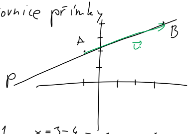
Napište všechny rovnice přímky



$A = [-1, 2] \quad B = [3, 4]$

$\vec{u} = \overrightarrow{AB} = (4, 2)$

$p: \; X = B + t \cdot \vec{u}$

$$\boxed{p: \begin{array}{l} x = 3 + t \cdot 4 \\ y = 4 + t \cdot 2 \end{array}}$$

$t = -1 \qquad \begin{array}{l} x = 3 - 4 = -1 \\ y = 4 - 2 = 2 \end{array} = A$

$p: \; ax + by + c = 0 \longrightarrow p: \; 2x - 4y + c = 0$

$\vec{u} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \quad \vec{n} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \qquad A: \; 2(-1) - 4 \cdot 2 + c = 0$

$c = 10$

$$\boxed{p: \; 2x - 4y + 10 = 0}$$

úsekový tvar: $\frac{2x}{-10} - \frac{4y}{-10} = 1$

$\boxed{p: \; \frac{x}{-5} + \frac{y}{\frac{5}{2}} = 1} \rightarrow \begin{array}{l} r = -5 \\ s = \frac{5}{2} \end{array}$

směrnicový tvar: $\boxed{p: \; y = \frac{1}{2}x + \frac{5}{2}} \qquad k = \frac{1}{2} = \tan \varphi$

$\varphi = \arctan\left(\frac{1}{2}\right)$

$\tan^{-1}$

Najděte parametr t odpovídající bodu $C \in q$

$q: A, \vec{u} \qquad A = [1, 1] \qquad C = [-5, -2]$

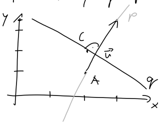
$\vec{u} = (2, 1)$

(tj. to samé jako: ověřte $C \in q$)

$q: \begin{array}{l} x = 1 + 2t \\ y = 1 + t \end{array} \qquad C: \begin{array}{l} -5 = 1 + 2t \xrightarrow{} t = -3 \qquad -5 = 1 - 6 \quad ✓ \\ -2 = 1 + t \longrightarrow \boxed{t = -3} \end{array}$

$D = [2, 1] \qquad D: \left. \begin{array}{l} 2 = 1 + 2t \longrightarrow t = \frac{1}{2} \\ 1 = 1 + t \longrightarrow t = 0 \end{array} \right\} \longrightarrow D \notin q$

Najděte obecnou rovnici přímky q tak, aby $q \perp p$ a $C \in q$.



$p: \begin{array}{l} x = 2 + t \\ y = 1 + \frac{5}{2}t \end{array} \qquad C = [2, 2]$

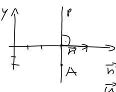
$\vec{u} \perp q \longrightarrow \vec{n}_q = \vec{u}$

$q: \; 1 \cdot x + \frac{5}{2}y + c = 0 \xrightarrow{C \in q} 1 \cdot 2 + \frac{5}{2} \cdot 2 + c = 0$

$c = -7$

$$\boxed{q: \; x + \frac{5}{2}y - 7 = 0}$$

Přímka p prochází bodem $A = [3, -2]$ kolmo k ose x. Zapište všechny rovnice.



$\vec{n} = (1, 0)$

$\vec{u} = (0, 1)$

$p: \; x = 3$

$p: \; 1 \cdot x + 0 \cdot y + c = 0$

$1 \cdot 3 + 0 \cdot (-2) + c = 0$

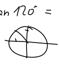
$c = -3$

$x - 3 = 0$

$p: \begin{array}{l} x = 3 + 0 \cdot t \\ y = -2 + t \end{array}$

Přímka p je dána bode $A = [1, 2\sqrt{3}]$ a směrovým úhlem $\varphi = 120°$

$p: \; y = k \cdot x + q \qquad k = \tan 120° = \frac{\sin \frac{2}{3}\pi}{\cos \frac{2}{3}\pi} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{3}$



$y = -\sqrt{3}x + q \xrightarrow{A \in p} 2\sqrt{3} = -\sqrt{3} \cdot 1 + q \longrightarrow q = 3\sqrt{3}$

$\boxed{p: \; y = -\sqrt{3}x + 3\sqrt{3}} \longrightarrow \boxed{p: \; \frac{x}{3} + \frac{y}{3\sqrt{3}} = 1}$

Vyšetřete vzájemnou polohu přímek p a q.

$p \nparallel q \Rightarrow$ najděte souřadnice průsečíku

1) $p = \{[1 + 2t, 2 - 3t], t \in \mathbb{R}\} \qquad \vec{u}_p = (2, -3)$

$q = \{[-1 + 2k, 7 - 3k], k \in \mathbb{R}\} \qquad \vec{u}_q = (2, -3)$

$\vec{u}_p = \vec{u}_q$

- jestliže $p = q \Rightarrow (A \in p \Rightarrow A \in q)$

$A = [1, 2] \in p. \qquad A \in q? \qquad \begin{array}{l} 1 = -1 + 2k \longrightarrow k = 1 \\ 2 = 7 - 3k \longrightarrow k = \frac{5}{3} \end{array}$

$A \notin q$

$\Rightarrow p \parallel q, \; p \neq q$

- $p: \begin{array}{l} x = 1 + 2t \\ y = 2 - 3t \end{array} \qquad q: \begin{array}{l} x = -1 + 2k \\ y = 7 - 3k \end{array}$

$1 + 2t = -1 + 2k$

$2 - 3t = 7 - 3k$

$\left. \begin{array}{l} 2t - 2k = -2 \quad / \cdot \frac{3}{2} \\ -3t + 3k = 5 \end{array} \right\} \oplus$

$0t + 0k = 2$

$0 = 2 \longrightarrow NŘ \Rightarrow p \parallel q$

---

2) $p = \{[1 + 2t, 2 - 3t], t \in \mathbb{R}\} \qquad \vec{u}_p = (2, -3)$

$q: \; 2x + y - 1 = 0 \qquad \vec{n}_q = (2, 1)$

$\vec{u}_p \cdot \vec{n}_q = 4 - 3 = 1$

$\vec{u}_p \neq \alpha \cdot \vec{n}_q \quad \forall \alpha \in \mathbb{C}$

$p: \left. \begin{array}{l} x = 1 + 2t \quad / \cdot 3 \\ y = 2 - 3t \quad / \cdot 2 \end{array} \right\} \oplus$

$3x + 2y = 7$

$\left. \begin{array}{ll} p: & 3x + 2y = 7 \\ q: & 2x + y = 1 \quad / \cdot (-2) \end{array} \right\} \oplus$

$-x + 0y = 5$

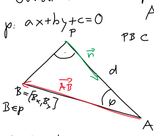
$x = -5 \longrightarrow p, q$ jsou různoběžné

$y = 11 \qquad p \cap q = P = [5, 11]$

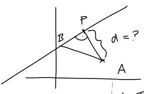
Odvoďte vzorec pro vzdálenost bodu od přímky

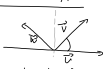
$A = [A_x, A_y]$

$p: \; ax + by + c = 0$



$PB \subset p$





$\vec{u} \cdot \vec{v} > 0$

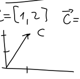
$\vec{u} \cdot \vec{w} < 0$

$d = |AB| \cdot \cos \varphi = |\overrightarrow{AB}| \left| \frac{\vec{n} \cdot \overrightarrow{AB}}{|\vec{n}| \cdot |\overrightarrow{AB}|} \right|$

$\overrightarrow{AB} = B - A \qquad = \frac{|\vec{n} \cdot \overrightarrow{AB}|}{|\vec{n}|} = \frac{|\vec{n} \cdot (B - A)|}{|\vec{n}|} = \frac{|\vec{n} \cdot \vec{B} - \vec{n} \cdot \vec{A}|}{|\vec{n}|}$

$\vec{n} = (a, b) \qquad = \frac{|a \cdot B_x + b \cdot B_y - a \cdot A_x - b \cdot A_y|}{|\vec{n}|} \overset{*}{=}$

$C = [1, 2] \quad \vec{C} = \overrightarrow{OC}$



$B \in p \Rightarrow a B_x + b B_y + c = 0$

$a B_x + b B_y = -c$

$\overset{*}{=} \frac{|-c - a \cdot A_x - b \cdot A_y|}{|\vec{n}|} = \frac{|a \cdot A_x + b \cdot A_y + c|}{\sqrt{a^2 + b^2}}$

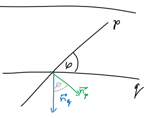
$$\boxed{d = \frac{|a \cdot A_x + b \cdot A_y + c|}{\sqrt{a^2 + b^2}}}$$

$A = [0, 0]$

$d = \frac{|c|}{\sqrt{a^2 + b^2}}$

Odchylka dvou přímek

$\varphi = ?$



$p: \; x + 2y - 1 = 0$

$q: \; 2x - y + 4 = 0$

$\vec{n}_p = (1, 2)$

$\vec{n}_q = (2, -1) \qquad \vec{n}_p \cdot \vec{n}_q = 0 \longrightarrow \vec{n}_p \perp \vec{n}_q \longrightarrow p \perp q$