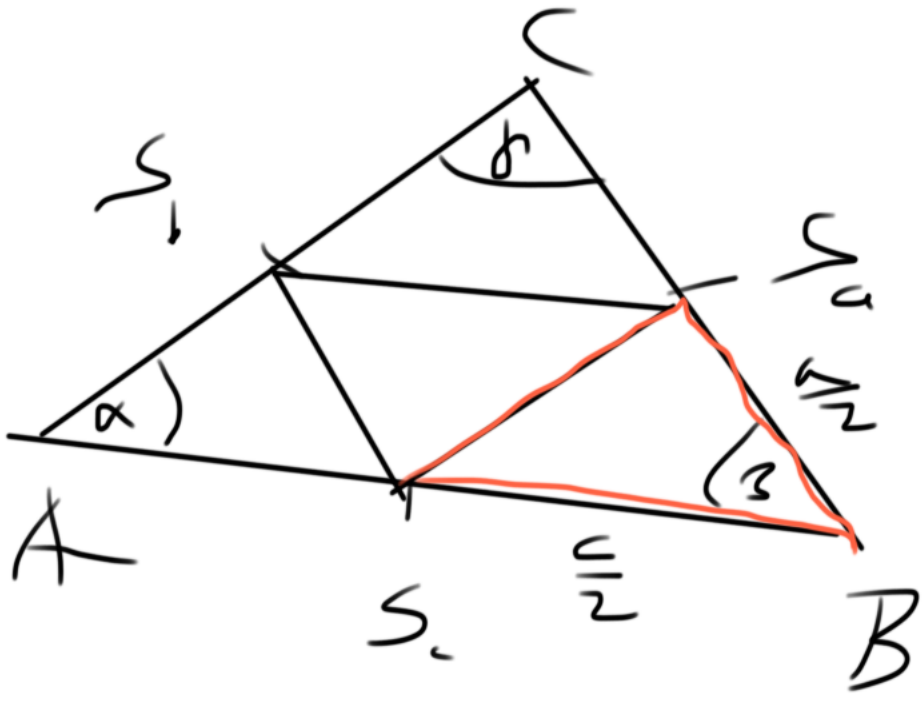
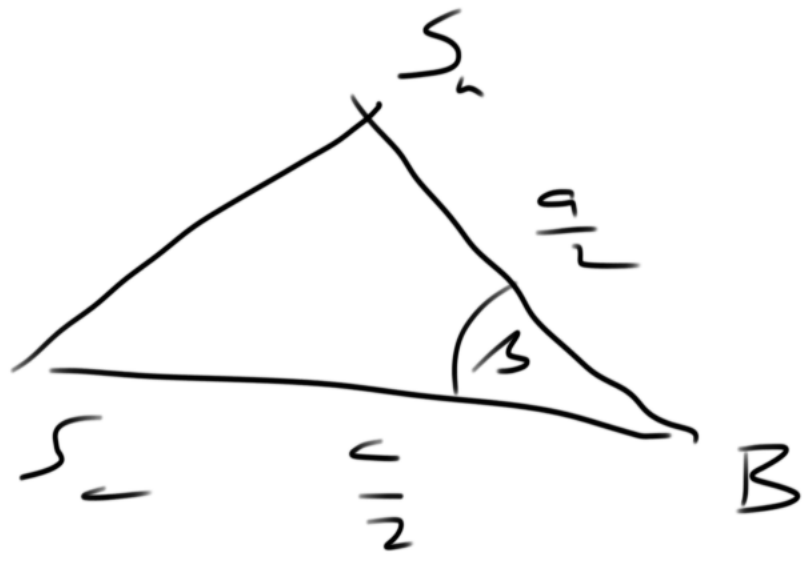
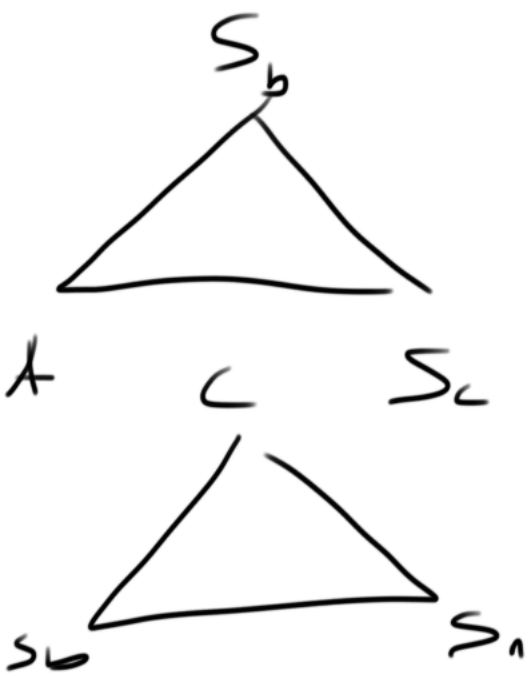
$\triangle ABC$, střední příčky

Ukažte podobnost







SUS

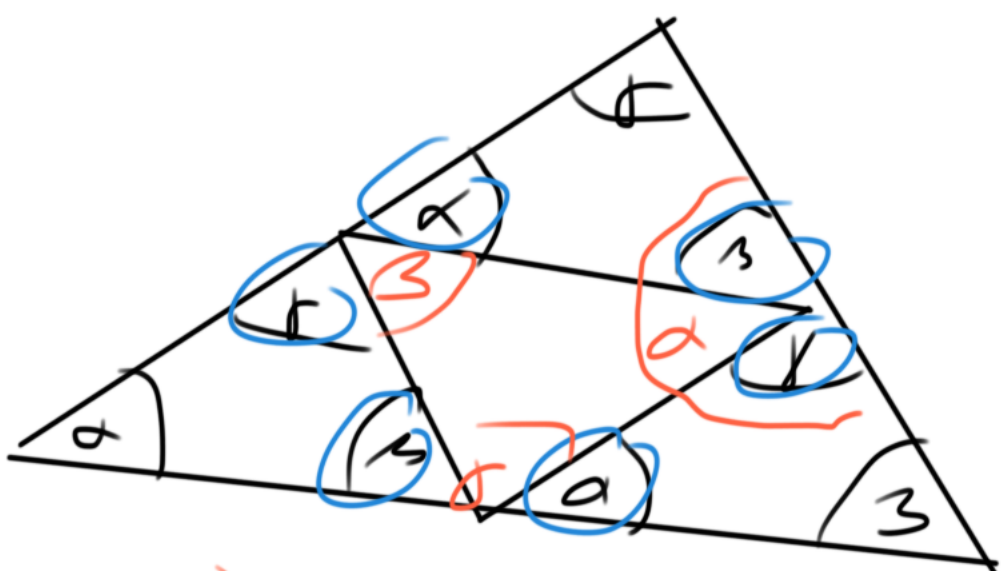
$\rightarrow \triangle S_c B S_1 \sim \triangle ABC$

$k = \frac{1}{2} : |BS_1| = k \cdot |BC|$

$|BS_c| = k \cdot |BA|$

stejně

SUS



+ podobnost

součet úh. 180°

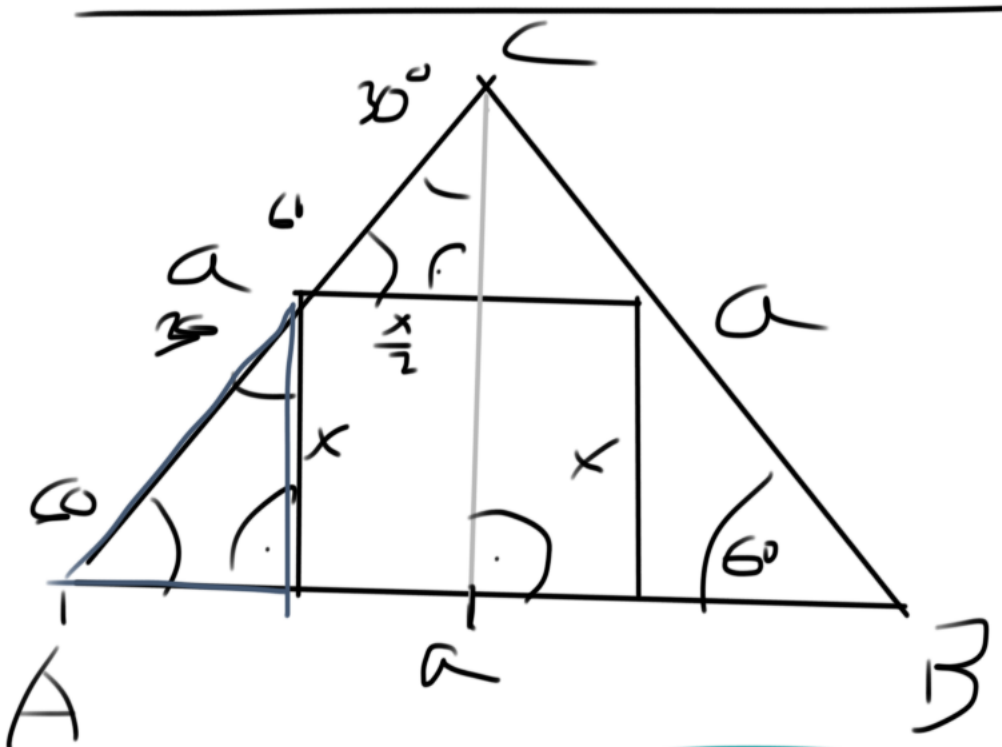
$\alpha + \beta + \gamma = 180°$ $\Rightarrow$ $\triangle S_a S_b S_c \sim \triangle ABC$

24

$\triangle ABC$ rovnostranný

vepsaný čtverec

→ strana čtverce = ?



$\frac{a}{2} - \frac{x}{2}$

$tg\,\vartheta = \frac{\text{protilehlá}}{\text{přilehlá}}$

$tg\,30° = tg\,\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

$\frac{\sin\frac{\pi}{6}}{\cos\frac{\pi}{6}} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$$\frac{\sqrt{3}}{3} = \frac{\frac{a}{2} - \frac{x}{2}}{x}$$

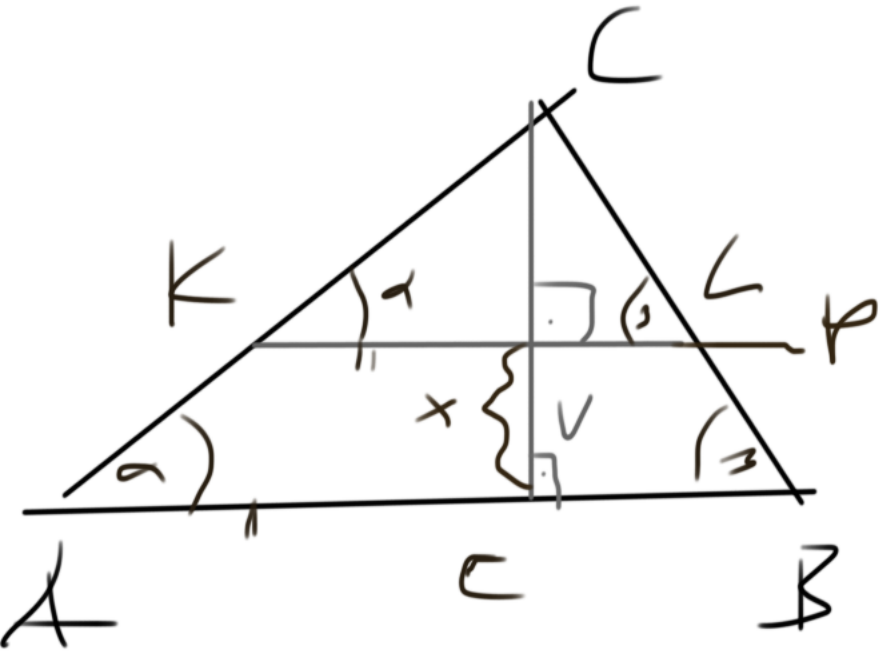
$x \neq 0$

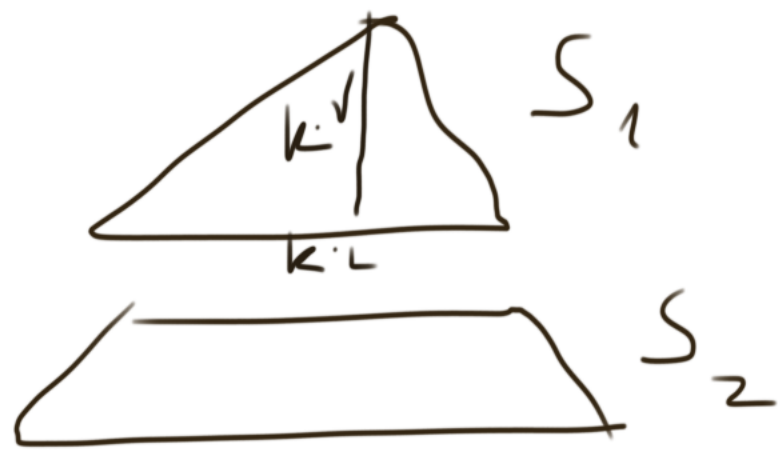
$$\frac{\sqrt{3}}{3}x = \frac{a}{2} - \frac{x}{2}$$

$$x\left(1 + \frac{2}{3}\sqrt{3}\right) = a$$

$$\frac{2}{3}\sqrt{3}x + x = a$$

$$x = \frac{a}{1 + \frac{2}{3}\sqrt{3}}$$





x tak, aby $S_1 = S_2$

$\triangle KLC \sim \triangle ABC$ věta uu s

$\Rightarrow \exists k: |KC| = k \cdot |AC|$

$|LC| = k \cdot |BC|$

$S = \frac{c \cdot v}{2}$ $\quad S_1 = \frac{1}{2}(k \cdot c)(k \cdot v)$

$\underline{S_1 = k^2 \cdot \frac{1}{2} c \cdot v = k^2 \cdot S}$

$\underline{S_2 = S - S_1 = S - k^2 S = S(1-k^2)}$

$S_1 = S_2$

$k^2 \cdot S = S(1-k^2)$

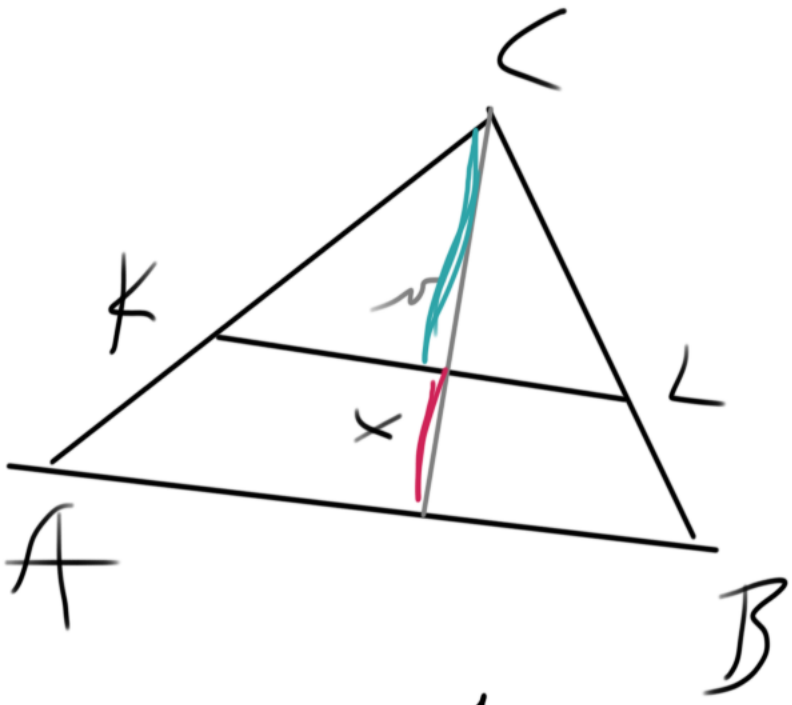
$k^2 = 1 - k^2$

$k^2 = \frac{1}{2}$

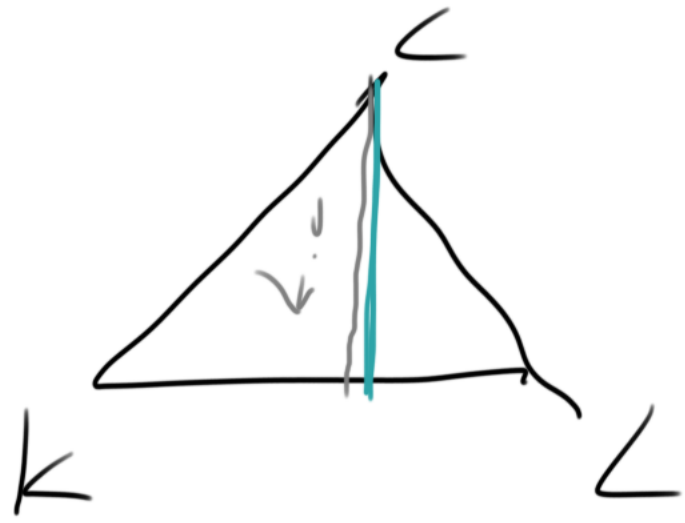
$k = \pm\sqrt{\frac{1}{2}}$

$k > 0$

nevyjímá



$$k = \frac{1}{\sqrt{2}}$$



$$k \cdot v = \frac{1}{\sqrt{2}} v$$

$$x + k \cdot v = v$$

$$x = v - k \cdot v$$

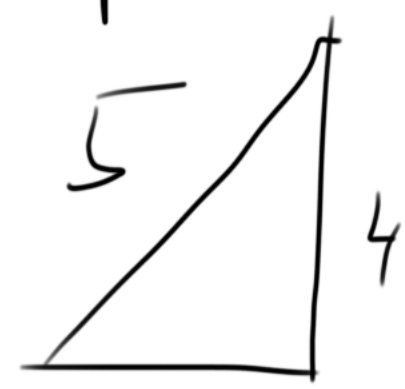
$$x = v(1-k)$$

$$x = v\left(1 - \frac{1}{\sqrt{2}}\right)$$

$$\boxed{x = v\left(1 - \frac{\sqrt{2}}{2}\right)}$$

Rozhodnite, zda jsou $\triangle$ pravoúhlé

a) 5, 3, 4

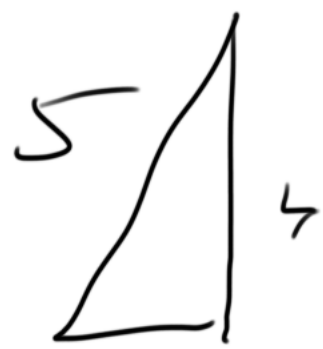


Pythagorova věta

$c^2 = a^2 + b^2$: $5^2 = 3^2 + 4^2$

LS PS $25 = 9 + 16 = 25$ ✓

b) 5, 1, 4

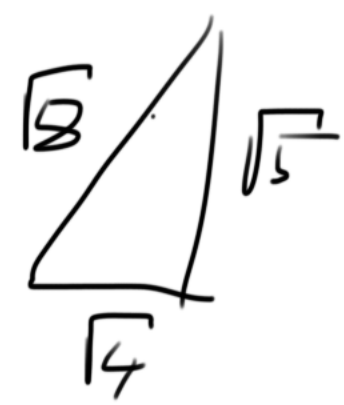


$LS = 25$

$PS = 1 + 4^2 = 17$

$LS \neq PS$

c) $\sqrt{5}$, $\sqrt{8}$, $\sqrt{4}$



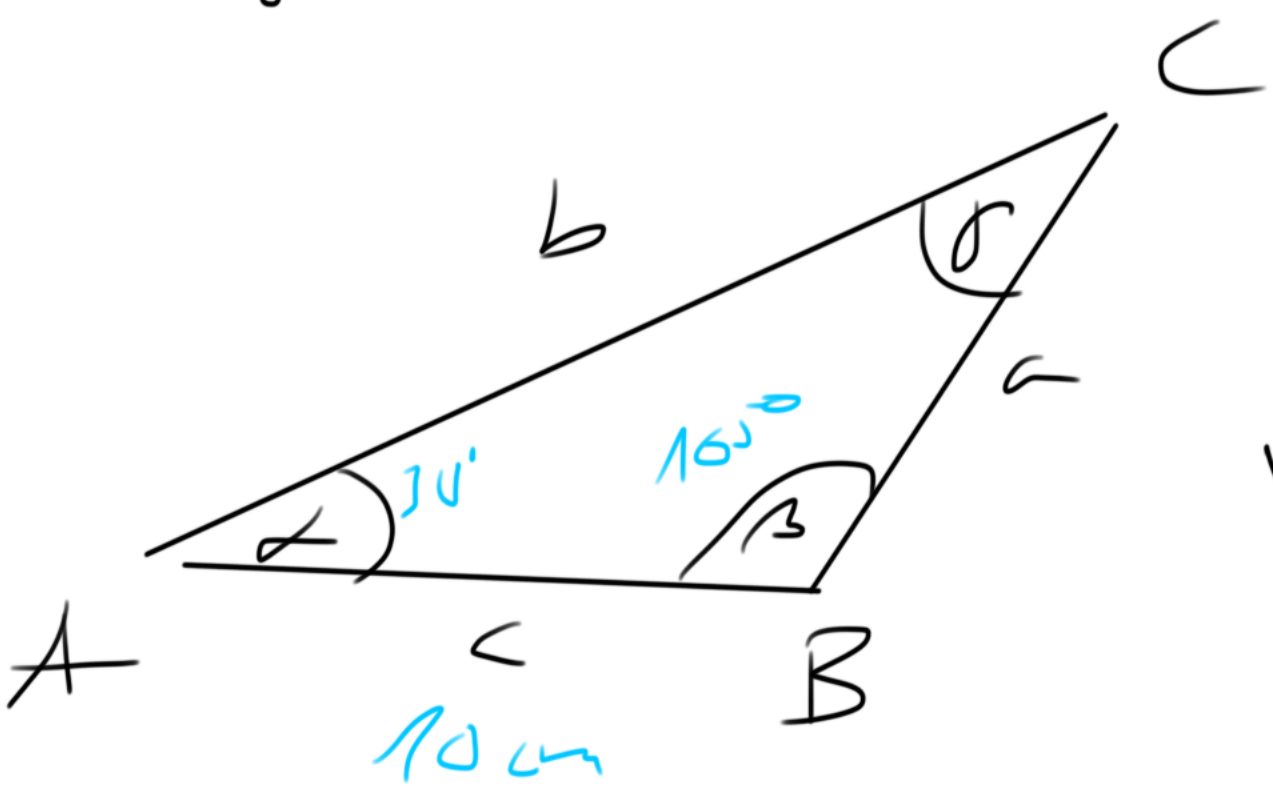
$LS = 8$

$PS = 5 + 4 = 9$

$LS \neq PS$

$\triangle ABC$: $\alpha = 30^\circ$ $\beta = 105^\circ$ $c = 10$ cm

$\gamma = ?$ $a, b = ?$



$\gamma = 180 - \alpha - \beta$

$= 45^\circ$

Sinová věta: $\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$

$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$

$b = \frac{c}{\sin\gamma} \cdot \sin\beta = \frac{10}{\frac{\sqrt{2}}{2}} \cdot \frac{\sqrt{2}}{4}(\sqrt{3}+1) = \frac{2 \cdot 5}{2}(\sqrt{3}+1)$

$\sin 45^\circ = \frac{\sqrt{2}}{2}$

$b = 5(\sqrt{3}+1)$

$\sin 105^\circ = \sin 60 \cos 45 + \sin 45 \cos 60 = \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2}$

$= \frac{\sqrt{2}}{4}(\sqrt{3}+1)$

$60 + 45$ $\quad \sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$$\frac{a}{\sin\alpha} = \frac{c}{\sin\gamma}$$

$$c = 10$$

$$\gamma = 45^\circ$$

$$\sin\gamma = \frac{\sqrt{2}}{2}$$

$$a = \frac{c}{\sin\gamma}\sin\alpha$$

$$\alpha = 30^\circ$$

$$\sin\alpha = \frac{1}{2}$$

$$a = \frac{10}{\frac{\sqrt{2}}{2}} \cdot \frac{1}{2}$$

$$a = \frac{10}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{10\sqrt{2}}{2} = 5\cdot\sqrt{2}$$

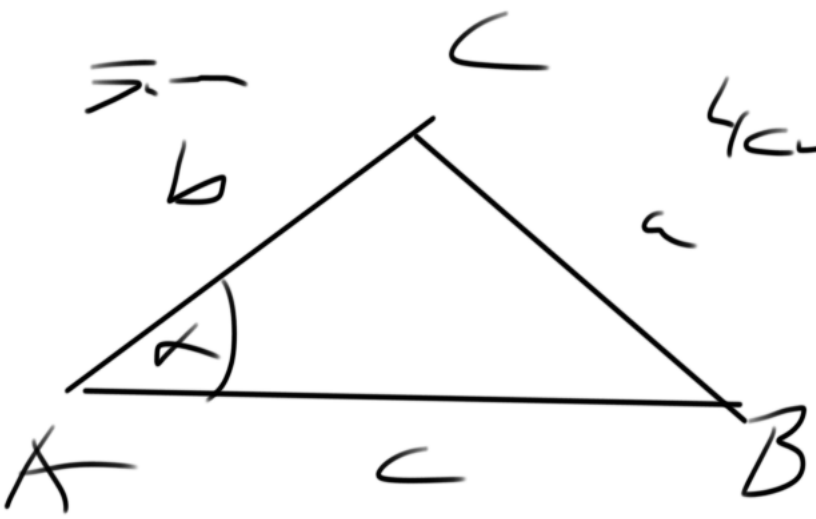
1

$$\boxed{\begin{array}{lll} a = 5\cdot\sqrt{2} & b = \frac{\sqrt{2}}{4}(1+\sqrt{3}) & c = 10 \\ \alpha = 30^\circ & \beta = 105^\circ & \gamma = 45^\circ \end{array}}$$

$\triangle ABC$: $a = 4\,cm$, $b = 5\,cm$

$\alpha = 45^\circ$

$c = ?$



2 strany + úhel

→ cosinová věta

$$\boxed{a^2 = b^2 + c^2 - 2bc\cdot\cos\alpha}$$

$$b^2 = a^2 + c^2 - 2ac\cdot\cos\beta$$

$\alpha = 45^\circ \quad \cos\alpha = \frac{\sqrt{2}}{2}$

$$16 = 25 + c^2 - 2\cdot 5\cdot c\cdot\frac{\sqrt{2}}{2}$$

$$0 = 9 + c^2 - 5\sqrt{2}\,c$$

$$c^2 - 5\sqrt{2}\,c + 9 = 0$$

$a \quad b \quad c$

$$\frac{5\sqrt{2} \pm \sqrt{50 - 36}}{2} = \frac{5\sqrt{2} \pm \sqrt{14}}{2}$$

$$c_1 = \frac{5\sqrt{2} + \sqrt{14}}{2} \qquad c_2 = \frac{5\sqrt{2} - \sqrt{14}}{2}$$

$\sqrt{2} \sim 1{,}5$ $\qquad \sqrt{14} \sim 3{,}5$

$$c_2 = \frac{7{,}5 - 3{,}5}{2} \leq 2$$

$$c_1 = \frac{7{,}5 + 3{,}5}{2} \geq 6$$

3,5
3,5
175
105
12,25

$a + b > c$ $\qquad a = 4\,cm$

$b + c > a$ $\qquad b = 5\,cm$

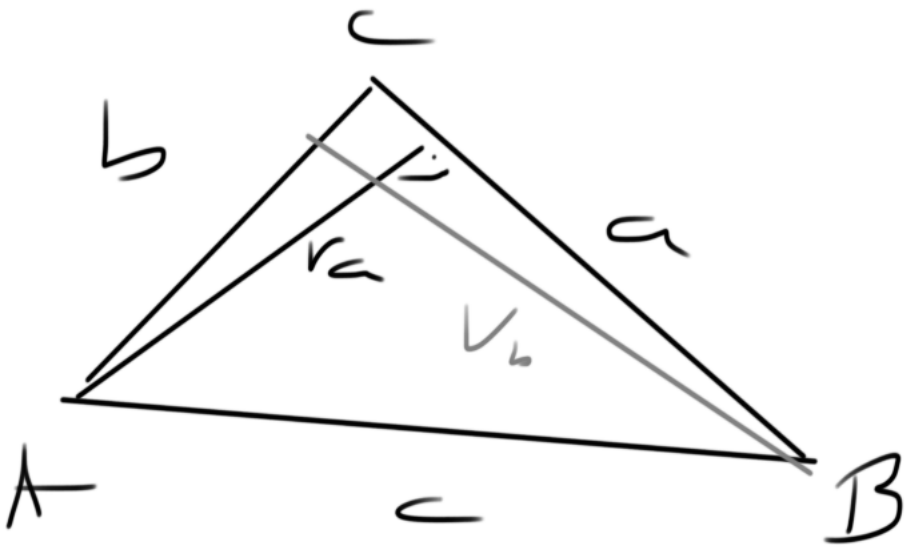
$c + a > b$ $\qquad c_1 = 6$ ✓ $\qquad c_2 = 2$ ✓

$a \rightarrow 3{,}5\,m \rightarrow \times$

$\triangle ABC \quad a = 5\,cm \quad b = 4\,cm$

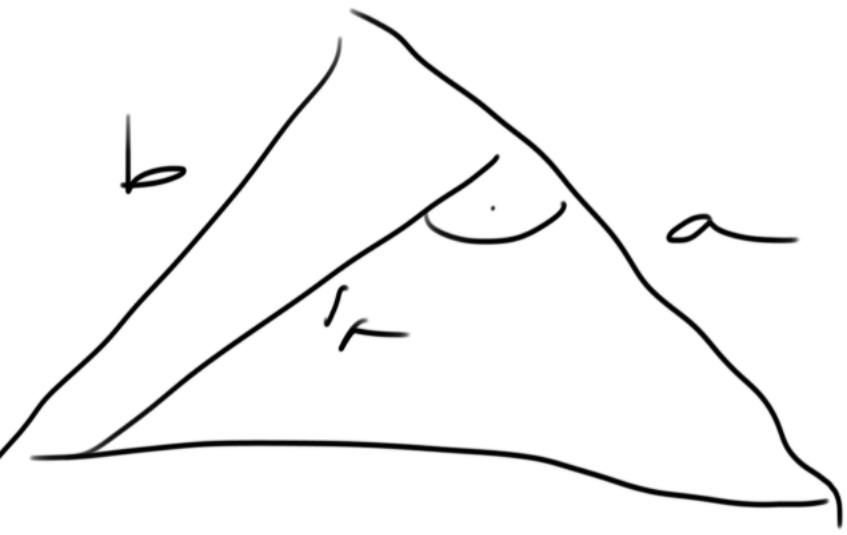
$v_a = 2\,cm$

$S = ? \quad v_b = ?$



$$S = \frac{1}{2} a \cdot v_a$$

$$S = \frac{1}{2} \cdot 2 \cdot 5 = 5\ cm^2$$



$$S = \frac{1}{2} b \cdot v_b$$

$$5 = \frac{1}{2} \cdot 4 \cdot v_b$$

$$5 = 2 \cdot v_b$$

$$\boxed{v_b = \frac{5}{2}}$$

délka m, cm, km

plocha m^2, cm^2, km^2

2. mocnina ⟶ „čtverec"

$\triangle ABC$ $a = 5\,cm$ $b = \ldots$

$V = ?$ $S = ?$

$V = 3\,cm$ $S = 25\,cm^2$

⟶ všechno do stejných jednotek.

$S = 6\,cm^2$

$\times S = 0{,}2 \cdot 50 = 10$ ($m \cdot cm$, ?)

$\checkmark S = 0{,}2 \cdot 0{,}5 = 0{,}1$ ($m \cdot m$, m^2)

$$S_{\triangle} = \frac{1}{2}(a + v)$$

délka ~ m

obsah m^2 ⟶ očividně hloupost

$$S_{\triangle} = \frac{1}{2} a \cdot v_a$$

$$m \cdot m = m^2$$

$a = 5\,cm \quad v = 3\,cm$

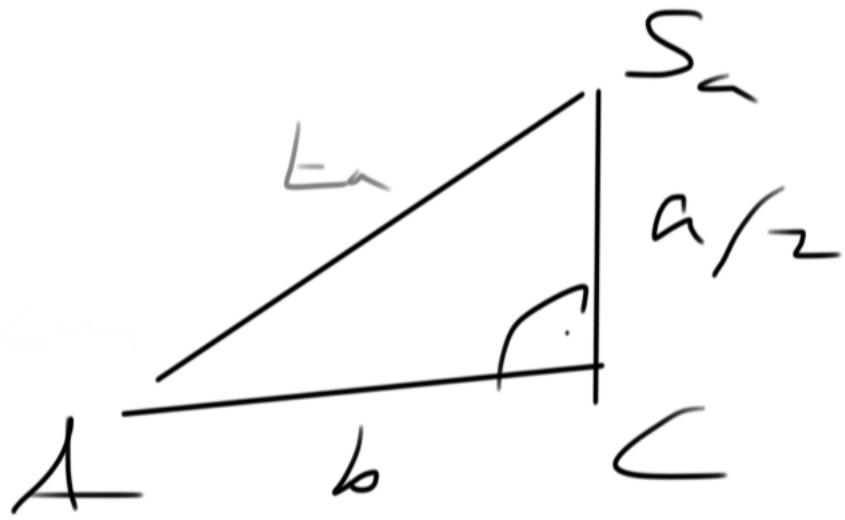
$$S_{\triangle} = \frac{1}{2} 5\,cm \cdot 3\,cm = \frac{1}{2} \cdot 5 \cdot 3\ cm \cdot cm$$

$$= \frac{15}{2}\,cm^2$$

$\triangle ABC$, pravoúhlý, c přepona

$a = 4\text{cm}$ $\quad t_a = 6\text{cm}$

$t_b = ?$



$$t_a^2 = b^2 + \left(\frac{a}{2}\right)^2$$

$$b = \sqrt{t_a^2 - \left(\frac{a}{2}\right)^2}$$

$$b = \sqrt{36 - 4} = \sqrt{32}$$

$$= 4\sqrt{2}$$

$$t_b = \sqrt{a^2 + \left(\frac{b}{2}\right)^2}$$

$$= \sqrt{16 + 8}$$

$$= \sqrt{24} = \boxed{2\sqrt{6} = t_b}$$