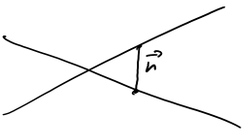
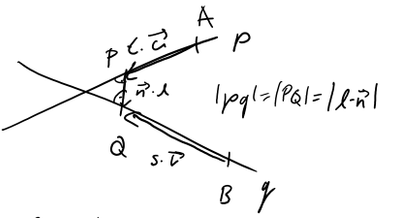
Vzdálenost mimoběžek





$\vec{n} \perp \vec{u} \wedge \vec{n} \perp \vec{v}$

$Q = A + t \cdot \vec{u} + t \cdot \vec{n}$

$Q = B + s \cdot \vec{v}$

$\longrightarrow$ proložit přímkou p rovinu σ $\quad A + t\vec{u} + t\vec{n} = B + s\vec{v}$

$p \subset \sigma : \sigma \parallel q$

$\longrightarrow$ sta ci vzdálenost od roviny

$p = \{[2,3,5+t], t \in \mathbb{R}\}$ $\quad q = \{[0,1+u,4-u], u \in \mathbb{R}\}$

$\vec{u} = (0,0,1)$ $\quad A = [2,3,5] \in p$ $\quad A + t\vec{u} + t\cdot\vec{n} = B + s\cdot\vec{v}$

$\vec{v} = (0,1,-1)$ $\quad B = [0,1,4] \in q$

$\vec{n} = \vec{u} \times \vec{v} = (-1,0,0)$ $\quad \vec{n}\cdot\vec{u} = 0$, $\vec{n}\cdot\vec{v} = 0$

$2 \quad -t = 0 \quad \longrightarrow t = 2$

$3 \quad = 1 + s \quad \longrightarrow s = 2$

$5 + t \quad = 4 - s \quad \longrightarrow t = -3$

$|t\vec{n}| = 2 \cdot |\vec{n}| = 2$ ✓

- $\sigma : ax + by + cz + d = 0$ $\quad \vec{n} = (-1,0,0) \longrightarrow \sigma : -x + d = 0$

Rovinu σ proložíme např. přímkou q: $B \in q \rightarrow B \in \sigma$: $\quad B = [0,1,4]$

$-0 + d = 0 \quad \longrightarrow \quad d = 0$

$\sigma : \; +x = 0$

$A \in p$

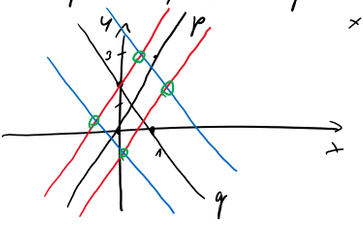
$|pq| = |p\sigma| = |A\sigma| = \dfrac{|a_\sigma A_x + b_\sigma A_y + c_\sigma A_z + d|}{\sqrt{a_\sigma^2 + b_\sigma^2 + c_\sigma^2}} = \dfrac{|2 + 0 + 0 + 0|}{1} = 2$ ✓ $\quad A = [2,3,5]$

Najděte všechny body, které mají od přímky p vzdálenost $\frac{\sqrt{10}}{5}$ a od přímky q $2\sqrt{5}$.

$A = \{X \in \mathbb{R}^2 : |pX| = \frac{\sqrt{10}}{5}\}$

$B = \{X \in \mathbb{R}^2 : |qX| = 2\sqrt{5}\}$

$p: 3x - y = 0$ $\quad q: 2x + y - 2 = 0$ $\quad x + \frac{y}{2} = 1$



$d = \dfrac{|ax + by + c|}{\sqrt{a^2 + b^2}}$

$|Xp| = \dfrac{|3x - y|}{\sqrt{10}} = \dfrac{\sqrt{10}}{5}$ $\quad |Xq| = \dfrac{|2x + y - 2|}{\sqrt{5}} = 2\cdot\sqrt{5}$

$|3x - y| = \frac{10}{5}$ $\quad |2x + y - 2| = 10$

$|3x - y| = 2$

$\longrightarrow \boxed{|2x + y - 2| = 5\,|3x - y|}$

$|3x - y| = 2$ $\quad$ $|2x + y - 2| = 10$

$3x - y = 2$ $\quad -3x + y = 2$ $\quad$ $2x + y - 2 = 10$ $\quad -2x - y + 2 = 10$

I: $y = 3x - 2$ $\quad$ II: $y = 3x + 2$ $\quad$ III: $y = -2x + 12$ $\quad$ IV: $y = -2x - 8$

$I \cap II = \emptyset$ $\quad III \cap IV = \emptyset$

$I \cap III$: $\quad y = 3x - 2$, $y = -2x + 12$ $\ominus$

$0 = 5x - 14$

$x = \frac{14}{5}$

$y = \frac{42}{5} - \frac{10}{5} = \frac{32}{5}$

$P_{13} = [\frac{14}{5}, \frac{32}{5}]$ ✓

$I \cap IV$: $\quad y = 3x - 2$, $y = -2x - 8$ $\ominus$

$0 = 5x + 6$

$x = -\frac{6}{5}$

$y = -\frac{18}{5} - \frac{10}{5} = -\frac{28}{5}$

$P_{14} = [-\frac{12}{5}, -\frac{28}{5}]$ ✓

$II \cap III$: $\quad y = 3x + 2$, $y = -2x + 12$ $\ominus$

$0 = 5x - 10$

$x = 2$

$y = 8$

$P_{23} = [2,8]$ ✓

$II \cap IV$: $\quad y = 3x + 2$, $y = -2x - 8$ $\ominus$

$0 = 5x + 10$

$x = -2$

$y = -4$

$P_{24} = [-2,-4]$ ✓