

Funkce

$x \rightarrow f$
- $f: \mathbb{R} \rightarrow \mathbb{R}$

$$f = \{(x, y) \in \mathbb{R} \times \mathbb{R} ; f(x) = y\}$$

vzor obrát

$$x_1 = x_2 \Rightarrow y_1 = y_2$$

Zadání funkce $\xrightarrow{\text{predpisem a } D_f}$

výkres (x, y) (graf)

1) $f: \mathbb{R} \rightarrow \mathbb{R}$

$$D_f \subset \mathbb{R} \quad f_f \subset \mathbb{R}$$
$$[2, 3] \rightarrow [0, 10]$$

2) $f: \mathbb{R} \rightarrow \mathbb{R}$

$$D_f = \mathbb{R} \quad f_f = \mathbb{R}$$

$$f: y = 2x + 3$$

$$D_f = \mathbb{R}$$



$$g: \boxed{y = 2x + 3}$$

$$D_f = \mathbb{R}^+$$

S. 1 Vlastnosti funkcií

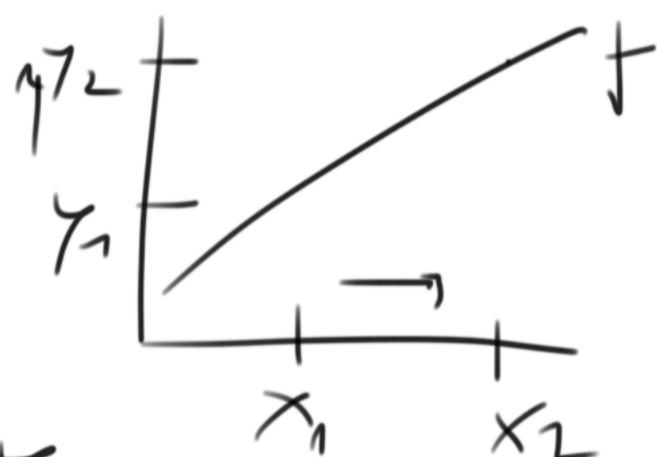
- Monotonie Onezenosť
- Periodicită Parita
- (Spojitosť Hlackosť)

Monotonie

$$f, D_f \quad x_1, x_2 \in I \subset D_+ \quad y_1 = f(x_1) \quad y_2 = f(x_2)$$

↑ interval

Funkce f je



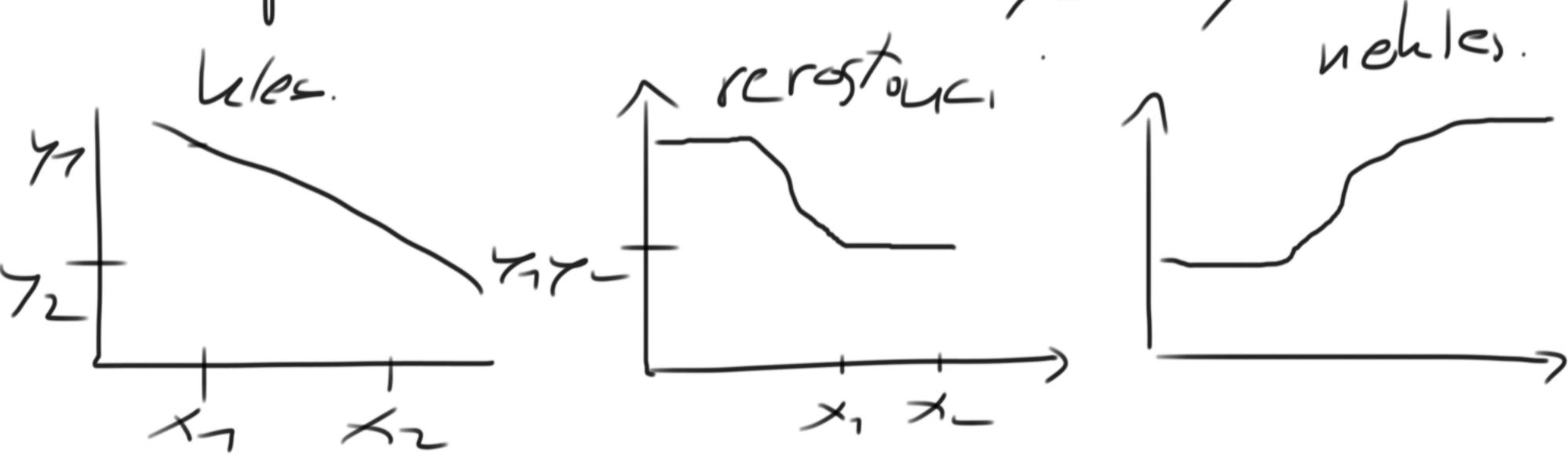
rostoucí $x_2 > x_1 \Rightarrow y_2 > y_1$

(růstec monotonie)

klouzající $x_2 > x_1 \Rightarrow y_2 < y_1$

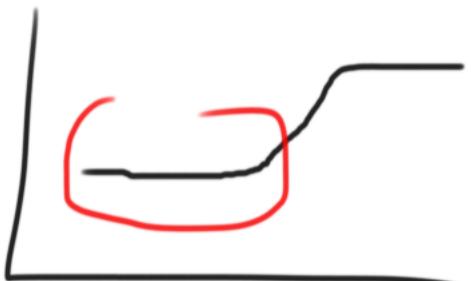
herastoucí $x_2 > x_1 \Rightarrow y_2 \leq y_1$

nehklouzající $x_2 > x_1 \Rightarrow y_2 \geq y_1$



f rostoucí $\Rightarrow f$ neklesající

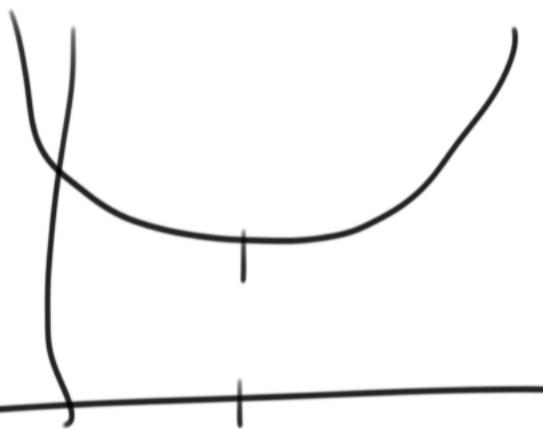
~~z~~



f klesající $\Rightarrow f$ nerostoucí

Interval monotonic

rost. $r(a, \infty)$



kles. $r(-\infty, a)$

klesající a rostoucí

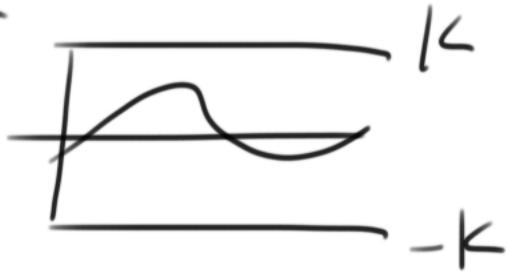
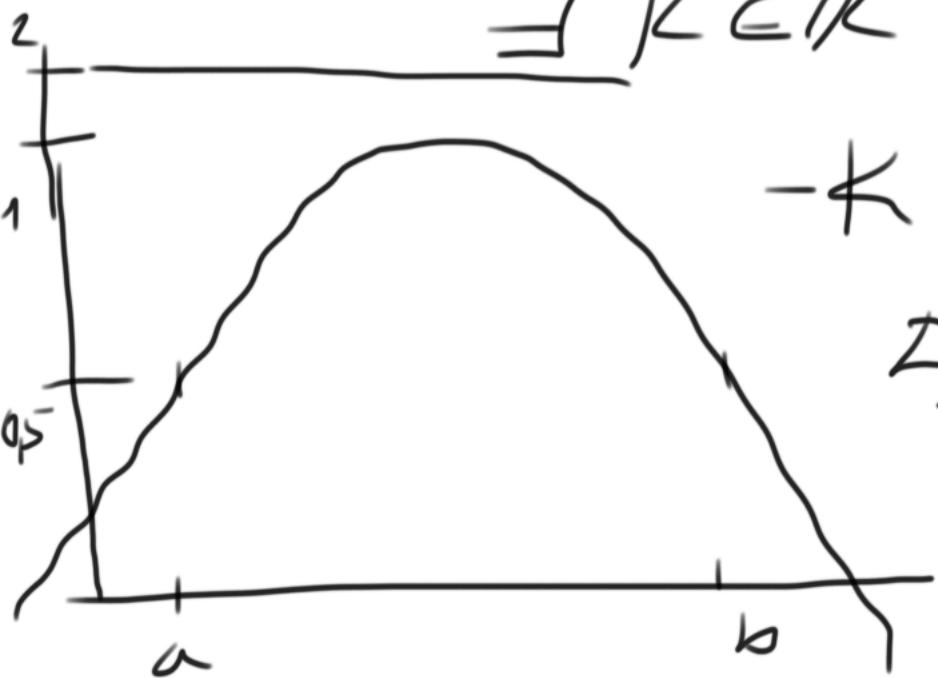
Omezenost

f omezená na $M \subset D_f \Leftrightarrow$

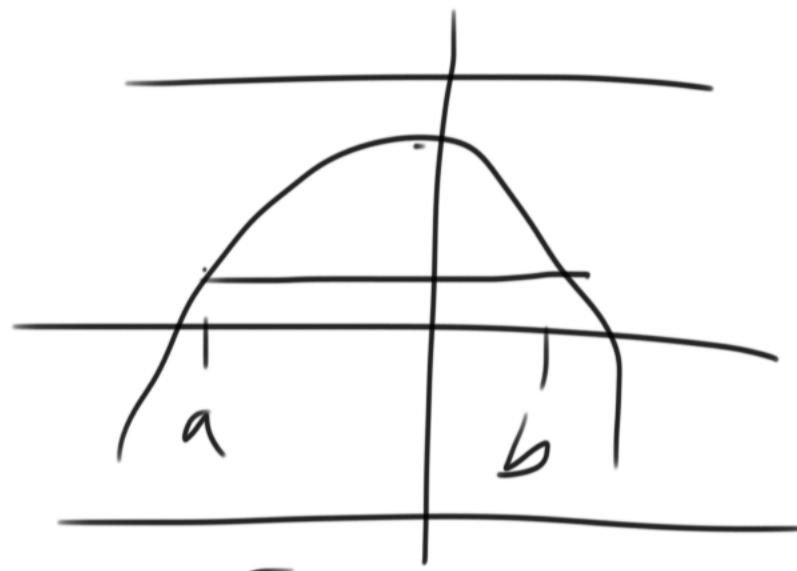
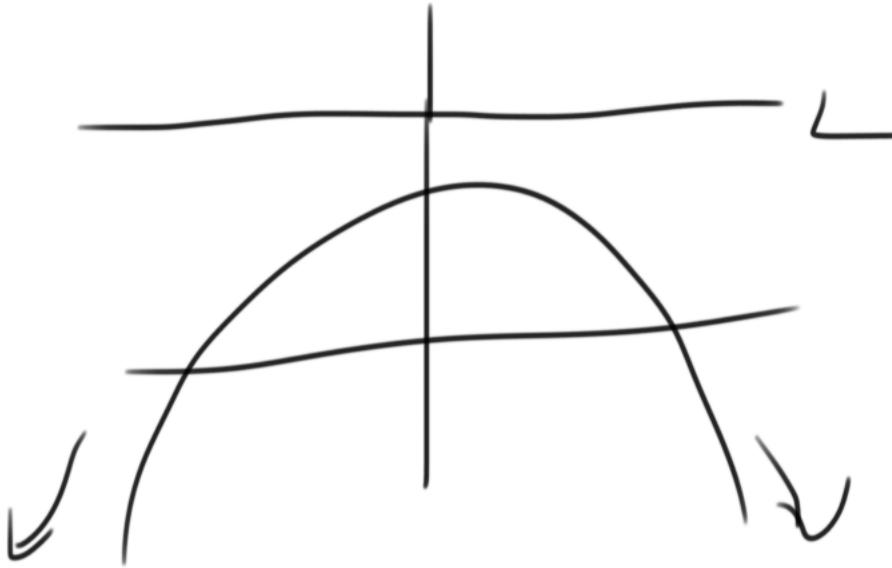
$\exists K \in \mathbb{R} : |f(x)| \leq K \quad \forall x \in M$

$-K \leq f(x) \leq K \quad \forall x \in M$

$$D_f = \mathbb{R}$$



f je om. slora / zdola na \mathbb{R}^+
 $\Leftrightarrow \exists K \in \mathbb{R} \quad f(x) \leq K \quad \forall x \in M$ slora
 $f(x) \geq K \quad \forall x \in M$ zdola

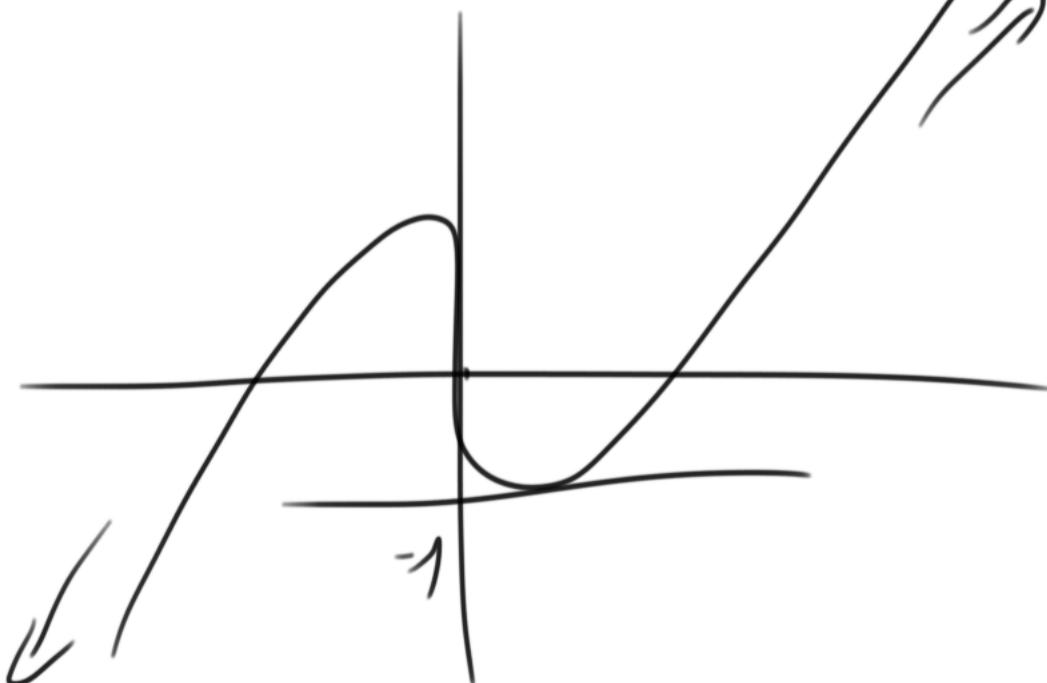


slora na \mathbb{R}^+

na $[a, b]$ je omezená

f je omezená \Leftrightarrow existuje slora i zdola

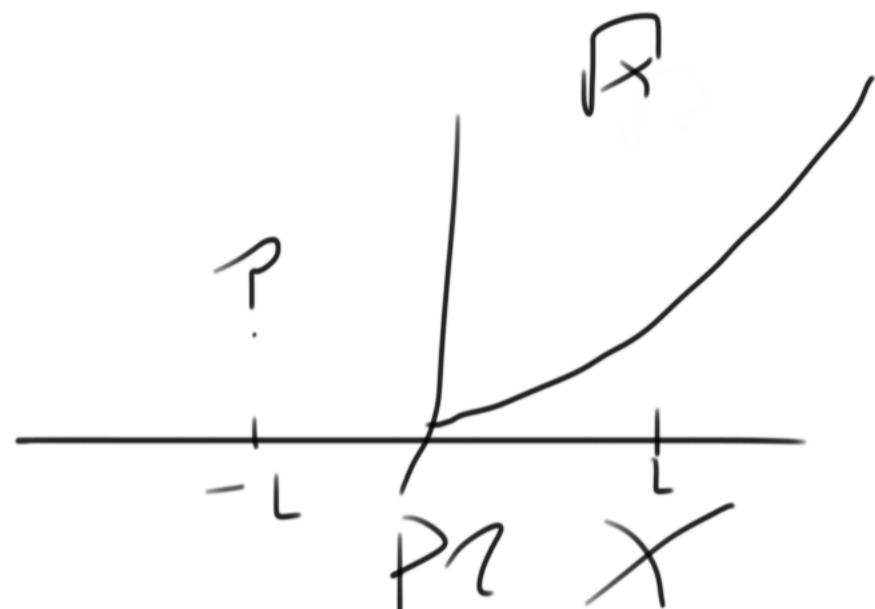
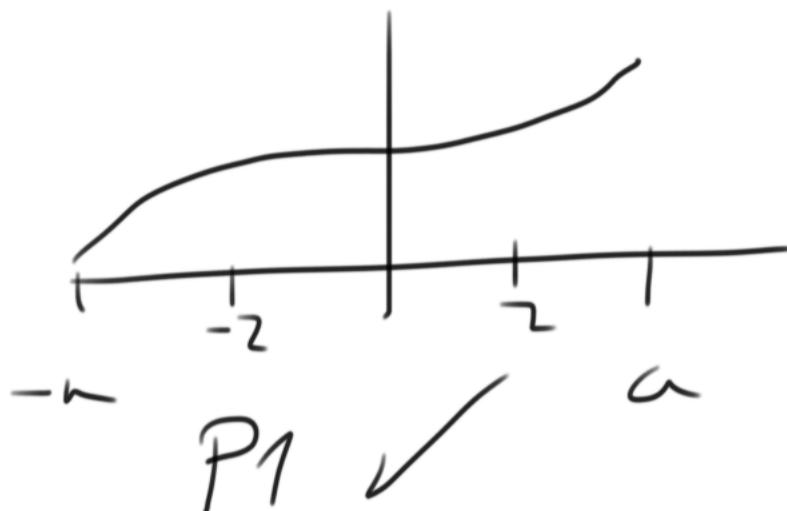
f je nemezená \Leftrightarrow ani slora ani zdola



\mathbb{R}^+ : zdola

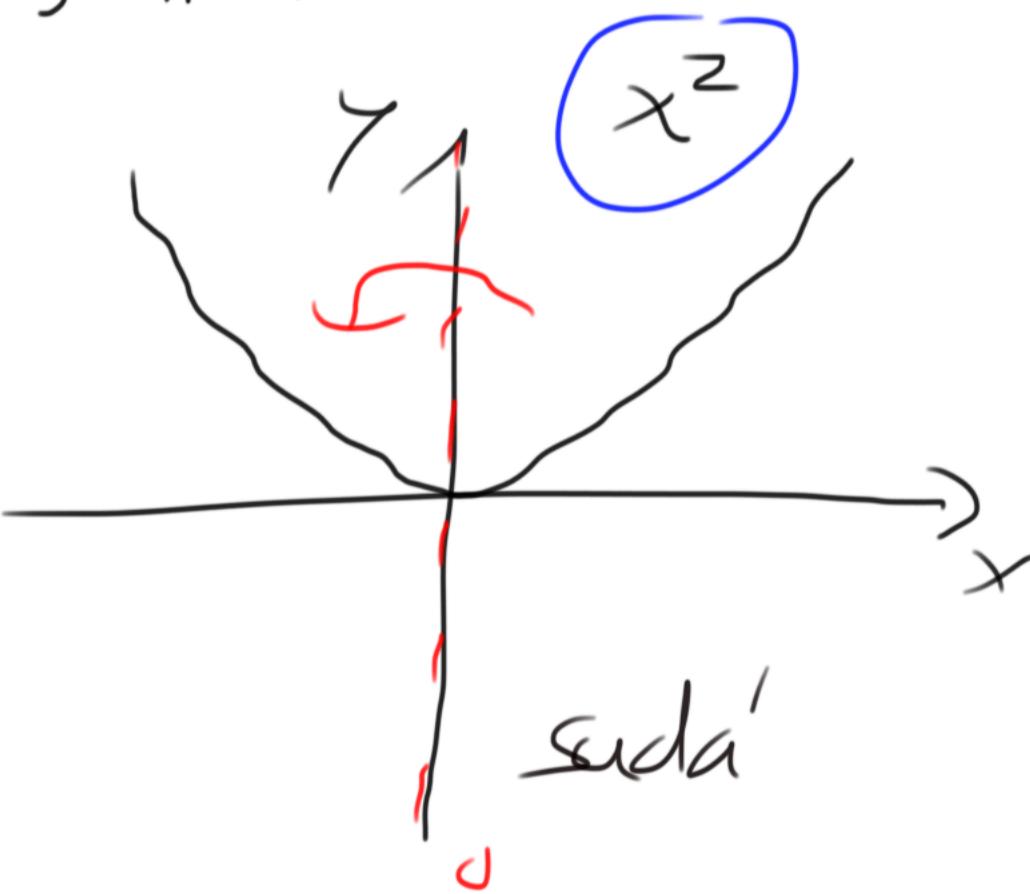
Pirita
 ↘ lichí
 ↘ suda'

$P_1: f \in \mathcal{P}_f, \forall x \in \mathbb{R} \rightarrow x \in \mathbb{R}$

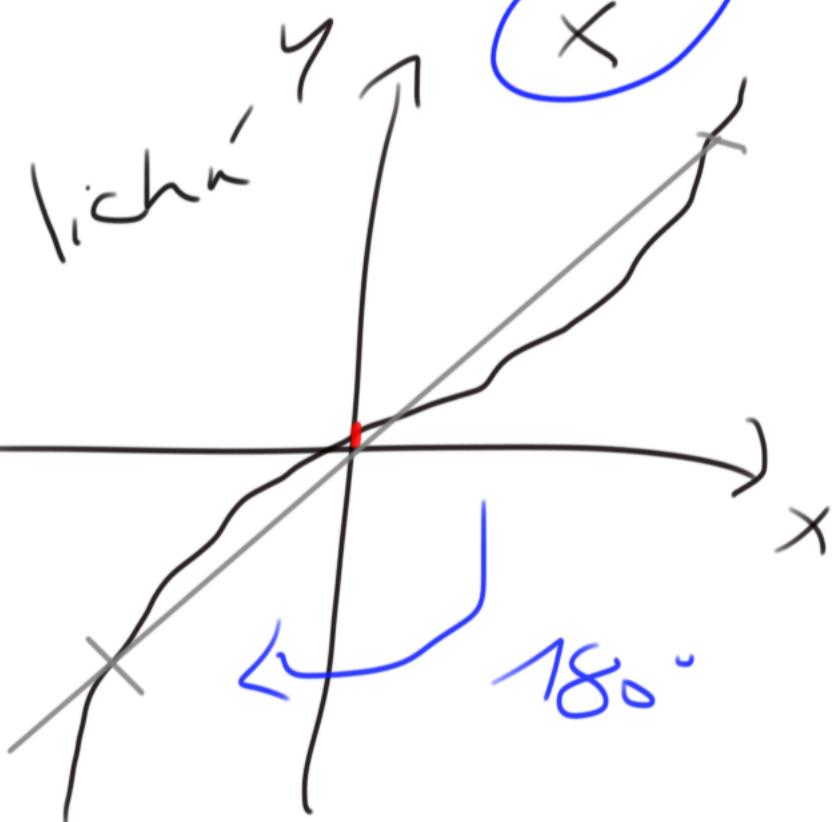


f suda' $\Leftrightarrow f(-x) = f(x)$

f lichí $\Leftrightarrow f(-x) = -f(x)$



f lichí $\Leftrightarrow f(-x) = -f(x)$



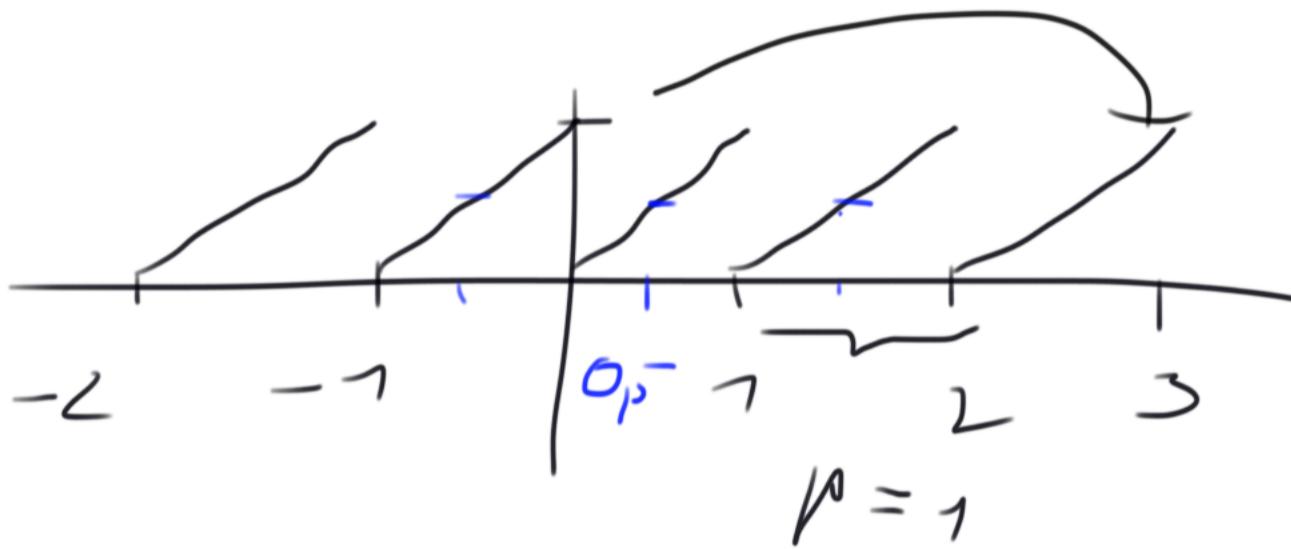
$$\begin{aligned}
 f(-x) &= (-x)^2 = x^2 = f(x) \\
 f(-x) &= (-x)^3 = -x^3 \\
 &= -f(x)
 \end{aligned}$$

Periodicität

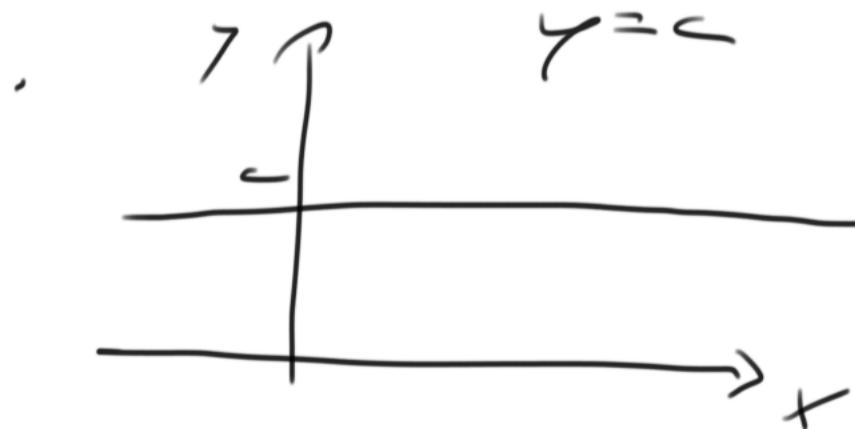
$$\frac{f_1 \in \mathcal{P}_f \quad \forall x \in D_f \quad \underline{x+p \in D_f}, \quad p \in \mathbb{R}^+}{p=3 \quad 2 \in D_f \Rightarrow 2+3 \in D_f}$$

\Leftrightarrow periodika' \rightarrow periodon p

$$\Leftrightarrow f(x+p) = f(x)$$



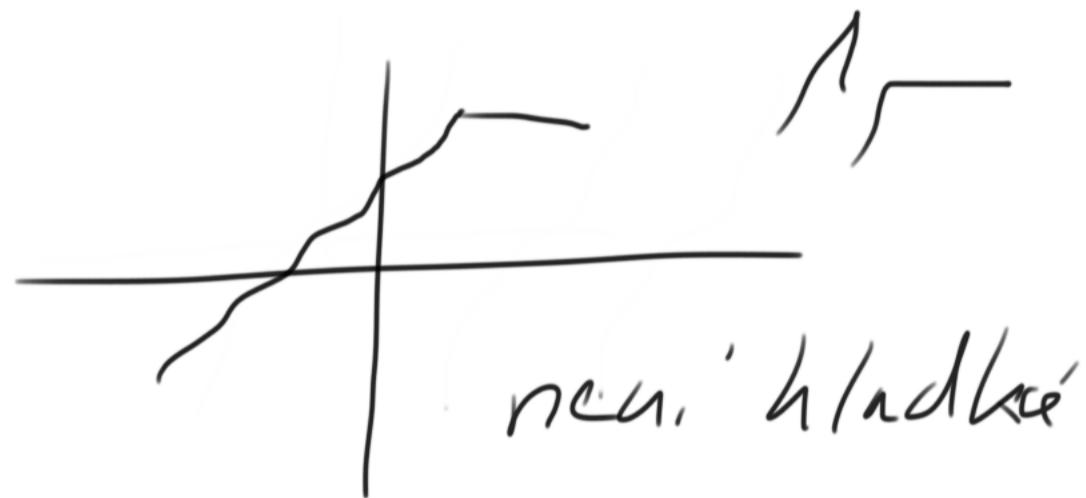
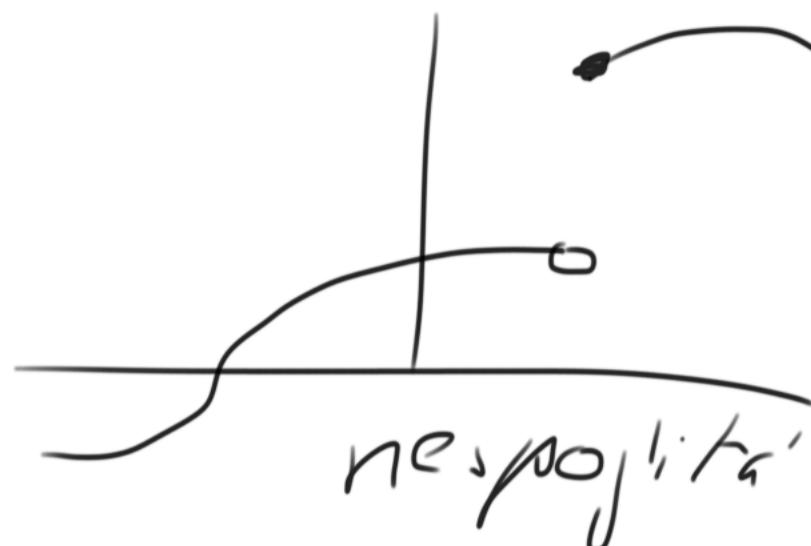
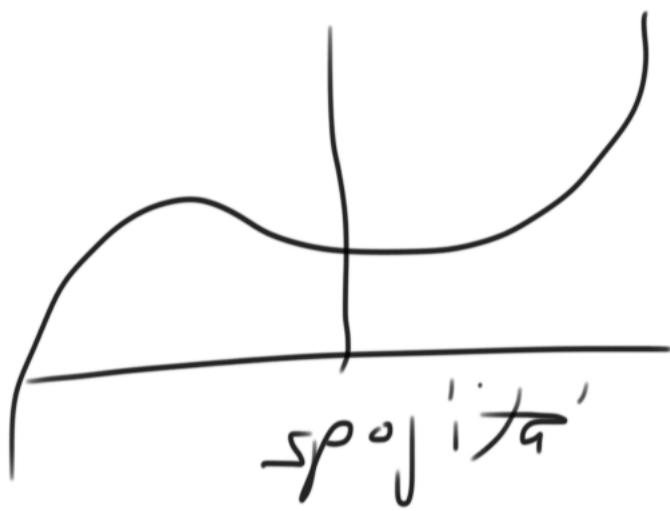
$$\cdot f(x+np) = f(x) \quad \forall n \in \mathbb{Z}$$



$$f(x+p) = f(x)$$

$$\forall p \in \mathbb{R}$$

Spojitost & Hladkosť



injektivé

surjektive

~~bijektive
(inj. \wedge surj.)~~

surjektívny - "na"

$$F: X \rightarrow Y$$



$$f: y = x^2 \quad \underline{R \rightarrow R} \quad \text{nech. surjektivé}$$

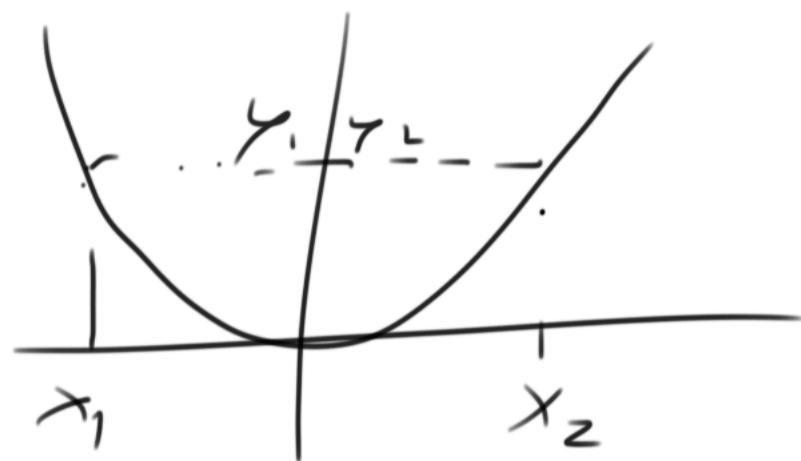
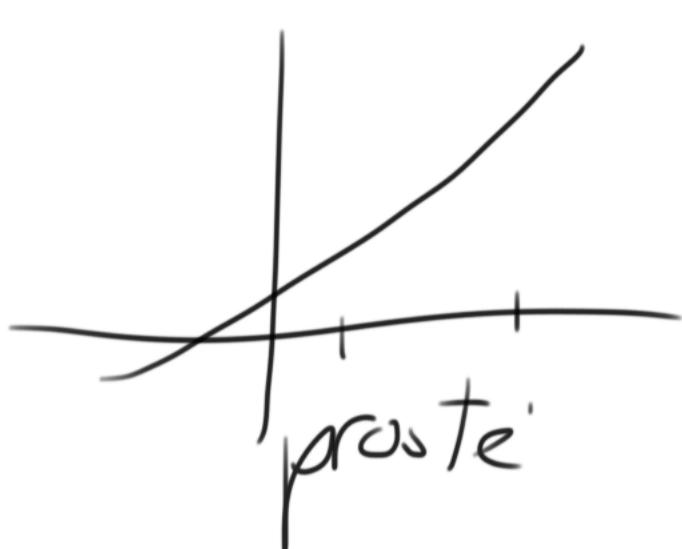


$$g: y = x^2 \quad \underline{R \rightarrow R^+} \quad \text{surjektivé}$$

injektiv - "do"

$$(x_1, y_1), (x_2, y_2) \in f$$

$$y_1 = y_2 \Rightarrow x_1 = x_2$$



$y_1 = y_2$ a/c $x_1 \neq x_2$
neni prostá'

f rostouc' nebo klesajic' na \mathbb{R}_+
 $\Rightarrow f$ prostc'.

5.2 Transf.-grafů

El. mat. Lin. a kadr. fce.

"Transf.-grafů fci"

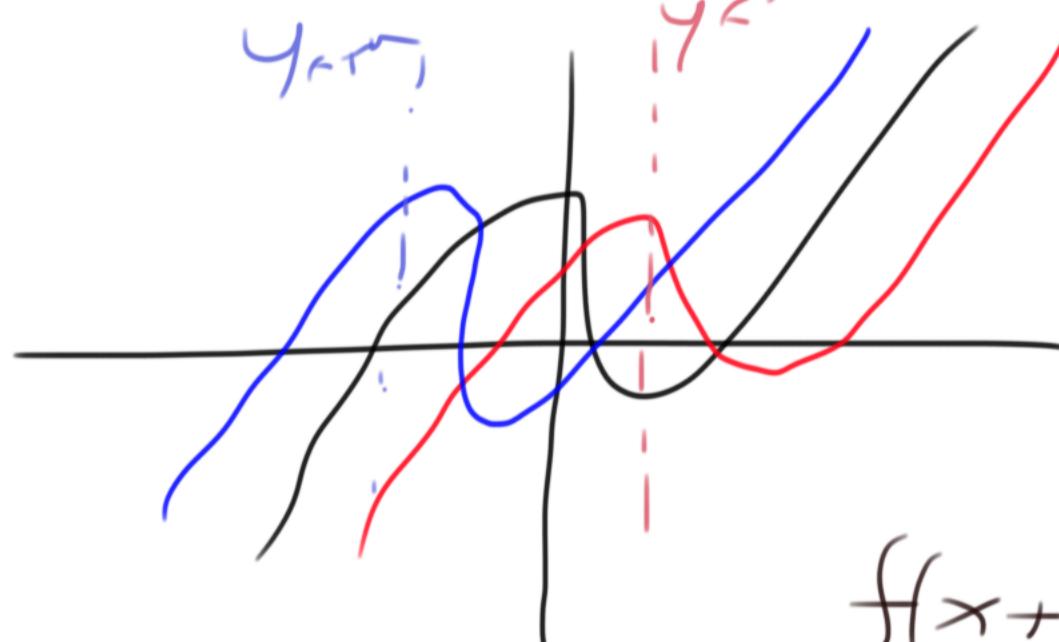
Heleňa Říhová

$$y = f(x)$$



1) Posun v x

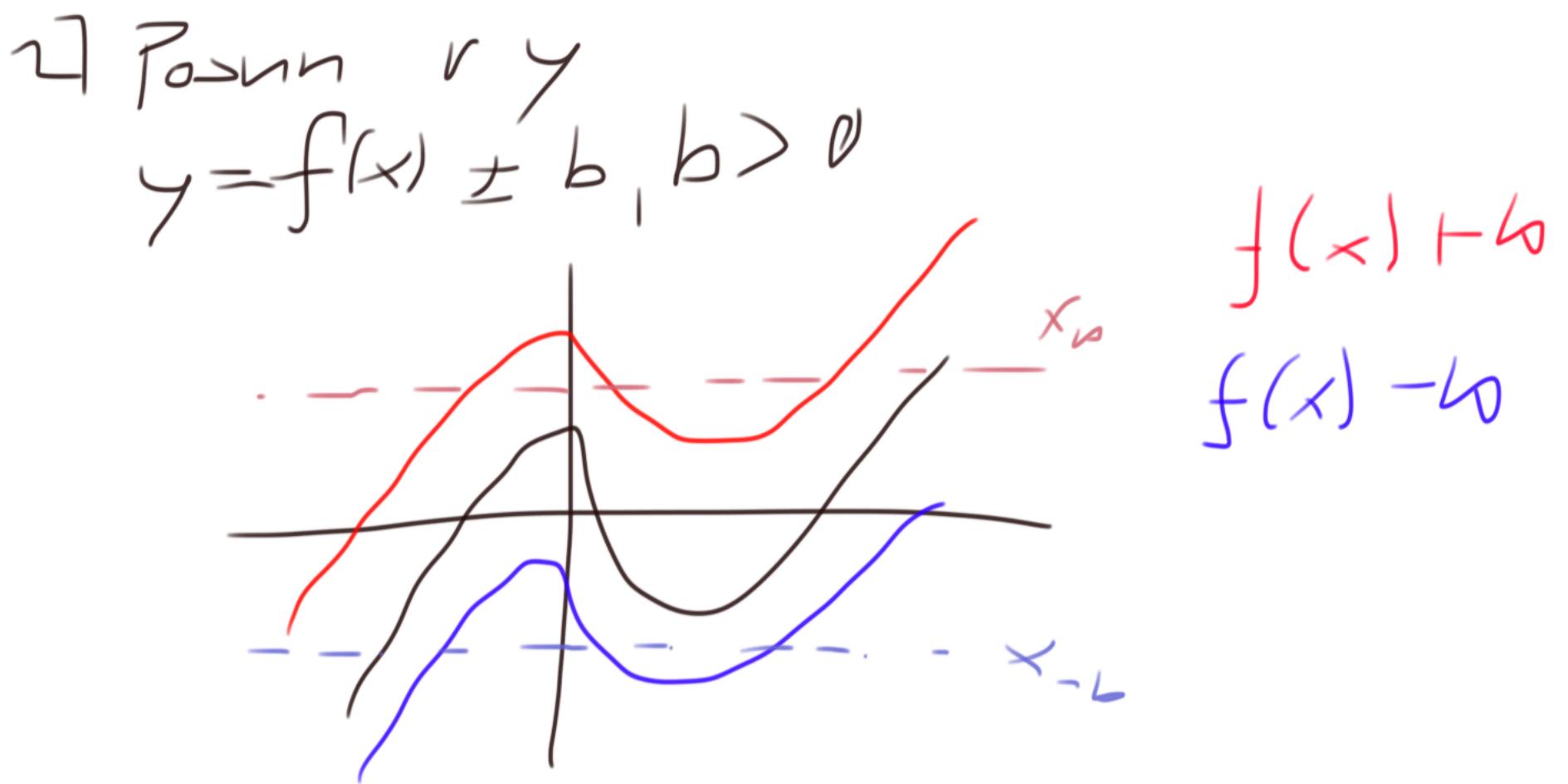
$$y = f(x \pm a), \underline{a > 0}$$



$$f(x+a)$$

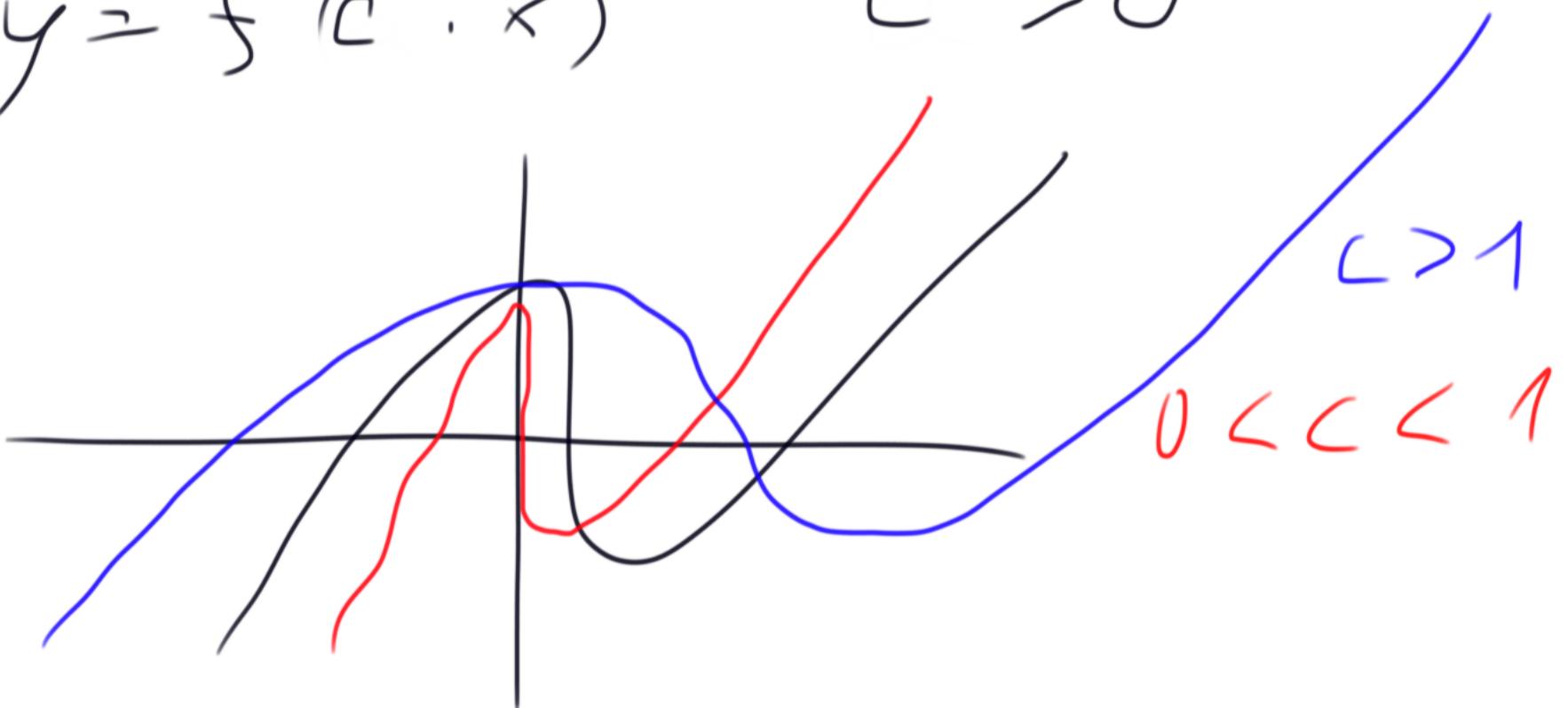
$$f(x-a)$$

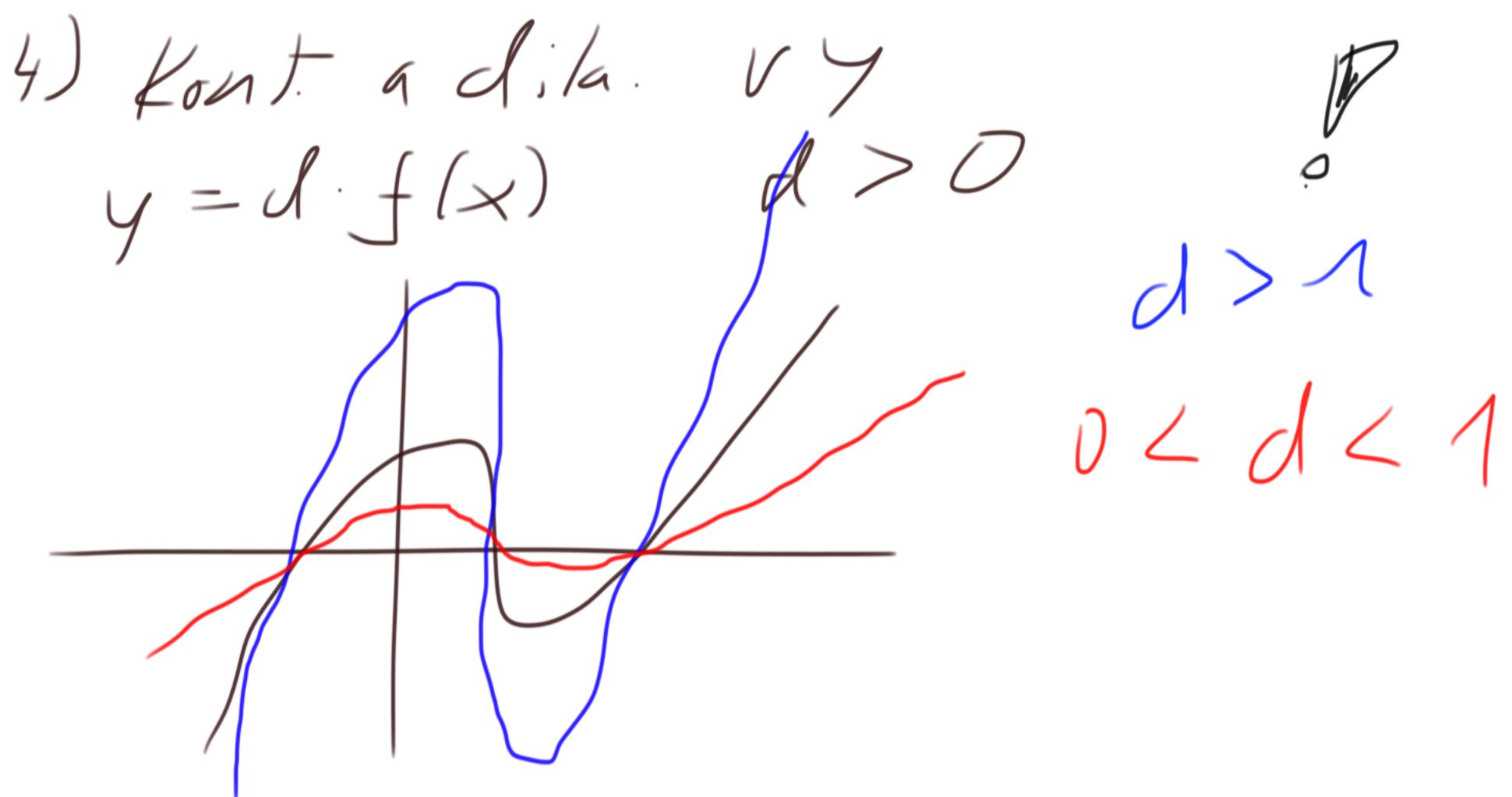
$$f(x+3)$$



3) Konfr. a dilatace $\checkmark x$

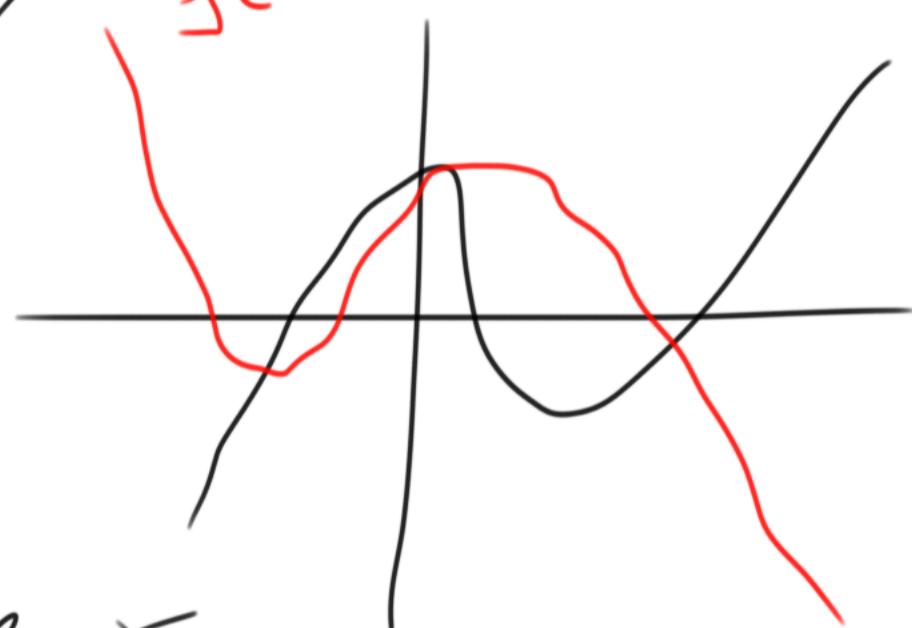
$$y = f(c \cdot x) \quad c > 0$$





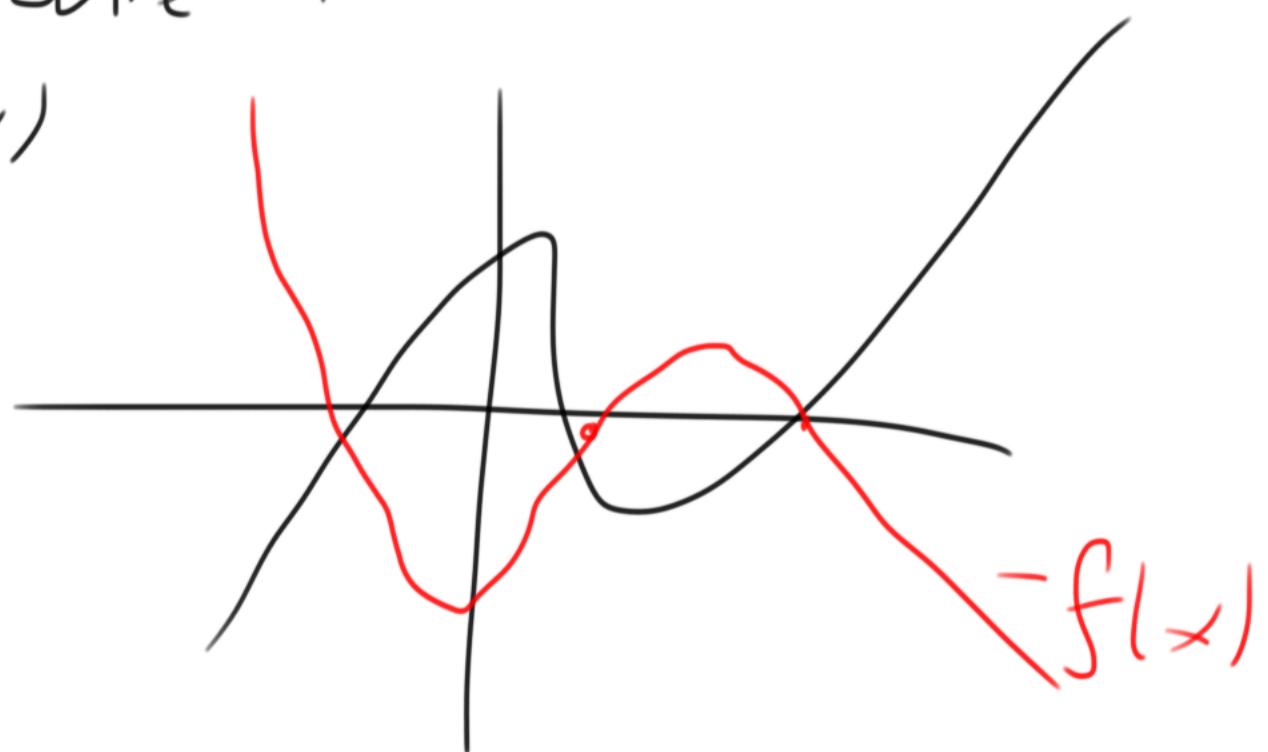
5) Préhl. podle y $s(-x)$

$$y = f(-x)$$



6) Préhl. podle x

$$y = -f(x)$$



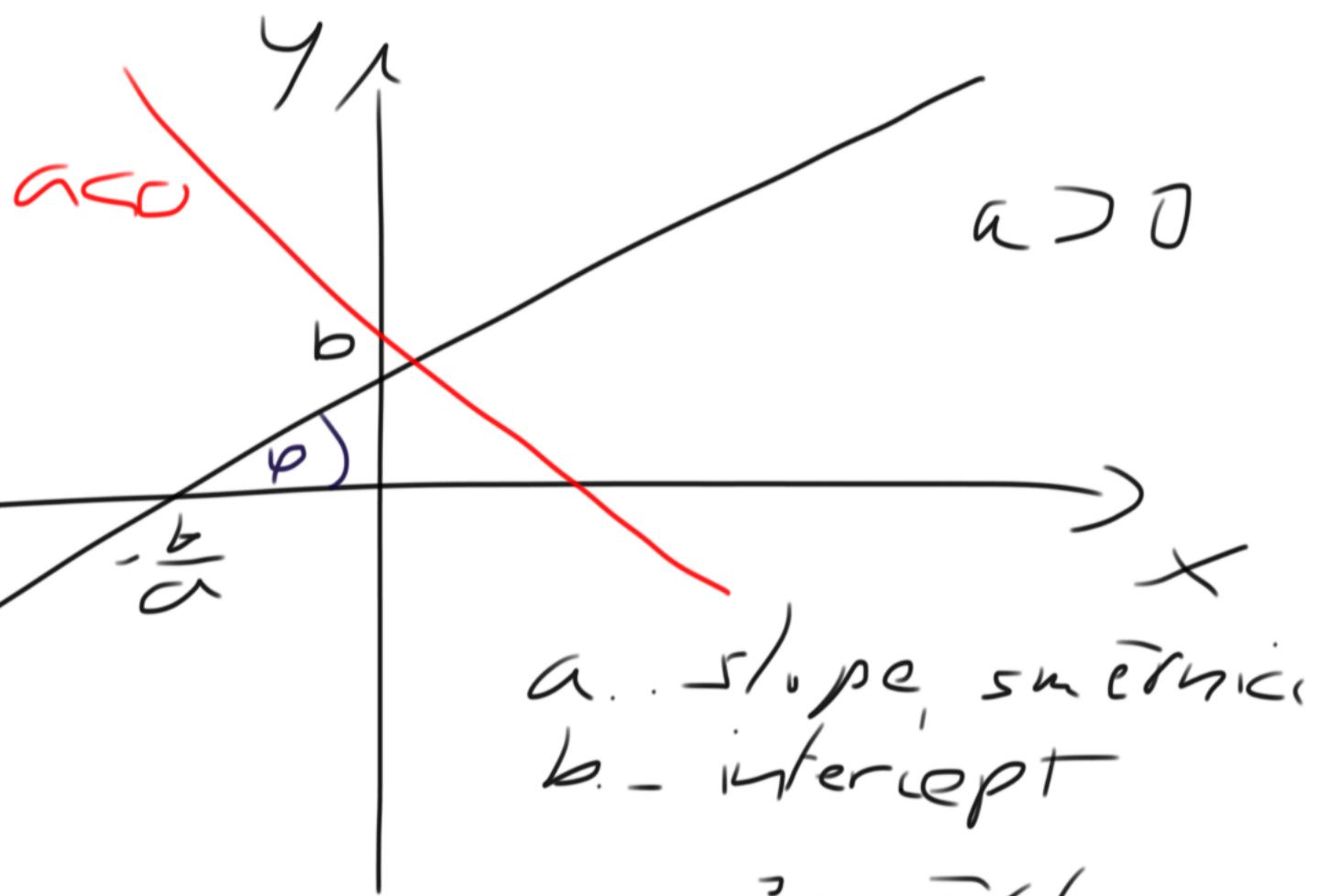
$$y = f(x)$$

5.3 Lineární funkce

$$y = a \cdot x + b \quad a, b \in \mathbb{R}$$

$$D_f = \mathbb{R} \quad K_f = \mathbb{R} \quad a \neq 0$$

průřezka



průsečík s x

$$\boxed{y=0}$$

$$0 = a \cdot x + b$$

$$x = -\frac{b}{a}$$

průsečík s y:

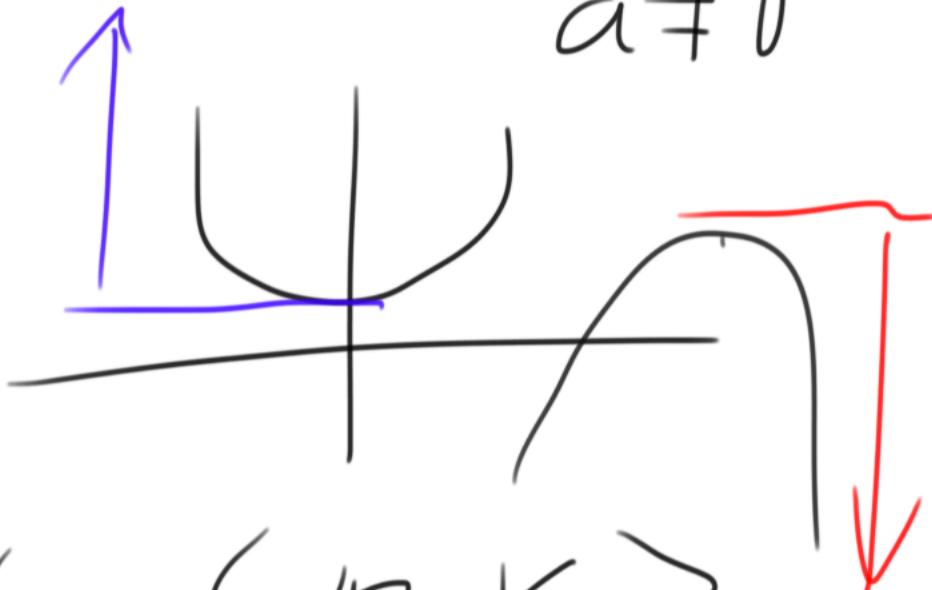
$$\boxed{x=0}$$

$$\underline{y = a \cdot 0 + b = b}$$

5.4. Kvadratická funkce

$$y = ax^2 + bx + c \quad a, b, c \in \mathbb{R} \quad a \neq 0$$

grafem: parabola



$$D = \mathbb{R}$$

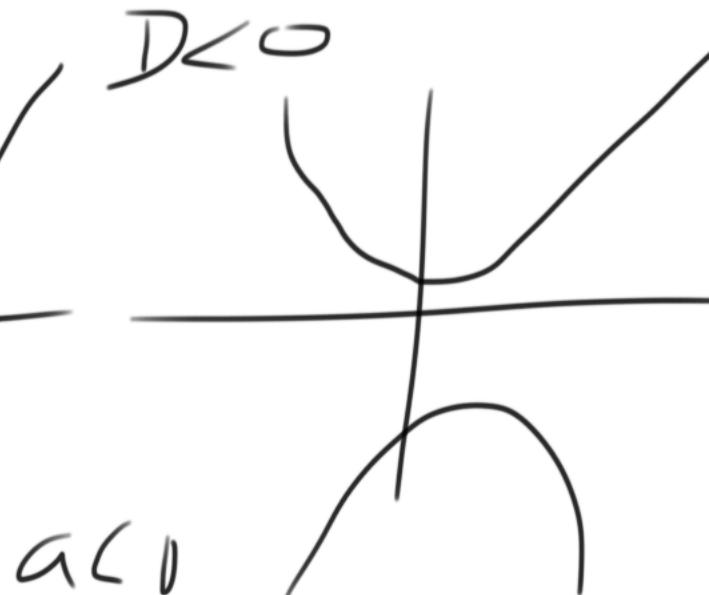
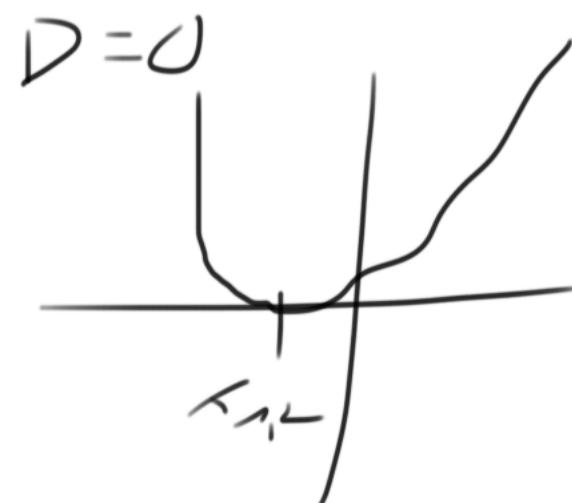
$$H_S = \langle V_S, \omega \rangle \text{ nebo } (-\omega, V_S)$$

•mezeha shora nebo zdola

průs. s y: $x=0 \quad y = a \cdot 0 + b \cdot 0 + c \quad y = c$

průs. s x: $y=0 \quad a \cdot x^2 + b \cdot x + c = 0$

$$D = b^2 - 4 \cdot a \cdot c$$

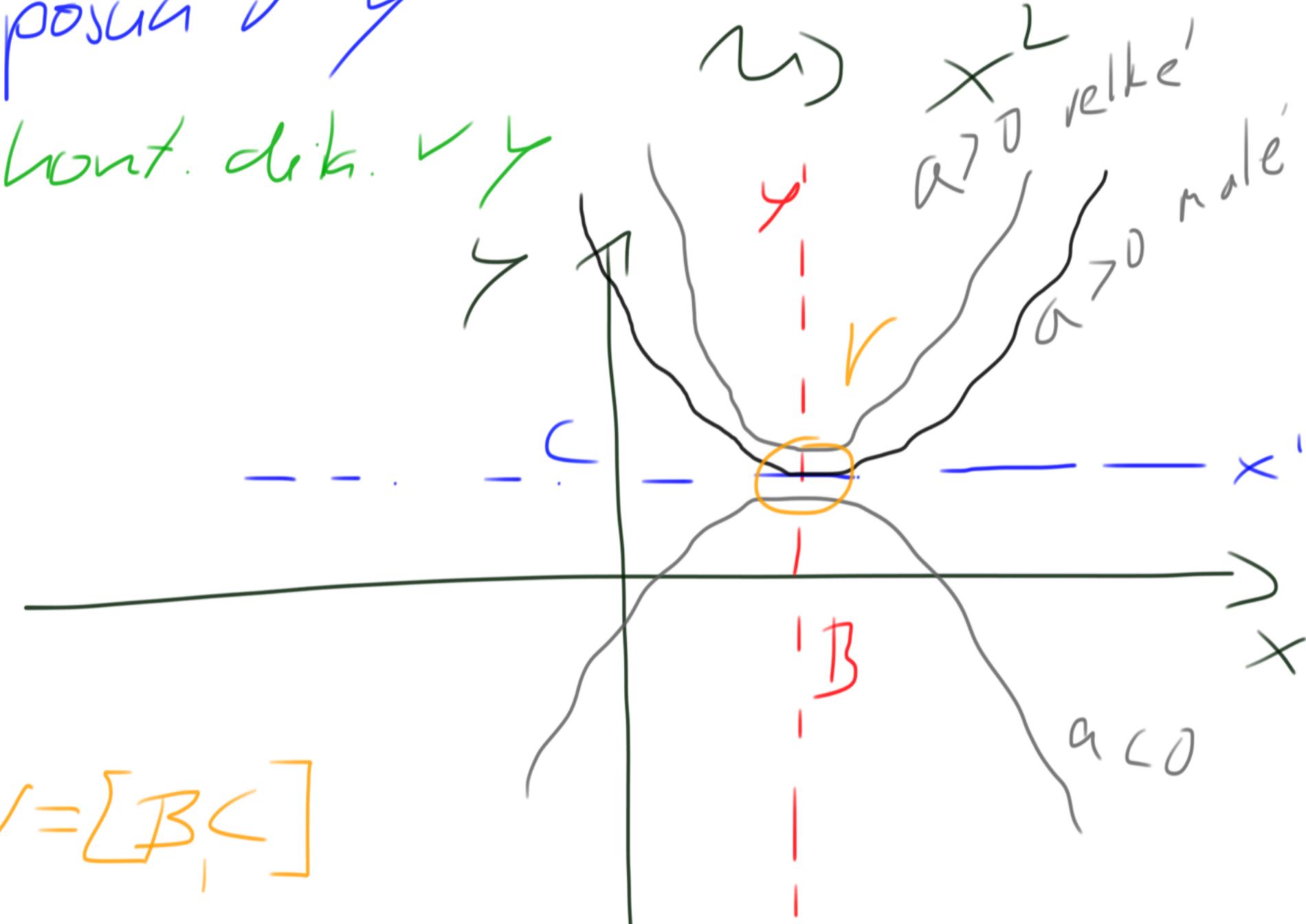


$$y = a \cdot (x - B)^2 + C \quad B = -\frac{b}{2a}$$

posun v x

posun v y

kont. dik. v y



$$V = [B \mid C]$$

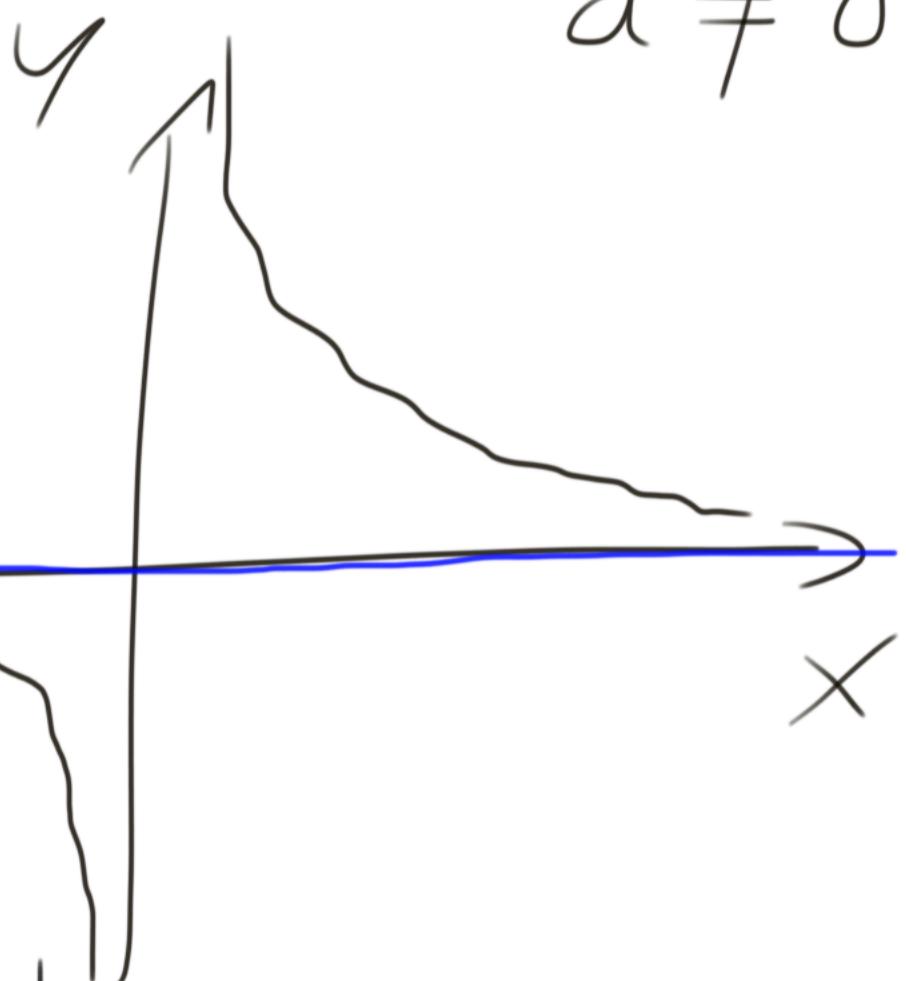
$$= \left[-\frac{b}{2a} \mid C - \frac{b^2}{4a} \right]$$

souřadnice
vrchol

5.5 Lineární funkce

$$y = \frac{a}{x+b} + c \quad a, b, c \in \mathbb{R} \quad a \neq 0$$

hyperbola
 $y = \frac{1}{x}$



$$D_f = \mathbb{R} \setminus \{-b\}$$

$$J_f = \mathbb{R} \setminus \{c\}$$

