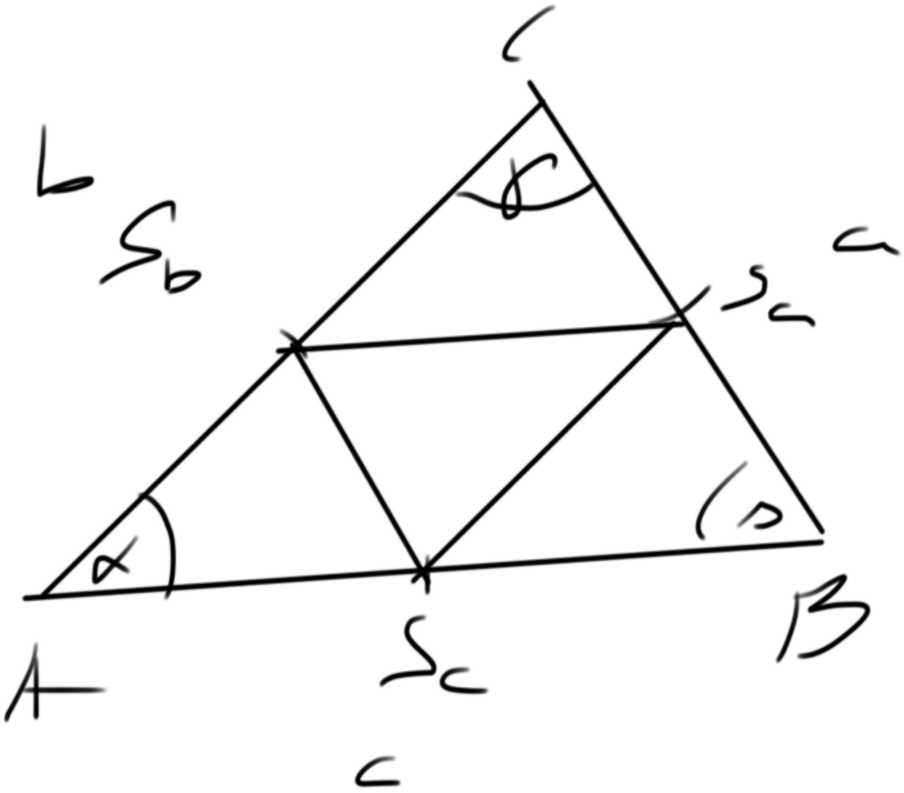
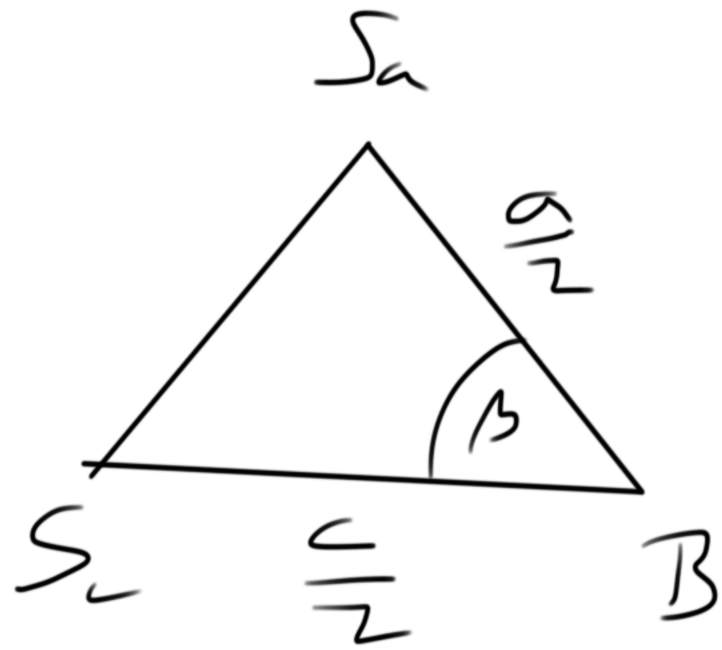
1) $\triangle ABC$, střední příčky

$\rightarrow$ ukažte podobnost



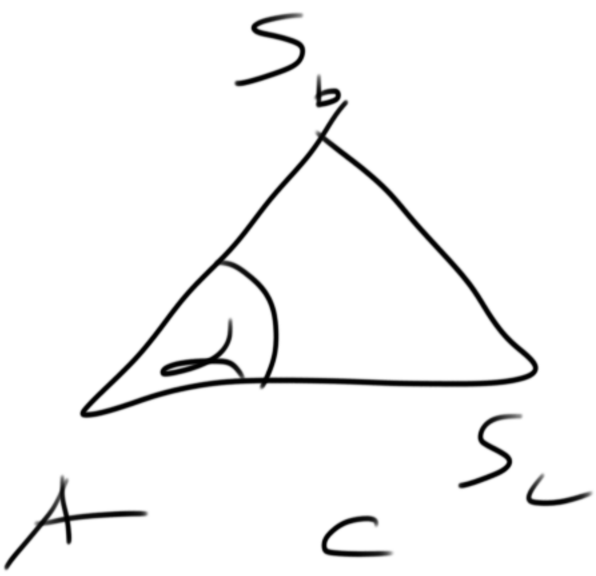


$\boxed{sus}$

k:
$$k = \frac{\frac{c}{2}}{c} = \frac{1}{2}$$
$$\frac{\frac{a}{2}}{a} = \frac{1}{2} = k$$

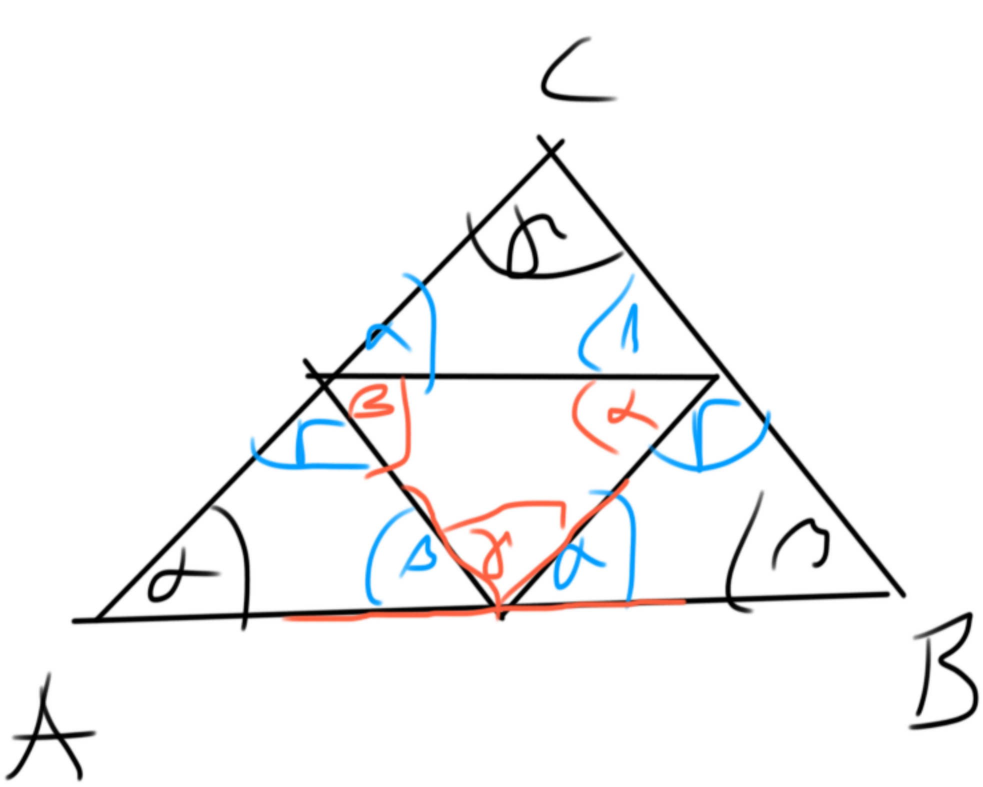
$\rightarrow \triangle BS_cS_a \sim \triangle ABC$

$k = \frac{1}{2}$



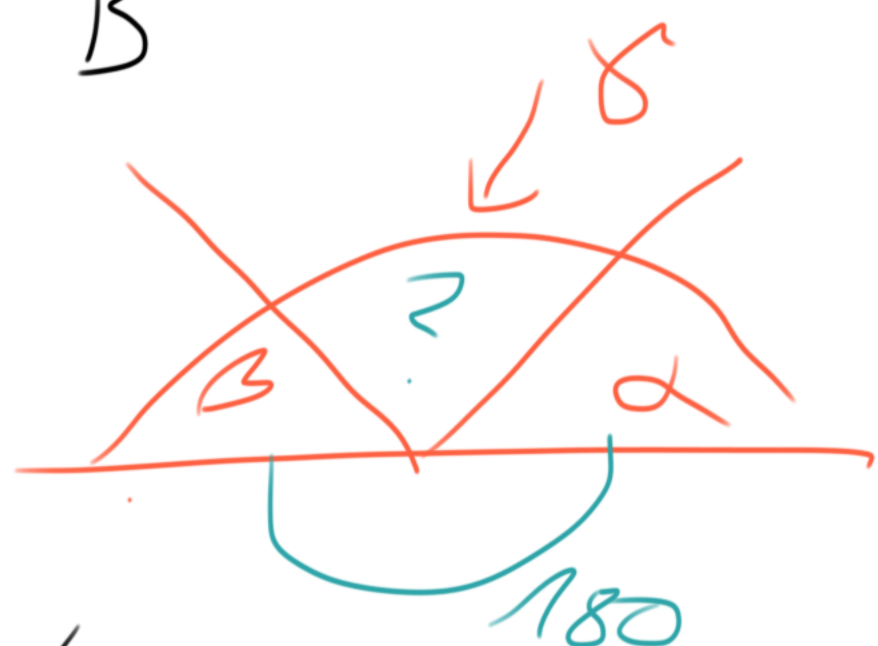
stejně

$\triangle AS_cS_b \sim \triangle ABC$ $\quad$ $\triangle CS_bS_a \sim \triangle ABC$



$$\alpha+\beta+\gamma=180^\circ$$

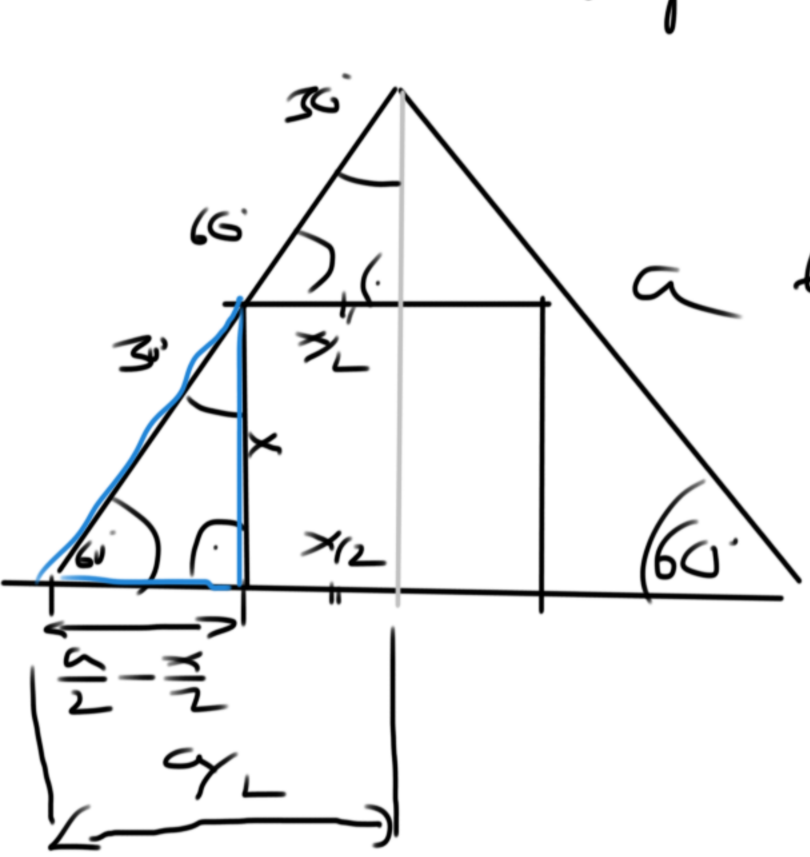
z podobnosti

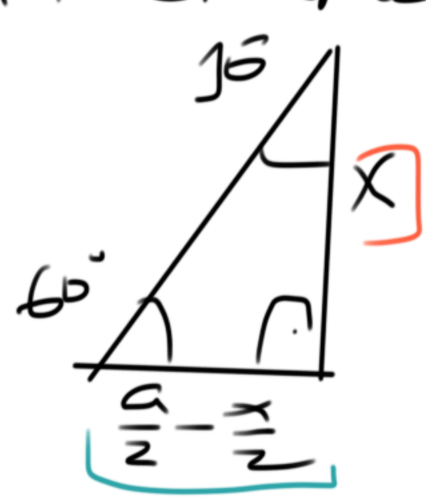


$\underline{uu} \Rightarrow$ podobné

$\triangle ABC$, rovnostranný a

vepsán čtverec x $\qquad x = ?$





$$\operatorname{tg}\alpha = \frac{\text{protilehlá}}{\text{přilehlá}}$$

$$\operatorname{tg} 30^\circ = \frac{\sqrt{3}}{3}$$

$$\frac{\sqrt{3}}{3} = \frac{\frac{a}{2}-\frac{x}{2}}{x} \quad / \cdot 6x$$

$$2\sqrt{3}x = 3a - 3x$$

$$x(2\sqrt{3}+3) = 3a$$

$$\boxed{x = \frac{3a}{2\sqrt{3}+3}}$$

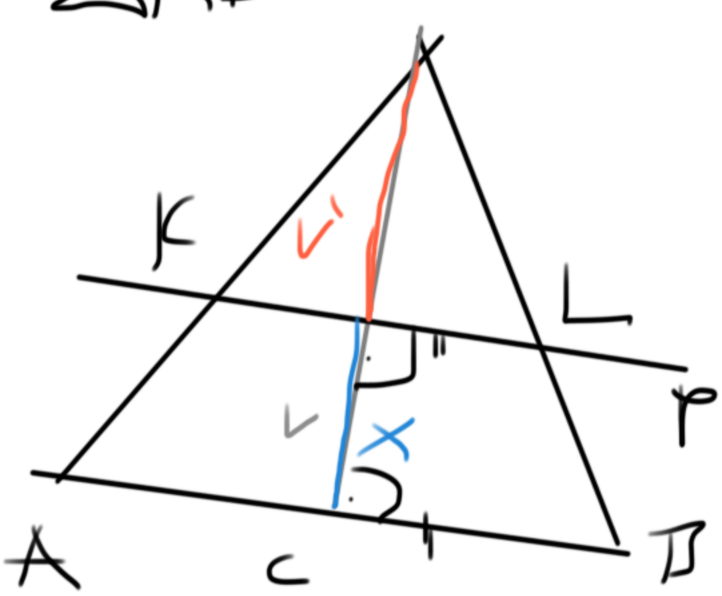
$$x = \frac{3a}{2\sqrt{3}+3} \qquad a^2-b^2=(a+b)\cdot(a-b)$$

$$x = a\cdot\frac{3}{\underbrace{2\sqrt{3}+3}_{a+b}}\cdot\frac{2\sqrt{3}-3}{\underbrace{2\sqrt{3}-3}_{a-b}}$$

$$x = a\cdot\frac{3\cdot(2\sqrt{3}-3)}{12-9} = a(2\sqrt{3}-3)$$

$S_{ABC} = S$

$\triangle ABC$ c



$x = ?$: $S_1 = S_2$

$\triangle KLC \sim \triangle ABC \Rightarrow \exists k: \ |KC| = k\cdot|AC|$

$|LC| = k\cdot|BC|$

$S = \frac{1}{2}c\cdot v \qquad S_2 = S - S_1 = S - k^2\cdot S = S(1-k^2)$

$$\underbrace{S_1}_{1} = \frac{1}{2}c'\cdot v' = \frac{1}{2}(k\cdot c)\cdot(k\cdot v) = k^2\cdot\underbrace{\frac{1}{2}v\cdot c}_{S} = k^2\cdot S$$

$c' = k\cdot c \quad v' = k\cdot v$

$S \qquad S_1 = k^2 \cdot S \qquad S_2 = S(1-k^2)$

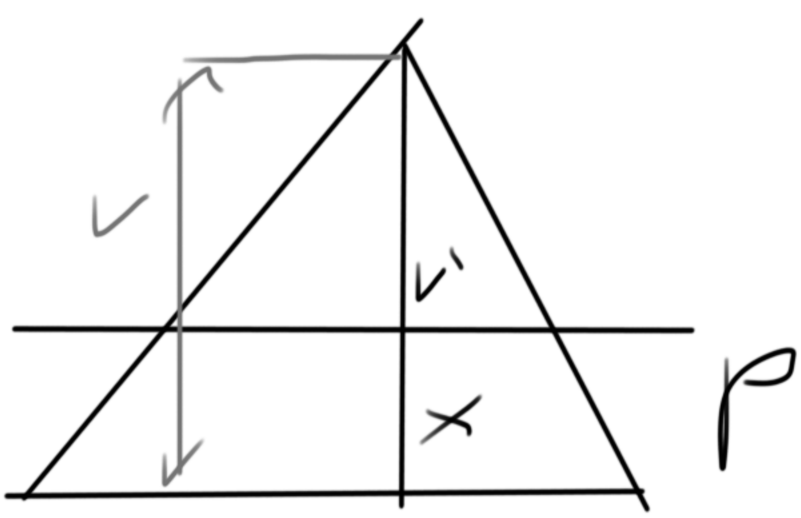
$$S_1 = S_2$$

$$k^2 \cdot \cancel{S} = \cancel{S} \cdot (1-k^2)$$

$$2k^2 = 1$$

$$k = \pm\sqrt{\tfrac{1}{2}}$$

nezajímá



$$v = v' + x$$

$$v' = k \cdot v$$

$$k = \sqrt{\tfrac{1}{2}}$$

$$x = v - v'$$

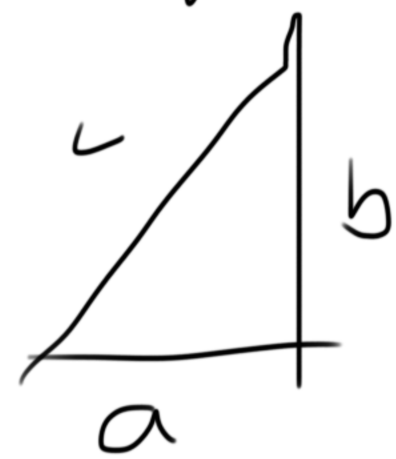
$$x = v - kv$$

$$x = v(1-k)$$

$$\boxed{x = v\left(1 - \sqrt{\tfrac{1}{2}}\right)}$$

Rozhodněte, zda △ je pravoúhlý

a) 5, 3, 4



$\underline{c^2 = a^2 + b^2}$,

5, 12, 13

9, 40, 41

$LS = 25$

$PS = 4^2 + 3^2 = 16 + 9 = 25$

$LS = PS$ ✓

b) 5, 1, 4

$LS = 25$

$PS = 1 + 16 = 17$

$LS \neq PS \implies$ △ není pravoúhlý

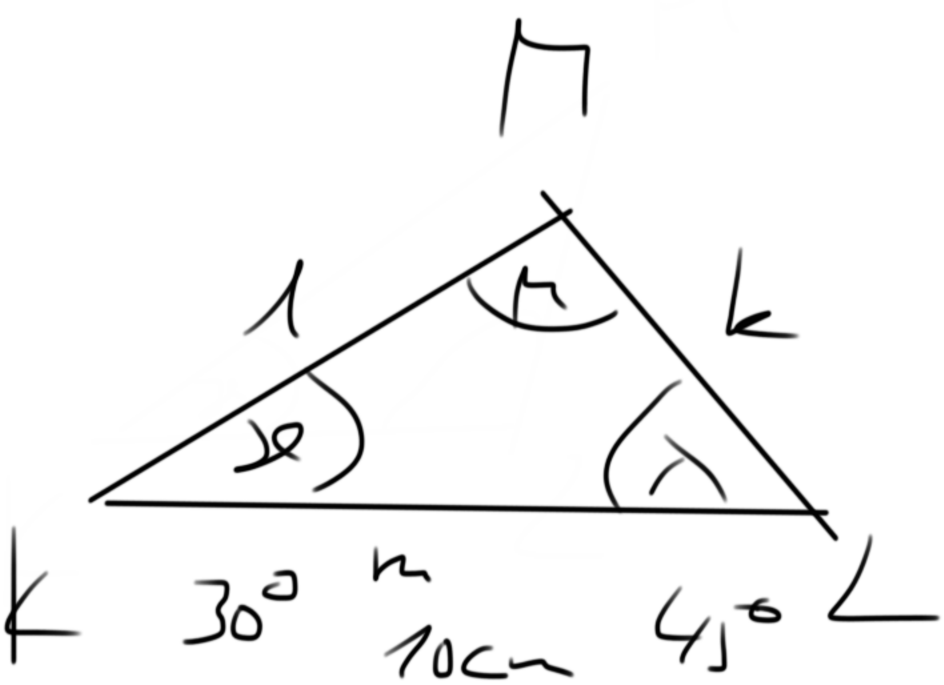
c) $\sqrt{5}, \underbrace{\sqrt{8}}, \sqrt{4}$

$LS = 8 \quad PS = 5 + 4 = 9 \quad LS \neq PS$

$\triangle KLM$ $\quad \varkappa = 30^\circ \quad \lambda = 45^\circ$

$\varkappa \lambda \mu \quad m = 10\text{ cm}$

$\mu = ? \quad k, l = ?$



$$\frac{\sin\varkappa}{k} = \frac{\sin\lambda}{l} = \frac{\sin\mu}{m}$$

$$\frac{k}{\sin\varkappa} = \frac{l}{\sin\lambda} = \frac{m}{\sin\mu}$$

$$\varkappa + \lambda + \mu = 180^\circ$$

$$\mu = 180^\circ - \varkappa - \lambda$$

$$\mu = 105^\circ$$

$\sin 105^\circ$

$60 + 45$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$$

$$\sin 105 = \sin(60+45) = \sin 60 \cdot \cos 45 + \sin 45 \cos 60$$

$$\frac{\sqrt{3}}{2} \quad \frac{\sqrt{2}}{2} \quad \frac{\sqrt{2}}{2} \quad \frac{1}{2}$$

$$= \frac{\sqrt{2}}{4}(\sqrt{3}+1)$$

$n = 10\,\text{cm} \qquad \kappa = 30^\circ \qquad \lambda = 45^\circ \qquad \mu = 105^\circ$

$\sin\kappa = \frac{1}{2} \qquad \sin\lambda = \frac{\sqrt{2}}{2} \qquad \sin\mu = \frac{\sqrt{2}}{4}(\sqrt{3}+1)$

$$\frac{k}{\sin\kappa} = \frac{n}{\sin\mu}$$

$$k = \frac{\sin\kappa}{\sin\mu} n = \frac{\frac{1}{2}}{\frac{\sqrt{2}}{4}(\sqrt{3}+1)} \cdot 10$$

$$= \frac{4 \cdot 5}{\sqrt{2}(\sqrt{3}+1)} = \frac{20}{\sqrt{2}(\sqrt{3}+1)} = k$$

$$\frac{l}{\sin\lambda} = \frac{n}{\sin\mu}$$

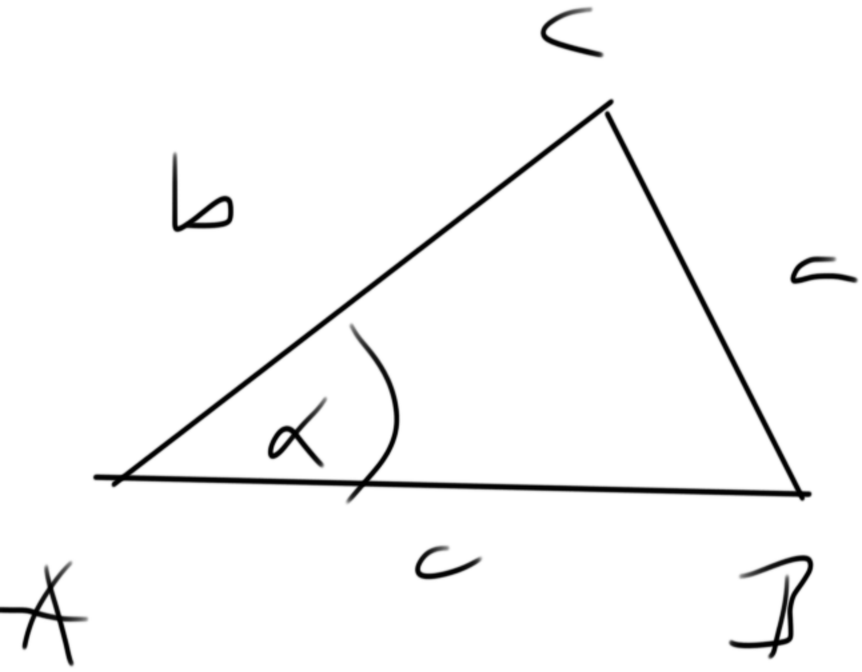
$$l = \frac{\sin\lambda}{\sin\mu} \cdot n = \frac{\frac{\cancel{\sqrt{2}}}{\cancel{2}}}{\frac{\cancel{\sqrt{2}}}{4}(\sqrt{3}+1)} \cdot \overset{5}{\cancel{10}}$$

$$= \frac{4 \cdot 5}{\sqrt{3}+1} = \underline{\frac{20}{\sqrt{3}+1}} = l$$

$$l = \frac{20}{\sqrt{3}+1} \cdot \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{20(\sqrt{3}-1)}{3-1} = \underline{10(\sqrt{3}-1)}$$

$\Delta ABC \quad a = 4\,cm \quad b = 5\,cm$

$\alpha = 45^\circ$



$$a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos\alpha$$

$$b^2 = a^2 + c^2 - 2 \cdot a \cdot c \cdot \cos\beta$$

$$(\sqrt{2} \cdot 5)^2 = 5^2 \cdot (\sqrt{2})^2$$
$$= 25 \cdot 2 = 50$$

$$\cos 45^\circ = \frac{\sqrt{2}}{2}$$

$$16 = 25 + c^2 - \cancel{2} \cdot 5 \cdot c \cdot \frac{\sqrt{2}}{\cancel{2}}$$

$$0 = c^2 - \sqrt{2} \cdot 5 \cdot c + 9$$

$$\frac{\sqrt{2} \cdot 5 \pm \sqrt{50 - 36}}{2} = \boxed{\frac{5\sqrt{2} \pm \sqrt{14}}{2}}$$

$\sqrt{2} \approx 1{,}5 \qquad \sqrt{14} \approx 3{,}5$

$$\begin{array}{r} 3{,}5 \\ 3{,}5 \\ \hline 175 \\ 105 \\ \hline 12{,}25 \end{array}$$

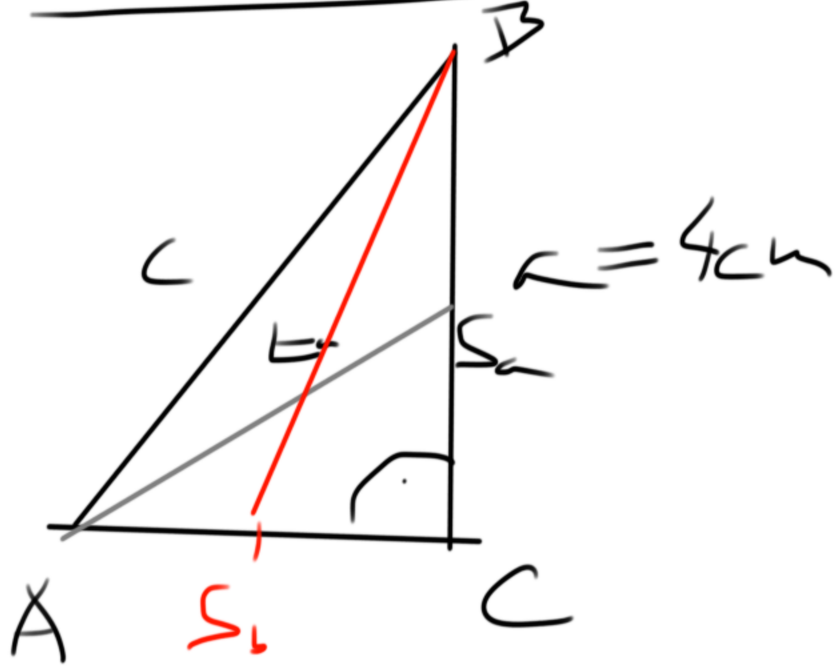
$$c_1 = \frac{5 \cdot 1{,}5 + 3{,}5}{2} = \frac{11}{2} = 5{,}5\,cm$$

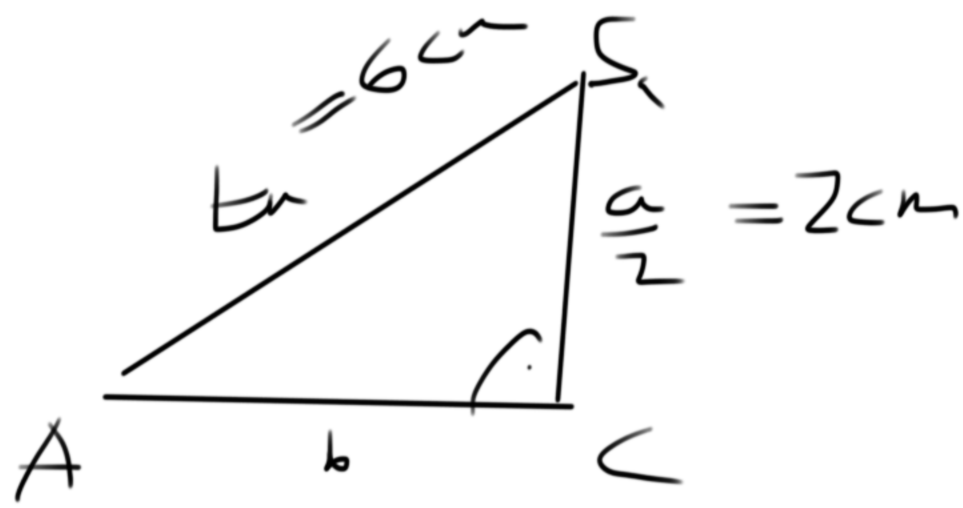
$$c_2 = \frac{7{,}5 - 3{,}5}{2} = 2\,cm$$

$\triangle ABC$, pravoúhlý, c přepona

$a = 4\,cm \qquad t_a = 6\,cm$

$t_b = ?$



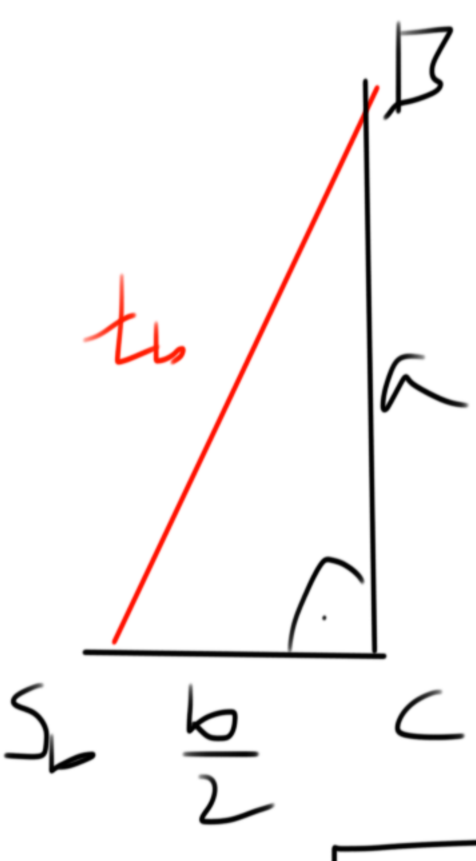


$$t_a^2 = b^2 + \left(\frac{a}{2}\right)^2$$

$$b = \sqrt{t_a^2 - \left(\frac{a}{2}\right)^2}$$

$$= \sqrt{36 - 4} = \sqrt{32}$$

$$\boxed{b = 4 \cdot \sqrt{2}}$$



$$t_b = \sqrt{a^2 + \left(\frac{b}{2}\right)^2} = \sqrt{16 + 8}$$

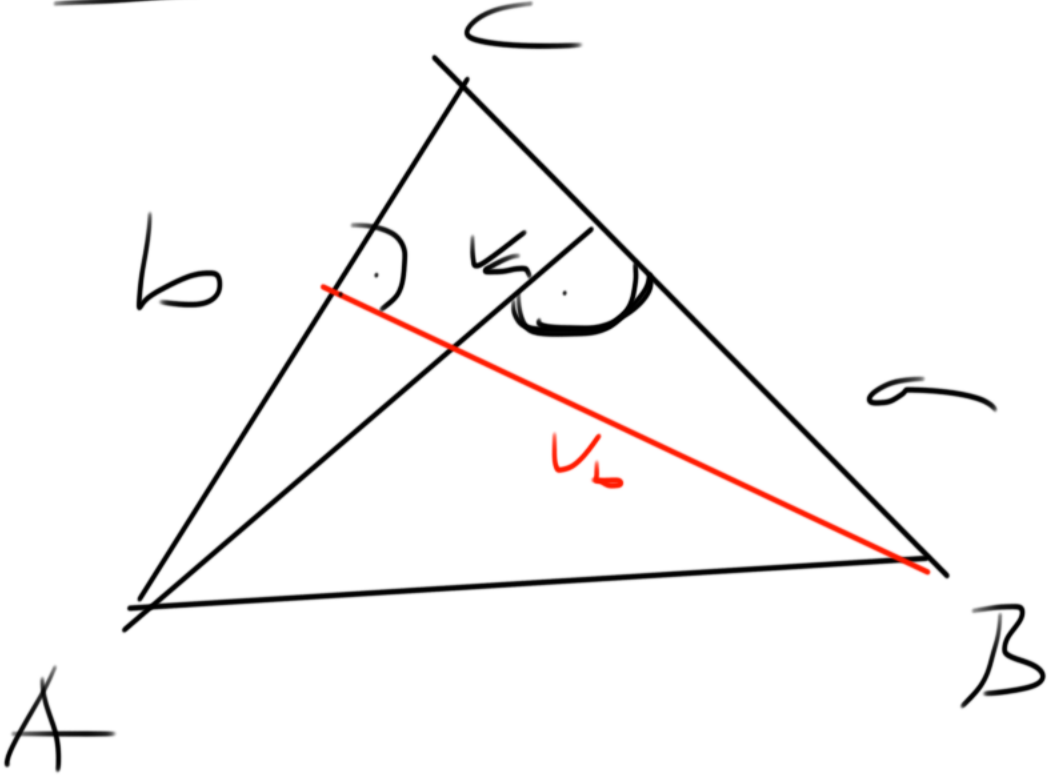
$$\left(2\sqrt{2}\right)^2 \qquad = \sqrt{24} = 2\sqrt{6}$$

$$\boxed{t_b = 2 \cdot \sqrt{6}}$$

$\triangle ABC \quad a = 5\,cm \quad b = 4\,cm$

$v_a = 2\,cm$

$S = ? \quad v_b = ?$



$S = \frac{1}{2} a \cdot v_a$

$= \frac{1}{2} 5 \cdot 2 = \underline{5\,cm^2}$

$S = \frac{1}{2} b\, v_b$

$5 = \frac{1}{2} 4 \cdot v_b$

$\boxed{v_b = \frac{5}{2}\,cm}$

---

délka: m, cm

plocha: m^2, cm^2 2. mocnina "čtvereční"

$S = \frac{1}{2} \underbrace{(a + v_a)}_{5cm + 3cm = 8cm}$ očividně nesmysl

$S = \frac{1}{2} a \cdot v_a = \frac{1}{2} 5cm \cdot 3cm = \frac{1}{2} 5 \cdot 3\,cm^2 = \frac{15}{2}\,cm^2$