Polynomial Kernels for Traveling Salesperson

Václav Blažej, **Pratibha Choudhary**, **Dušan Knop**, **Šimon Schierreich**, **Ondřej Suchý**, and **Tomáš Valla**

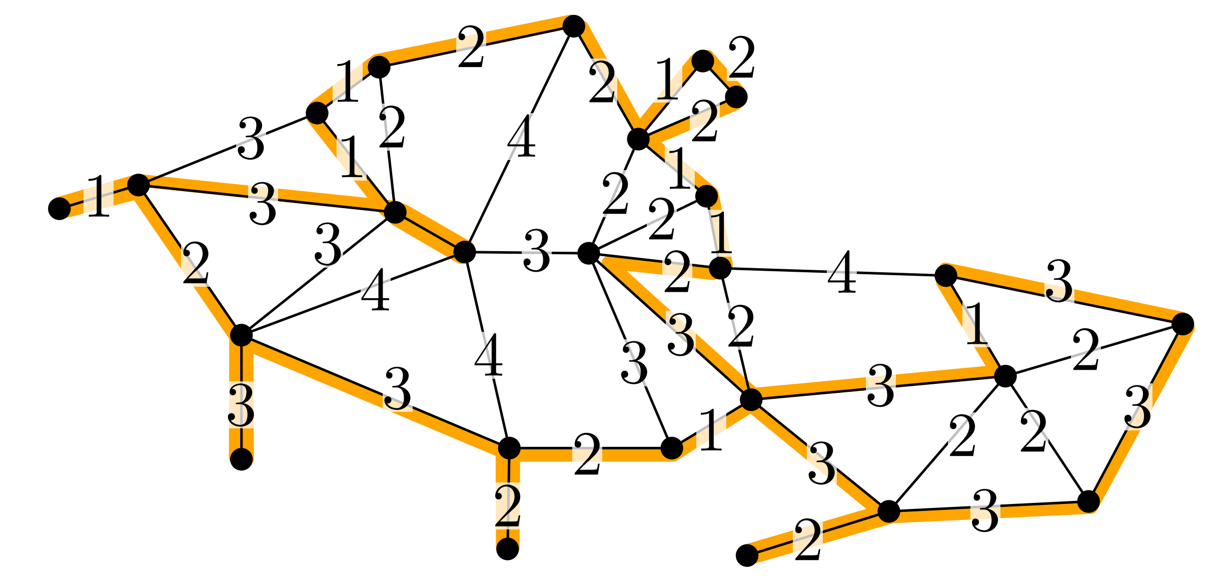
Faculty of Information Technology, Czech Technical University in Prague, Prague, Czech Republic

Traveling Salesperson Problem (TSP)

Input: Simple **weighted** undirected graph $G = (V, E, \omega)$, where $\omega\colon E \to \mathbb{N}$ and a **budget** $B \in \mathbb{N}$.

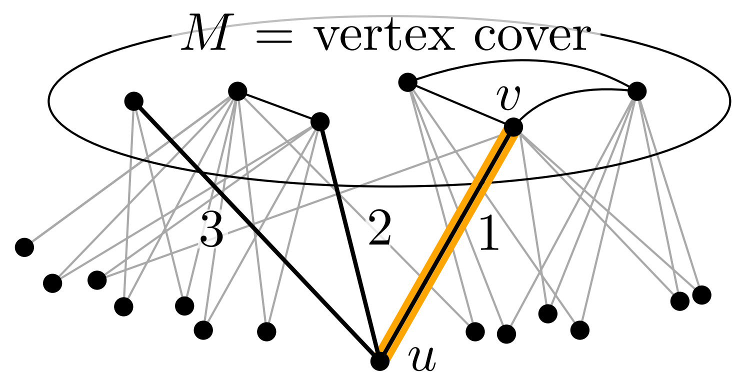
Output: Is there a **closed walk** R that visits all vertices and has the total weight at most B?

- TSP is an NP-hard problem
- it is FPT with respect to treewidth



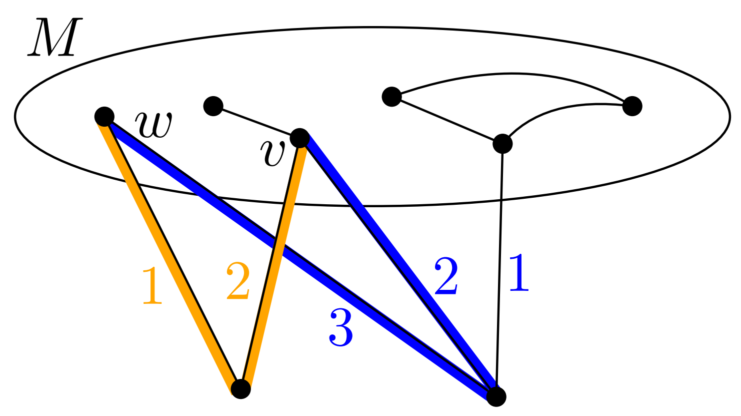
Vertex Cover Number

- vertices outside of the vertex cover M have a cheapest way to connect to M



connect u with v using a total weight 2

- connecting all vertices in the cheapest way may not give a connected solution
- "pay" an additional fee to some vertices to change their connections so that the solution is connected



pay 1 or **pay 3** to connect w with v

- retain M and a polynomial number of such vertices for each (v, w) pair
- $\rightarrow$ polynomial kernel

Our results

Vertex Cover Number
Remove k vertices to obtain an independent set.
TSP has $\mathcal{O}(k^{16})$ kernel.

Feedback Edge Set No.
Remove k edges so that no cycles are left.

Mod. to Const. Paths
Remove k vertices to obtain contant-length paths
kernel from $\downarrow$ result

Mod. to Const. Comps.
TSP has $k^{\mathcal{O}(r)}$ kernel where r is size of left connected components.

Feedback Vertex Set No.
Remove k vertices so that no cycles are left.

Fractioning Number
Remove k vertices so that components of size $\leq k$ remain.
no polynomial kernel

Mod. to Disjoint Cycles
Remove k vertices so that disjoint cycles are left.

Treewidth

arXiv

Feedback Edge Set No.

- leaves always have a clear solution
- chains of degree 2 vertices have the number of possibilities small and can be modelled with smaller subgraphs
- similar reductions also work for the generalized TSP (see box at the bottom)
- exhaustive application gives a polynomial kernel

Negative results

- no polynomial kernel for TSP with respect to the **fractioning number** unless polynomial hierarchy collapses
- no polynomial kernel with respect to the combined parameter **treewidth and maximum degree** unless polynomial hierarchy collapses
- unweighted Subset TSP with respect to the **modulator to disjoint cycles** is WK[1]-hard $\Rightarrow$ no polynomial kernel

EUROPEAN UNION
European Structural and Investment Funds
Operational Programme Research, Development and Education

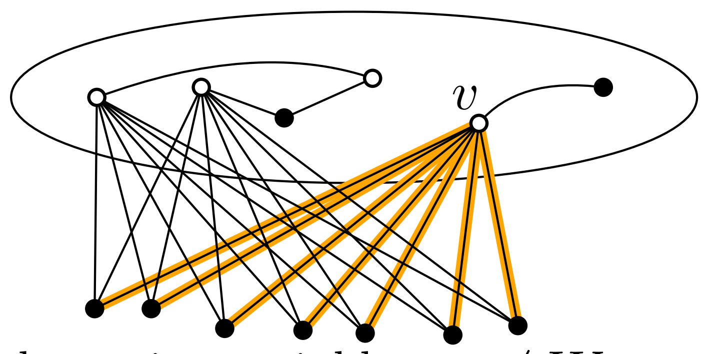
MINISTRY OF EDUCATION, YOUTH AND SPORTS

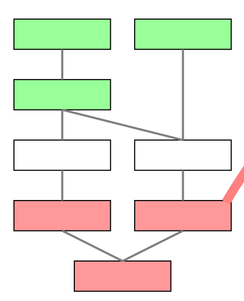
The authors acknowledge the support of the OP VVV MEYS funded project CZ.02.1.01/0.0/0.0/16.019/0000765 "Research Center for Informatics" and the Grant Agency of the Czech Technical University in Prague funded grant No. SGS20/208/OHK3/3T/18.

Generalizations

Subset TSP

- has a set of waypoints $W \subseteq V$ (full) that need to be traversed

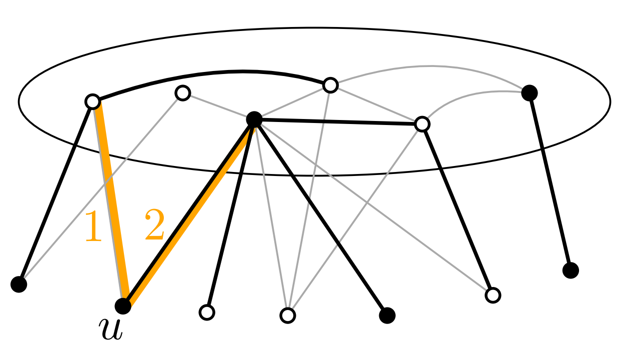


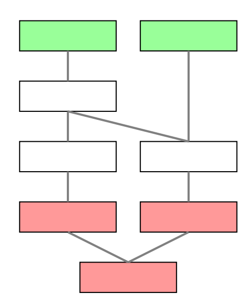


- enough vertices neighbor $v \notin W \rightarrow$ reroute the solution through it
- polynomial kernel w.r.t. the modulator to constant paths

Waypoint Routing Problem

- has a capacity $c\colon E \to \mathbb{N}$ for every edge





- WRP can be reduced to capacities 1 (thin) and 2 (thick)
- polynomial kernel with respect to the vertex cover number