$$

$\sin x = \frac{1}{2}$ Řešte v $\mathbb{R}$




$\pi - \frac{\pi}{6} = \frac{5}{6}\pi$

$$x_1 = \frac{\pi}{6} + 2\pi \cdot k$$

$$x_2 = \frac{5}{6}\pi + 2\pi \cdot k$$

$$k \in \mathbb{Z}$$

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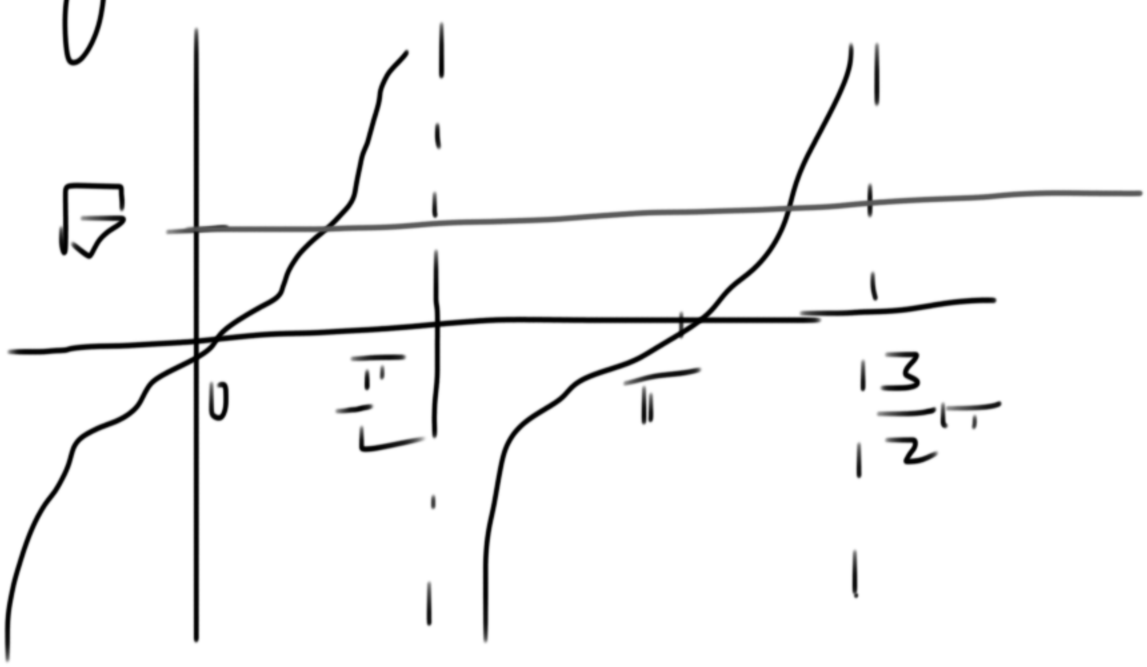
$\operatorname{tg} x = \sqrt{3}$

$\operatorname{tg} \frac{\pi}{6} = \sqrt{3}$




$$x = \frac{\pi}{6} + k \cdot \pi$$

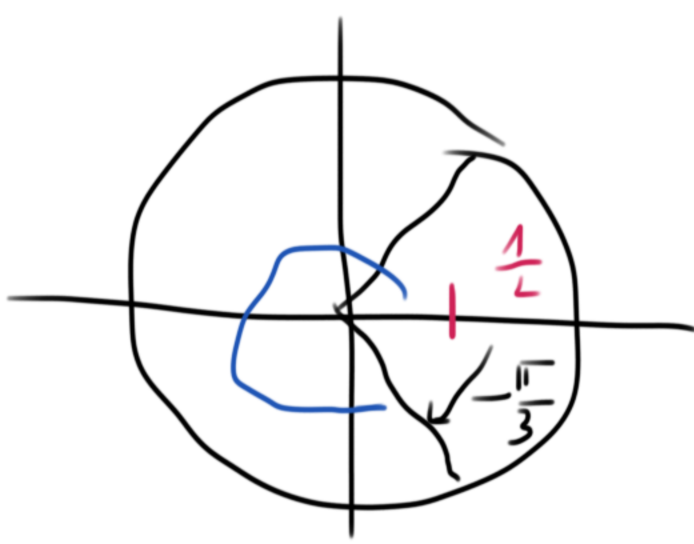
$$k \in \mathbb{Z}$$

$$2 \cdot \frac{\cos x + 1}{3} - \frac{4\cos x - 1}{2} = 1 - \cos x \quad / \cdot 6$$

$$4\cos x + 4 - 12\cos x + 3 = 6 - 6 \cdot \cos x$$

$$-2\cos x = -1$$

$$\cos x = \frac{1}{2}$$




$$\boxed{x_1 = \frac{\pi}{3} + k \cdot 2\pi}$$

$$\cos(x_2) = \frac{1}{2}$$

$$\cos\left(-\frac{\pi}{3}\right) = \cos\left(-\frac{\pi}{3} + 2\pi\right)$$

$$= \cos\left(\frac{5}{6}\pi\right)$$

$$\boxed{x_2 = \frac{5}{6}\pi + k \cdot 2\pi}$$

$$\frac{5\sin x+4}{10\sin x+4}=1$$ Řešit

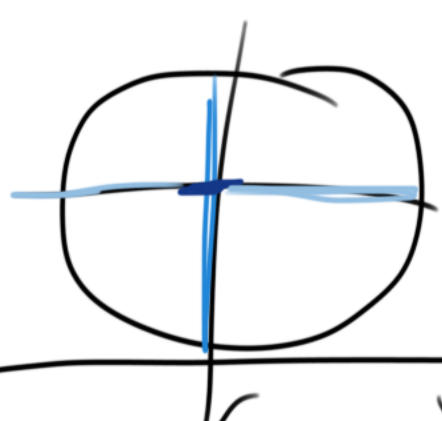
$$10\sin x+4\neq 0$$

$$\sin x\neq \underbrace{-\frac{2}{5}}$$

$$5\sin x+4=10\sin x+4$$

$$5\sin x=0$$

$$\sin x=0$$



$$x=k\cdot\pi \qquad k\in\mathbb{Z}$$

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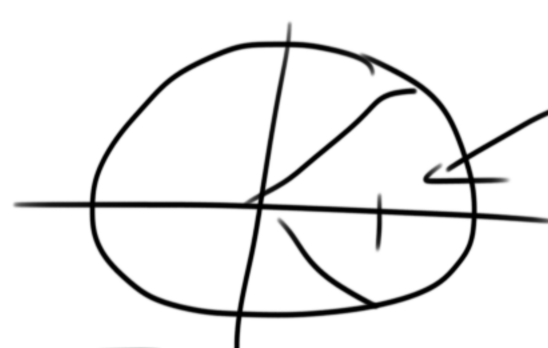
$$\cos(10x)=\frac{\sqrt{2}}{2}$$



$$\underbrace{\frac{\sqrt{2}}{2}}$$

substituce

$$z=10x$$

$$\cos z=\frac{\sqrt{2}}{2}$$

$$z_1=\frac{\pi}{4}+k\cdot 2\pi$$

$$z_2=\frac{7}{4}\pi+k\cdot 2\pi$$

$$10x_1=\frac{\pi}{4}+k\cdot 2\pi$$

$$\boxed{x_1=\frac{\pi}{40}+k\cdot\frac{1}{5}\pi}$$

$$\boxed{x_2=\frac{7}{40}\pi+k\cdot\frac{1}{5}\pi}$$

$$\sin\left(4x - \frac{\pi}{3}\right) = \sqrt{2}$$

$$z = 4x - \frac{\pi}{3}$$

$$\underline{\sin(z) = \sqrt{2}}$$ $\sqrt{2} \doteq 1{,}41 > 1$

$\sin, \cos\ H = \langle -1, 1 \rangle$

$\underline{NR}$

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$$\cot g\left(\frac{\pi}{6} - x\right) = \frac{\sqrt{3}}{3}$$

$$z = \frac{\pi}{6} - x$$ $\cot g\, z = \frac{\cos z}{\sin z}$

$$\cot g(z) = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$$

$$z = \frac{\pi}{3} + k\cdot\pi$$

$$\frac{\pi}{6} - x = \frac{\pi}{3} + k\cdot\pi \quad / -\frac{\pi}{6}$$

$$-x = \frac{\pi}{6} + k\cdot\pi \quad / \cdot(-1)$$

$$\boxed{x = -\frac{\pi}{6} + k\cdot\pi \quad \forall k \in \mathbb{Z}}$$

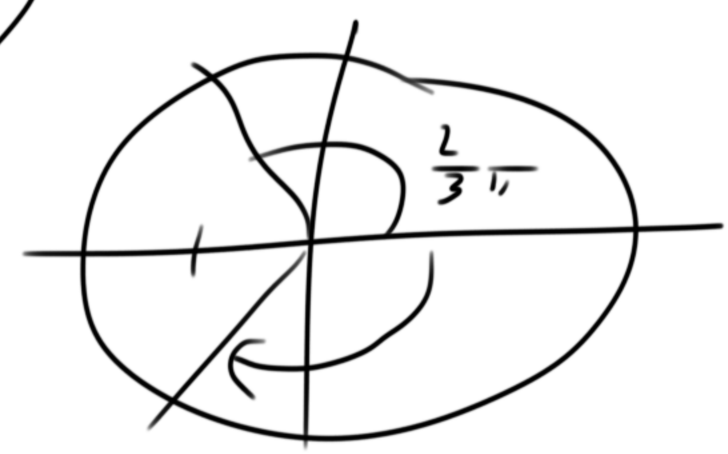
$$2\cos^2 x - \cos x - 1 = 0$$ Reste vR

$$y = \cos x$$

$$2y^2 - y - 1 = 0$$

$$y_{1,2} = \frac{1 \pm \sqrt{1+8}}{4} = \begin{cases} 1 \\ -\frac{1}{2} \end{cases}$$

$$\left( \begin{array}{l} \cos x_1 = 1 \\ \cos x_2 = -\frac{1}{2} \end{array} \right)$$




$$\boxed{\begin{array}{l} x_1 = k \cdot 2\pi \\ x_2 = \frac{2}{3}\pi + k \cdot 2\pi \\ x_3 = \frac{4}{3}\pi + k \cdot 2\pi \end{array}}$$ $\forall k \in \mathbb{Z}$

$$2\cdot\sin^2 x + 3\cos x = 0$$

$$\cos^2 x + \sin^2 x = 1 \quad \forall x \in \mathbb{R}$$

$$\sin^2 x = \underline{1-\cos^2 x}$$

$$2(1-\cos^2 x) + 3\cos x = 0$$

$$-2\cos^2 x + 3\cos x + 2 = 0$$

$$y = \cos x$$

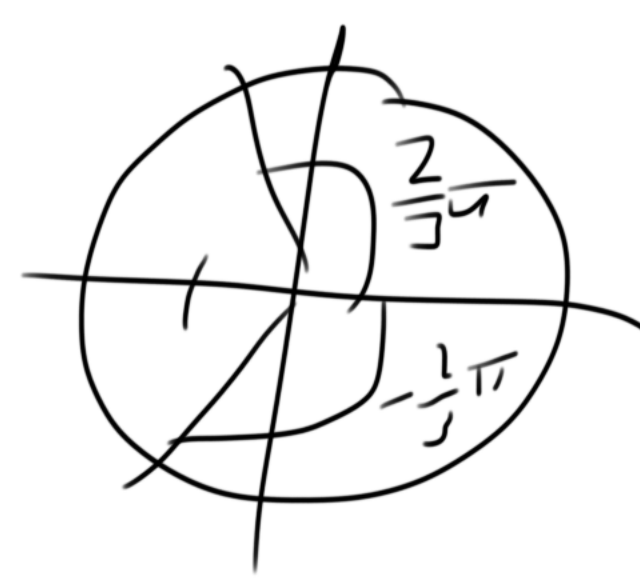
$$-2y^2 + 3y + 2 = 0$$

$$y_{1,2} = \frac{-3 \pm \sqrt{9+16}}{-4} = \begin{cases} \dfrac{-3+5}{-4} = -\dfrac{1}{2} \\[2ex] \dfrac{-3-5}{-4} = +2 \end{cases}$$

$$\cos x_2 = 2 > 1 \quad \text{NR}$$

$$\cos x_1 = -\frac{1}{2}$$

$$\boxed{\begin{aligned} x_1 &= \frac{2}{3}\pi + 2\pi\cdot k \\ x_2 &= \frac{4}{3}\pi + 2\pi\cdot k \end{aligned}}$$

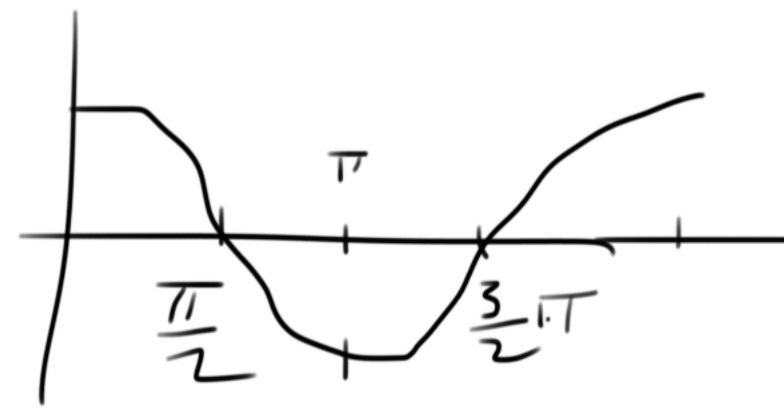


$\cos\left(\frac{5}{2}x\right)=0$

Řešte v $\langle 0,\pi\rangle$

$\cos(z)=0$

$z=\frac{\pi}{2}+k\cdot\pi$




$\frac{5}{2}x=\frac{\pi}{2}+k\cdot\pi$

$x=\frac{\pi}{5}+k\cdot\frac{2}{5}\pi \quad \forall k\in\mathbb{Z}$

chyba

v $\langle 0,\pi\rangle$:

$$\boxed{\begin{aligned} x&=\frac{\pi}{5}\\ x&=\frac{3}{5}\pi\\ x&=\frac{5}{5}\pi=\pi \end{aligned}}$$

Spočítejte $\sin(2x)$ jestliže víte, že $\sin(x) = \frac{2}{5}$

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$$\sin x = \frac{2}{5}$$

$$\sin(2x) = \underbrace{2}\,\underbrace{\sin x}\,\underbrace{\cos x}$$

$$\sin^2 x + \cos^2 x = 1$$

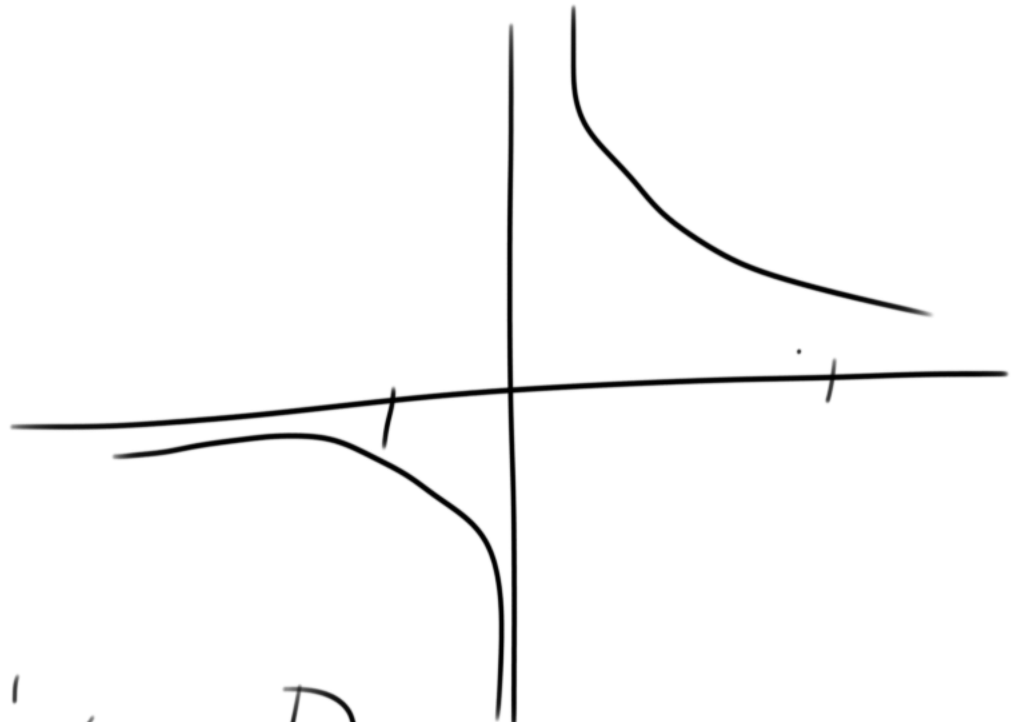
$$\cos x = \sqrt{1 - \sin^2 x}$$

$$= \sqrt{1 - \frac{4}{25}}$$

$$= \sqrt{\frac{21}{25}} = \frac{\sqrt{21}}{5}$$

$$\sin(2x) = 2 \cdot \frac{2}{5} \cdot \frac{\sqrt{21}}{5} = \frac{4\sqrt{21}}{25}$$ ✓

$\frac{1}{x}$



klesající na $D_f$

klesající na $(-\infty, 0)$

a na $(0, \infty)$

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$\cos\left(\frac{7}{12}\pi\right)$

$\cos(x \pm y) = \cos x \cos y \mp \sin x \cdot \sin y$

$\frac{\pi}{4}$ $\frac{\pi}{3}$

$\frac{7}{12}\pi - \frac{4}{12}\pi + \frac{3}{12}\pi = \frac{\pi}{3} + \frac{\pi}{4}$

$\cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \cos\frac{\pi}{3} \cdot \cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \frac{1}{2}\,\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$