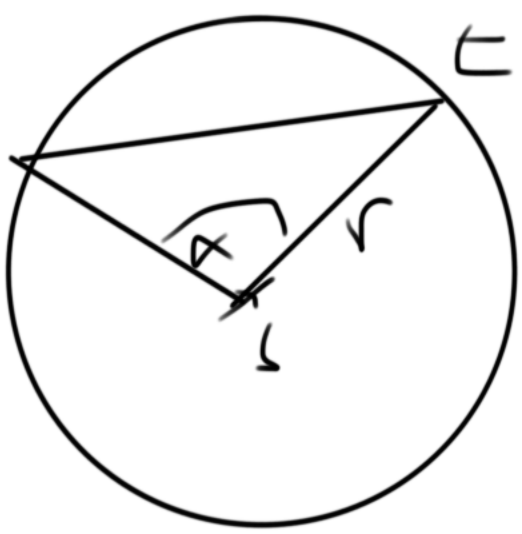
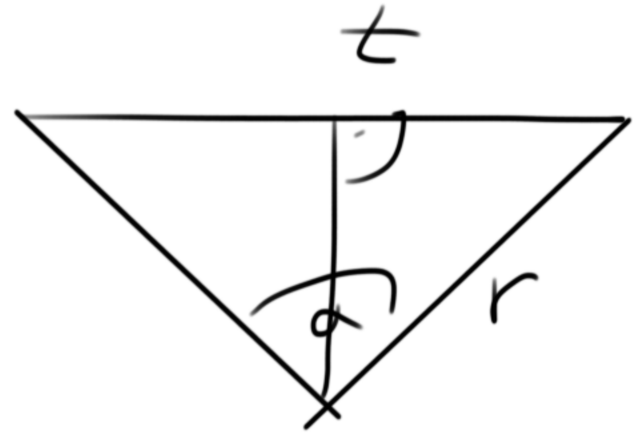
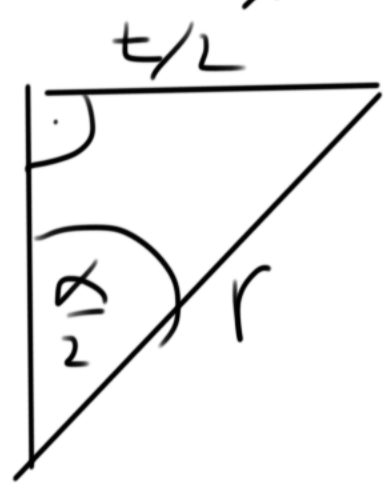
1) Vypočtěte O, S ○, tětiva $t = 4$ cm a příslušný středový úhel $\alpha = 60°$







$$\sin\frac{\alpha}{2} = \frac{\frac{t}{2}}{r}$$

$$r = \frac{t}{2 \cdot \sin\left(\frac{\alpha}{2}\right)}$$

$\alpha = 60°$

$\frac{\alpha}{2} = 30°$

$\sin 30° = \frac{1}{2}$

$r = t = \underline{4\text{ cm}}$

$O = 2\pi r$ $S = \pi r^2$

$O = 8\pi$ cm $S = 16\pi$ cm^2

$\cancel{3{,}14 \cdot 8 = 21{,}1\ldots}$

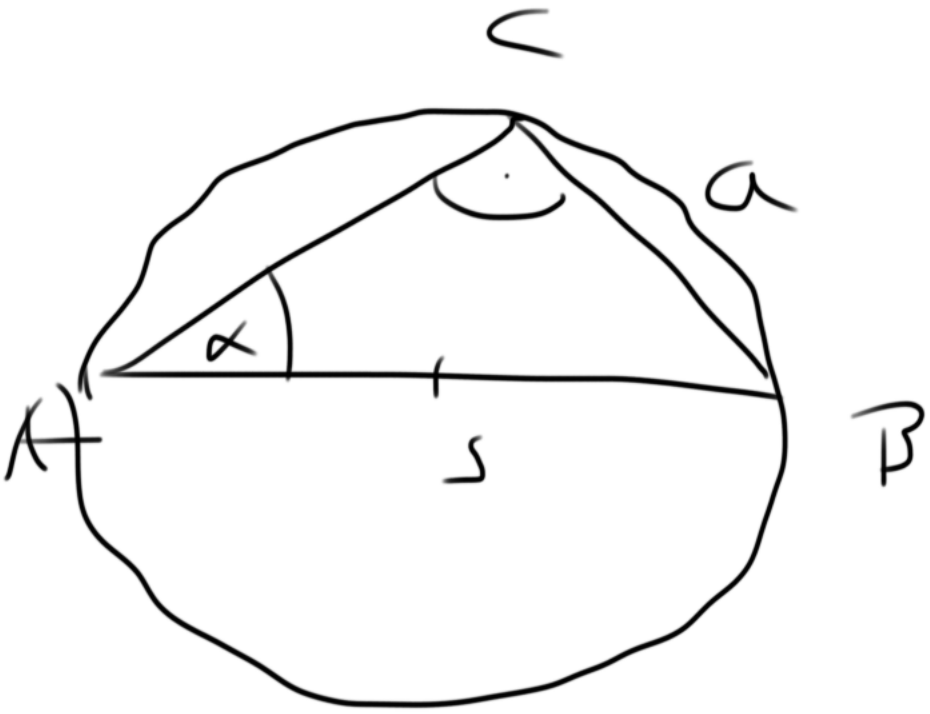
$\underline{\sqrt{8} = 2\sqrt{2}}$

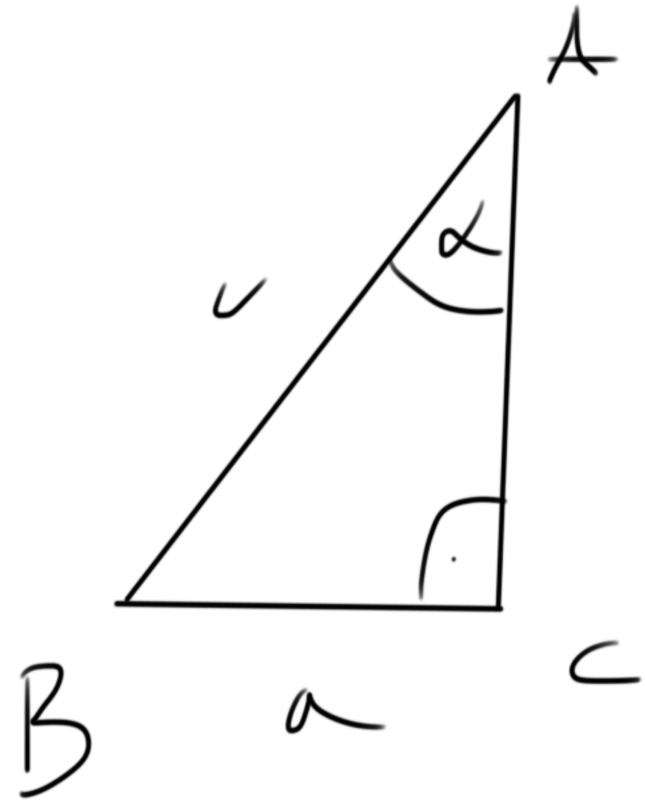
$\doteq 2 \cdot 1{,}41$

$\doteq 2{,}8\ldots$

2) Počítejte O, S ○ opsané pravoúhlému △

$\alpha = 15°$, $a = \frac{1}{\sqrt{2+\sqrt{3}}}$





c ~ průměr kružnice

$\sin\alpha = \frac{a}{c}$

$$\boxed{c = \frac{a}{\sin\alpha}}$$

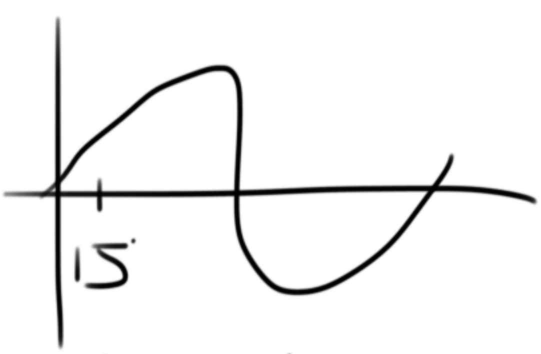
$$c = \frac{a}{\sin \alpha} \qquad a = \frac{1}{\sqrt{2+\sqrt{3}}} \qquad \alpha = 15^\circ$$

$$\left|\sin \frac{\phi}{2}\right| = \sqrt{\frac{1-\cos\phi}{2}} \qquad \alpha \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \checkmark$$

$$\alpha = 15^\circ \qquad 2\alpha = 30^\circ$$

$$\frac{\phi}{2} = 15^\circ \qquad \phi = 30^\circ$$

$$\sin 15^\circ = \sqrt{\frac{1-\cos 30^\circ}{2}} \qquad \cos 30^\circ = \frac{\sqrt{3}}{2}$$



$$= \sqrt{\frac{1-\sqrt{3}/2}{2}} = \sqrt{\frac{\frac{2-\sqrt{3}}{2}}{2}}$$

$\sin 15^\circ > 0$

$$= \sqrt{\frac{2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$$

$$\boxed{\sin 15^\circ = \frac{\sqrt{2-\sqrt{3}}}{2}}$$

$$c = \frac{a}{\sin\alpha}$$

$$c = \frac{a}{\sin\alpha} \qquad a = \frac{1}{\sqrt{2+\sqrt{3}}}\,cm$$

$$\sin\alpha = \frac{\sqrt{2-\sqrt{3}}}{2}$$

$$c = \frac{\frac{1}{\sqrt{2+\sqrt{3}}}}{\frac{\sqrt{2-\sqrt{3}}}{2}} = \frac{2}{\sqrt{2+\sqrt{3}}\cdot\sqrt{2-\sqrt{3}}}$$

$$= \frac{2}{\sqrt{(2+\sqrt{3})\cdot(2-\sqrt{3})}} = \frac{2}{\sqrt{4-3}} = \underline{\underline{2}}$$

$$(A+B)\cdot(A-B) = A^2 - B^2$$

$$\boxed{c = 2}\ cm$$ průměr kružnice

$$r = 1\,cm$$

$$\boxed{O = 2\pi\,cm \qquad S = \pi\,cm^2}$$

3) Odvoďte vzorec pro obvod a obsah pravidelného mnohoúhelníku.

n-úhelník: n vnitřních úhlů

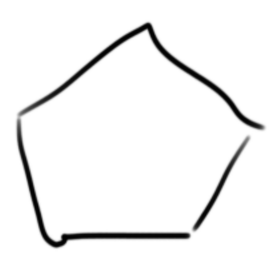
obecný 5-úhelník

Pravidelný

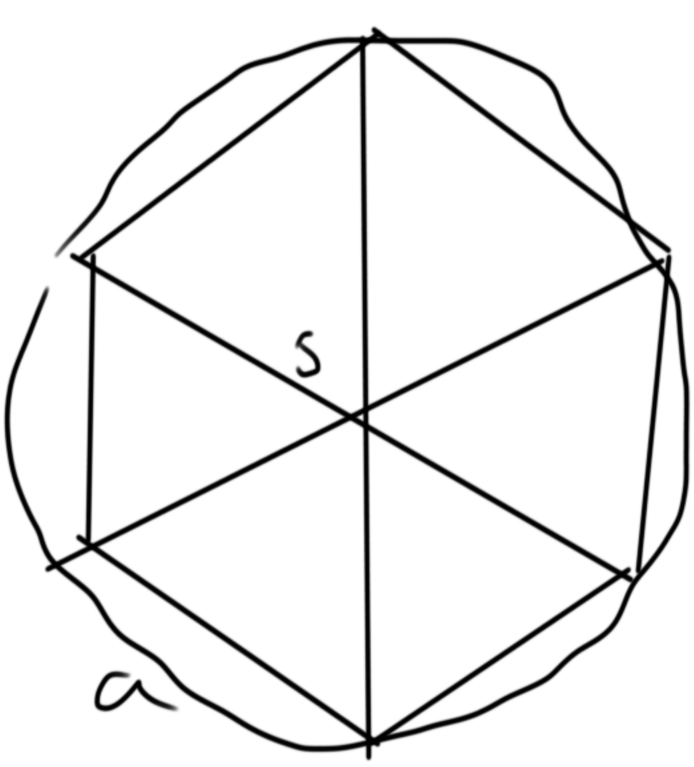
→ má všechny strany stejně dlouhé

⇒ má všechny vnitřní úhly stejně veliké.

př. 5-úhelník



$S_n = n \cdot S_\triangle$



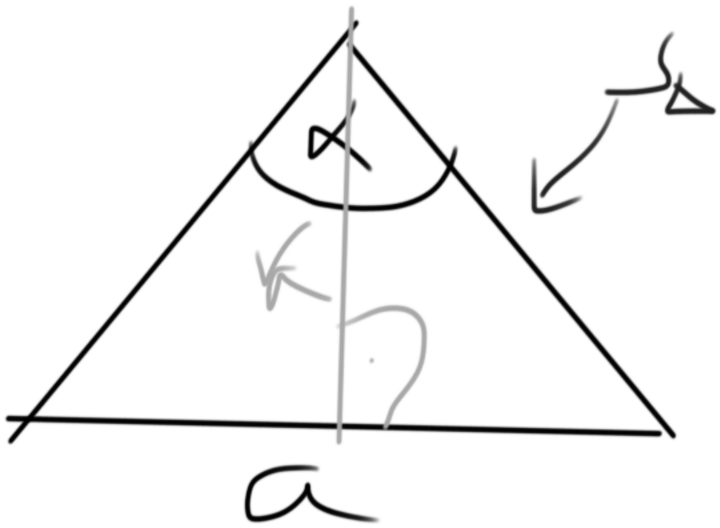
úhlopříčky ⇒ $\triangle$

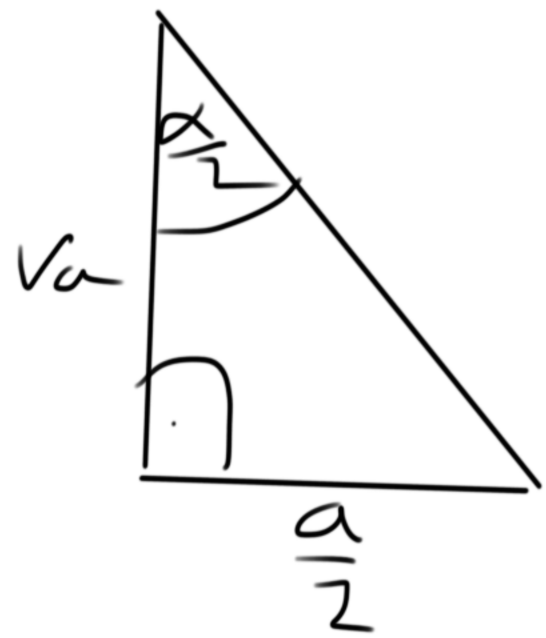
pravidelný ⇒ $\triangle$ stejné

rovnoramenný

$\alpha = \frac{2\pi}{n}$

n-úhelník, pravidelný o straně a





$$S_n = n \cdot S_\triangle$$

$$S_\triangle = \frac{a \cdot v_a}{2}$$

$$\cot g \frac{\alpha}{2} = \frac{v_a}{\frac{a}{2}}$$

$$v_a = \cot g\left(\frac{\alpha}{2}\right) \cdot \frac{a}{2}$$

$$S_\triangle = \frac{1}{2} \cdot a \cdot v_a = \frac{1}{2} a \cdot \frac{a}{2} \cdot \cot g\left(\frac{\alpha}{2}\right)$$

$$\boxed{S_\triangle = \frac{1}{4} a^2 \cot g\left(\frac{\alpha}{2}\right)}$$

$$S_n = n \cdot S_\triangle = \frac{n}{4} a^2 \cot g\left(\frac{\alpha}{2}\right)$$

$$\boxed{\alpha = \frac{2\pi}{n}}$$

$$\frac{\alpha}{2} = \frac{\pi}{n}$$

$$\boxed{S_n = \frac{n}{4} a^2 \cot g\left(\frac{\pi}{n}\right)}$$

$$\underline{S_n = \frac{n}{4} a^2 \operatorname{cotg}\left(\frac{\pi}{n}\right)} = n \cdot \underline{S_\Delta}$$

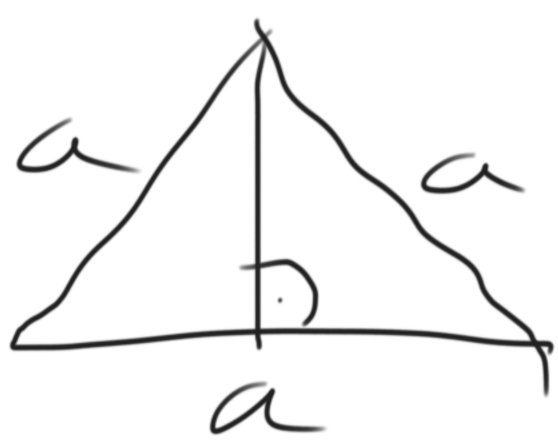
$n=4$: čtverec

$$\frac{\pi}{n} \to \frac{\pi}{4}$$

$$\operatorname{cotg}\left(\frac{\pi}{4}\right) = 1$$

$$\underline{S_4 = 1 \cdot a^2 \cdot 1 = a^2}$$

$n=3$:



rovnostranný trojúhelník

$$\frac{\pi}{n} \to \frac{\pi}{3} \qquad \operatorname{cotg}\left(\frac{\pi}{3}\right) = \frac{\cos\frac{\pi}{3}}{\sin\frac{\pi}{3}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$$

$$S_3 = \frac{3}{4} \cdot a^2 \cdot \frac{\sqrt{3}}{3} = \frac{\sqrt{3}}{4} a^2$$

$n=6$: $\quad \cot g\left(\frac{\pi}{6}\right) = \frac{\cos\frac{\pi}{6}}{\sin\frac{\pi}{6}} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

$$S_6 = \frac{6}{4}a^2\cdot\sqrt{3} = \frac{3}{2}\sqrt{3}a^2 \quad ✓$$

---

4) Vzájemná poloha kružnic.

chyba na přednášce:

iii) $r_1+r_2 > v > r_1$ × špatně

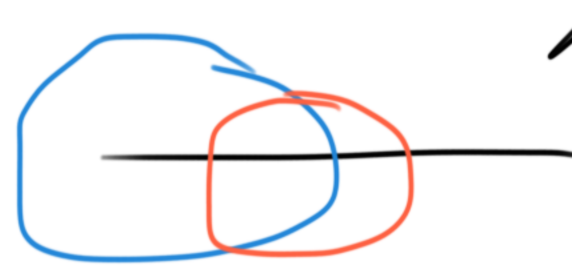
$r_1+r_2 > v > r_1-r_2$ správně

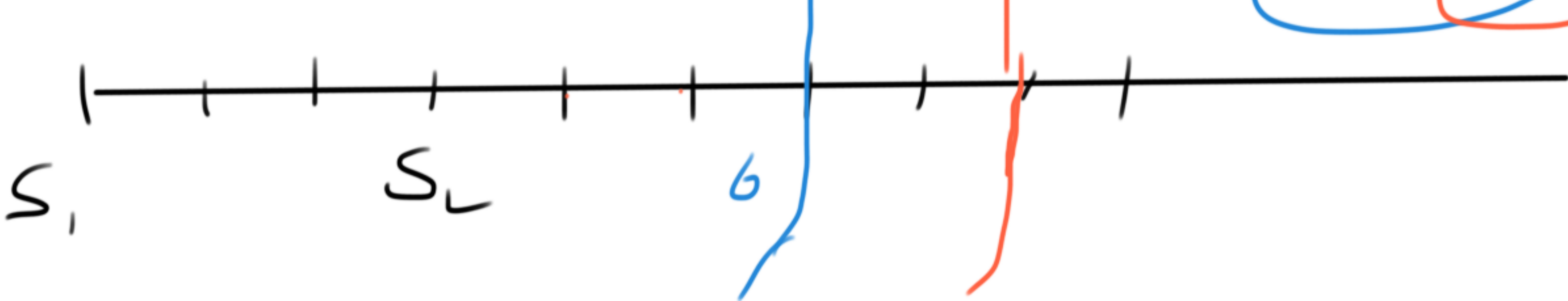
$r_1 = 6$ cm, $r_2 = 5$ cm, $v = 3$ cm

$r_1 - r_2 = 1$ cm $\qquad r_1+r_2 > v > r_1-r_2$

$r_1 + r_2 = 11$ cm

$\Rightarrow$ 2 společné body

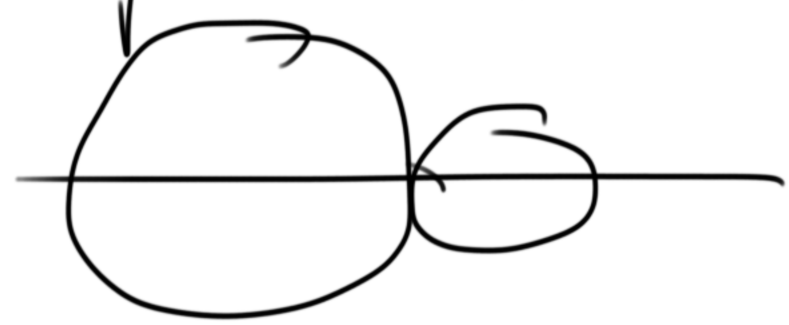


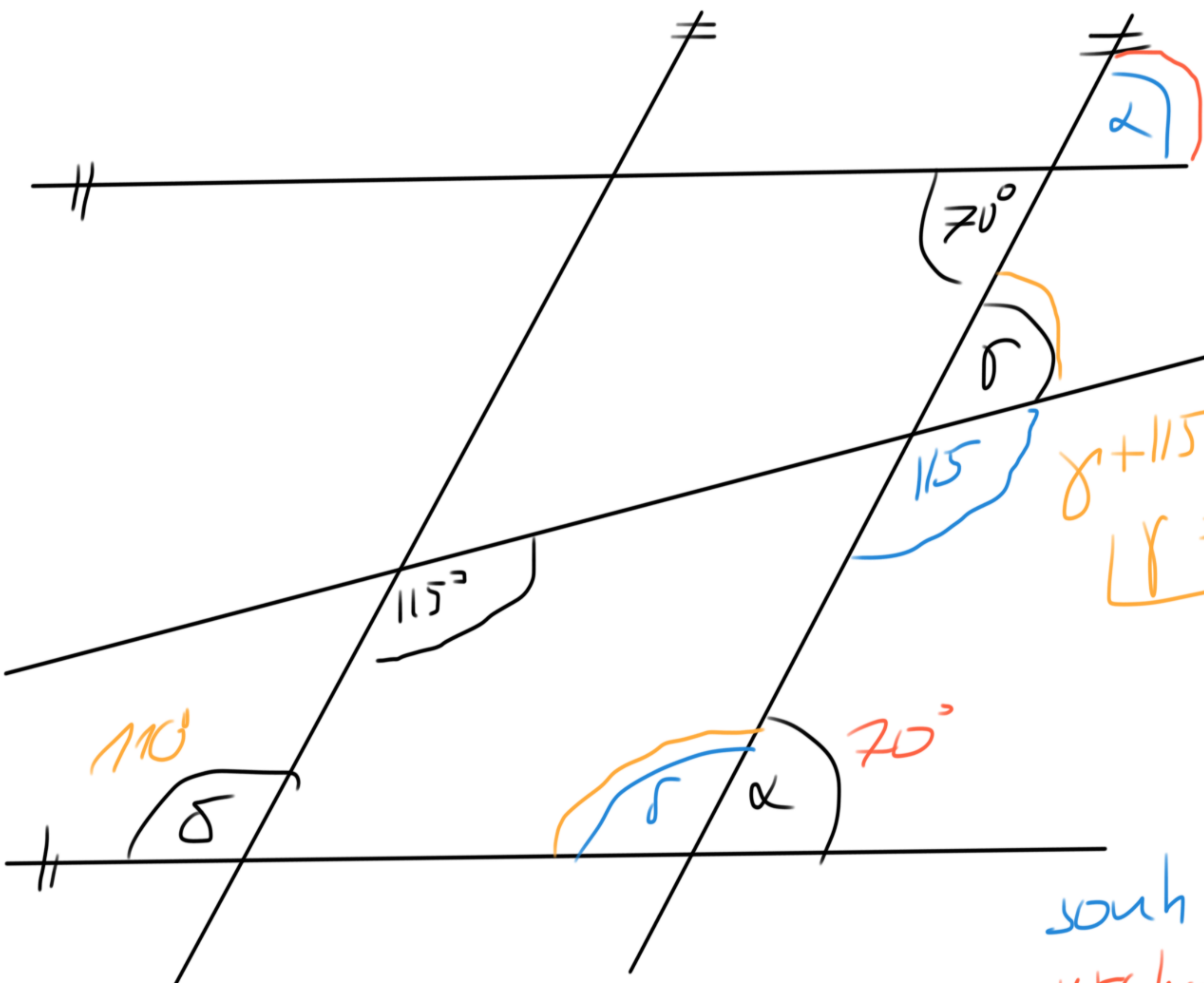


$r_1 = 32$ cm, $r_2 = 15$ cm, $v = 47$ cm

$r_1 + r_2 = 47$ cm

$\longrightarrow$ vnější dotyk





$\gamma + 115 = 180$

$\boxed{\gamma = 65}$

$\alpha + \delta = 180°$

$\Rightarrow \delta = 110°$

souhlasné

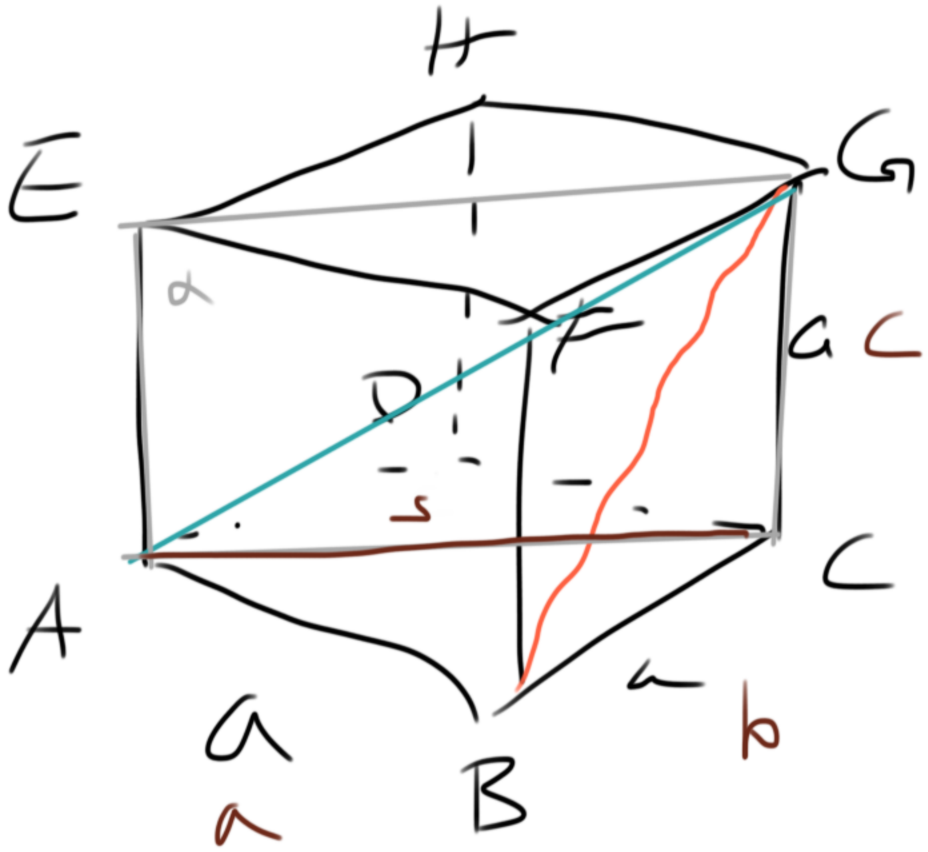
vrcholové

vedlejší

Objemy a povrchy těles

5) délka tělesové úhlopříčky krychle je $3\sqrt{6}$.

a) a b) V c) S



$V_{kvádr} = a \cdot b \cdot c$

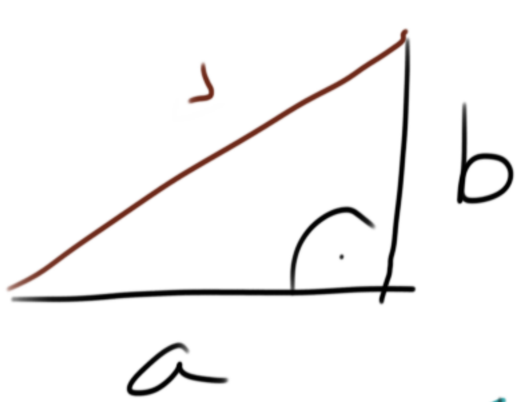
$V_{krychle} = a^3$

$S = 6 \cdot S_{\square} = 6 \cdot a^2$

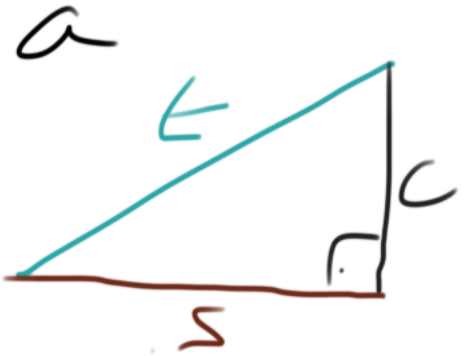
$\alpha \ldots ACG$

$t \in \alpha$

stěnová úhlopříčka



$s = \sqrt{a^2 + b^2}$



$t = \sqrt{s^2 + c^2}$

$t = \sqrt{a^2 + b^2 + c^2}$

tělesová úhlopříčka

$t = 3\sqrt{6}$ cm $\quad a = ?$ $\quad V = ?$ $\quad S = ?$

Krychle

$a = b = c \quad t = \sqrt{a^2 + a^2 + a^2}$

$= \sqrt{3a^2} = \underline{\sqrt{3}\,a}$

$\sqrt{3}\,a = 3\sqrt{6}$ $\quad s = \sqrt{2}\,a$

$\sqrt{3}\,a = 3\sqrt{3}\cdot\sqrt{2}$

$\boxed{a = 3\cdot\sqrt{2}}$ cm

$\underline{V = a^3 = 27\cdot 2^{\frac{3}{2}}}$ cm^3

$S = 6\cdot S_{\square} = 6\cdot a^2 = 9\cdot 2 = 18$ cm^2

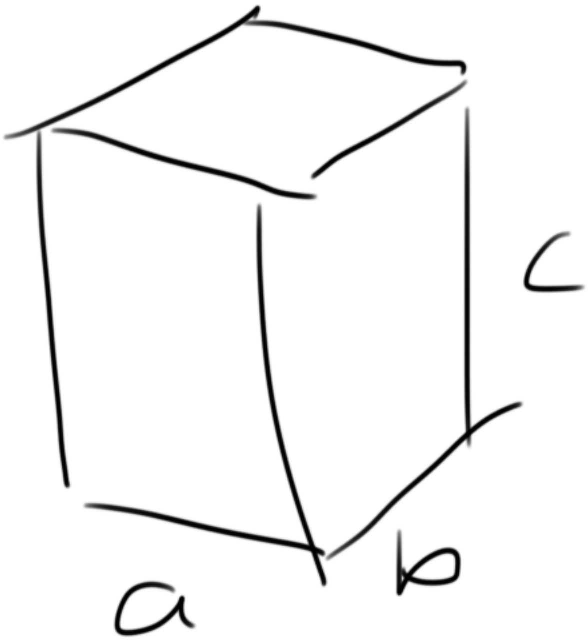
Krychle o straně a $\quad a \rightarrow a' = 2a$

$V' = ?$

$V = a^3 \quad V' = a'^3 = (2\cdot a)^3 = 8a^3$

$= 8V$

$S' = 6a'^2 = 6(2a)^2 = 4\cdot 6a^2 = 4\cdot S$

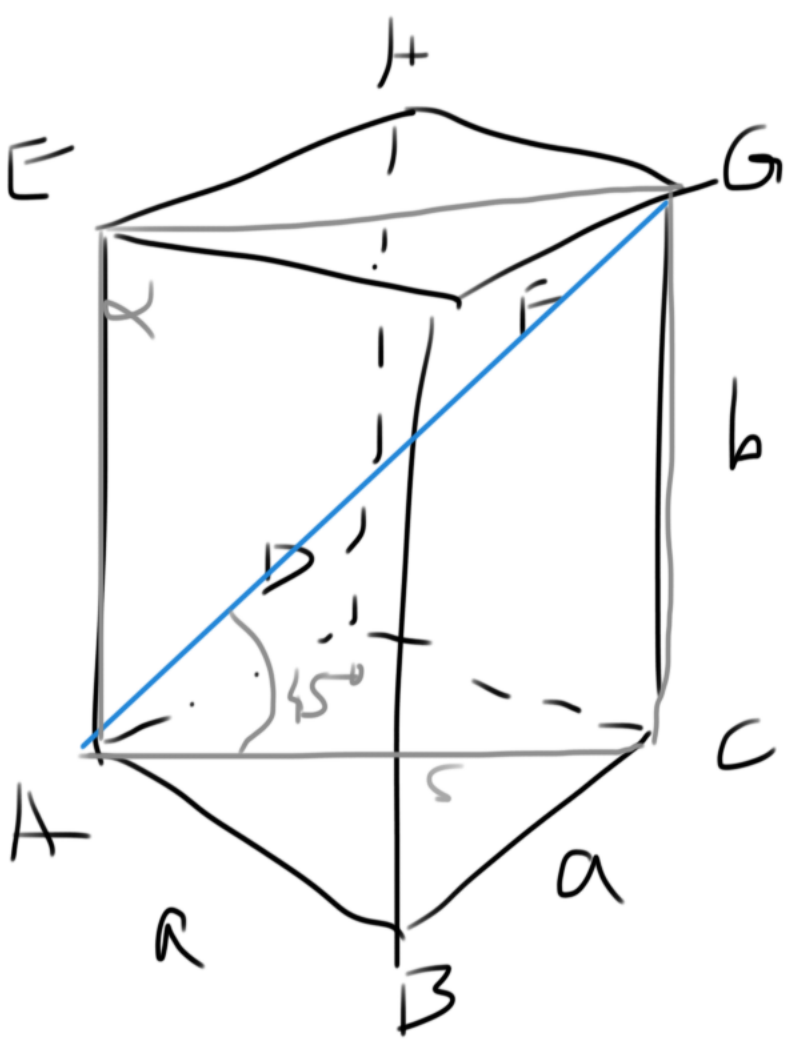


$V = a \cdot b \cdot c$

$a' = 2a$

$b' = b \quad c' = c$

$V' = a' \cdot b' \cdot c' = 2a \cdot b \cdot c = 2 \cdot V$



$V = 64\ cm^3$

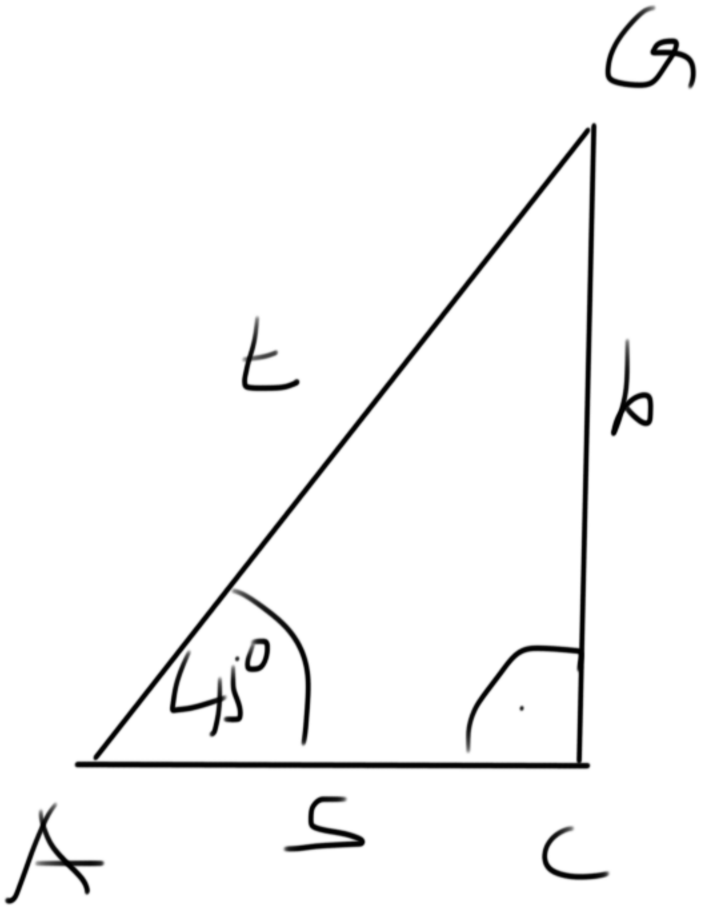
čtvercová podstava

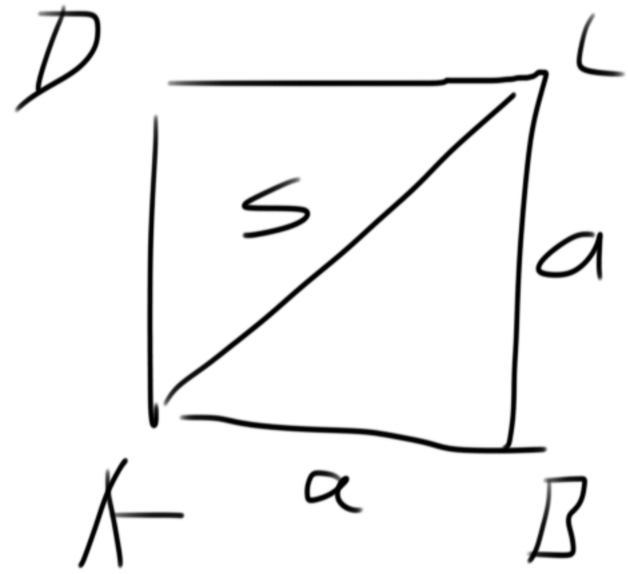
odchylka $t = \overleftrightarrow{AG}$ od roviny ABC je $45°$

$S = ?$

$\alpha \perp ABC \wedge \overleftrightarrow{AG} \in \alpha$

$\Rightarrow$ to nám určuje odchylku





$V = a \cdot a \cdot b$

Čtverec $\Rightarrow$ $s = \sqrt{2}\, a$

$\operatorname{tg} 45° = \frac{b}{s} \Rightarrow b = s \cdot \underbrace{\operatorname{tg} 45°}_{} \overset{1}{=}$

$\boxed{b = \sqrt{2}\, a}$

$\left.\begin{aligned} V &= 64\ \text{cm}^3 \\ &= a^2 \cdot b \end{aligned}\right\}$

$64 = a^2 \cdot \sqrt{2}\, a$

$a^3 = \frac{64}{\sqrt{2}}$

$a^3 = \frac{2^6}{2^{1/2}} = 2^{\frac{11}{2}}$

$\boxed{a = 2^{11/6}}$ $\quad 2^{\frac{11}{6}}$