6. Goniometrie

1. úloha

funkce ostrého úhlu

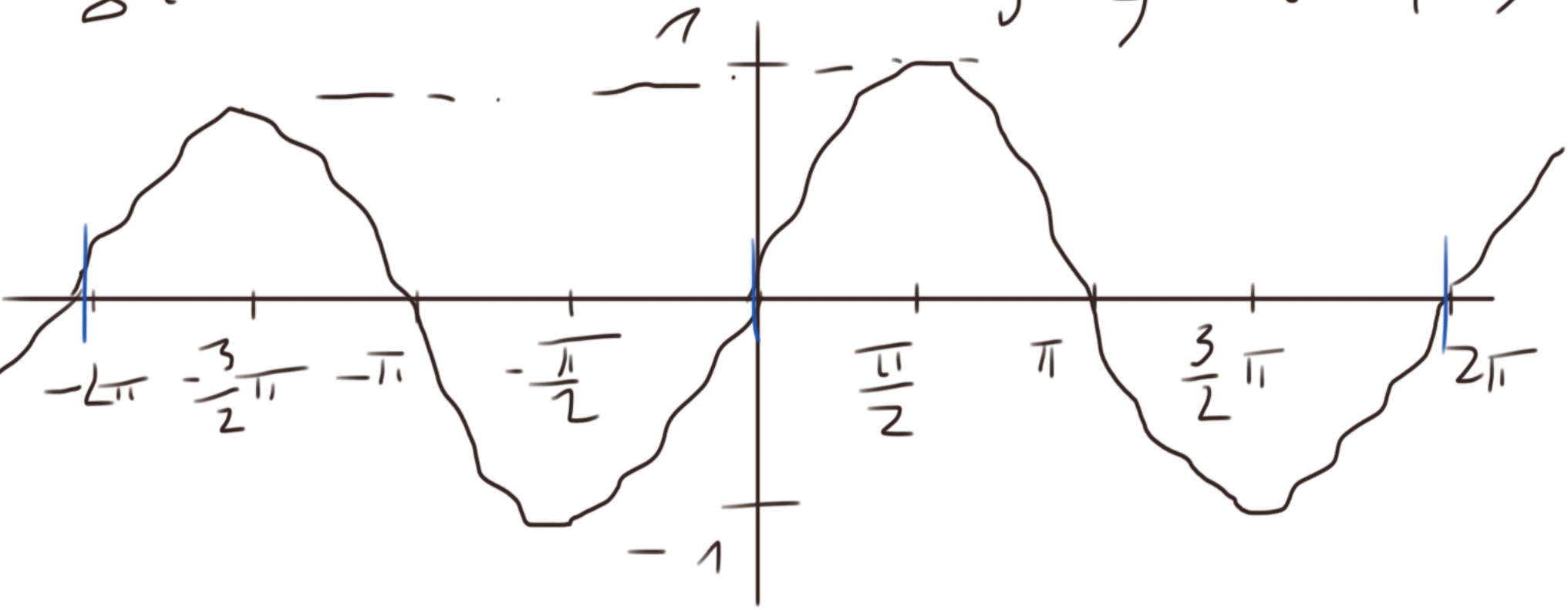
2. úloha

6.1 Goniometrické funkce

$\sin x$, $\cos x$, $\operatorname{tg} x$, $\operatorname{cotg} x$

$\sin x$

$f: y = \sin(x)$



$D_f = R$

$H_f = \langle -1, 1 \rangle$

periodická s periodou $T = 2\pi$

lichá

sudá

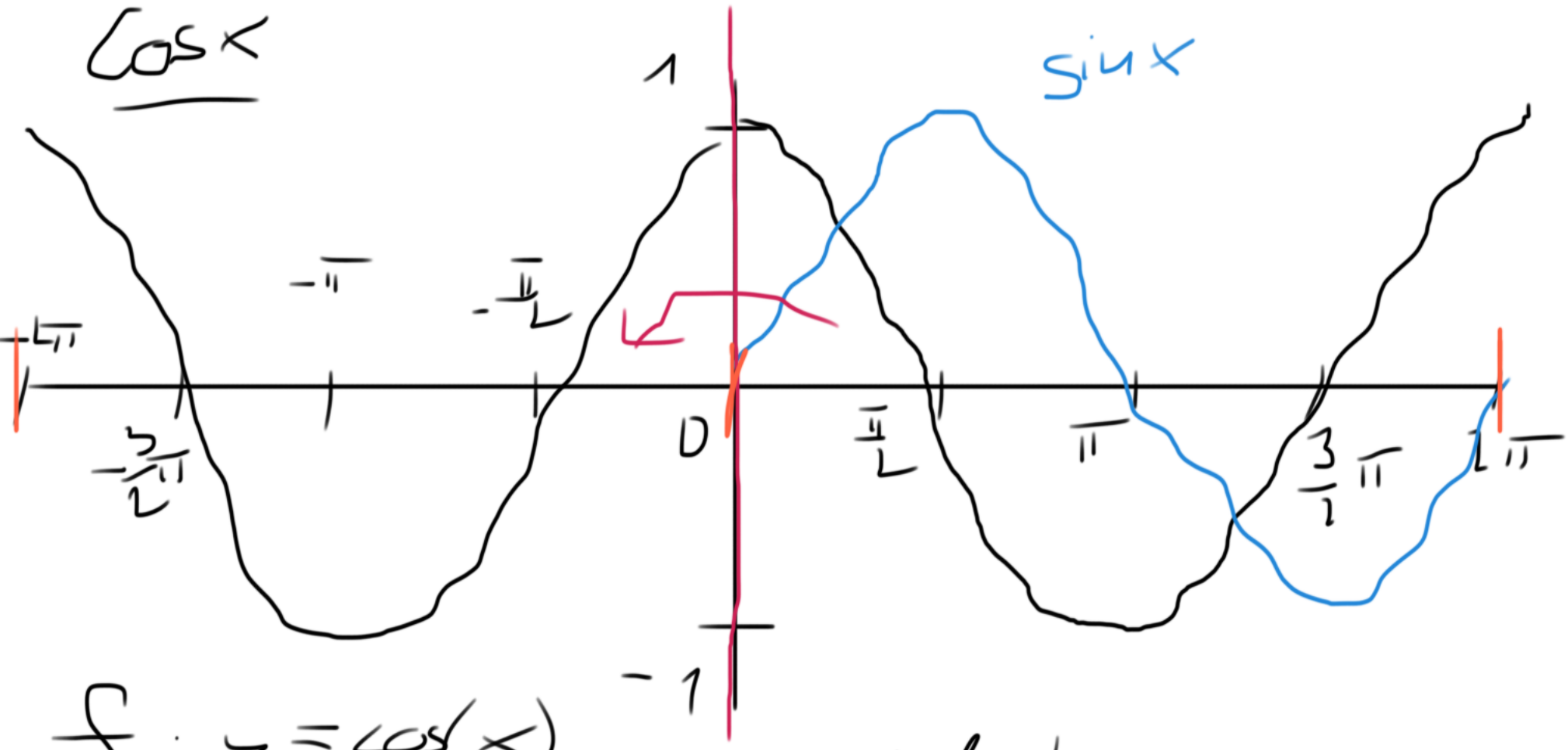
spojitost: na D_f

maxima: $\frac{\pi}{2} + k \cdot 2\pi$, $k \in \mathbb{Z}$

minima: $\frac{3}{2}\pi + k \cdot 2\pi$, $k \in \mathbb{Z}$

nuly: $k \cdot \pi$

cos x



$f: y = \cos(x)$

$D_f = \mathbb{R}$

$H_f = \langle -1; 1 \rangle$

ograniczona

parzysta

okresowa

okres podstawowy $T = 2\pi$

$\therefore$ spełnia na D_f

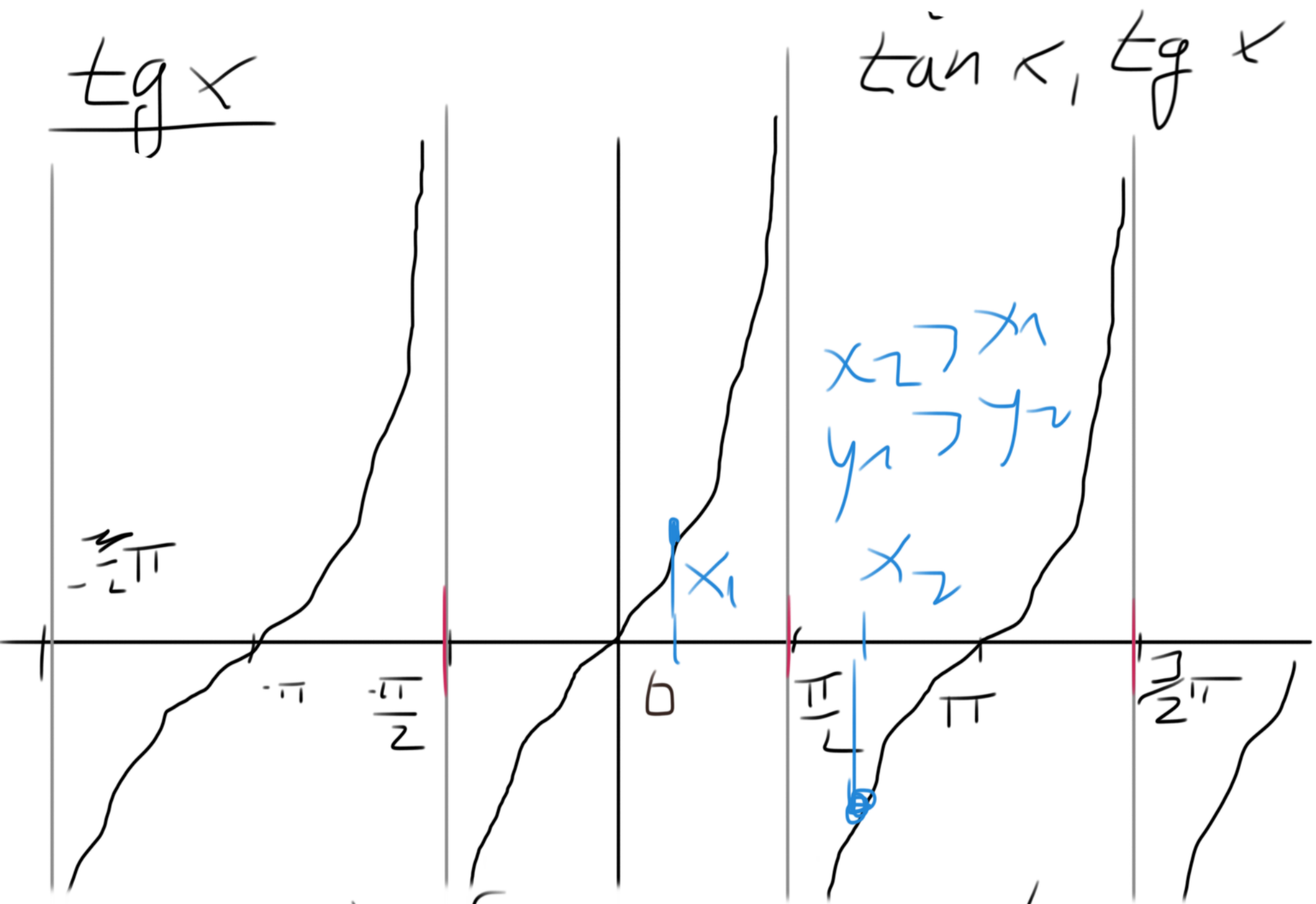
maxima: $k \cdot 2\pi$

minima $\pi + k \cdot 2\pi$

miejsca zerowe $\frac{\pi}{2} + k \cdot \pi$

$\operatorname{tg} x$

$\tan x$, tg x



$D_f = \mathbb{R} \setminus \{\frac{\pi}{2} + k \cdot \pi ; k \in \mathbb{Z}\}$

$H_f = \mathbb{R}$ periodická $T = \pi$

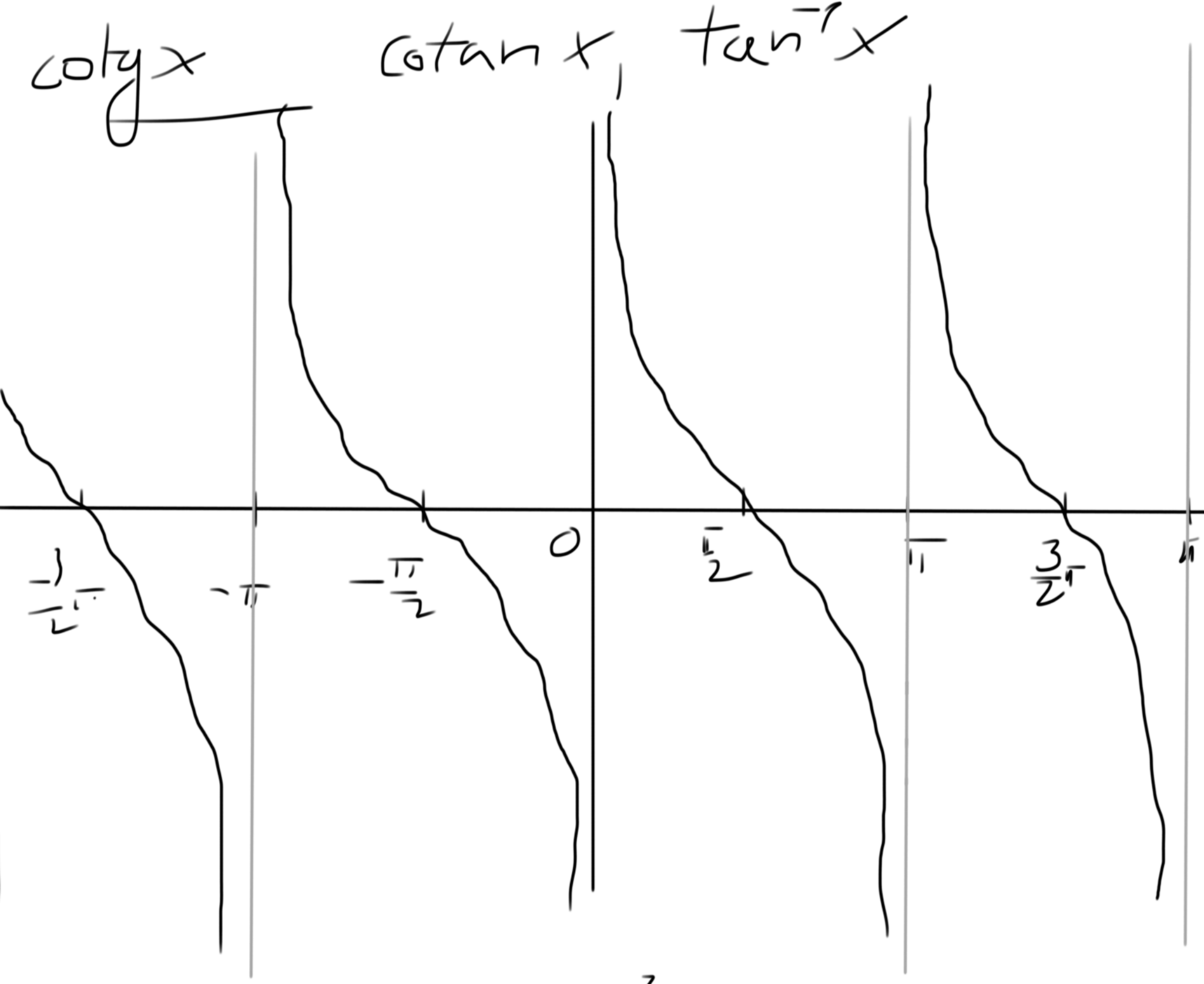
neomezená nuly: $k \cdot \pi$, $k \in \mathbb{Z}$

lichá asymptoty: $\frac{\pi}{2} + k \cdot \pi$

rostoucí $\forall (-\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi)$

$k \in \mathbb{Z}$

cotg x, $\tan^{-1} x$



$D_f = \mathbb{R} \setminus \{k \cdot \pi,\ k \in \mathbb{Z}\}$

$H_f = \mathbb{R}$

nemetická

lichá

periodická $T = \pi$

spojitá v $(0 + k \cdot \pi,\ \pi + k \cdot \pi)$

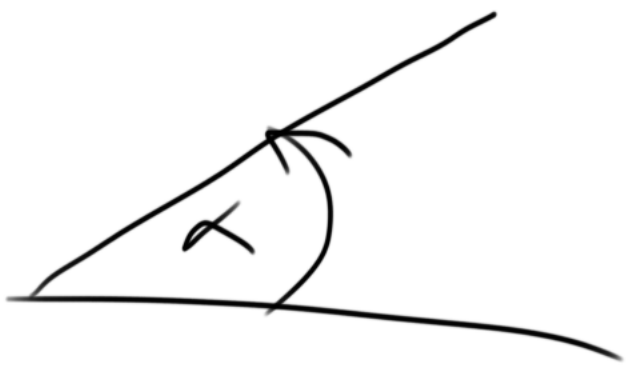
klesající v $\nearrow$

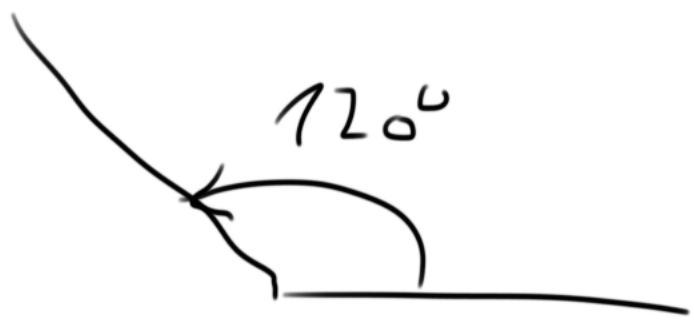
nuly $\frac{\pi}{2} + k \cdot \pi$

asymptoty $k \cdot \pi$ $k \in \mathbb{Z}$

sin 30°

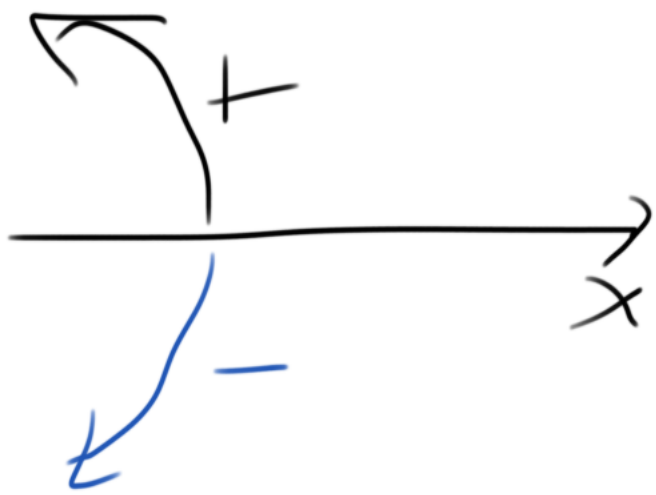
Obloukova míra





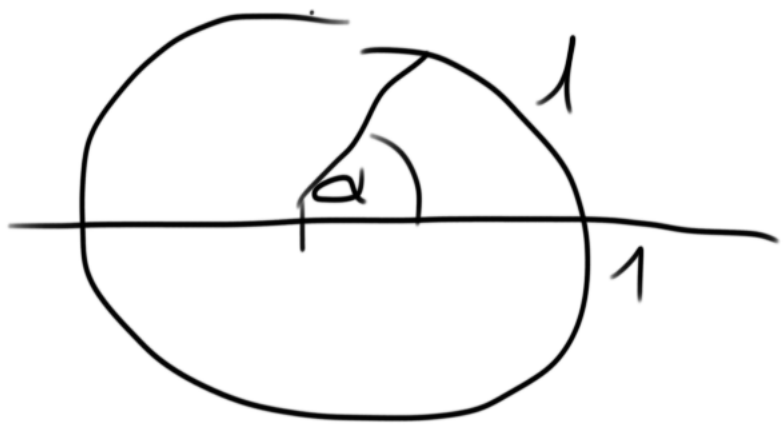
$|\alpha| = 30°$

$45°$



plný 360°

přímý 180



$r = 1$

l ... délka oblouku

$l = 1$ ... $\alpha = ?$

$\alpha = 1$ rad

Úhel $\alpha = 1$ rad vymezuje oblouk délky $l = 1$ $(r = 1)$

2π rad. . . . 360°	r $l = r \cdot \alpha$
π rad . . . . 180°	360° $l = r \cdot 2\pi$

$2\pi \ldots 360^\circ$

$\pi \ldots 180^\circ$

$\alpha = 30^\circ$ $\qquad$ $\alpha = x$ rad

$\uparrow$ $2\pi \ldots 360 \uparrow$

$x \ldots 30$

$$\frac{x}{2\pi} = \frac{30}{360}$$

$$x = 2\pi \frac{1}{12} = \frac{\pi}{6}$$

$\alpha = \frac{\pi}{4}$ $\qquad$ $\alpha = x^\circ$

$\uparrow$ $2\pi \ldots 360 \uparrow$

$\frac{\pi}{4} \ldots x$

$$\frac{x}{360} = \frac{\frac{\pi}{4}}{2\pi}$$

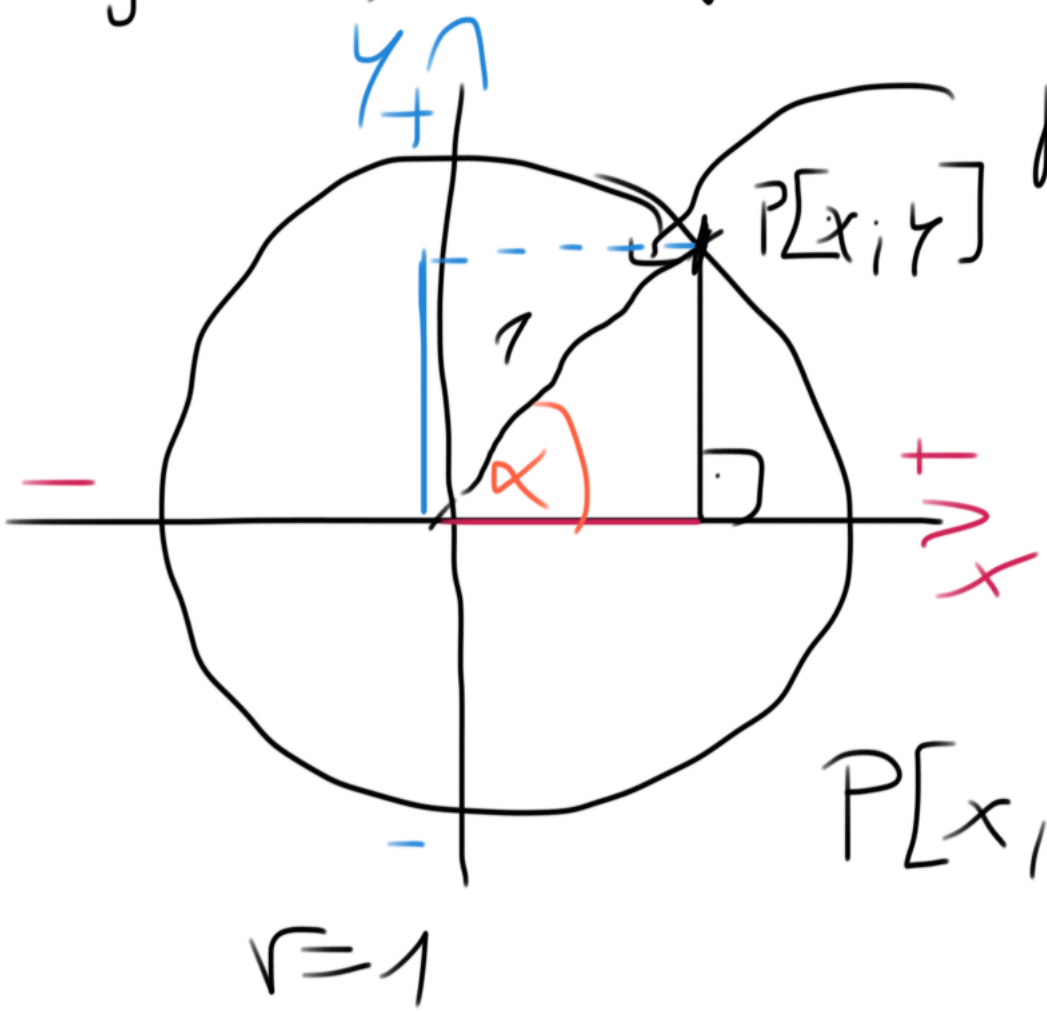
$$x = \frac{1}{8} 360 = 45^\circ$$

30°	45°	60°	90°
$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$

$165^\circ = 2 \cdot 60^\circ + 45^\circ \sim 2 \cdot \frac{\pi}{3} + \frac{\pi}{4} = \ldots$

6.3 Hodnoty GF

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\sin x$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\mathrm{tg}\, x$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	X
$\mathrm{cotg}\, x$	X	$\sqrt{3}$	1	$\sqrt{3}/3$	0



průvodič

$\cos\alpha$

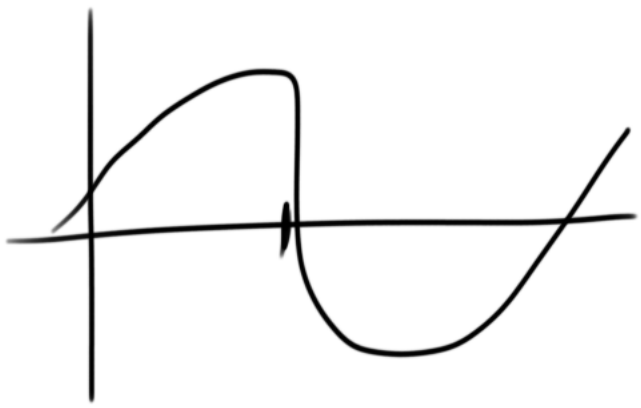
$\sin\alpha$

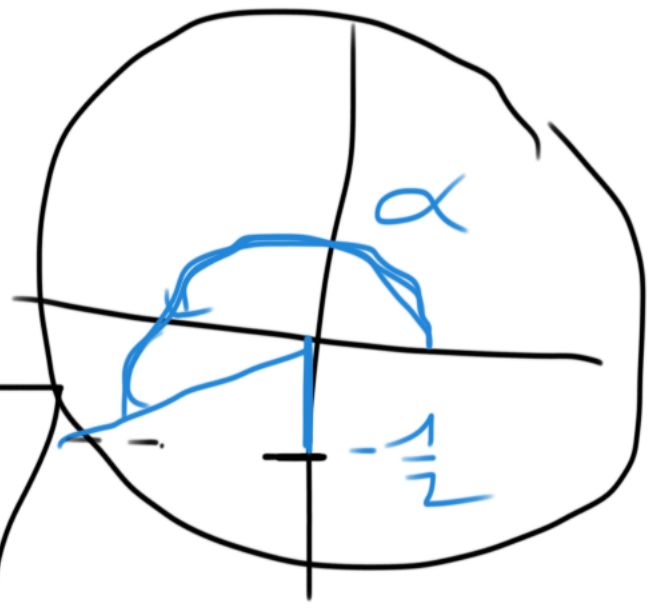
$P[x,y] = P[\cos\alpha, \sin\alpha]$

Vypočítěte $\sin\left(\frac{7}{6}\pi\right)$

$$\alpha = \frac{7}{6}\pi = 1\pi + \frac{1}{6}\pi$$

$180° \quad 30° \quad = 210°$





$$\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}$$

$$\sin\left(\frac{7}{6}\pi\right) = \frac{1}{2}$$

✓

6.4 Goniometrické vzorce

$$\sin(x) = \cos\left(x - \frac{\pi}{2}\right)$$
$$\cos(x) = \sin\left(x + \frac{\pi}{2}\right)$$

$$\operatorname{tg}(x) = \frac{\sin x}{\cos x}$$

$$\operatorname{cotg}(x) = \frac{\cos x}{\sin x} = \operatorname{tg}^{-1} x = \frac{1}{\operatorname{tg} x}$$

tg^{-1} = tag^{-1}

$\sin(-x) = -\sin(x)$ — lichá

$\cos(-x) = \cos(x)$ — sudá

$\operatorname{tg}(-x) = -\operatorname{tg}(x)$
$\operatorname{cotg}(-x) = -\operatorname{cotg}(x)$
} dokázat DDÚ

$\sin(x + 2\pi k) = \sin(x)$ — periodicita.

$$\sin^2 x + \cos^2 x = 1$$

$\sin^2 x = (\sin x)^2$

$\sin^2(x) \neq \sin(x^2) = \sin x^2$

$$\sin(2x) = 2 \cdot \sin x \cdot \cos x$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$\sin(x \pm y) = \sin x \cdot \cos y \pm \sin y \cos x$$

$$\cos(x \pm y) = \cos x \cdot \cos y \mp \sin x \sin y$$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ $\qquad \cos\left(\frac{\pi}{6}\right) = ?$

$\cos^2 x + \sin^2 x = 1$

$$\cos\left(\frac{\pi}{6}\right) = \sqrt{1 - \sin^2\left(\frac{\pi}{6}\right)}$$

$$= \sqrt{1 - \frac{1}{4}}$$

$$= \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$ ✓

6.5 Goniometrické rovnice

nalezení $x \in \mathbb{R}$
$f(x) = 0$
} Rovnice

$f(x)$ - goniometrická nebo
jejich kombinace

Strategie

1. převést do tvaru
goniometrická(výraz s x) = číslo/výraz

2. vyjádření všech podmínek
řešení

3. zkouška správnosti úpravy

$\sin(3x) = 1 - \sin(3x)$ $/+\sin(3x)$

Reste v R

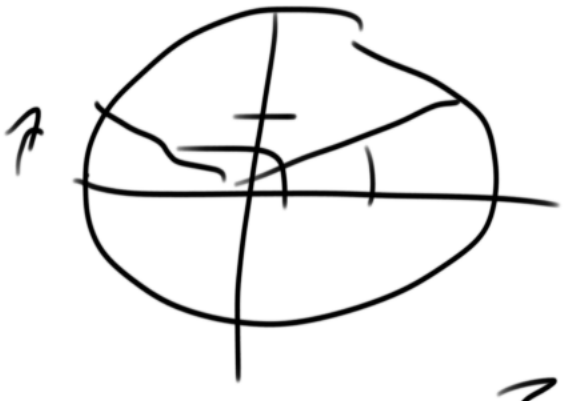
1. $2\sin(3x) = 1$

$\underline{\sin(3x) = \frac{1}{2}}$

2. $z = 3x$ $\sin(z) = \frac{1}{2}$

$z_1 = \frac{\pi}{6} + k \cdot 2\pi$

$z_2 = \frac{5\pi}{6} + k \cdot 2\pi$]



3. $3x_1 = \frac{\pi}{6} + k \cdot 2\pi$

$x_1 = \frac{\pi}{18} + k \cdot \frac{2}{3}\pi$

$3x_2 = \frac{5\pi}{6} + k \cdot 2\pi$

$x_2 = \frac{5\pi}{18} + k \cdot \frac{2}{3}\pi$

Reste v $\mathbb{R}$.

$$\cos^2 x + \cos x = 0$$

$$y = \cos x$$

$$\longrightarrow \quad y^2 + y = 0$$

$$y \cdot (y+1) = 0$$

$$y_1 = 0$$

$$y_2 = -1$$

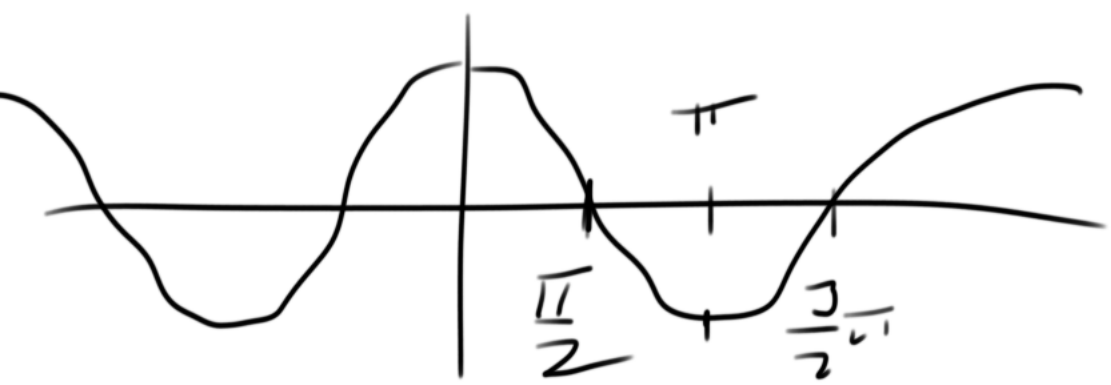
$$\cos x_1 = 0$$

$$\cos x_2 = -1$$

$$x_1 = \frac{\pi}{2} + k \cdot \pi$$

$$x_2 = \pi + k \cdot 2\pi$$

$k \in \mathbb{Z}$



Reste v $\langle 0, \frac{\pi}{2} \rangle$

$$x = \frac{\pi}{2}$$