Nerovnice a rovnice s abs. hodnotou

$2x < 6 \quad /:2$

$x < 3$

$-2x < 6 \quad /:(-2)$

$x > -3$

$\frac{1}{x-1} < 3$

Pozor na znaménko $(x-1)$

$\rightarrow$ nutno rozdělit na jednotlivé případy

$x-1$: < 0 | > 0

$\ominus$ 1 $\oplus$

$x \in (-\infty, 1)$

$(x-1) < 0$

$\frac{1}{x-1} < 3 \quad / \cdot (x-1)$ (zápor!)

$1 > 3(x-1)$

$x \in (1, \infty)$

$(x-1) > 0$

$1 < 3(x-1)$

$\frac{1}{x-2} < \frac{1}{x+3}$ Nulové body: $-3, 2$

$x \neq 2$

$x = -3$

I) $x \in (-\infty, -3)$

$\frac{1}{x-2} < \frac{1}{x+3} \quad / \cdot (x-2)$

$1 > \frac{x-2}{x+3} \quad / \cdot (x+3)$

$x+3 < x-2$

$0 < -5$

NEPLATÍ NIKDY

NŘ

II) $x \in (-3, 2)$

$\frac{1}{x-2} < \frac{1}{x+3} \quad / \cdot (x-2)$

< 0

$1 > \frac{x-2}{x+3} \quad / \cdot (x+3)$

> 0

$x+3 > x-2$

$0 > -5$

$x \in (-3, 2)$

III) $x \in (2, \infty)$

$x+3 < x-2$

$0 < -5$

NŘ

Řešení: sjednocení

$x \in (-3, 2)$

$$\frac{1}{x-2} < \frac{1}{x+3} \quad / - \frac{1}{x+3} \qquad x \neq 2,\ x \neq -3$$

$$\frac{1}{x-2} - \frac{1}{x+3} < 0$$

$$\frac{x+3-(x-2)}{(x-2)\cdot(x+3)} < 0$$

$$\frac{5}{(x-2)\cdot(x+3)} < 0$$

$$\Rightarrow \quad x \in (-3, 2)$$

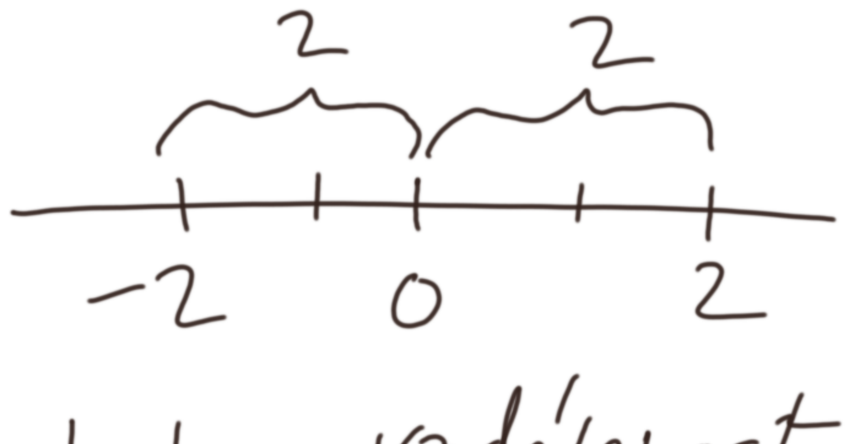
Abs. hodnota:

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$|5| = 5, \ |0| = 0$

$|-2| = 2$

$|x| = 2$

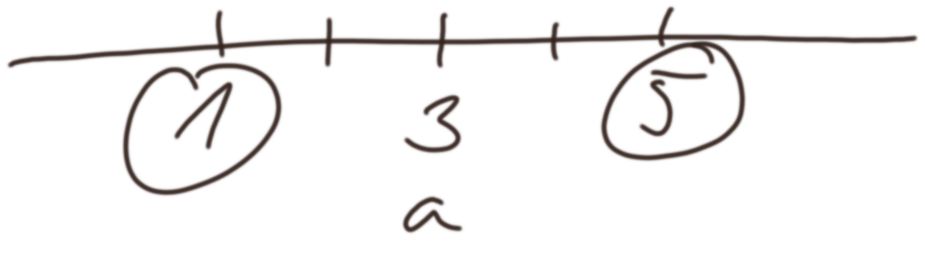


$x_1 = -2$
$x_2 = 2$

$|x|$ ... vzdálenost č. x od 0.

$|x-3| = 2$

$|x-a|$ ... vzdálenost č. x od č. a



$x_1 = 1$
$x_2 = 5$

$|x+3|+|x-2|=3$

$|x-(-3)|+|x-2|=3$

$|x-a|$

Těžké řešit geometricky

$|x-2|$: $\underset{I}{(x-2)<0}$ | $\underset{II}{(x-2)>0}$

$|x-2|=-(x-2)$ 2 $|x-2|=x-2$

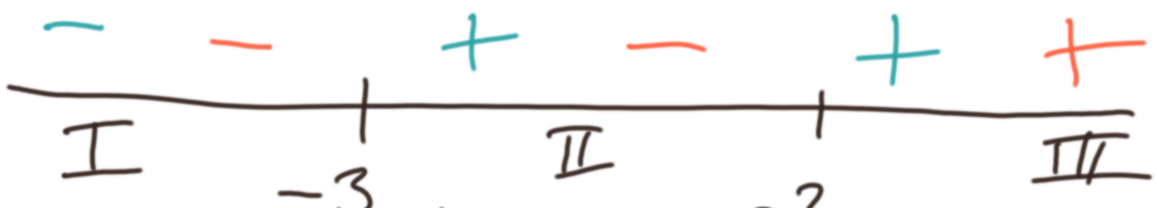
$=-2x+2$

$|x|=\begin{cases} x, & x\geq 0 \\ -x, & x<0 \end{cases}$

I: $(-\infty, 2)$

II: $\langle 2, \infty)$

$|x+3|+|x-2|=3$



$|x+3|=-(x+3)$ $|x+3|=x+3$ $|x+3|=x+3$

$|x-2|=-(x-2)$ $|x-2|=-(x-2)$ $|x-2|=x-2$

I) $x\in(-\infty,-3)$

$-(x+3)-(x-2)=3$

$-x-3-x+2=3$

$-2x-1=3$

$x=-2$

$-2 \notin I \Rightarrow NŘ.$

II) $x\in\langle -3, 2)$

$x+3-(x-2)=3$

$3+2=3$

$5=3$

$NŘ.$

III) $x\in\langle 2,\infty)$

$x+3+x-2=3$

$2x+1=3$

$x=1$

$1 \notin III \Rightarrow NŘ.$

$K=\{\}=\emptyset$

$NŘ.$

Např.

$K=\{2,3\}$

$\cup(4,\infty)$

Lineární fce: $y = ax + b$, $a \neq 0$

$a, b \in \mathbb{R}$

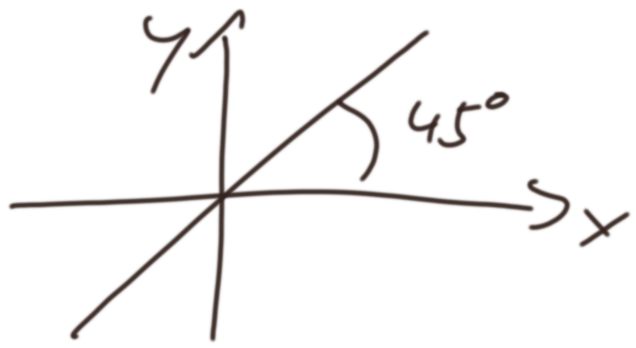
Kvadratická fce $y = ax^2 + bx + c$

$a \neq 0$, $a, b, c \in \mathbb{R}$

Lineární lomená fce: $y = \frac{a}{x-b} + c$

$a \neq 0$, $a, b, c \in \mathbb{R}$

$y = x$:



přímka

$y = x^2$

parabola

$y = \frac{1}{x}$

rovnoosá hyperbola

Načrtněte a popište funkci

$g(x) = |-2x^2 - 12x - 18|$ parabola

$g'(x) = -2x^2 - 12x - 18$

Průsečíky s x:

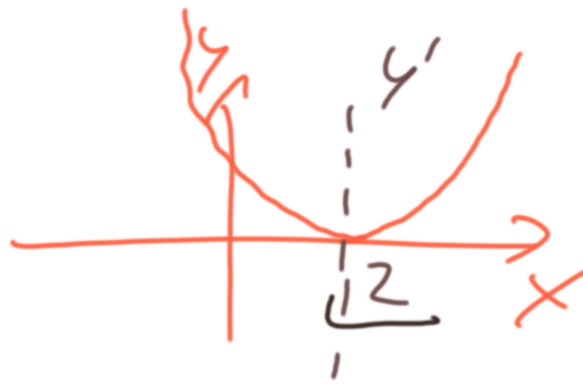
$0 = -2x^2 - 12x - 18 \quad / :(-2)$

$0 = x^2 + 6x + 9 = (x+3)^2$

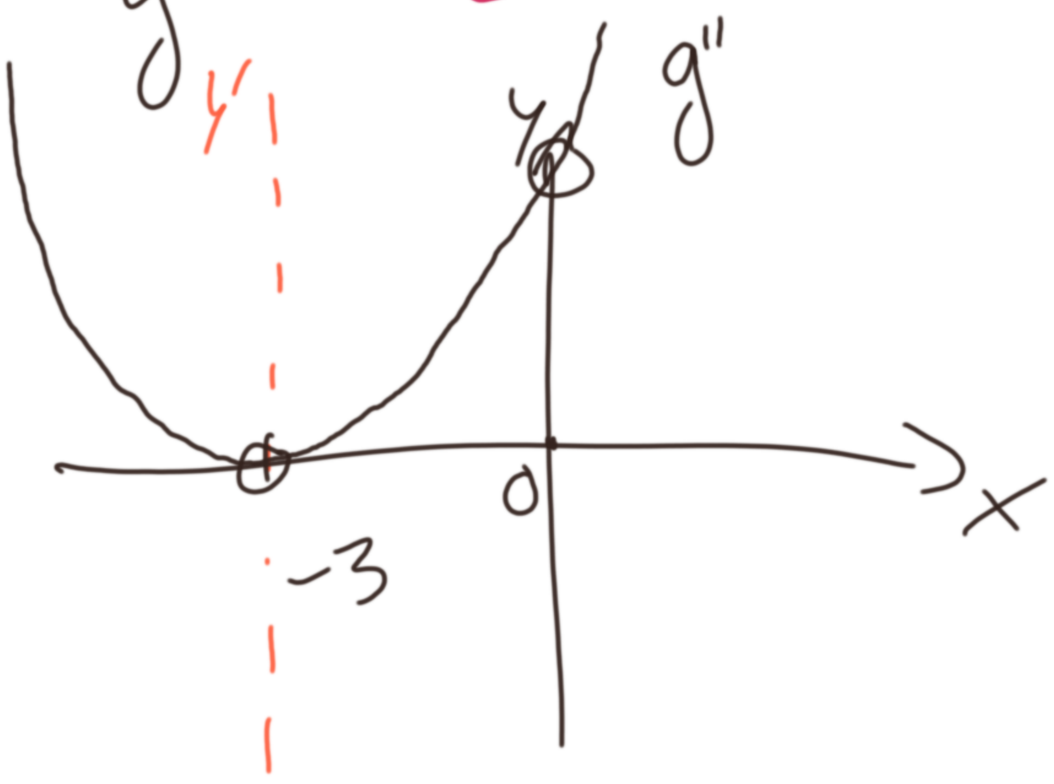
$x_{1,2} = \frac{-6 \pm \sqrt{36 - 4 \cdot 9}}{2} = -\frac{6 \pm 0}{2} = -3$

posun v ose x

$y = x^2 \rightarrow y = (x-2)^2$



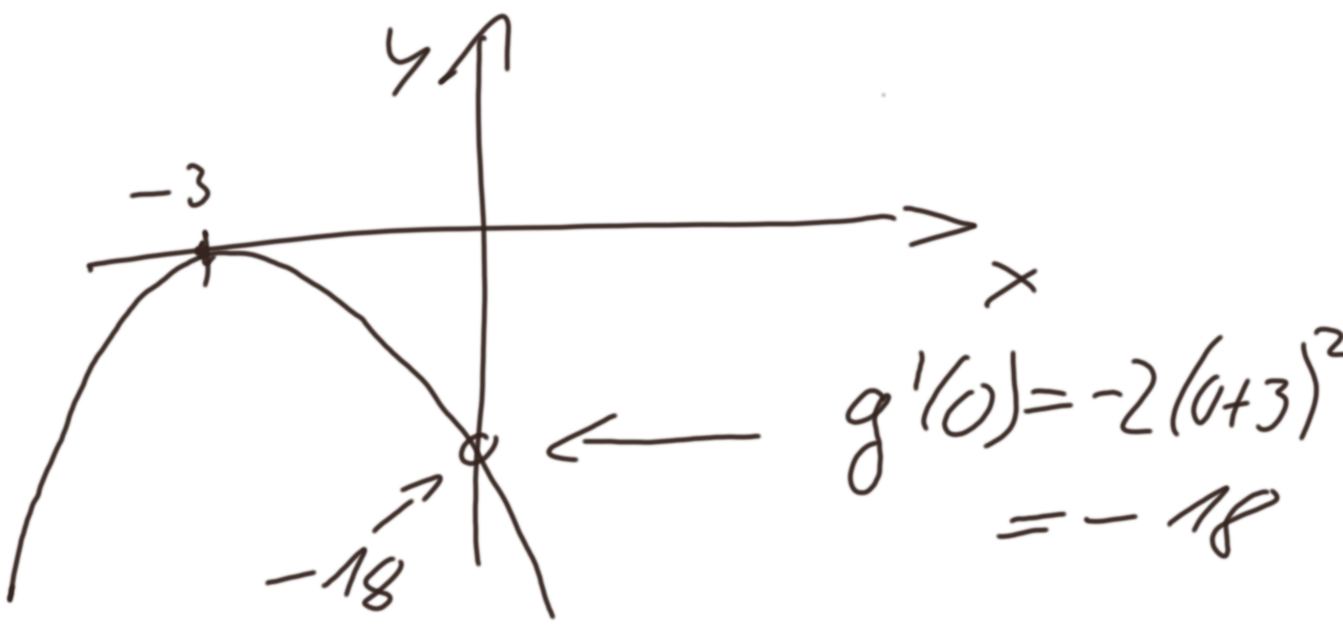
$g''(x) = |(x+3)^2|$



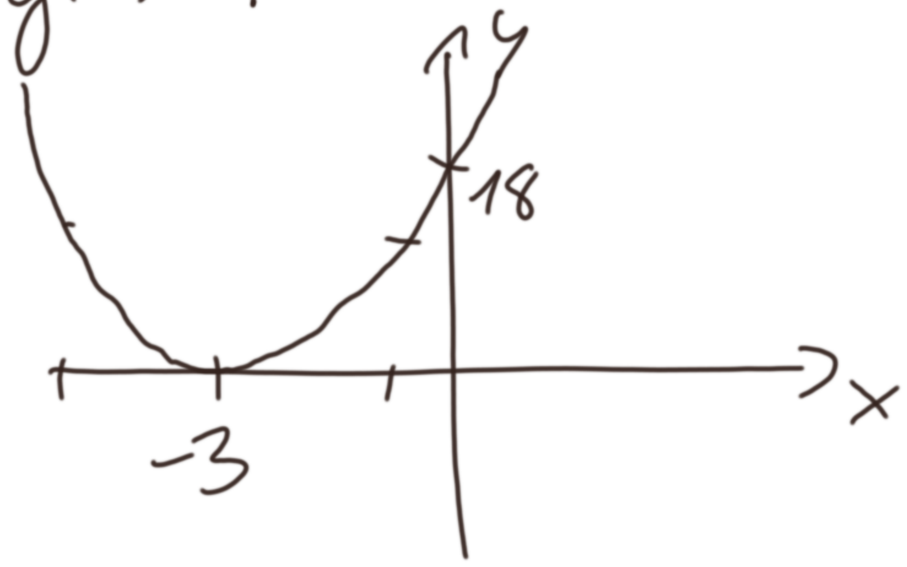
$g'(x) = -2x^2 - 12x - 18$

$= -2 \cdot (x^2 + 6x + 9)$

$= -2 \cdot (x+3)^2$



$g(x) = |-2 \cdot (x+3)^2| = 2 \cdot (x+3)^2$



$D_g = \mathbb{R}$

$H_g = \langle 0, \infty)$

minimum v -3: [-3, 0]

klesající v $(-\infty, -3)$

rostoucí v $(-3, \infty)$

není prostá

není ani lichá

ani sudá

Načrtněte a popište funkci:

$f(x) = |x-4| - |x|$

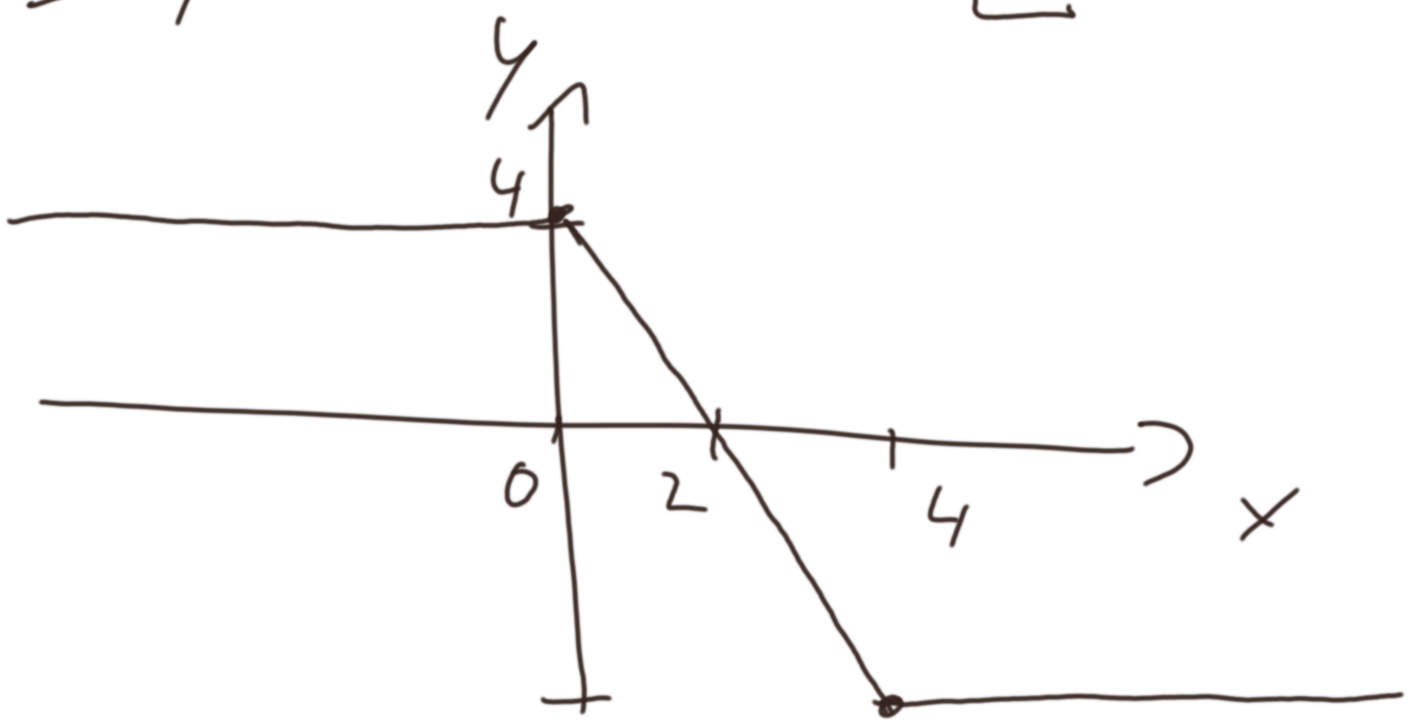
$x \in (-\infty, 0)$: $f(x) = -(x-4) - (-x) = -x+4+x = 4$

$x = 0$

$x \in \langle 0, 4)$: $f(x) = -(x-4) - x = -x+4-x = -2x+4$

$x = 4$: $f(x) = -4$

$x \in (4, \infty)$: $f(x) = x-4-x = -4$



Průsečík s x:

$0 = -2x+4$

$2x = 4$

$x = 2$

$f(0) = 4$

$f(4) = -4$

$D_f = \mathbb{R}$

$H_f = \langle -4, 4 \rangle$

omezená

konstantní v $(-\infty, 0)$ a $(4, \infty)$

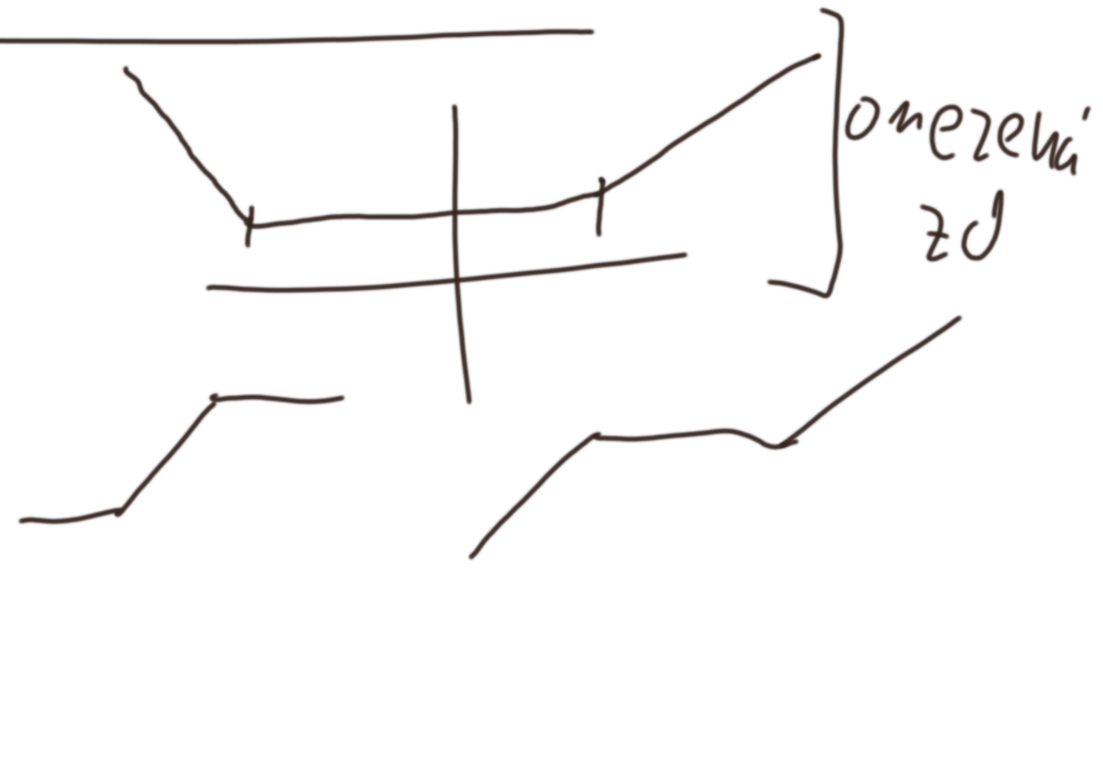
klesající v $(0, 4)$

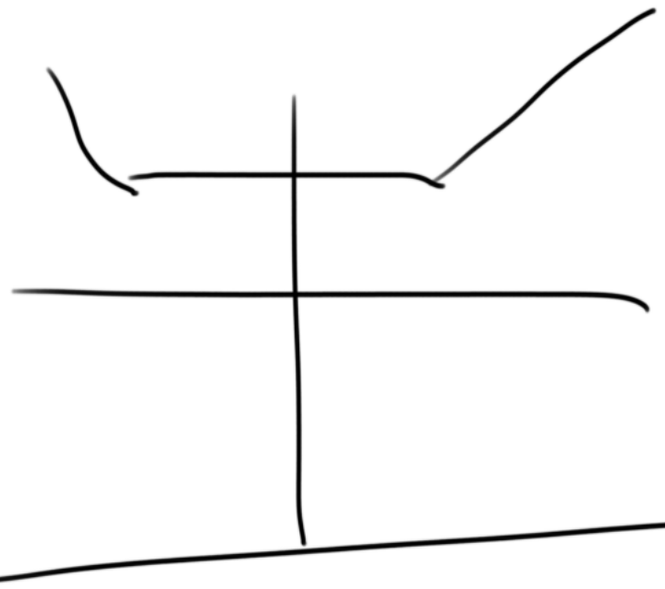
není ani sudá ani lichá

není prostá

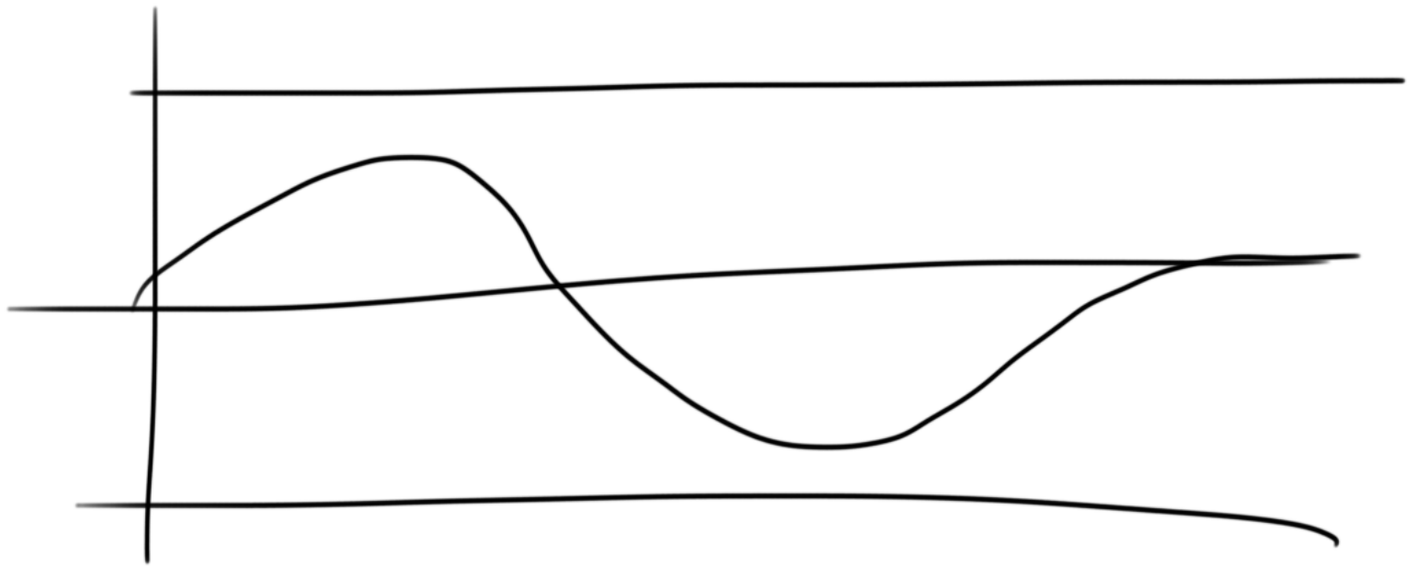
---

můžete v principu dostat graf





omezená zdola



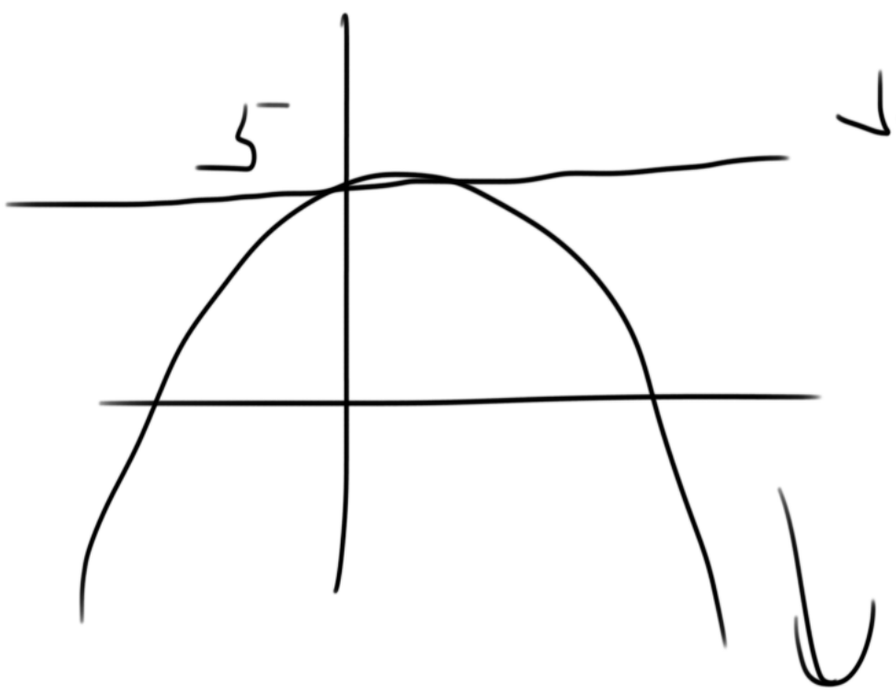
omezená $\Leftrightarrow \exists C \in \mathbb{R} : -C \leq f(x) \leq C$

$\forall x \in D_f$

$\Leftrightarrow$ H_f omezený

Např. $H_f = \langle -3, 5 \rangle$

$\Rightarrow$ f je omezená



shora omezená

$H_f = (-\infty, 5]$

Speciální rovnice

$\sqrt{5x+6} - 2 = x \quad / +2$ umocnění není ekvivalentní úprava

$\sqrt{5x+6} = x+2 \quad /^2$

$5x+6 = (x+2)^2$

$5x+6 = x^2+4x+4$

$0 = x^2 - x - 2$

$0 = (x-2)(x+1)$

$x_1 = 2$

$x_2 = -1$

Nutná kontrola

Zk. x_1: $LS = \sqrt{10+6} = \sqrt{16} = 4$

$PS = 2+2 = 4$ $LS = PS$ ✓

x_2: $LS = \sqrt{-5+6} = 1$

$PS = -1+2 = 1$ $LS = PS$

$K = \{-1, 2\}$

$\sqrt{5x+6} - 2 = x \quad /^2$ $(A \pm B)^2 = A^2 \pm 2AB + B^2$

$(5x+6) - 2 \cdot 2\sqrt{5x+6} + 4 = x^2$ ✓ správně, ale je to moc pracné

✗ $(5x+6) - 4 = x^2$ ✗

$(A-B)^2 \neq A^2 - B^2$

Soustavy lin. rovnic

$$\begin{aligned} 4x - 7y &= 20 \\ 7x + 4y &= 35 \end{aligned}$$

$$4x = 20 + 7y$$

$$\boxed{x = 5 + \frac{7}{4}y}$$

$$7\left(5 + \frac{7}{4}y\right) + 4y = 35$$

$$\cancel{35} + \frac{49}{4}y + 4y = \cancel{35}$$

$$\left(\frac{49}{4} + 4\right) \cdot y = 0$$

$$y = 0$$

$$x = 5 + \frac{7}{4} \cdot 0 \Rightarrow x = 5$$

$$K = \{[5, 0]\}$$

$$\begin{aligned} -3x - 5y &= 1 \quad /\cdot 2 \\ -6x + 10y &= -2 \end{aligned}$$

$$\begin{aligned} -6x - 10y &= 2 \\ -6x + 10y &= -2 \end{aligned} \quad \Big\} \oplus$$

$$-12x + 0 \cdot y = 0 \Rightarrow x = 0$$

$$-3 \cdot 0 - 5y = 1$$

$$y = -\frac{1}{5}$$

$$K = \left\{\left[0, -\frac{1}{5}\right]\right\}$$

Řešíme v $\mathbb{R}^2$

$\mathbb{R} \times \mathbb{R} = \mathbb{R}^2$

$[x, y] \in \mathbb{R}^2$

↑ uspořádaná dvojice

$$K = \{[x, y_0]\}$$

$$x+y=3 \cdot 2$$
$$2x+2y=6$$
$$2x+2y=6 \quad \ominus$$
$$0=0$$

$$x=3-y$$
$$6-2y+2y=6$$
$$6=6$$

→ soustava má nekonečně mnoho řešení

$$x+y=3 \quad \cdot 2$$
$$2x+2y=3$$

$$x+y=3$$
$$x+y=\frac{3}{2}$$

$$2x+2y=6$$
$$2x+2y=3 \quad \ominus$$
$$0+0=3$$
$$0=3$$

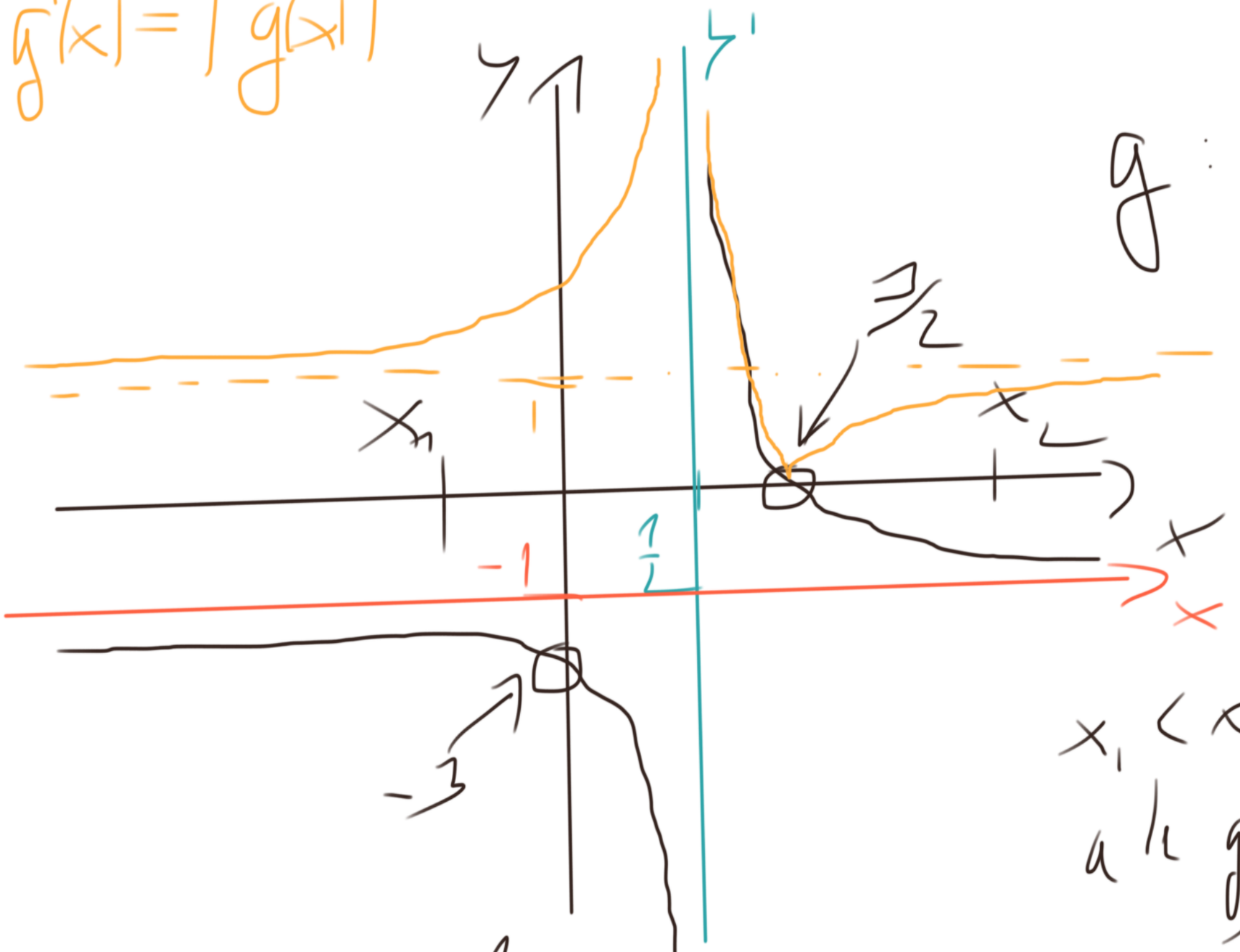
→ soustava nemá žádné řešení

$$g(x) = \frac{2}{2x-1} - 1$$

$$= \frac{1}{x-\frac{1}{2}} - 1$$

lin. lomená
$\longrightarrow$ hyperbola
$\uparrow\downarrow$ (x-a)

$|g'(x)| = |g(x)|$



$g: x \longrightarrow y$

$x' \longrightarrow y'$

$y' = \frac{1}{x'}$

$x_1 < x_2$, $y_1 < y_2$
a tj. g není rostouc. v D_g

$P_x: 0 = \frac{1}{x-\frac{1}{2}} - 1$

$x - \frac{1}{2} = 1$

$x = \frac{3}{2}$

$P_x = [\frac{3}{2}, 0]$

$P_y: y = \frac{1}{0-\frac{1}{2}} - 1 = -3$

$P_y = [0, -3]$

$D_g = \mathbb{R} \setminus \{\frac{1}{2}\}$ neomezená

$H_g = \mathbb{R} \setminus \{-1\}$ není shora ani zdola

klesající v $(-\infty, \frac{1}{2})$ a v $(\frac{1}{2}, \infty)$