$\sqrt{x} + x = 2$

$\sqrt{x} = 2 - x \quad /^2$

$x = 4 - 4x + x^2$

$0 = x^2 - 5x + 4$

$0 = (x-4)\cdot(x-1)$

$x_1 = 1 \qquad x_2 = 4$

Zk: $x = 1$

$LS = \sqrt{1} = 1$

$PS = 2 - 1 = 1$ ✓

$x = 4$

$LS = 2$

$PS = -2$ ✗

$\to 2 - x \geq 0$

$x \leq 2$

Algebraické výrazy

$$\frac{x^2+x}{x} = \frac{x(x+1)}{x} = x+1$$

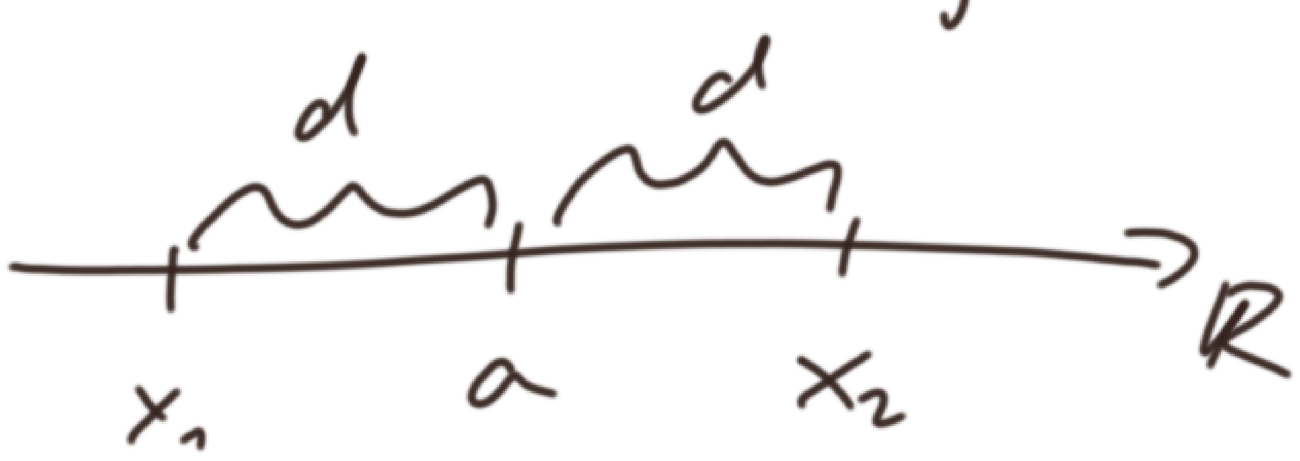
Rovnice

$AV_1(x) = AV_2(x)$

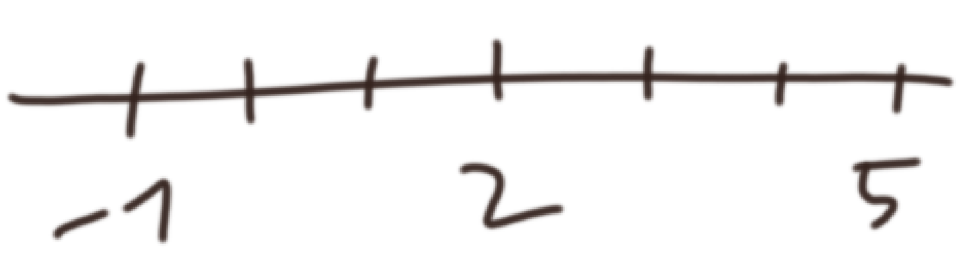
$$0 = \frac{x^2+x}{x} \left(= \frac{x(x+1)}{x} = \underline{x+1} \right)$$

Rovnice s abs. hodnotou

$|x-a| = d \qquad x$ je od a vzdáleno d



Př. $\quad |x-2| = 3$

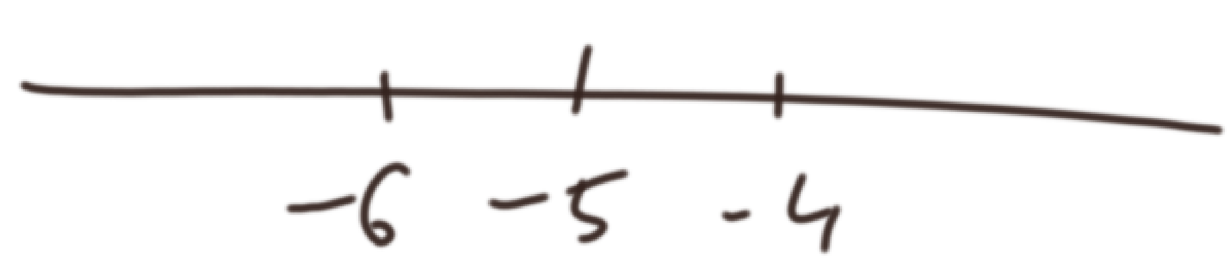


$\to x_1 = -1$

$x_2 = 5$

$|x-(-5)|$

$|x+5| = 1$



Řešte v $\mathbb{R}$: $|x-2| + |-x+1| = 1$

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

I	II	III
$x-2<0$		$x-2>0$
$\lvert x-2\rvert = -x+2$	$\lvert x-2\rvert = -x+2$	$\lvert x-2\rvert = x-2$
$-x+1>0$	$-x+1<0$	
$\lvert -x+1\rvert = -x+1$	$\lvert -x+1\rvert = x-1$	$\lvert -x+1\rvert = x-1$

I) $x \in (-\infty, 1\rangle$

$-x+2-x+1=1$

$-2x=-2$

$\boxed{x=1}$

II) $x \in (1, 2)$

$-x+2+x-1=1$

$1=1$

$x \in (1,2)$

III) $x \in \langle 2, \infty)$

$x-2+x-1=1$

$2x=4$

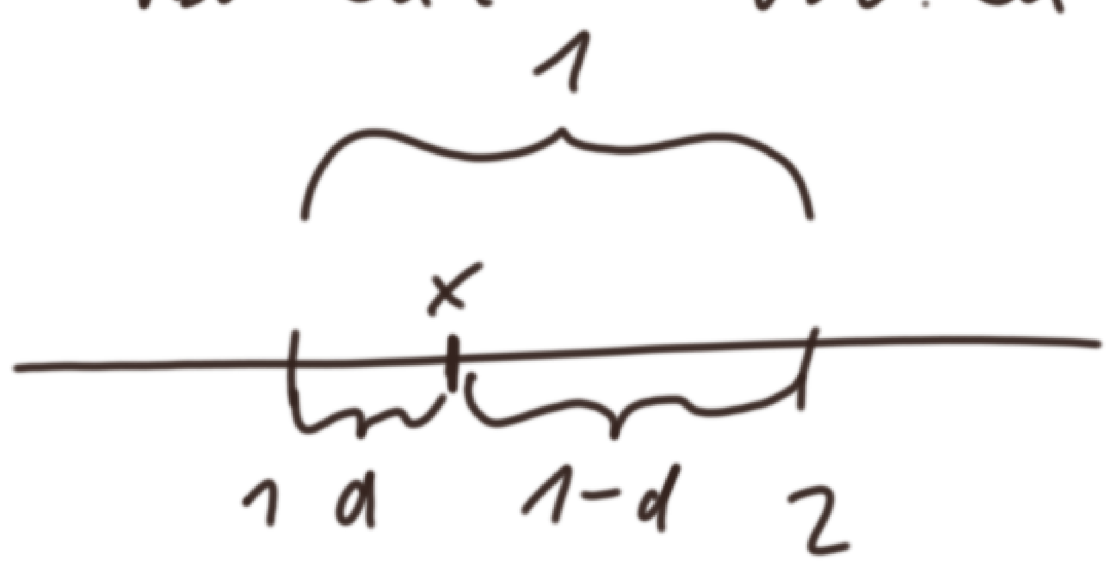
$\boxed{x=2}$

$\longrightarrow x \in \langle 1, 2\rangle \qquad K = \langle 1, 2\rangle$

Geometricky: $|x-2| + |-x+1| = 1$

$|x-2| + |x-1| = 1$

vzd. od 2   vzd. od 1

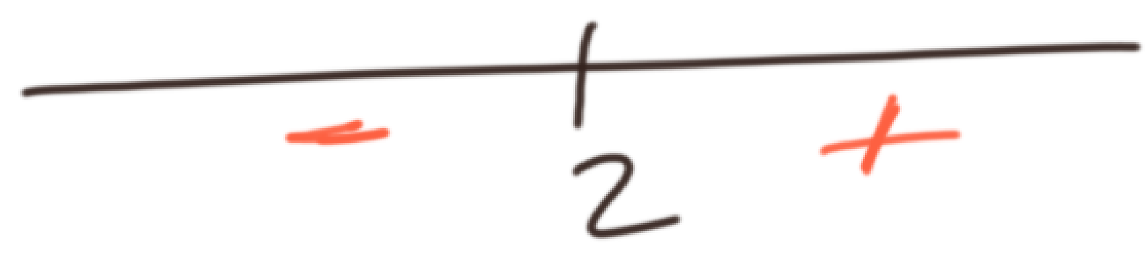


$d+1-d=1$

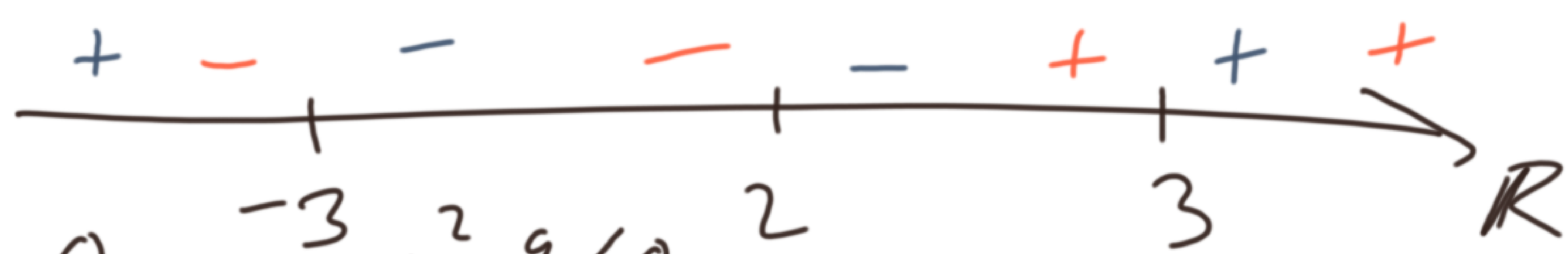
$\longrightarrow x \in \langle 1, 2\rangle$

Řešte v $\mathbb{R}$:

$$|x^2-9|+|x-2|=5$$



$$x^2-9=(x+3)\cdot(x-3)$$



$x^2-9>0$ $\qquad$ $x^2-9<0$

$|x^2-9|=x^2-9$ $\qquad$ $|x^2-9|=-x^2+9$

$|x-2|=-x+2$

$$|x-x^2-1|=|2x-3-x^2|$$

$$|x^2-x+1|=|x^2-2x+3|$$

$$x^2-x+1=\left(x-\tfrac{1}{2}\right)^2+1-\tfrac{1}{4}=\underbrace{\left(x-\tfrac{1}{2}\right)^2}_{\geq 0}+\underbrace{\tfrac{3}{4}}_{>0}>0$$

Soustavy lineárních rovnic

Řešte v $\mathbb{R}^2$

$$\left.\begin{aligned} 7x - 3y &= 15 \quad /\cdot 2 \\ 5x + 6y &= 27 \end{aligned}\right\} \oplus$$

$$19x + 0y = 57$$

$$\underline{x = 3}$$

$$21 - 3y = 15$$

$$3y = 6$$

$$\underline{y = 2}$$

$$\boxed{\begin{aligned} x &= 3 \\ y &= 2 \end{aligned}} \checkmark \qquad K = \{3, 2\} \times$$

$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{[a, b] : a \in \mathbb{R} \wedge b \in \mathbb{R}\}$$

$$\vec{X} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \in \mathbb{R}^2 \qquad K = \{(3, 2)\} \checkmark$$

$$\left.\begin{aligned} 2x + 3y &= -3 \quad /\cdot(-3) \\ 6x + 9y &= 9 \end{aligned}\right\} \oplus$$

$$0x + 0y = 18 \qquad \rightarrow 2x + 3y = 3$$

$$0 = 18 \quad \times$$

$\rightarrow$ NŘ.

$$\left.\begin{aligned} 2x + 3y &= -3 \quad /(-2) \\ 4x + 6y &= -6 \end{aligned}\right\} \oplus$$

$$0 = 0$$

$\rightarrow$ nekonečně mnoho řešení

$$2x + 3y = -3$$

$$y = -1 - \frac{2}{3}x$$

$$K = \left\{\left(x, -1 - \frac{2}{3}x\right), x \in \mathbb{R}\right\}$$

např. $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ není řešením:

$$2 \cdot 0 + 3 \cdot 0 = -3$$

$$0 = -3$$

Lineární, kvadratické, lin. lomenné a fce s abs. h

- D, H, monotonie, prostá?, parita, význačné body, omezenost

$f(x)$, D_f, $x_1, x_2 \in D_f$

$y_1 = f(x_1)$

$y_2 = f(x_2)$

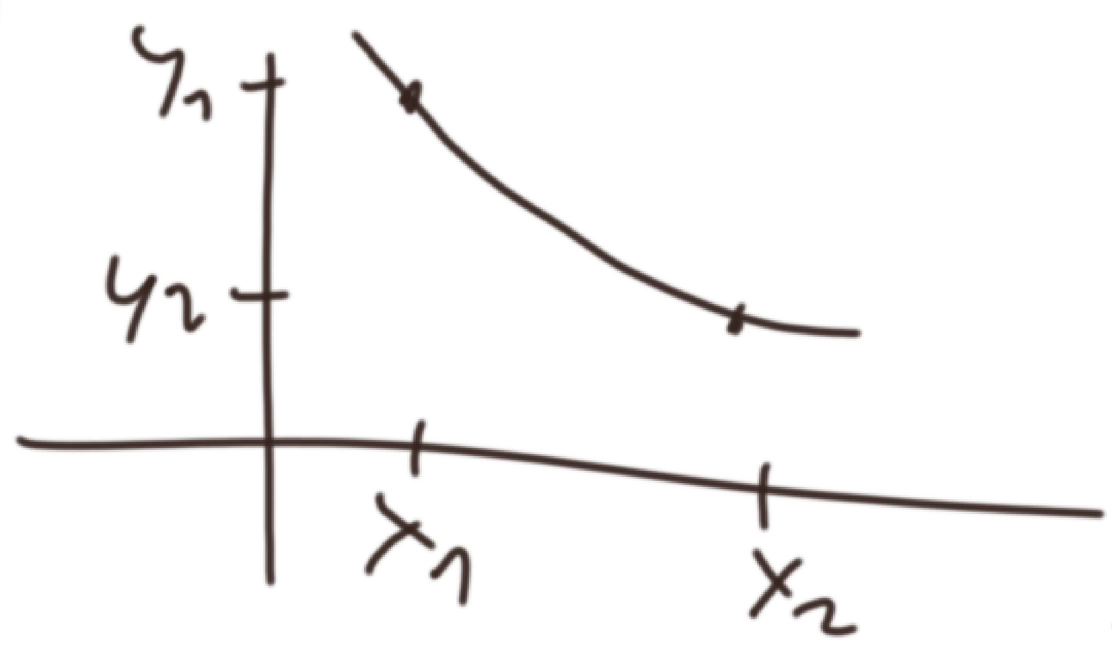
klesající: $x_1 < x_2 \Rightarrow y_1 > y_2$

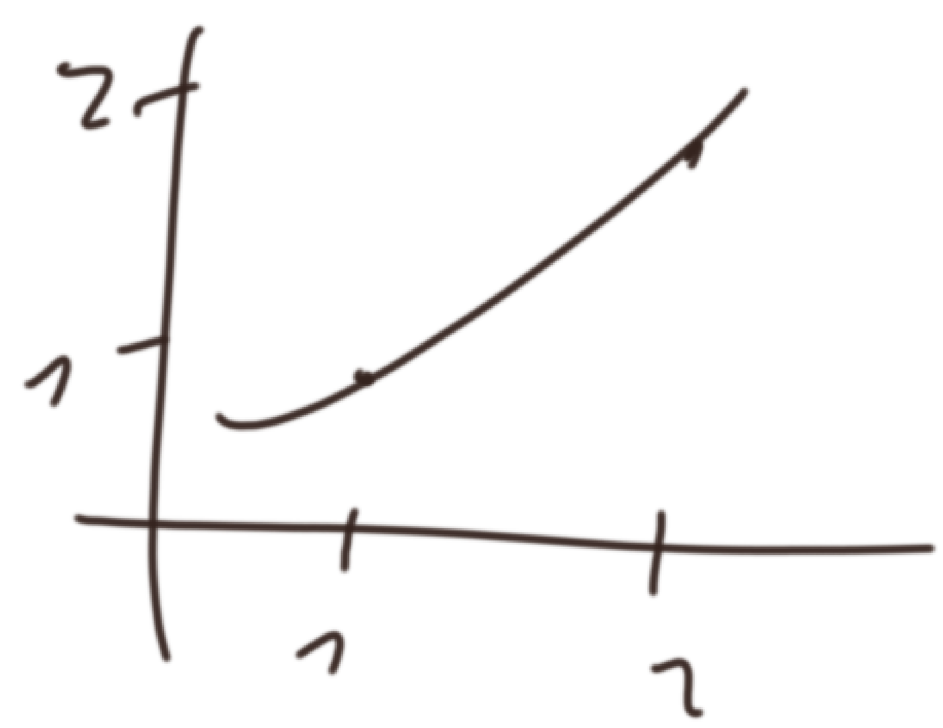
rostoucí: $x_1 < x_2 \Rightarrow y_1 < y_2$

neklesající: $x_1 < x_2 \Rightarrow y_1 \leq y_2$

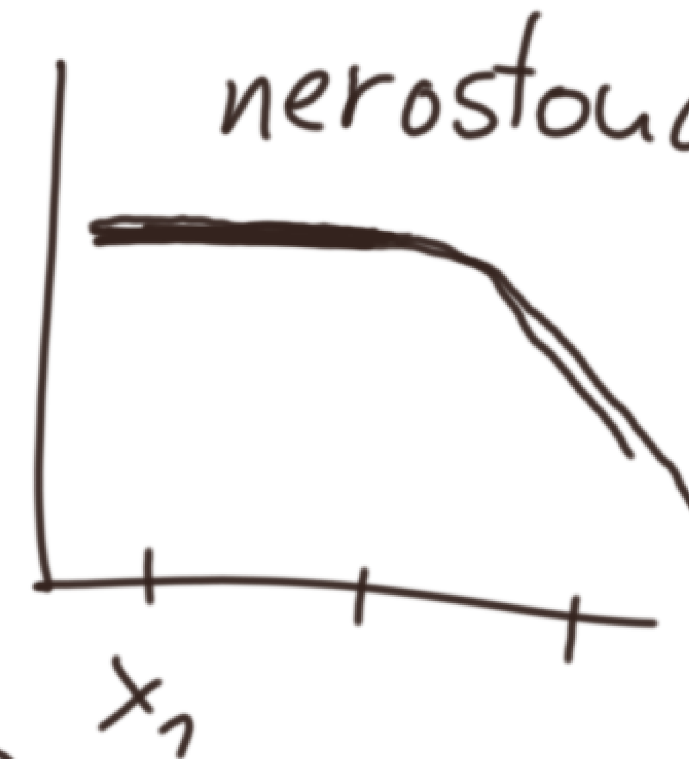
nerostoucí: $x_1 < x_2 \Rightarrow y_1 \geq y_2$

neklesající





nerostoucí



f je rostoucí $\Rightarrow$ f je i neklesající

f je klesající $\Rightarrow$ f je i nerostoucí

Nakreslete graf funkce a určete . . .

$f: y = |x^2 - 3x| + 2$

x^2:

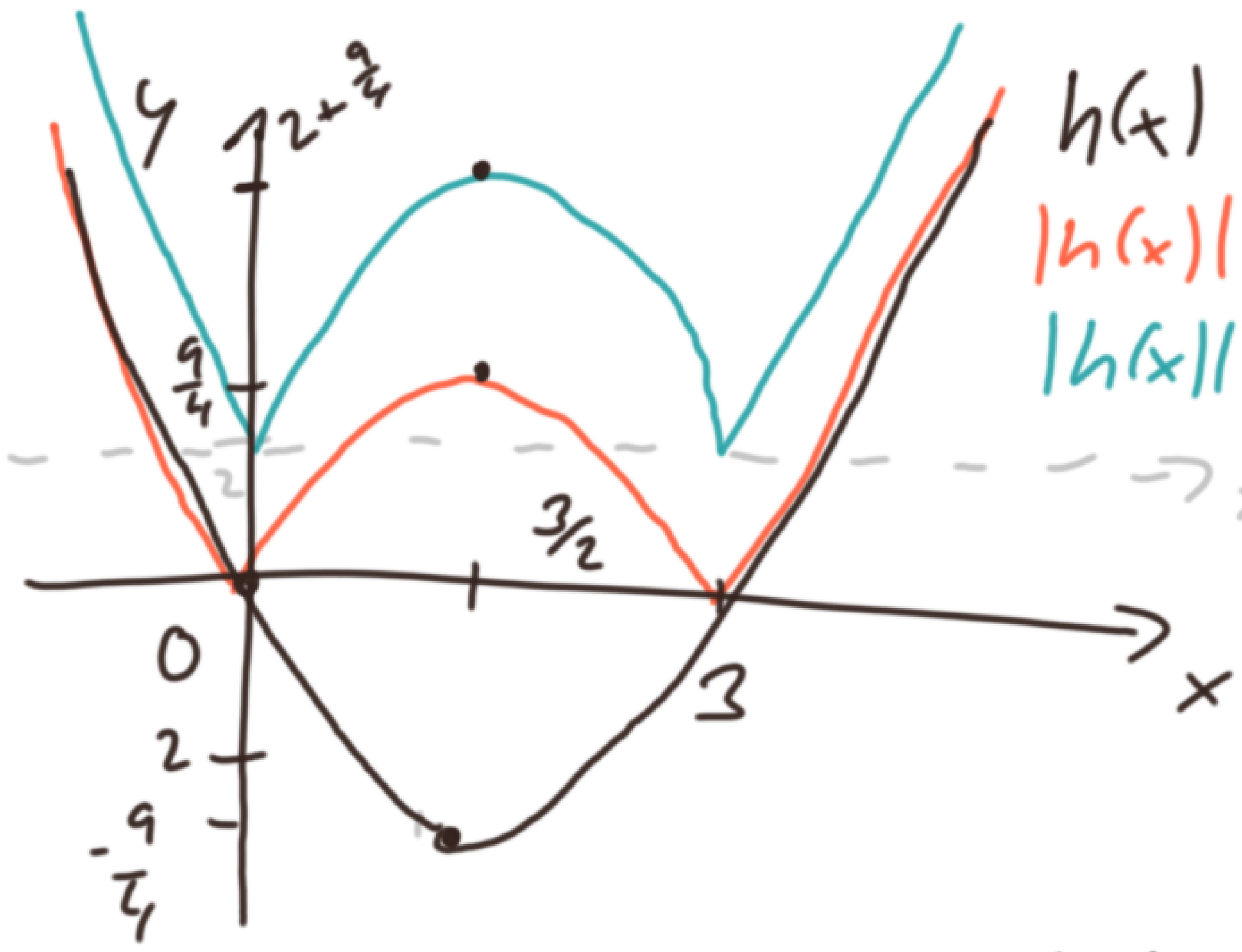
1. $x^2 - 3x$ parabola
2. $|x^2 - 3x|$ abs. h.
3. $|x^2 - 3x| + 2$ posun v y ↑

$(x-a)^2$

$h: y = x^2 - 3x + 0$ $\qquad f(x) = |h(x)| + 2$

$= x \cdot (x-3) \longrightarrow x_1 = 0, \; x_2 = 3$ $\qquad a = 1 > 0 \Longrightarrow$

$= \left(x - \frac{3}{2}\right)^2 - \frac{9}{4} \Longrightarrow V = \left[\frac{3}{2}, -\frac{9}{4}\right]$



$h(x)$

$|h(x)|$

$|h(x)| + 2 = f(x)$

$D_f = \mathbb{R}$

$H_f = \langle 2, \infty)$

$\rightarrow$ zdola omezená

↘ v $(-\infty, 0)$ a v $\left(\frac{3}{2}, 3\right)$

↗ v $\left(0, \frac{3}{2}\right)$ a v $(3, \infty)$

ani sudá, ani lichá, není prostá

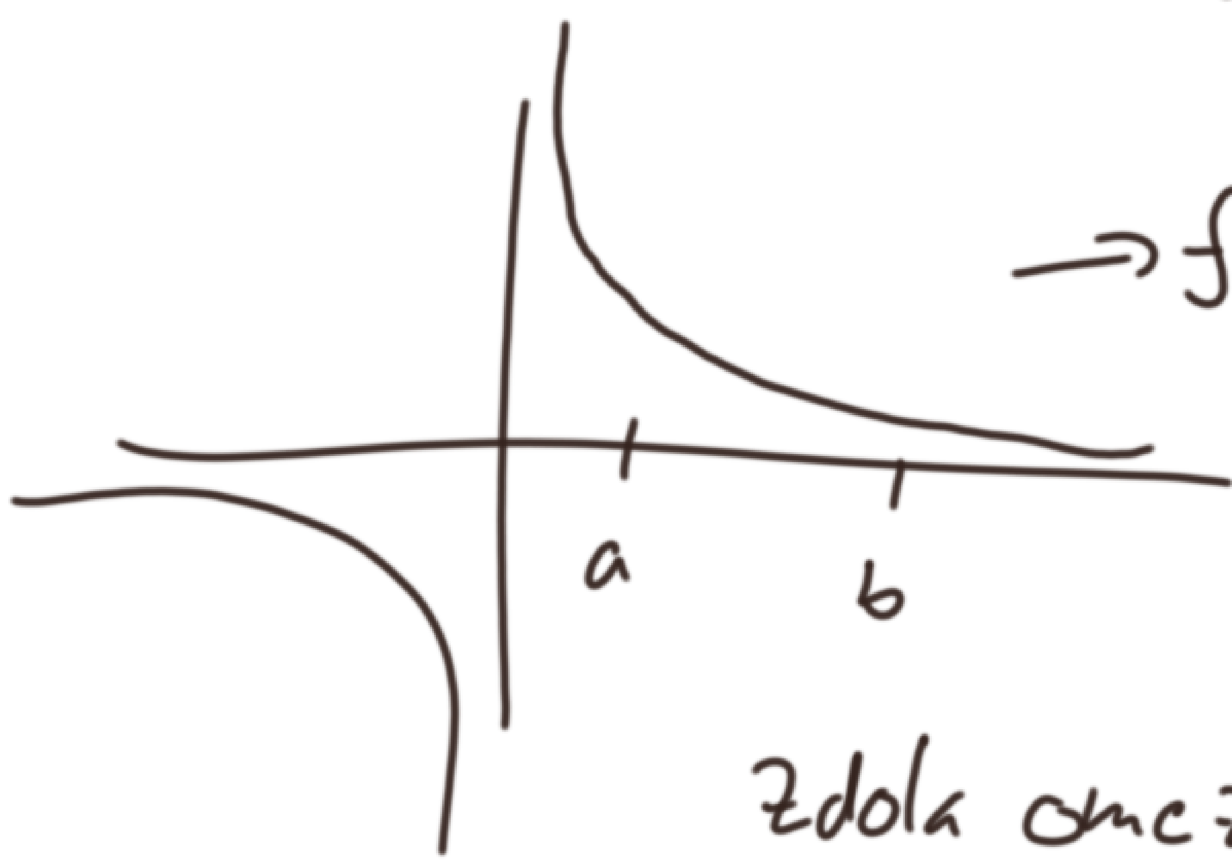
Lineární lomená

$f: y = \frac{a}{x-b} + c$

není omezená (na D_f)

$\frac{1}{x}$

rovnoosá hyperbola



$\rightarrow f(x)$ je omezená na $\langle a, b \rangle$

$|\frac{1}{x}|$

zdola omezená na $\mathbb{R}^+$

shora omezená na $\mathbb{R}^-$

$f: y = \frac{r \cdot x + s}{p \cdot x + q}$ lineární lomená funkce

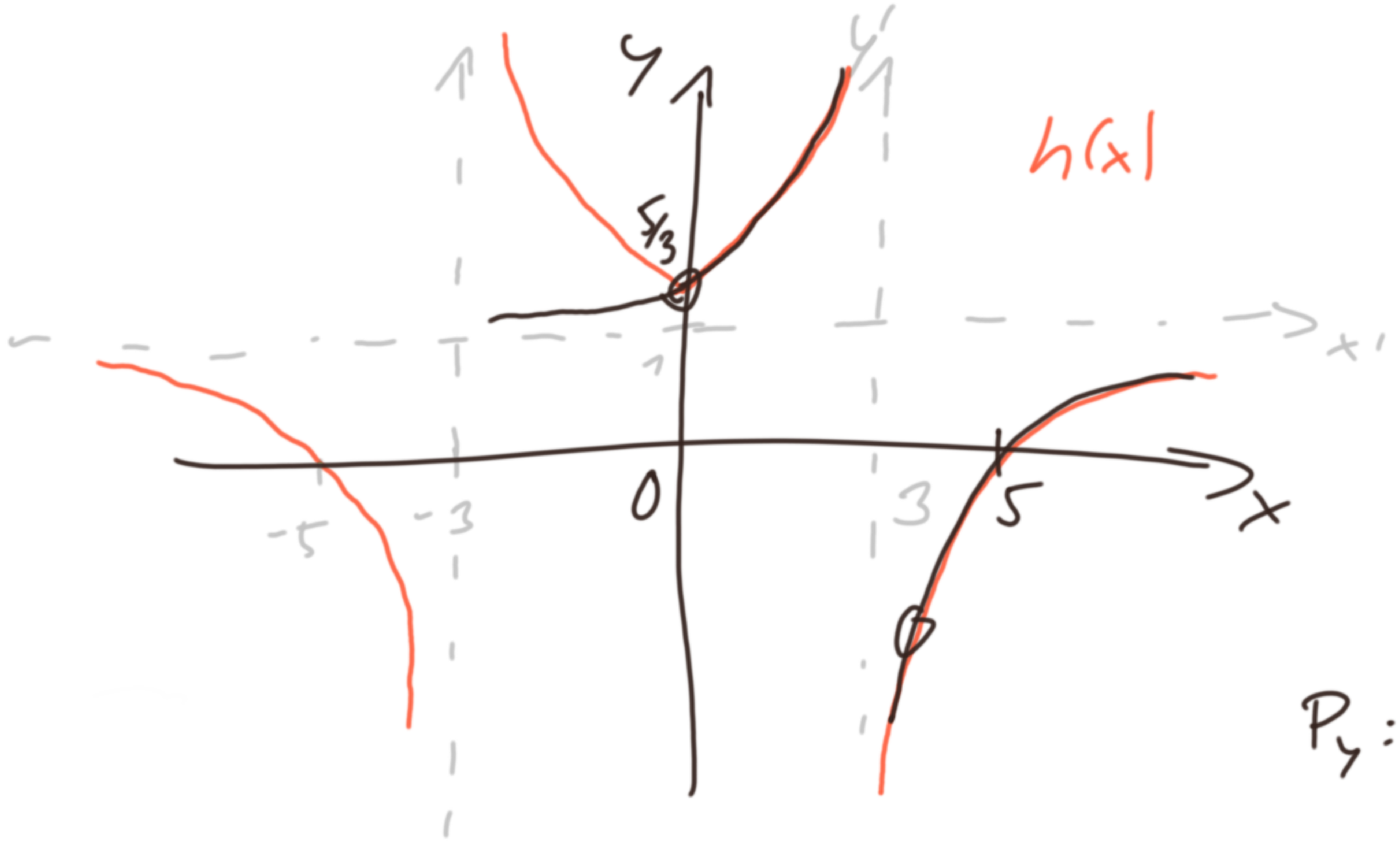
$$\frac{3x+1}{x+2} = \frac{3\cdot(x+2)+1-6}{x+2} = \frac{3\cdot\cancel{(x+2)}}{\cancel{x+2}} + \frac{-5}{x+2}$$

$$= \frac{-5}{x+2} + 3$$

$h: y = \frac{-2}{|x|-3} + 1$

$f(|x|)$ vždy sudá: $f(|-x|) = f(|x|)$

$i: y = \frac{-2}{x-3} + 1 \longrightarrow h(x) = i(|x|)$



$-\frac{1}{x}$

Průsečíky:

P_x: $y = 0$

$$\frac{-2}{x-3} + 1 = 0$$

$$x - 3 = 2$$

$$\underline{x = 5}$$

P_y: $x = 0$

$$y = \frac{-2}{-3} + 1 = \frac{5}{3}$$

D_h: $|x| - 3 \neq 0$

$|x| \neq 3$

$x \neq \pm 3$

$D_h = \mathbb{R} \setminus \{-3, 3\}$

$H_h = \mathbb{R} \setminus \{1\}$

$\searrow$ v $(-\infty, -3)$ a v $(-3, 0)$

$\nearrow$ v $(0, 3)$ a v $(3, \infty)$

sudá, není prostá, neomezená, lokální min $\frac{5}{3}$ v $x = 0$

$y = -x^2 + 4x + 5$ $\qquad a = -1 < 0$

$= -[x^2 - 4x - 5]$

$= -[(x-2)^2 - 5 \underbrace{- 4}_{}^{-B^2}] = -[(x-2)^2 - 9]$

$(A - B)^2 = A^2 - 2AB + \underbrace{B^2}_{}$

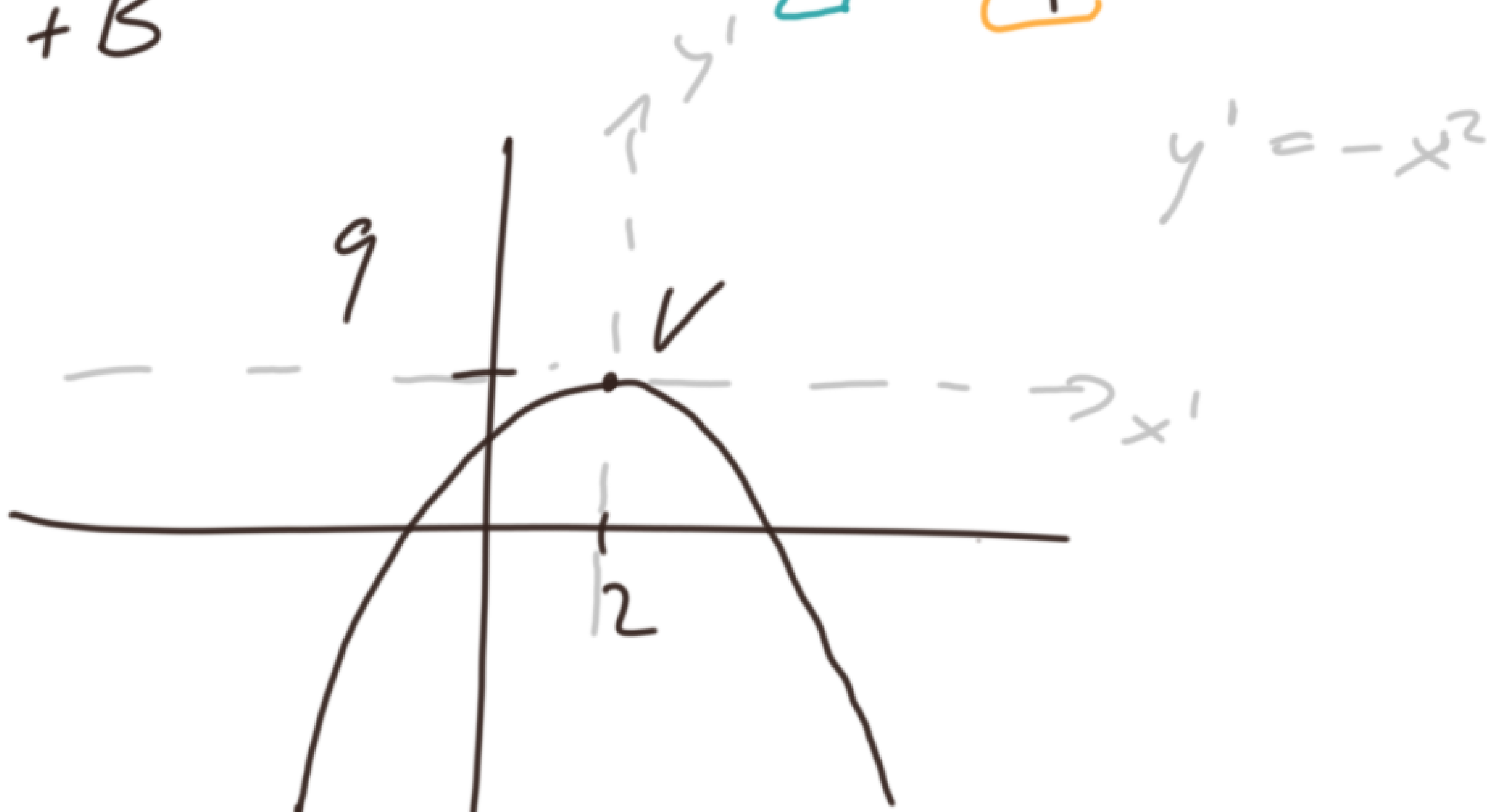
$(A \pm B)^2 = A^2 \pm 2 \cdot A \cdot B + B^2$

$= -(x-\underbrace{2}_{})^2 + \underbrace{9}_{}$

$A = x$

$4x = 2x \cdot B$

$B = 2$



$\longrightarrow V = [\underbrace{2}_{}, \underbrace{9}_{}]$ – lok. max

$\nearrow (-\infty, 2)$ $\qquad \searrow (2, \infty)$

lokální maximum