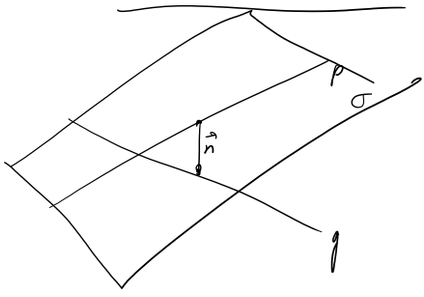
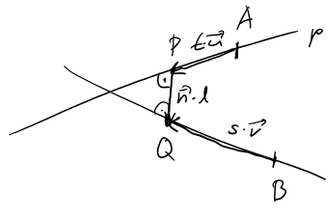
Vzdálenost mimoběžek





$|pq| = |PQ| = |t \cdot \vec{n}|$

$Q = A + t \cdot \vec{u} + t \cdot \vec{n}$

$Q = B + s \cdot \vec{v}$

$A + t \cdot \vec{u} \cdot t \cdot \vec{n} = B + s \cdot \vec{v}$

proložíme např. p rovinu σ:

$p \subset \sigma$, $\vec{n}$ je normálový vektor roviny

$\rightarrow q \parallel \sigma$

$\rightarrow |pq| = |q\sigma| = |B\sigma|$

$p = \{[2,3,5+t],\ t \in \mathbb{R}\}$ $\quad q = \{[0,1+k,4-k],\ k \in \mathbb{R}\}$

$\vec{u} = (0,0,1)$

$\vec{v} = (0,1,-1)$

$\vec{n} = \vec{u} \times \vec{v} = (-1,0,0)$ $\quad \vec{n} \cdot \vec{u} = 0$, $\vec{n} \cdot \vec{v} = 0$

$A + t \cdot \vec{u} + t \cdot \vec{n} = B + k \cdot \vec{v}$

$2 - t = 0 \rightarrow t = 2$

$3 = 1 + k \rightarrow k = 2$

$5 + t = 4 - k \rightarrow t = -3$

$$|pq| = |t \cdot \vec{n}| = 2 \cdot 1 = 2 \quad \checkmark$$

$\sigma: ax + by + cz + d = 0 \xrightarrow{\vec{n}_\sigma = \vec{n}} \sigma: -x + d = 0$

$A = [2,3,5] \in p \Rightarrow A \in \sigma: -2 + d = 0$

$d = 2$

$\Rightarrow \boxed{\sigma: -x + 2 = 0}$

$$|pq| = |q\sigma| = |B\sigma| = \frac{|0+2|}{1} = 2 \quad \checkmark$$

$B = [0,1,4] \in q$

Najděte všechny body, které mají od přímky p vzdálenost $\frac{\sqrt{10}}{5}$ a od přímky q $2\sqrt{5}$.

$p: 3x - y = 0$

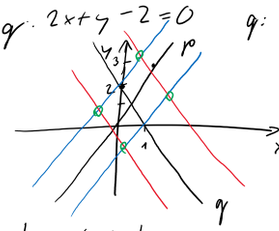
$y = 3x$

$A = \{X \in \mathbb{R}^2: |Xp| = \frac{\sqrt{10}}{5}\}$

$B = \{X \in \mathbb{R}^2: |Xq| = 2\sqrt{5}\}$

Hledáme $A \cap B$

$q: 2x + y - 2 = 0$ $\quad q: x + \frac{y}{2} = 1$



Vzdálenost bodu od přímky: $d = \frac{|ax + by + c|}{\sqrt{a^2 + b^2}}$ $\quad X = [x,y]$

$$|Xp| = \frac{|3x - y|}{\sqrt{10}} = \frac{\sqrt{10}}{5} \qquad |Xq| = \frac{|2x + y - 2|}{\sqrt{5}} = 2\sqrt{5}$$

$|3x - y| = 2$ $\quad |2x + y - 2| = 10$

$\rightarrow |2x + y - 2| = 5 \cdot |3x - y|$

tudy ne

$|3x - y| = 2$

$3x - y \geq 0$ $\quad -3x + y < 0$

$3x - y = 2$ $\quad -3x + y = 2$

I: $y = 3x - 2$ $\quad$ II: $y = 3x + 2$

$|2x + y - 2| = 10$

$2x + y - 2 = 10$ $\quad -2x - y + 2 = 10$

III: $y = -2x + 12$ $\quad$ IV: $y = -2x - 8$

$I \cap III$: $\quad y = 3x - 2$, $y = -2x + 12$ $\ominus$

$0 = 5x - 14$

$x = \frac{14}{5}$

$y = \frac{42}{5} - \frac{10}{5} = \frac{32}{5}$

$P_{13} = [\frac{14}{5}, \frac{32}{5}]$

$I \cap IV$: $\quad y = 3x - 2$, $y = -2x - 8$ $\ominus$

$0 = 5x + 6$

$x = -\frac{6}{5}$

$y = -\frac{18}{5} - \frac{10}{5} = -\frac{28}{5}$

$P_{14} = [-\frac{6}{5}, -\frac{28}{5}]$

$II \cap III$: $\quad y = 3x + 2$, $y = -2x + 12$ $\ominus$

$0 = 5x - 10$

$x = 2$

$y = 8$

$P_{23} = [2,8]$

$II \cap IV$: $\quad y = 3x + 2$, $y = -2x - 8$ $\ominus$

$0 = 5x + 10$

$x = -2$

$y = -4$

$P_{24} = [-2,-4]$