Václav Alt

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heslo: nemamradlekorici

Průběžný test: 18.11. 18:00 - 20:00 čtvrtek
27.11. 9:00 - 11:00 sobota

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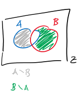
- Množiny
- Číselné obory a vlastnosti čísel
- algebraické výrazy
- Rovnice a nerovnice

$A \cap B$, $A \cup B$, $A \setminus B$, ...

$A = \{1, 2, 3\}$ $B = \{2, 3, 4\}$

$A \setminus B = \{x \in A;\ x \notin B\}$

$A \setminus B = \{1\}$



Zadání množiny pomocí charakt. vlastnosti

$M = \{x \in Z;\ V(x)\}$

Z ... základní množina $V(x)$ .. nějaký výrok o x

$A = \{n \in \mathbb{N};\ n^2 \leq 16\} = \{1, 2, 3, 4\}$

$B = \{x \in \mathbb{R};\ x^2 \leq 16\} = \langle -4, 4 \rangle$ Uzavř. int $[\ ,\ ]$ $\langle\ ,\ \rangle$

$\langle a, b \rangle = \{x \in \mathbb{R};\ a \leq x \leq b\}$ Otevřený $(\ ,\ )$

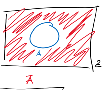
× $\langle 1, 3 \rangle = \{1, 2, 3\}$ ×

$D_f = (1, \infty)$

Doplněk množiny A

$\bar{A} = \{x \in Z;\ x \notin A\} = Z \setminus A$

A'

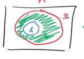


doplněk mn. A vzhledem k B

$\bar{A}_B = B \setminus A$

$B = A \cup \bar{A}_B$

$\bar{A}$



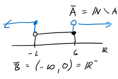
$\bar{A} \sim B$

Obecně $Z = A \cup \bar{A}$

$A = \{n \in \mathbb{N};\ n^2 \leq 16\}$ $\bar{A} = \{n \in \mathbb{N};\ n^2 > 16\}$

$\bar{A} = \mathbb{N} \setminus A = \{5, 6, 7, \dots\}$

$A = (-2, 6\rangle$



$\bar{A} = (-\infty, -2\rangle \cup (6, \infty)$

$B = \mathbb{R}_0^+ = [0, \infty)$ $\bar{B} = (-\infty, 0) = \mathbb{R}^-$

Bookhit, Množiny 3.6

Určete doplněk množiny B v mn. A.

$A = \mathbb{R}$ ✓ $B = \{x \in \mathbb{R};\ \sqrt{x^2} = -x\}$

$\sqrt{a} \geq 0$ $\forall a \in \mathbb{R}^+$ odmocnina je vždy nezáporná!

$x^2 \geq 0$ $\forall x \in \mathbb{R}$ $(-3)^2 = 9$

$\sqrt{x^2}$ ... definované $\forall x \in \mathbb{R}$ $\sqrt{(-3)^2} = \sqrt{9} = 3$

kladné číslo $\sqrt{(3)^2} = 3$

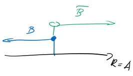
záporné číslo $\sqrt{(-3)^2} = 3 = -(-3)$

$\sqrt{x^2} = |x|$ $|x| = \begin{cases} x; & x \geq 0 \\ -x; & x < 0 \end{cases}$

$\sqrt{x^2} = -x \implies |x| = -x$ $\forall x \leq 0$

$B = \{x \in \mathbb{R};\ x \leq 0\} = \mathbb{R}_0^-$

$\bar{B} = \{x \in \mathbb{R};\ x > 0\} = \mathbb{R}^+$



Algebraické výrazy

$(x-)$ není AV

Zjednodušte výraz a určete podmínky

$\left(\frac{1}{a+b} + \frac{1}{a-b}\right) \cdot \left(\frac{1}{a} - \frac{1}{b}\right) = \frac{a-b+a+b}{(a+b)\cdot(a-b)} \cdot \frac{b-a}{ab}$

$= \frac{2\cancel{a}}{(a+b)\cancel{(a-b)}} \cdot \frac{-\cancel{(a-b)}}{\cancel{a}b} = \boxed{-\frac{2}{b\cdot(a+b)}}$

$\boxed{\begin{matrix} a, b \neq 0 \\ a \neq \pm b \end{matrix}}$

$b \neq 0$
$a \neq -b$
2 podm. chybí!

- $\sqrt[5]{\left(\frac{c^{1/2} \cdot c^{-1/3}}{c^{-5/6}}\right)^{-3}} = \left(\frac{c^{1/6}}{c^{-5/6}}\right)^{-\frac{3}{5}} = \left(c^{6/6}\right)^{-\frac{3}{5}} = c^{-3/5} = \frac{1}{c^{3/5}} = \sqrt[5]{\frac{1}{c^3}}$

$\sqrt{c} \implies c \geq 0$ $f(x) = \sqrt{x}$ $D_f = \mathbb{R}_0^+$ $\frac{1}{\square} \to c \neq 0 \implies \boxed{c > 0}$

Vzorečky: $(A \pm B)^2 = A^2 \pm 2A \cdot B + B^2$

$A^2 - B^2 = (A+B) \cdot (A-B)$

$ax^2 + bx + c \implies a \cdot (x - x_1) \cdot (x - x_2)$ $x_{1,2}$ kořeny

- $\frac{(\sqrt[4]{u} + \sqrt[4]{v})^2 + (\sqrt[4]{u} - \sqrt[4]{v})^2}{u - v} : \frac{2}{\sqrt{u} - \sqrt{v}} = \frac{2\sqrt{u} + 2\sqrt{v}}{u - v} : \frac{2}{\sqrt{u} - \sqrt{v}}$

$= \frac{\cancel{2}\cancel{(\sqrt{u} + \sqrt{v})}}{\cancel{(\sqrt{u}+\sqrt{v})} \cdot \cancel{(\sqrt{u}-\sqrt{v})}} \cdot \frac{\cancel{\sqrt{u} - \sqrt{v}}}{\cancel{2}} = 1$

$\boxed{\begin{matrix} u, v \geq 0 \\ u \neq v \end{matrix}}$

Rovnice a nerovnice

$ax^2 + bx + c = \bar{a} \cdot (x + \bar{b})^2 + \bar{c}$ Doplnění na čtverec dvojčlenu

$a, b, c \in \mathbb{R}$
$a \neq 0$

$\bar{a} = a$ $\bar{b} = \frac{b}{2a}$ $\bar{c} = c - \frac{b^2}{4a}$

$(A+B)^2 = A^2 + 2A \cdot B + B^2$

- $x^2 + 3x + 2 = \left(x + \frac{3}{2}\right)^2 + 2 - \left(\frac{3}{2}\right)^2 = \left(x + \frac{3}{2}\right)^2 - \frac{1}{4}$

A^2 $2A \cdot B$ $(A+B)^2$ $-B^2$ $A^2 - B^2$

$2 \cdot B = 3$
$B = 3/2$

$= \left(x + \frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2$

$= \left(x + \frac{3}{2} + \frac{1}{2}\right) \cdot \left(x + \frac{3}{2} - \frac{1}{2}\right)$

$= (x+2) \cdot (x+1)$

$\longrightarrow x_1 = -2$ $x_2 = -1$

Racionální nerovnice

~ obsahuje podíl polynomů

$\frac{(x+2)x}{x^2 - 3x}$ racionální výraz

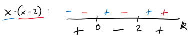
Řešte v $\mathbb{R}$:

$\frac{1}{x-2} < \frac{1}{x}$ $/ \cdot x \cdot (x-2)$

$x < x - 2$
$0 < -2$
NŘ

ŠPATNĚ

$2 < 5$ $/ \cdot (-1)$
$-2 < -5$ ŠPATNĚ
$-2 > -5$



1) $x \in (-\infty, 0) \cup (2, \infty)$: $x \cdot (x-2) > 0$

$x < x - 2$
$0 < -2$ NŘ.

2) $x \in (0, 2)$: $x \cdot (x-2) < 0$

$x > x - 2$
$0 > -2$

PLATÍ $\forall x \in (0, 2)$

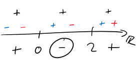
$\implies \boxed{x \in (0, 2)}$

- alternativa $\frac{1}{x-2} < \frac{1}{x}$ $/ -\frac{1}{x}$

$\frac{1}{x-2} - \frac{1}{x} < 0$

$\frac{x - (x-2)}{x(x-2)} < 0$

$\frac{2}{x(x-2)} < 0$



$\boxed{x \in (0, 2)}$

Rovnice s odmocninou

- $\sqrt{x} + x = 2$ $/^2$ $\xrightarrow{/-x}$ $\sqrt{x} = 2 - x$ $/^2$

$x + 2 \cdot \sqrt{x} \cdot x + x^2 = 4$ $/^2$ $x = 4 - 4x + x^2$

$x^4 + \dots = 16$ $x^2 - 5x + 4 = 0$

$(x-4) \cdot (x-1) = 0$

$\implies x_1 = 1$ $x_2 = 4$

Zk: $x = 1$: $LS = \sqrt{1} = 1$ $PS = 2 - 1 = 1$ $LS = PS$

$x = 4$: $LS = \sqrt{4} = 2$ $PS = 2 - 4 = -2$ $LS \neq PS$

$\boxed{x = 1}$

$\sqrt{x}$: $x \geq 0$

- $\overset{1}{\sqrt{2x+7}} + \overset{2}{\sqrt{x-5}} = \overset{3}{\sqrt{3x+2}}$ $/^2$

1) $x \geq -\frac{7}{2}$
2) $\boxed{x \geq 5}$
3) $x \geq -\frac{2}{3}$

$(\cancel{2x}\cancel{+7}) + 2 \cdot \sqrt{(2x+7) \cdot (x-5)} + (\cancel{x}\cancel{-5}) = \cancel{3x}\cancel{+2}$

$\cancel{2} \cdot \sqrt{(2x+7) \cdot (x-5)} = 0$ $/^2$

$(2x+7) \cdot (x-5) = 0$

$\boxed{x_1 = -\frac{7}{2}}$ není kořen $x_2 = 5$

Zk: $x = 5$: $LS = \sqrt{17} > 0$ $PS = \sqrt{17}$ $LS = PS$

$\boxed{x = 5}$