Modeling and Simulation — Lesson 4
Simple Rules, Complex Worlds: Cellular Automata

Václav B. Petrák

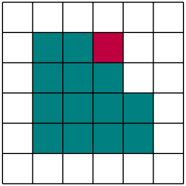
Faculty of Biomedical Engineering
Czech Technical University

April 17, 2025

Introduction

Cellular Automata

- *Automaton* (plural: *automata*) is a term used in for a theoretical machine that changes its internal state based on inputs and its previous state.
- The state set is usually defined as finite and discrete, which often causes nonlinearity in the system's dynamics
- *Cellular automata* are a set of such automata arranged in a regular spatial grid, whose states are simultaneously updated by a uniformly applied state-transition function that refers to the states of their neighbors.



Example of cellular automata for modelling forrest fire. Green represents trees, white space empty space, and red starting fire.

Introduction cellular automata

- **Definition:** Cellular automata (CA) are discrete, computational models used to simulate complex systems and processes. They consist of a grid of cells, each in one of a finite number of states, updating simultaneously according to a set of local rules.
- **History and Origin:**
  - Introduced by mathematicians Stanislaw Ulam and John von Neumann.
  - Initially conceptualized as a mathematical abstraction for self-replication.
- **Characteristics:**
  - **Grid:** Usually 2D, but can be 1D or 3D.
  - **States:** Common examples include binary states (0 and 1) but can be more complex.
  - **Neighborhoods:** Typically include adjacent cells (e.g., Moore or von Neumann neighborhoods).
  - **Rules:** Simple local interactions that determine the state of a cell in the next generation.

Significance

- Demonstrates how complex patterns and behaviors can emerge from simple rules applied at local levels.
- Used as a theoretical tool to study self-organization in natural systems.

John von Neumann

John von Neumann (1903–1957): Hungarian-American polymath made pioneering contributions across multiple scientific disciplines.

- **Mathematics:** significant contributions to set theory, functional analysis, and the foundations of mathematics.
- **Quantum Mechanics:** mathematical formulation of quantum mechanics, the axiomatic Hilbert space framework.
- **Computer Science:** Developed the Von Neumann architecture, the basis for most modern computer designs.
- **Game Theory:** founder of game theory, co-authored "Theory of Games and Economic Behavior," .
- **Nuclear Physics:** Played a key role in the Manhattan Project and in the development of the hydrogen bomb.
- **Statistics and Economics:** profoundly influenced economic theories .
- **Cellular Automata:** Proposed a self-replicating machine model using cellular automata, foundation for the field of artificial life.

Example of patter evolution

- The initial pattern constitutes the first generation of the system.
- The second generation is created by applying the above rules simultaneously to every cell in the first generation in parallel– births and deaths happen simultaneously.
- The rules continue to be applied repeatedly to create further generations.
- The rules determine the new states of each of the cells inp from the states of that cells and its neighbours
- The size of neighbourhoods determine the extent of the interaction between cells in the grid

Properties of Cellular Automata

Important properties which distinguis Cellular automata

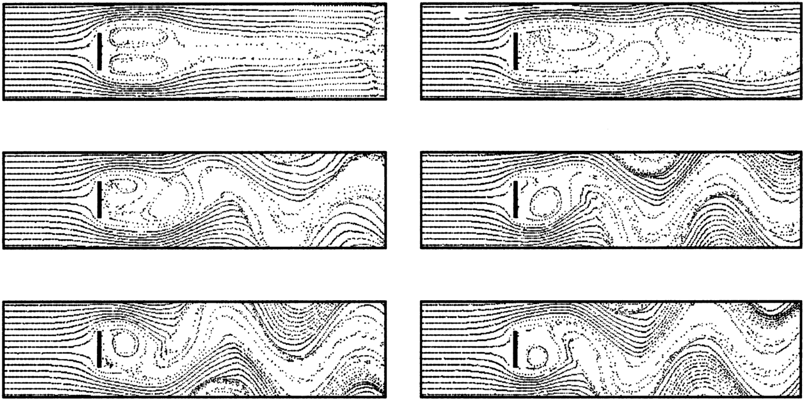
- **Localism:** States are updated based on the properties of the neighbourhood
- **Parallelism:** The state of every cell is updated in parallel
- **Homogeneity:** The same set of rules is applied across the automaton

These properties distinguish cellular automata from other types of automata or algorithm.

Properties of Cellular Automata

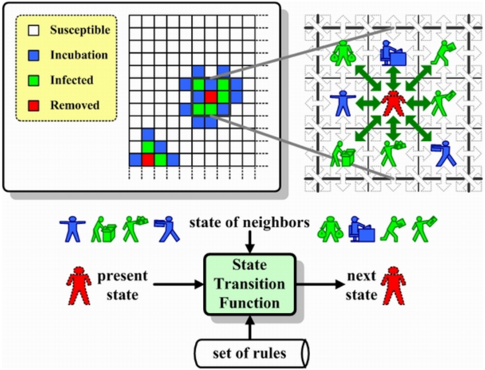
- It is **impossible** to predict, in advance, what behavior will be displayed by the Cellular Automata given a set of rules.
- There are a number of possible states into which a CA can descend into
- Stephen Wolfram proposed a classification scheme based on these criteria:
  - Evolution leads to a homogeneous state.
  - Evolution leads to a set of separated simple stable or periodic structures.
  - Evolution leads to a chaotic pattern.
  - Evolution leads to complex localized structures, sometimes long-lived.

Simulation of Fluids: Cellular automata are used to model fluid dynamics by discretizing the fluid space and applying rules for the interaction of particles, mimicking the behavior of real fluids.

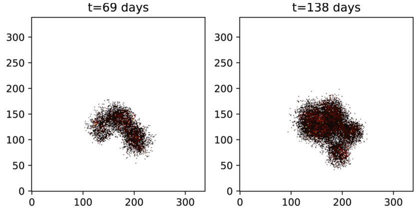


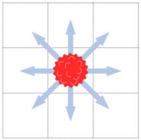
Applications of Cellular Automata

Epidemic Models: Cellular automata help simulate the spread of diseases by modeling individuals as cells and defining rules for infection transmission and recovery.

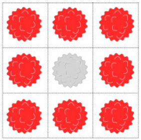


Simulation of Cancer Cell Growth: Cellular automata model the proliferation of cancer cells and their interactions with the surrounding tissue to understand and predict growth patterns.





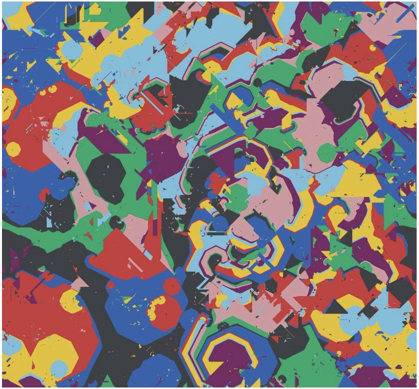
(a) If there are available spaces, the cell can move or proliferate to any of the adjacent positions.



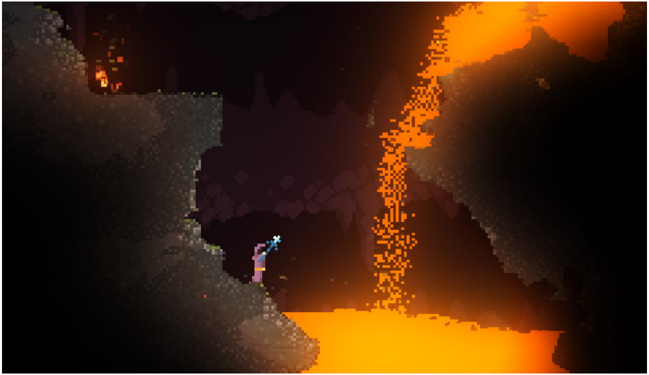
(b) If there are no available spaces, the cell becomes quiescent until an adjacent cell either moves or dies.

Applications of Cellular Automata

Art and design: Some artists experiments cellular automata to create complex, evolving patterns and textures that are used in digital art and visual effects.

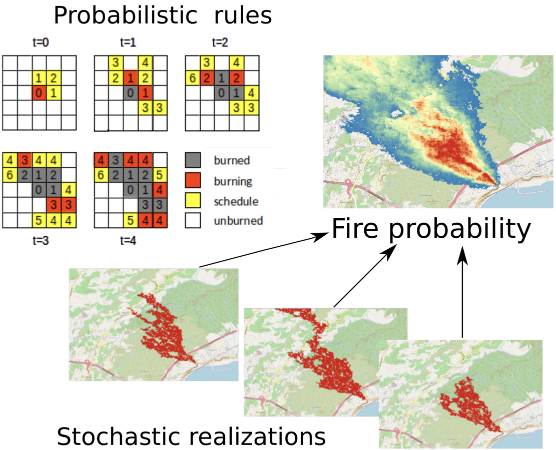


Art and design: Cellular automata were also used for visual effect in Noita game



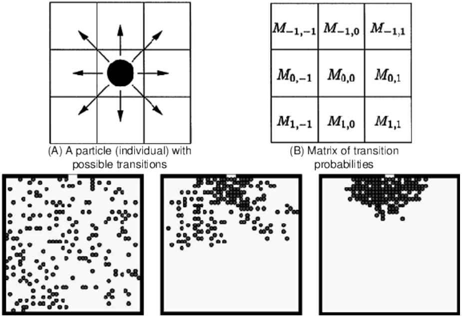
Applications of Cellular Automata

Simulation of Forest Fires: Cellular automata model the spread of fire through a forest based on wind, vegetation, and other factors, useful in understanding and managing wildfires.



Applications of Cellular Automata

Simulations of Social Movements: These models use cellular automata to represent social dynamics and group behaviors, aiding in the study of phenomena like crowd movements and protests.



(C) Simulation of pedestrians leaving room with single door

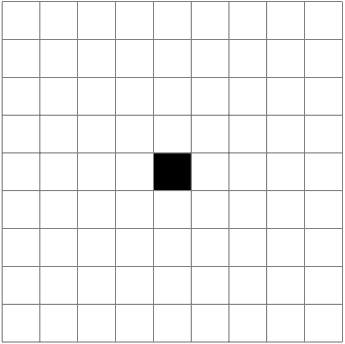
Conway's Game of Life

Try to google: "Conway's Game of Life"

Overview of Conway's Game of Life

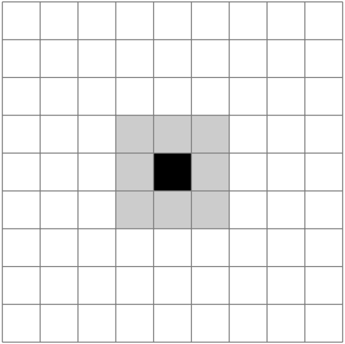
- **Overview:**
  - Created by the mathematician John Horton Conway in 1970.
  - A zero-player game where evolution is determined by its initial state, without stochastic elements.
  - The game unfolds on an infinite grid.
  - Each cell can be either alive or dead.
- **Rules of Evolution:**
  1. Any live cell with fewer than two live neighbors dies.
  2. Any live cell with two or three live neighbors lives on to the next generation.
  3. Any live cell with more than three live neighbors dies.
  4. Any dead cell with exactly three live neighbors becomes a live cell.

Each cell can be either alive or dead.



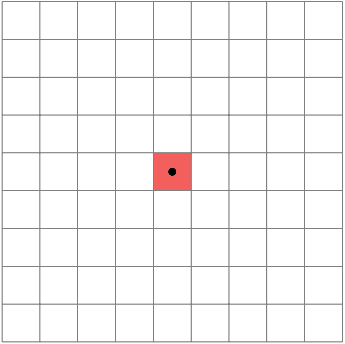
The status of each cell changes each generation.

Status depends on the status of the cell and its 8 neighbors.



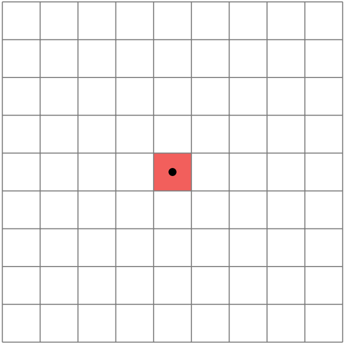
At each step, transitions occur according to four rules

Rule 1: Any live cell with fewer than two live neighbors dies, as if by underpopulation.



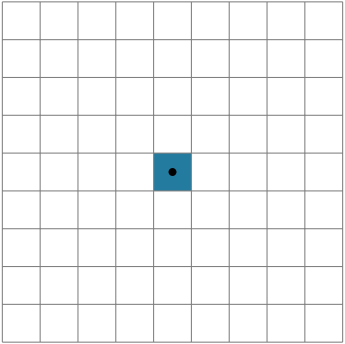
At each step, transitions occur according to four rules

Rule 1: Any live cell with fewer than two live neighbors dies, as if by underpopulation.



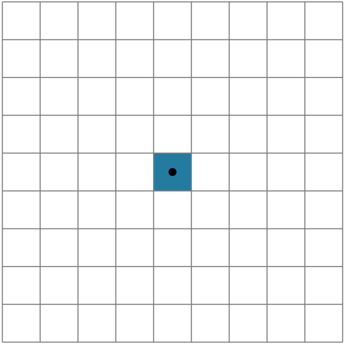
At each step, transitions occur according to four rules

Rule 2: Any live cell with two or three live neighbors lives on to the next generation.



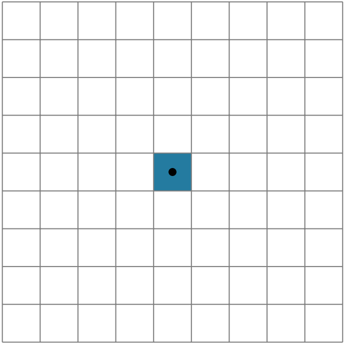
At each step, transitions occur according to four rules

Rule 2: Any live cell with two or three live neighbors lives on to the next generation.



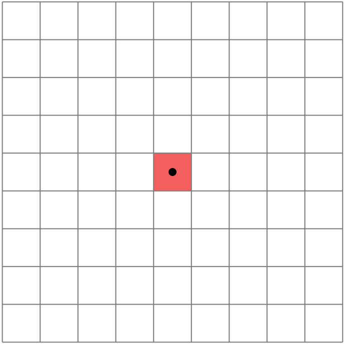
At each step, transitions occur according to four rules

Rule 2: Any live cell with two or three live neighbors lives on to the next generation.



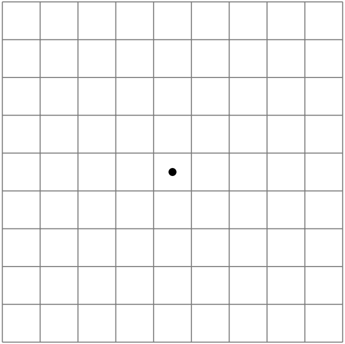
At each step, transitions occur according to four rules

Rule 3: Any live cell with more than three live neighbors dies, as if by overpopulation.



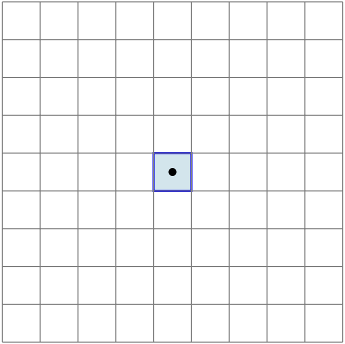
At each step, transitions occur according to four rules

Rule 4: Any dead cell with exactly three live neighbors becomes a live cell, as if by reproduction.



At each step, transitions occur according to four rules

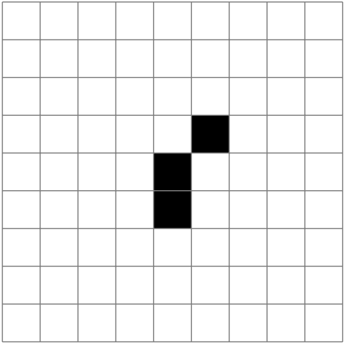
Rule 4: Any dead cell with exactly three live neighbors becomes a live cell, as if by reproduction.



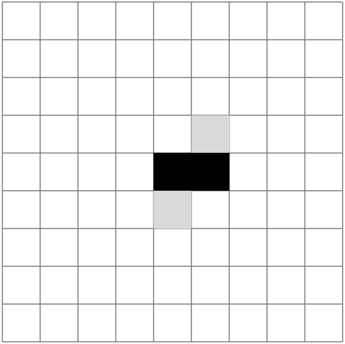
Frame Title

Starting from the initial configuration, these rules are applied, and the game board evolves, playing the game by itself.

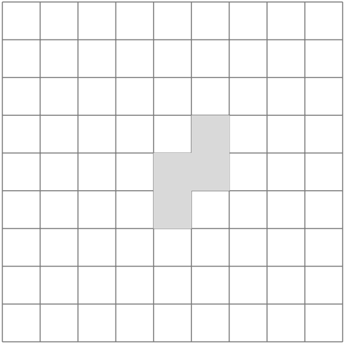
Example of pattern evolution



Example of pattern evolution

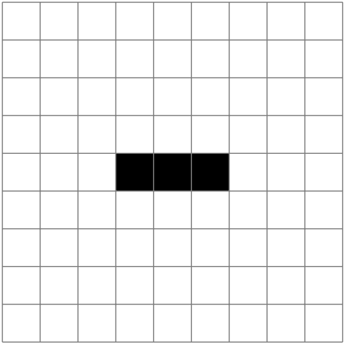


Example of pattern evolution



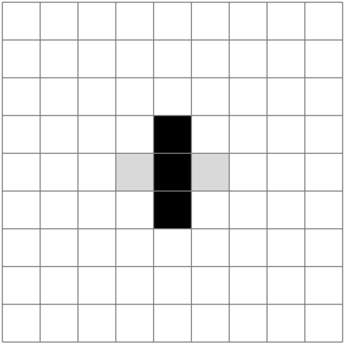
Oscillators

Periodic Life Forms or Oscillators are life forms that oscillate periodically.



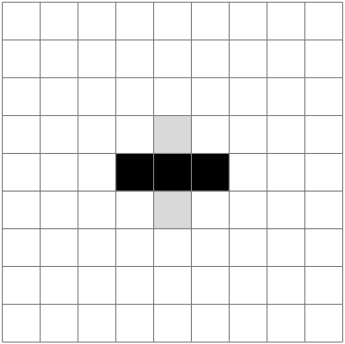
Oscillators

Periodic Life Forms or Oscillators are life forms that oscillate periodically.



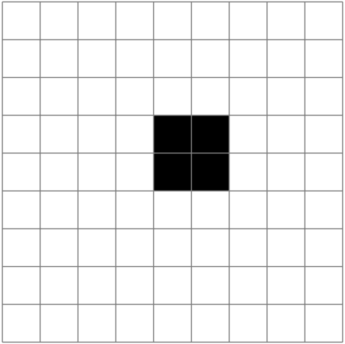
Oscillators

Periodic Life Forms or Oscillators are life forms that oscillate periodically.



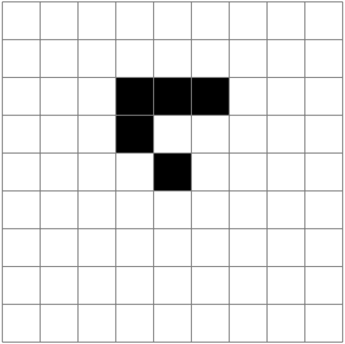
Still Life

Still Life: a stable, finite, and nonempty pattern.



Glider

A **glider** is a simple five-cell pattern that repeats itself every four generations and is offset diagonally by one cell. It is the smallest and most common type of spaceship. A spaceship is a pattern that moves across the game board.



Some Patterns in Conway's Game of Life

- **Oscillators:** Patterns that return to their initial state after a certain number of generations.
- **Spaceships:** Patterns that translate themselves across the board.
- **Gardens of Eden:** Configurations that cannot be reached from any other, meaning they have no predecessors.
- **Methuselahs:** Start with a small configuration and evolve over many generations before stabilizing.
- **Eaters:** Stationary or oscillating patterns that can destroy other patterns. Used to 'eat' gliders and other spaceships.
- **Rakes:** Patterns that move like spaceships but also emit other patterns, typically gliders.
- **Guns:** Stationary patterns that periodically emit spaceships or other patterns.
- **Memory Cell:** Stores information that can be read out.

Constructing Logical Gates in Conway's Game of Life

- **Computational Universality:** Conway's Game of Life is Turing complete, meaning it can simulate a universal constructor or any Turing machine.
- **Logic Gates:** Using gliders and eaters, specific logic gates like AND, OR, NOT can be constructed.
  - **AND Gate:** Two gliders collide in a way that if both are present, a new glider is formed (output).
  - **OR Gate:** Gliders are sent such that if at least one arrives, a new glider is formed.
  - **NOT Gate:** Utilizes a glider stream that is blocked unless another glider interferes, allowing a glider to pass through if no input glider is present.
- **Application:** These gates can be used to build more complex circuits and perform computations.
- A computer with memory, a processor, and a display can be built in Conway's Game of Life.

Task: How Will This Pattern Evolve?

- It is **impossible** to predict, in advance, what behavior will be displayed by the Cellular Automata given a set of rules.
- We will find out how the following shape evolves in the exercise.