NSD

1) $D(78,66)=2\cdot3=6$ $\qquad$ $78=2\cdot39=\boxed{2\cdot3}\cdot13$

$n(78,30)=2\cdot3\cdot5\cdot13$ $\qquad$ $66=2\cdot33=\boxed{2\cdot3}\cdot11$

nsn $\quad =390$ $\qquad$ $30=2\cdot3\cdot5$

$78=6\cdot13$

$66=6\cdot11$

$$\frac{1}{30}-\frac{7}{78}=\frac{78-7\cdot30}{30\cdot78}=\frac{78-210}{2540}$$

$$=\frac{13-7\cdot5}{390}=\frac{13-35}{390}=-\frac{22}{390}=-\frac{11}{195}$$

2) Usměrnění zlomku

$$\frac{1}{\sqrt2}=\frac{1}{\sqrt2}\cdot1=\frac{1}{\sqrt2}\cdot\frac{\sqrt2}{\sqrt2}=\frac{\sqrt2}{2}$$

3) Zjednošte výraz a určete jeho podmínky

$$(u^2\cdot v^3\cdot w\cdot x^{-7}):(5uv^{-3}w^2x^{-2})=\frac{u^2\cdot v^3\cdot w\cdot x^{-7}}{5\cdot u\cdot v^{-3}\cdot w^2\cdot x^{-2}}$$

$$=\frac{1}{5}\cdot\frac{u^2}{u}\cdot\frac{v^3}{v^{-3}}\cdot\frac{w}{w^2}\cdot\frac{x^{-7}}{x^{-2}}$$

$\frac{1}{a^s}=a^{-s}$

$a^r\cdot a^s=a^{r+s}$

$$=\frac{1}{5}u\cdot v^6\cdot w^{-1}\cdot x^{-5}$$

$$=\frac{uv^6}{5wx^5}$$

$u\neq0 \quad v\neq0 \quad w\neq0 \quad x\neq0$

$v^{-3}=\frac{1}{v^3}$

4) $$\left(\frac{1}{a+b}+\frac{1}{a-b}\right)\cdot\left(\frac{1}{a}-\frac{1}{b}\right)=$$

$a,b\neq0$

$a\neq b$

$a\neq-b$

$$=\frac{a-b+a+b}{(a+b)\cdot\cancel{(a-b)}}\cdot\frac{\overset{-(a-b)}{b-a}}{ab}=\frac{2a}{a+b}\cdot\frac{-1}{ab}$$

$$=\frac{-2}{(a+b)\cdot b}$$

5) $$\frac{xy-y-x^2+x}{xy+y-x^2-x}=\frac{y(x-1)-x\cdot(x-1)}{y(x+1)-x(x+1)}$$

$$=\frac{\cancel{(y-x)}\cdot(x-1)}{\cancel{(y-x)}\cdot(x+1)}=\frac{x-1}{x+1}$$

$(y-x)\cdot(x+1)\neq0$

$(y-x)\neq0$

$x+1\neq0$

$\boxed{\begin{matrix}y\neq x\\ x=-1\end{matrix}}$

Častá chyba: $$\frac{x-\cancel{y}}{1-(x-\cancel{y})}=\frac{1}{1-1}$$

$$=\frac{1\cdot\cancel{(x-y)}}{\left(\frac{1}{x-y}-1\right)\cancel{(x-y)}}=\frac{1}{\frac{1}{x-y}-1}$$

6) $$\frac{\sqrt{x+y}}{\frac{\sqrt{x-y}}{\sqrt{x^2-y^2}}}-x-y=\frac{\sqrt{x+y}}{\sqrt{\frac{\cancel{x-y}}{\cancel{(x-y)}\cdot(x+y)}}}-x-y$$

$x-y\neq0$

$y\neq y$

$x^2-y^2\neq0$

$(x+y)\cdot(x-y)\neq0$

$x\neq-y$

$\boxed{\begin{matrix}x+y\geq0\\ x-y\geq0\\ x^2-y^2\geq0\end{matrix}}$

Def $\sqrt{x}=y \iff x=y^2$

$$=\frac{\sqrt{x+y}}{\frac{1}{\sqrt{x+y}}}-x-y$$

$$=\sqrt{x+y}\cdot\frac{\sqrt{x+y}}{1}-x-y$$

$$=x+y-x-y=0$$

7) $$\sqrt[5]{\left(\frac{c^{1/2}\cdot c^{-1/3}}{c^{-5/6}}\right)^{-3}}=\left(\frac{c^{1/6}}{c^{-5/6}}\right)^{-\frac{3}{5}}=\left(c^{6/6}\right)^{-3/5}$$

$$=c^{-3/5}$$

$\sqrt[r]{a}=a^{\frac{1}{r}}$

$\sqrt[r]{a^s}=a^{\frac{s}{r}}$

$a^{-r}=\frac{1}{a^r}$

$\sqrt{c}\Rightarrow c\geq0$

$c\neq0$

$c>0$

$c\in\mathbb{R}^+$

8) $$\left[\left(\frac{x}{y}\right)^2-\frac{x}{y^2}\right]:\left(\frac{x-1}{y}\right)^2=\frac{x^2-x}{y^2}\cdot\left(\frac{y}{x-1}\right)^2$$

$y\neq0 \quad x\neq1$

$$=\frac{x\cdot(x-1)}{(x-1)^2}=\frac{x}{x-1}$$

$(A+B)^2=A^2+2A\cdot B+B^2$

$A^2-B^2=(A+B)\cdot(A-B)$

9) $$\frac{\left(\sqrt[4]{u}+\sqrt[4]{v}\right)^2+\left(\sqrt[4]{u}-\sqrt[4]{v}\right)^2}{u-v}:\frac{2}{\sqrt{u}-\sqrt{v}}=$$

$(A+B)\cdot(A-B)=A^2-B^2$

$(A+B)^2=A^2+\cancel{2\cdot AB}+B^2$

$(A-B)^2=A^2-\cancel{2\cdot A\cdot B}+B^2$ $\quad$ $+$

$(A+B)^2+(A-B)^2=2A^2+2B^2$

$$=\frac{2\sqrt{u}+2\sqrt{v}}{u-v}\cdot\frac{\sqrt{u}-\sqrt{v}}{2}$$

$$=\frac{(\sqrt{u}+\sqrt{v})\cdot(\sqrt{u}-\sqrt{v})}{u-v}$$

$$=\frac{(\sqrt{u})^2-(\sqrt{v})^2}{u-v}=1$$

$u,v\geq0 \qquad \sqrt{u}\neq\sqrt{v}\ /^2$

$u\neq v \qquad u\neq v$

Polynomy

$$p_n(x)=\sum_{i=0}^{n}a_ix^i$$

$n=0$: a_0 $\quad$ konstantní

$n=1$: a_1x+a_0 $\quad$ lineární (dvojčlen)

$n=2$: $a_2x^2+a_1x+a_0$ $\quad$ kvadratický (trojčlen)

$n=3$: $\quad$ kubický

Kořen $a\in\mathbb{R}$: $p_n(a)=0$

Každý polynom st. n má n komplexních kořenů

Každý polynom: $p_n(x)=a_n(x-x_1)\cdot(x-x_2)\cdots(x-x_n)$

$x_{1,\dots,n}$ jsou kořeny

n=2 $\quad ax^2+bx+c$ $\qquad$ $x_{1,2}=\frac{-b\pm\sqrt{D}}{2a}$

$D=b^2-4\cdot a\cdot c$ $\quad$ "diskriminant"

- $D>0$ ... 2 reálné kořeny
- $D=0$ ... 1 reálný dvojnásobný kořen
- $D<0$ ... nemá žádný reálný kořen

Př. $p(x)=x^2+5x-6$ $\qquad$ $D=25+4\cdot1\cdot6=49$

$a=1$

$b=5$

$c=-6$

$$x_{1,2}=\frac{-5\pm\sqrt{49}}{2}=\frac{-5\pm7}{2}=\begin{cases}1\\-6\end{cases}$$

$$p(x)=1\cdot(x-1)\cdot(x+6)$$

$$p(x)=x^2+6x+9=(x+3)^2=(x+3)\cdot(x+3)$$

$(A+B)^2=A^2+2A\cdot B+B^2$

$x_{1,2}=-3$

$$D=36-4\cdot9=0. \qquad x_{1,2}=\frac{-6\pm\sqrt0}{2}=-3$$

Doplnění na čtverec dvojčlenu

$$p(x)=ax^2+bx+c \longrightarrow \bar a(x+\bar b)^2+\bar c$$

$$x^2+5x-6=\Big(\underbrace{x^2+2\cdot x\frac{5}{2}+\left(\frac{5}{2}\right)^2}_{A^2+2\cdot A\cdot B+B^2}\underbrace{-\left(\frac{5}{2}\right)^2}_{-B^2}\Big)-6=$$

$(A+B)^2=A^2+2\cdot A\cdot B+B^2$

$A=x$

$B=\frac{5}{2}$

$$=\left(x+\frac{5}{2}\right)^2-6-\left(\frac{5}{2}\right)^2=\left(x+\frac{5}{2}\right)^2-\frac{49}{4}$$

$$x^2+5x-6=\underbrace{\left(x+\frac{5}{2}\right)^2}_{A^2}-\underbrace{\left(\frac{7}{2}\right)^2}_{B^2}=\underbrace{\left(x+\frac{5}{2}+\frac{7}{2}\right)}_{A+B}\underbrace{\left(x+\frac{5}{2}-\frac{7}{2}\right)}_{A-B}$$

$$=(x+6)\cdot(x-1)$$

$$x^2+2x+3=(x^2+2x+1)+3-1=$$

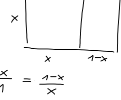
$D=4-4\cdot3=-8$

$$=(x+1)^2+2$$

$A^2-B^2=(A+B)(A-B)$

$$=(x+1+\sqrt2 i)\cdot(x+1-\sqrt2 i)$$

Zlatý řez



$x^2=1-x$

$x^2+x-1=0$

$$\frac{x}{1}=\frac{1-x}{x}$$

$$x^2+x-1=\left(x+\frac{1}{2}\right)^2-1-\left(\frac{1}{2}\right)^2$$

$A=x$

$2A\cdot B=x$

$2B=1$

$B=\frac{1}{2}$

$$=\left(x+\frac{1}{2}\right)^2-\frac{5}{4}=\left(x+\frac{1}{2}\right)^2-\left(\frac{\sqrt5}{2}\right)^2$$

$$=\left(x+\frac{1}{2}+\frac{\sqrt5}{2}\right)\cdot\left(x+\frac{1}{2}-\frac{\sqrt5}{2}\right)$$

$$x_{1,2}=\frac{-1\pm\sqrt5}{2}=\varphi$$