Celá čísla

$n \in \mathbb{N} \longrightarrow \exists -n$:

$\mathbb{Z} = \mathbb{N} \cup \{0\} \cup \{-n ; n \in \mathbb{N}\}$

$= \{-5, -4, \ldots, 0, 1, 2, \ldots\}$

$10 - 20 = -10 \in \mathbb{Z}$

Rac. čísla

$\mathbb{Q} = \{\frac{p}{q} ; p, q \in \mathbb{Z}, q \neq 0\}$

$q \in \mathbb{N}$

$\mathbb{Q} = \{$ čísla s konč. nebo periodic. des. rozvoj $\}$

$0{,}1 \quad 0{,}256 \quad -5{,}17$

$0{,}\overline{3} \qquad 0{,}211\overline{3} \qquad 0{,}\overline{123} = 0{,}123123\ldots$

$$\frac{a}{b}, \frac{c}{d} \in \mathbb{Q} \Rightarrow \frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a \cdot d}{b \cdot c} \in \mathbb{Q}$$

$$p = \frac{a}{100} \qquad \% = \frac{1}{100}$$

120 kg, 60 kg

$$\frac{60}{120} \cdot 100 = 50\,\%$$

$p \qquad q \in \mathbb{Q}$

$$\frac{p+q}{2}, \frac{p+q}{3}$$

Reálná č.

$\mathbb{R} = \{$nekonč. a neperiod. des. rozvoj$\} \cup \mathbb{Q}$

$\mathbb{R} \setminus \mathbb{Q}$ ... iracionální

$3{,}1415\ldots$ $\quad \pi, e$

$\sqrt{2}, \sqrt[4]{6^{-7}}$

$\sqrt{-1} \notin \mathbb{R}$ $\quad \mathbb{C}$

$a \in \mathbb{R}^+ \Rightarrow \sqrt{a} \in \mathbb{R}$

$a \in \mathbb{R}$

$$a + 0 = a$$

$$a \cdot 1 = a$$

$$a + (-a) = 0$$

$$a \cdot \frac{1}{a} = 1$$

$$a + (b + c) = (a + b) + c$$

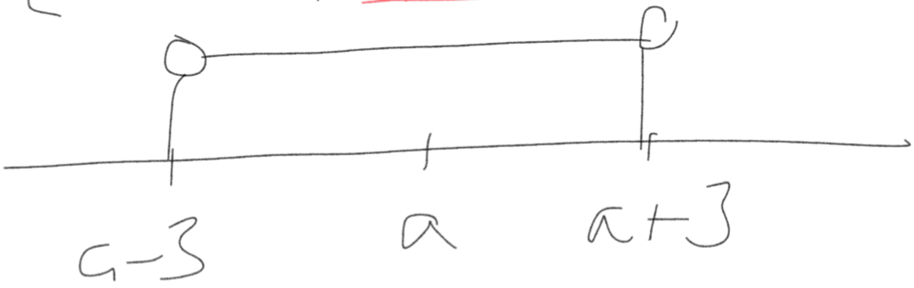
$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

$|-2| = 2 \qquad |2| = 2$

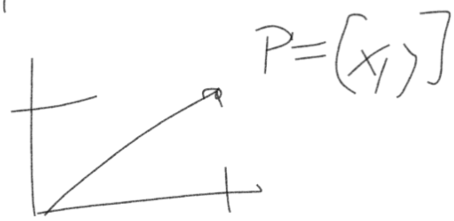
$|x-5|$ — vzd. x od. čísla 5

$M = \{x \in \mathbb{R}; \underline{|x-a| < 3}\}$



$\mathbb{R}^n = \mathbb{R} \times \mathbb{R} \times \dots \mathbb{R}$

$\mathbb{R}^2 \qquad (x, y)$



$\frac{x+2}{x+\sqrt{y}}$ x^2+3 $(\sqrt{2}-\sqrt{3})$

$\underbrace{a, b, x, y}_{\text{proměnné}}$ $\frac{\pi, e, 2}{\text{konstanty}}$

$x-y$ $y + + \sqrt{2}$ $x+(2-1)$

zlomky

mocniny a odmocniny

polynomy

$\sin 2x = 2 \cdot \sin x \cdot \cos x$

Zlomky

$\frac{1}{3}$ ← čitatel
← jmenovatel

$$\frac{x^{1}+1}{x-1} \cdot \frac{\sqrt{x}}{\sqrt[3]{xy}}$$

Sl. zlomky $$\frac{\frac{a}{b}}{\frac{c}{d}} \neq \frac{a}{\frac{b}{\frac{c}{d}}}$$

$\pm$ $$\frac{a}{b} \pm \frac{c}{d} = \frac{a \cdot d + c \cdot b}{b \cdot d}$$

$\cdot$ $$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$$

$:$ $$\frac{a}{b} : \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}$$

Krácení $$\frac{a \cdot \cancel{b}}{c \cdot \cancel{b}} \qquad \cancel{\frac{a}{b} + \frac{b}{c}}$$

Rozšíření $$\frac{a}{b} \cdot 1 = \frac{a}{b} \cdot \frac{c}{c} = \frac{a \cdot c}{b \cdot c}$$

Usměrňování: $$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Mocniny

$$a^r = \underbrace{a \cdot a \cdot \ldots \cdot a}_{r} \qquad a \in \mathbb{R},\ r \in \mathbb{N}$$

a .. mocněnec/základ

r .. mocnitel/exponent

$$a^r \cdot a^s = \underbrace{a \cdot a \cdot \ldots \cdot a}_{r}\ \underbrace{a \cdot a \cdot \ldots \cdot a}_{s} = a^{r+s}$$

$$\frac{a^r}{a^s} = a^{r-s} \qquad \frac{1}{a^s} = a^{-s}$$

$$\left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$$

$$a^0 = 1 \quad \forall a \in \mathbb{R} \setminus \{0\}$$

0^0 = není definována $(0^0 = 1)$

$$1^r = 1 \qquad a^{1,5}$$

$$\boxed{a^r \qquad \forall a, r \in \mathbb{R}}$$

Odmocniny

$$\sqrt[n]{a} = x \iff x^n = a$$

$$\left(\sqrt{2}\right)^2 = 2$$

$$\sqrt[n]{a} = a^{\frac{1}{n}} \Longrightarrow \sqrt[r]{a \cdot b} = \sqrt[r]{a} \cdot \sqrt[r]{b}$$

$$\sqrt{14400} = \sqrt{144 \cdot 100} = \sqrt{144} \cdot \sqrt{100}$$

$$= 12 \cdot 10 = 120$$

$$\sqrt{2x^2} = \sqrt{2}\sqrt{x^2} = \sqrt{2} \cdot x$$

! $a^r + b^r \neq (a+b)^r$

Podmínky

$\frac{a}{b}$ $\quad b \neq 0 \quad$ $\frac{1}{x-1}$ $\quad x \neq 1$

$\sqrt{-2} \notin \mathbb{R}$ $\quad \boxed{\sqrt{x} \quad x \geq 0}$

$\sqrt[3]{x} \quad x \in \mathbb{R} \quad \sqrt[3]{-27} = -3$ liché OK

Polynomy

Polynom st. n $\qquad n \in \mathbb{N}$

$$a_n x^n + a_{n-1} x^{n-1} + \dots a_1 x^1 + a_0 = p_n(x)$$

$$\text{st } p(x) = n$$

$$a_i, \; i = 0, \dots n \in \mathbb{R} \quad a_n \neq 0$$

$x^3 - 2x + 1$ st. 3

$x^2 - 2$

$n = 0 \qquad p_0(x) = a_0$

$n = 1 \qquad p_1(x) = a_0 + a_1 x$

$n = 2 \qquad p_2(x) = a_0 + a_1 x + a_2 x^2$

$$p_n(x) = \sum_{i=0}^{n} a_i x^i = a_0 x^0 + a_1 x^1 + \dots$$

$\pm$

$$p(x) \pm q(x) = \sum_{i=0}^{n} a_i x^i \pm \sum_{i=0}^{n} b_i x^i$$

$$= \sum_{i=0}^{n} (a_i \pm b_i) x^i$$

$p(x) = x^2 + 1 \qquad q(x) = x^3 + x^2$

$$p + q = x^3 + 2x^2 + 1$$

násobení:

$$(x^2+x+1)\cdot(x-1) = \underline{x^3-x^2}+x^2-x+x-1$$
$$= x^3-1$$

dělení

$$p(x) = x^2+1 \qquad p(2) = 4+1 = 5$$

$p(a) = 0 \qquad a$ ... kořen polynomu

$p_n(x)$ má n kořenů $\in \mathbb{C}$

$$x_{1,2} = \frac{-b \pm \sqrt{\underline{b^2-4ac}}}{2a} \qquad ax^2+bx+c$$

$<0 \rightarrow x_{1,2} \notin \mathbb{R}$

$$x^2-3x+2 = (x-1)\underbrace{(x-2)}_{\text{kořenový činitel}}$$

$x_1 = 1$
$x_2 = 2$

$$(x-x_1)(x-x_2)\cdots(x-x_n)$$

$$ax^2+bx+c = a\left(x^2+\frac{b}{a}x+\frac{c}{a}\right) = a(x-x_1)\cdot(x-x_2)$$

$$-\frac{b}{a} = x_1+x_2 \qquad \frac{c}{a} = x_1\cdot x_2$$

$$\frac{x-1}{x^2-3x-2} = \frac{\cancel{x-1}}{\cancel{(x-1)}(x-2)} = \frac{1}{x-2}$$

$$x^2-3x-2 \neq 0$$

$$x \neq 1,2$$

$$\boxed{\begin{aligned}(a\pm b)^2 &= a^2-2a\cdot b+b^2\\ (a+b)(a-b) &= a^2-b^2\\ (a\pm b)(a^2\mp ab+b^2) &= a^3-b^3\end{aligned}}$$

!

$$(a+b)^n = \sum_{i=0}^{n}\binom{n}{i} a^{n-i} b^{i} \qquad \binom{n}{i} = \frac{n!}{(n-i)!\,i!}$$

$n=3$

0					1					
1				1		1				
2			1		2		1			
3		1		3		3		1		
4	1		4		6		4		1	

$$(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$$

$$(a-b)(a^n+a^{n-1}b+a^{n-2}b^2+\ldots+a\cdot b^{n-1}+b^n) =$$

$$= a^{n+1}-b^{n+1}$$

$$ax^2+bx+c \longrightarrow \bar{a}\underbrace{(x-\bar{b})}_{\text{dvojčlen}}{}^2+\bar{c}$$

čtverec

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$x^2 \underbrace{-3}_{2b}x + 2 = \left(\underbrace{x}_{a} \underbrace{+}_{+} \underbrace{\left(-\frac{3}{2}\right)}_{b}\right)^2 + 2 \underbrace{-\left(-\frac{3}{2}\right)^2}_{-b^2}$$

$$\left(x-\frac{3}{2}\right)^2 - \frac{1}{4}$$