Distributed Derandomization via Network Decomposition

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joint work with
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We will see...

...distributed deterministic algorithm for network decomposition.

This is a key technical tool for a lot of theory of the **LOCAL** model built in past few years.

Teaser:

“For locally checkable problems, **P-RLOCAL = P-LOCAL**, i.e., **all** randomized distributed algorithms can be efficiently derandomized. ”

“$\Delta+1$ coloring problem (and other problems) can now be solved in poly(log log n) rounds by a randomized algorithm. “

Plan

1. See why is network decomposition a very handy technique.
2. Formulate general framework for turning sequential algorithms into distributed ones. See how it relates to derandomization.
3. See a simple deterministic algorithm for network decomposition.
4. See what this tells us about deterministic **CONGEST** algorithms and randomized **LOCAL**/**MPC** algorithms.

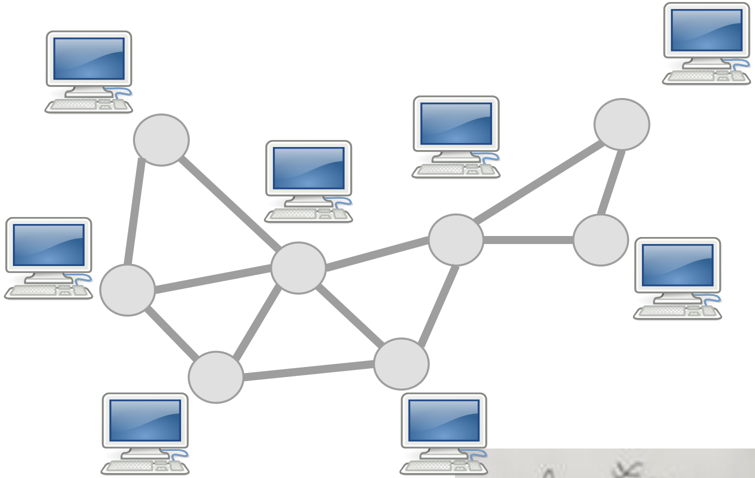
The **LOCAL** model of distributed graph algorithms

- Undirected graph $G=(V,E)$ with n nodes
- One computer in each node

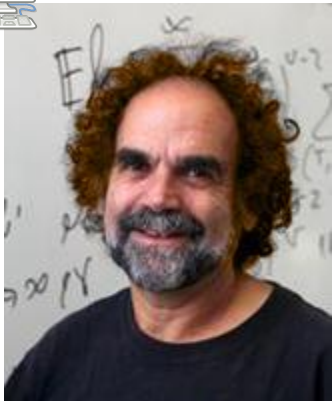
- Synchronous message passing rounds
- Unbounded message size and $\boxed{\text{computation!}}$ (more honest version: **CONGEST** model)

- Initially, nodes know only (upper bound on) n and their $\boxed{\text{unique } O(\log n) \text{ bit label}}$
- In the end, each node should know its part of output

- Time complexity: number of rounds



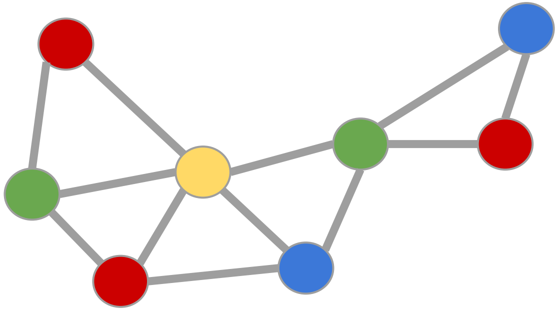
LOCAL model
[Linial FOCS'87]



Network decomposition: why we like it

Our running example: Δ+1 coloring

- greedy sequential algorithm
- efficient randomized solution



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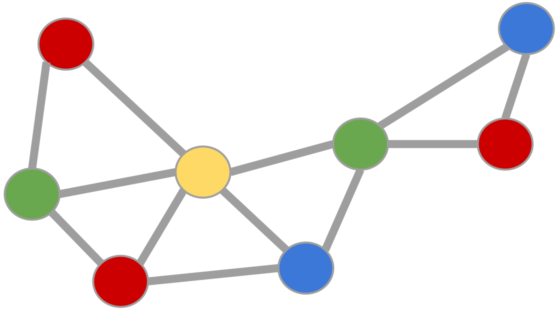
Q: Is there a principled approach to the problem that would also work for maximal independent set, matching, …?



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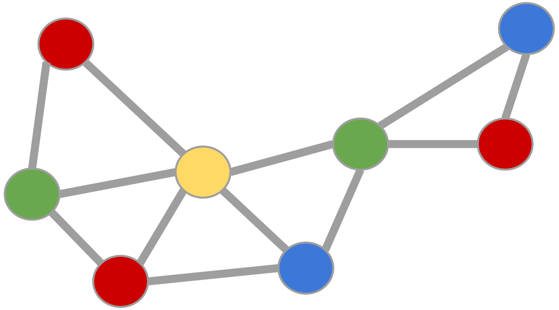

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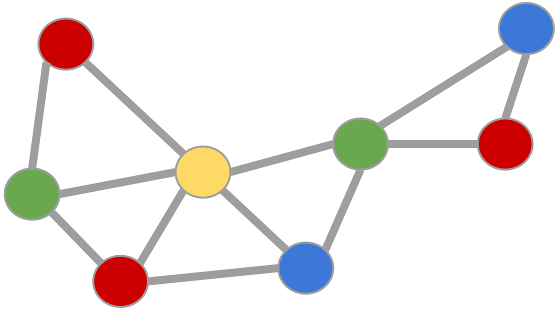

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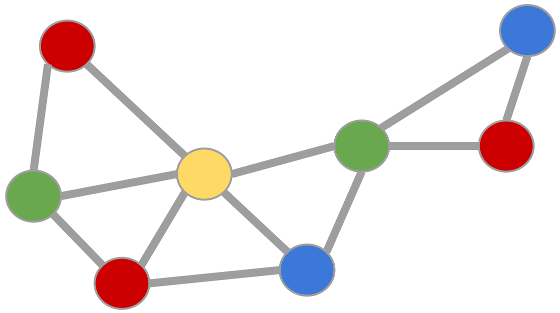

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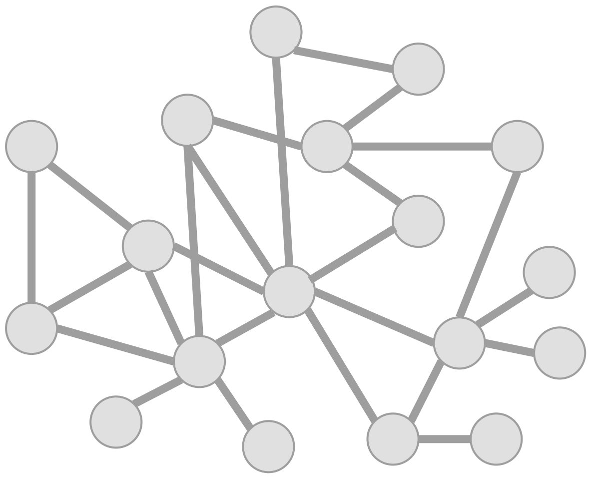
Principled approach

Easy case for our quest: the underlying graph has poly(log n) diameter.



Solution: bruteforce

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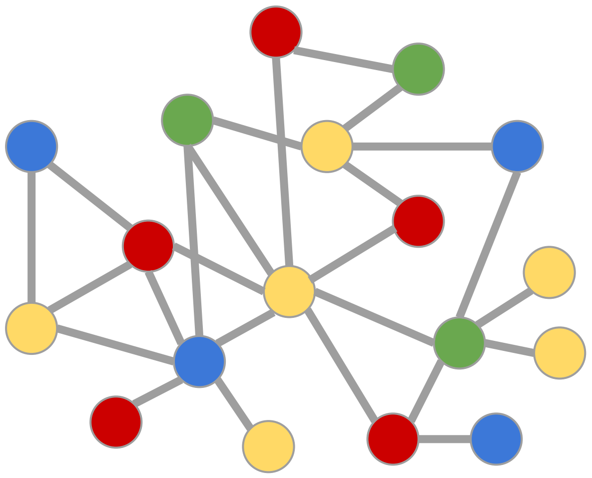

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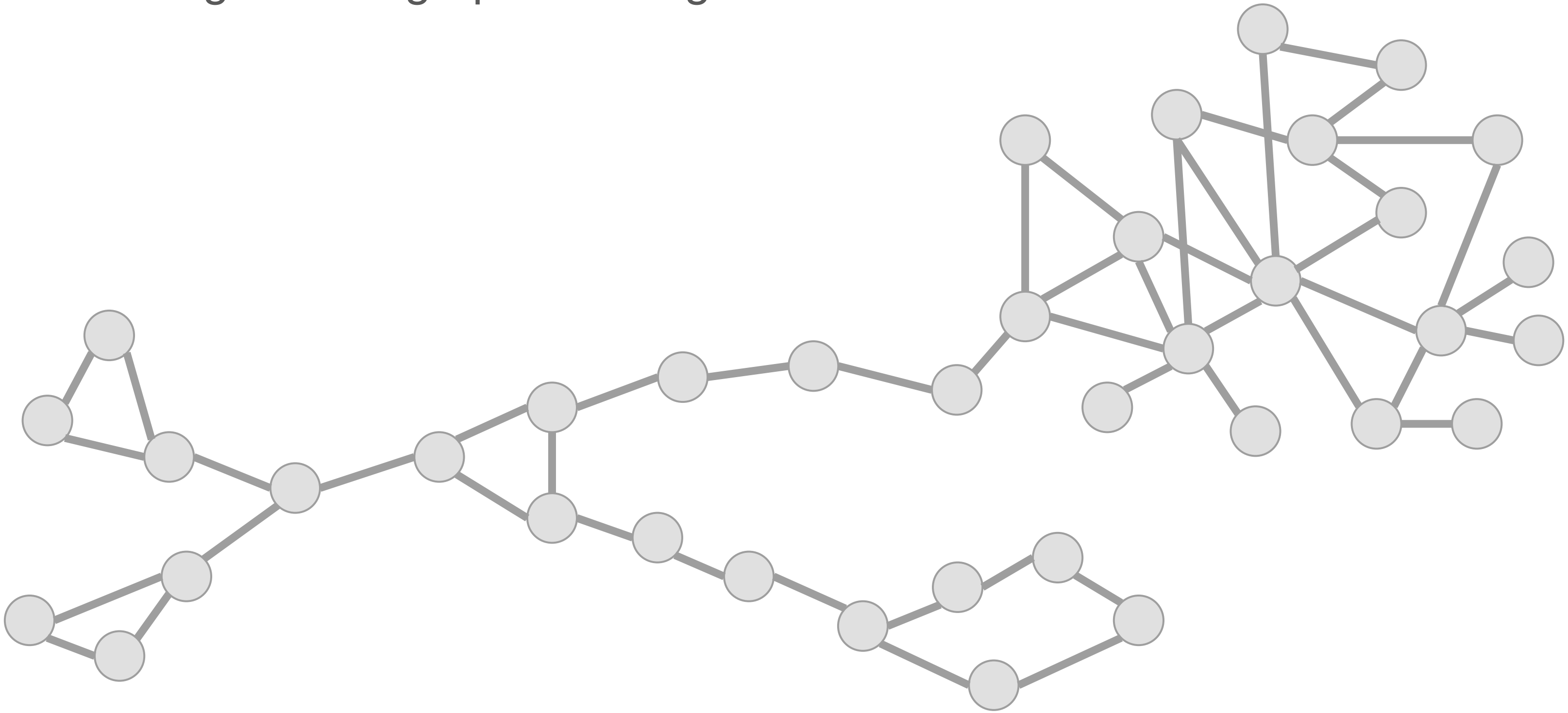
Interesting case: a graph with huge diameter.



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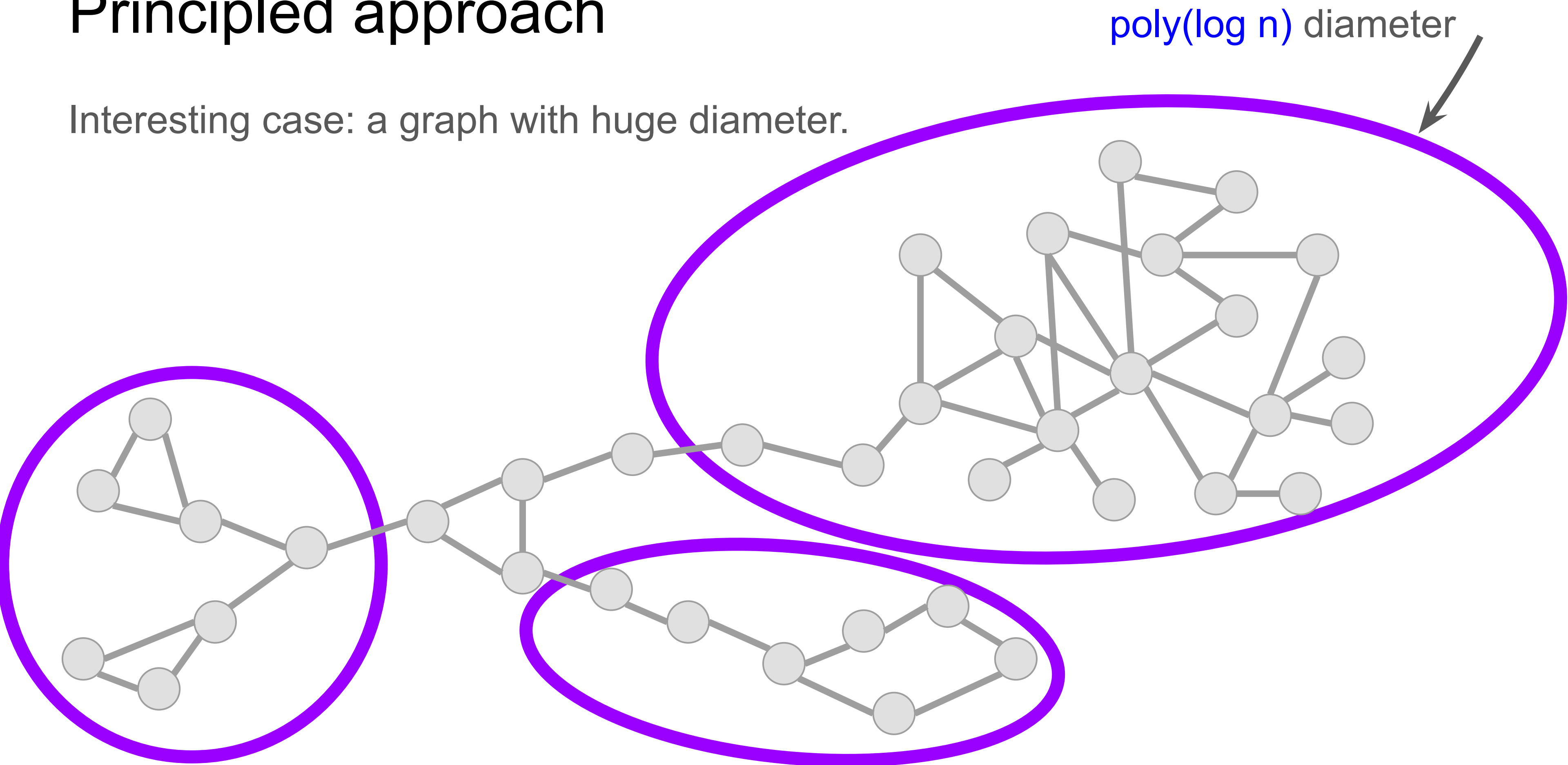

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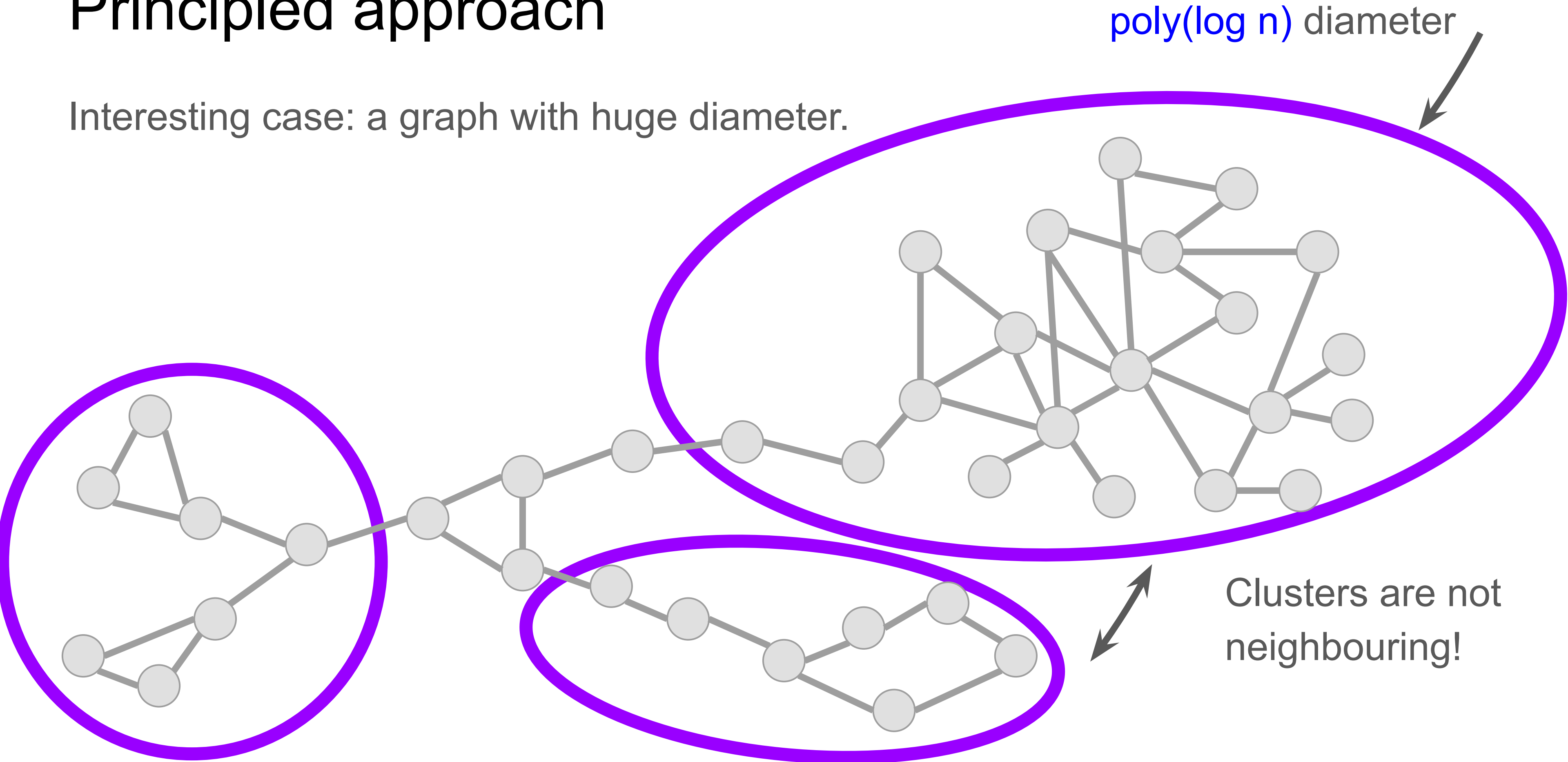

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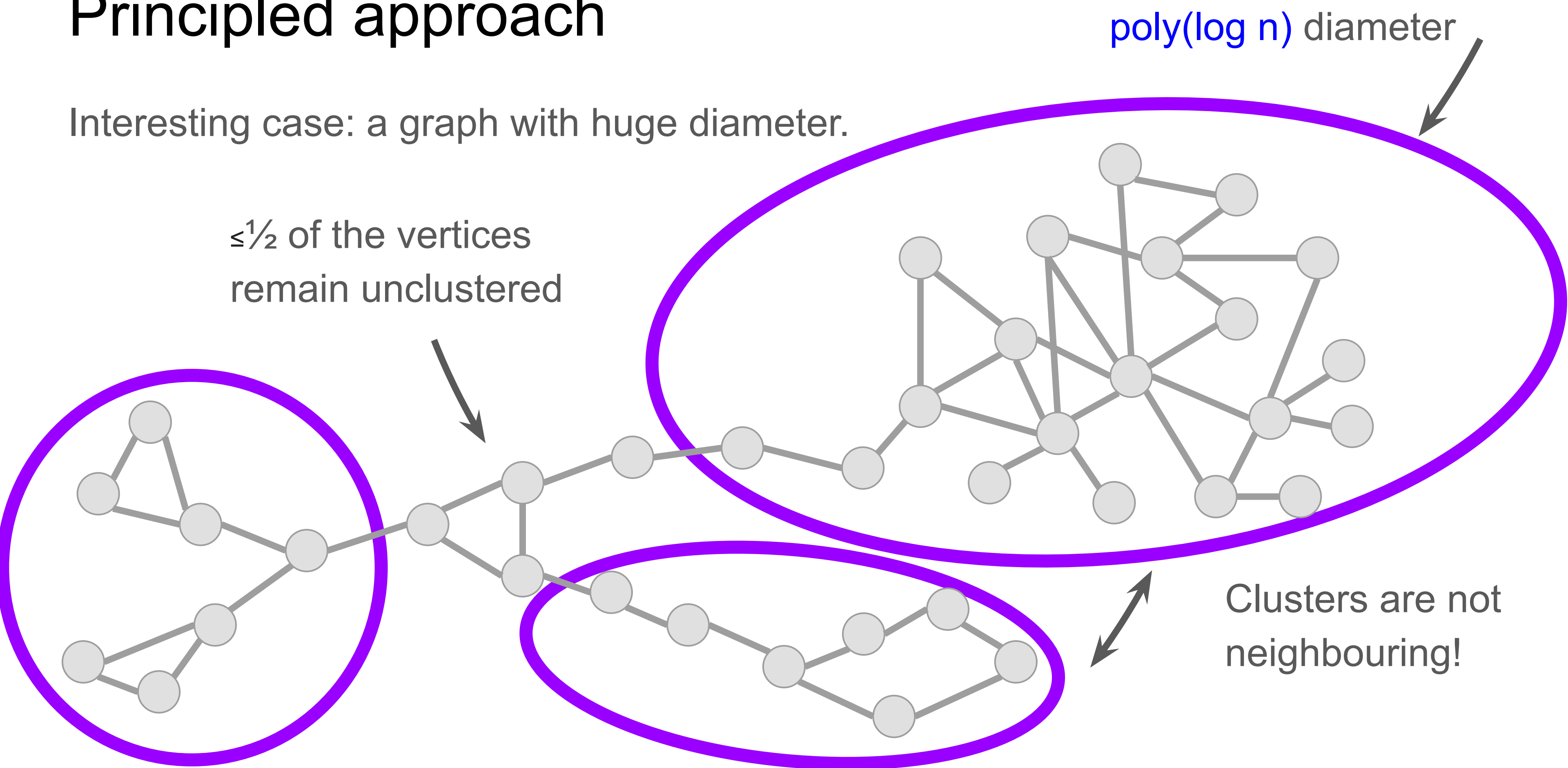

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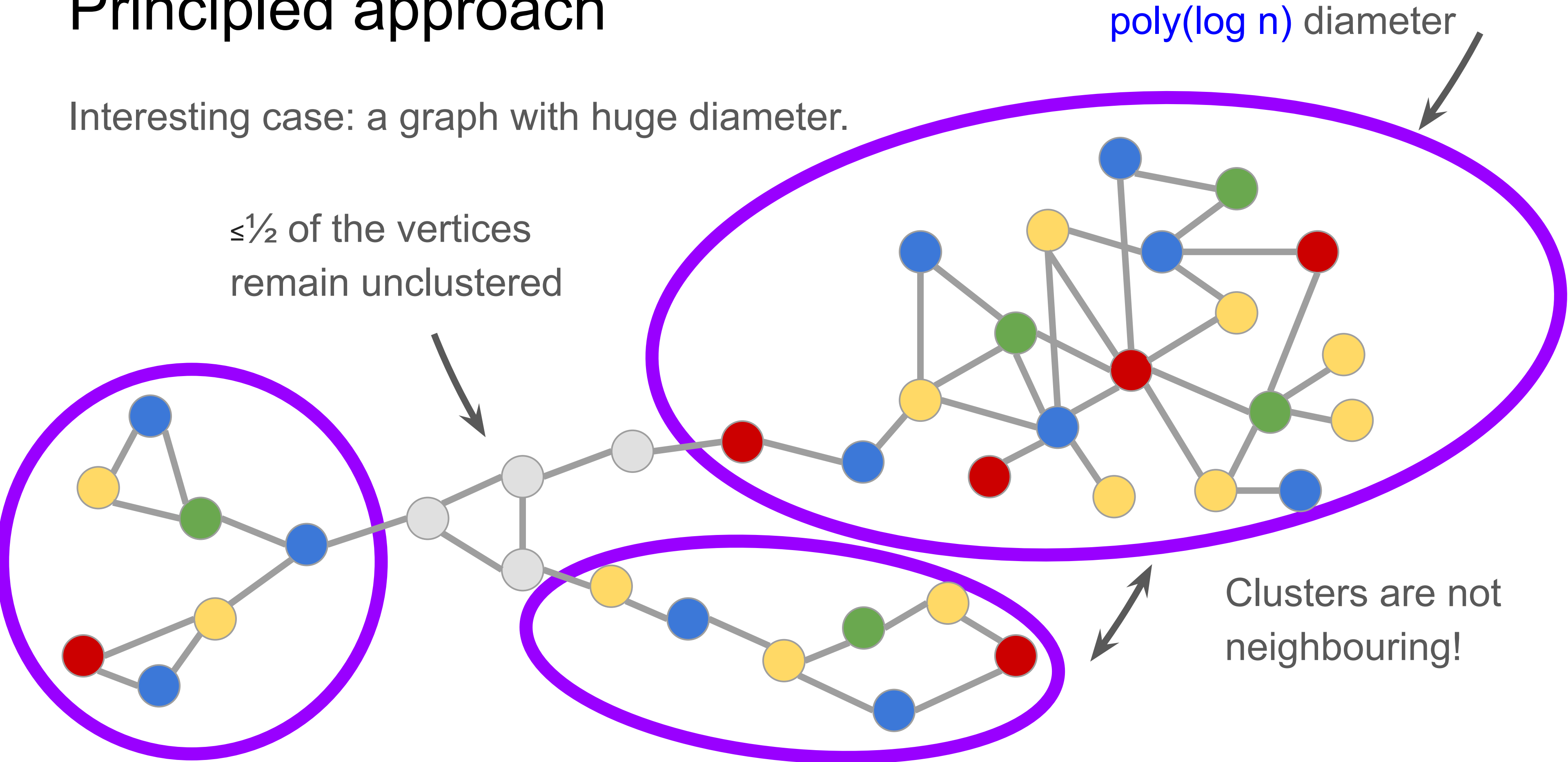

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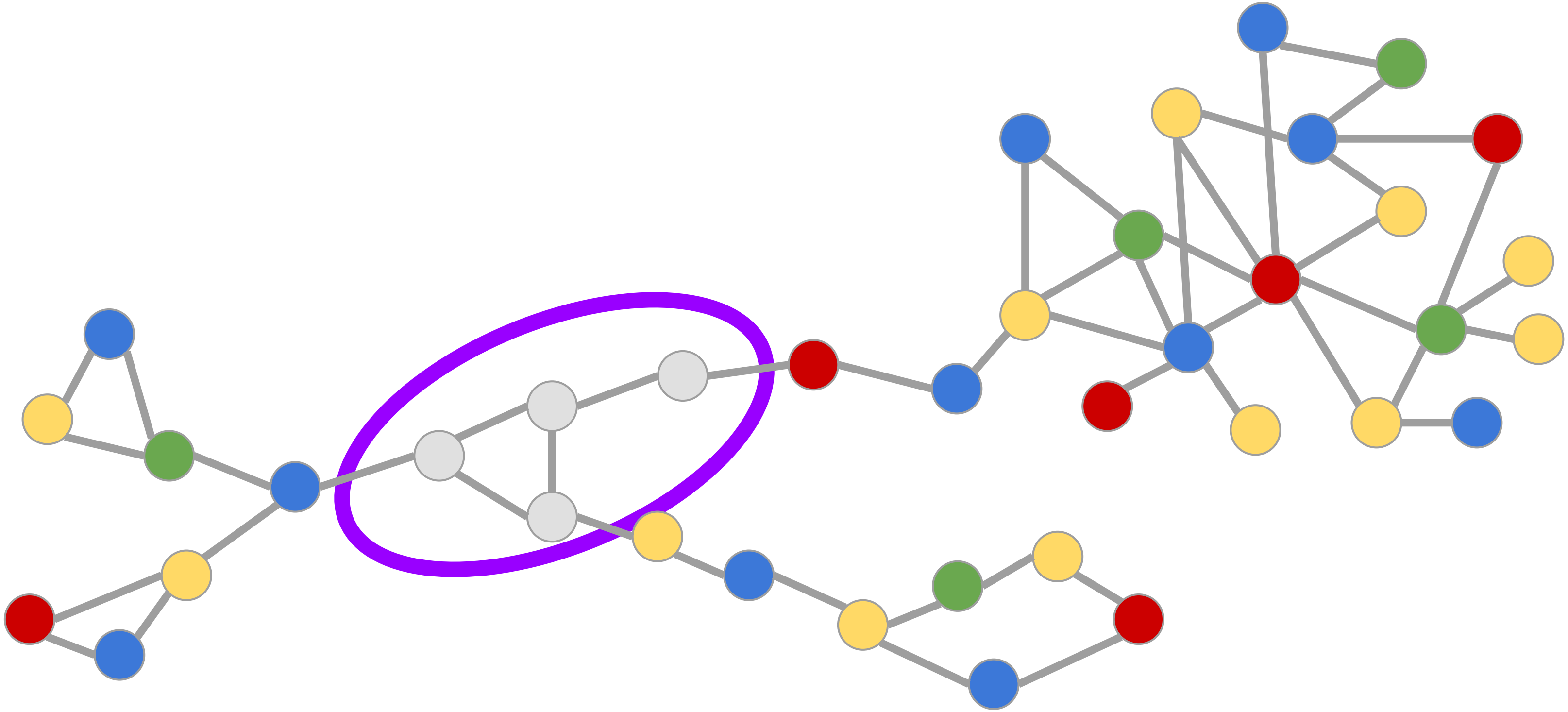

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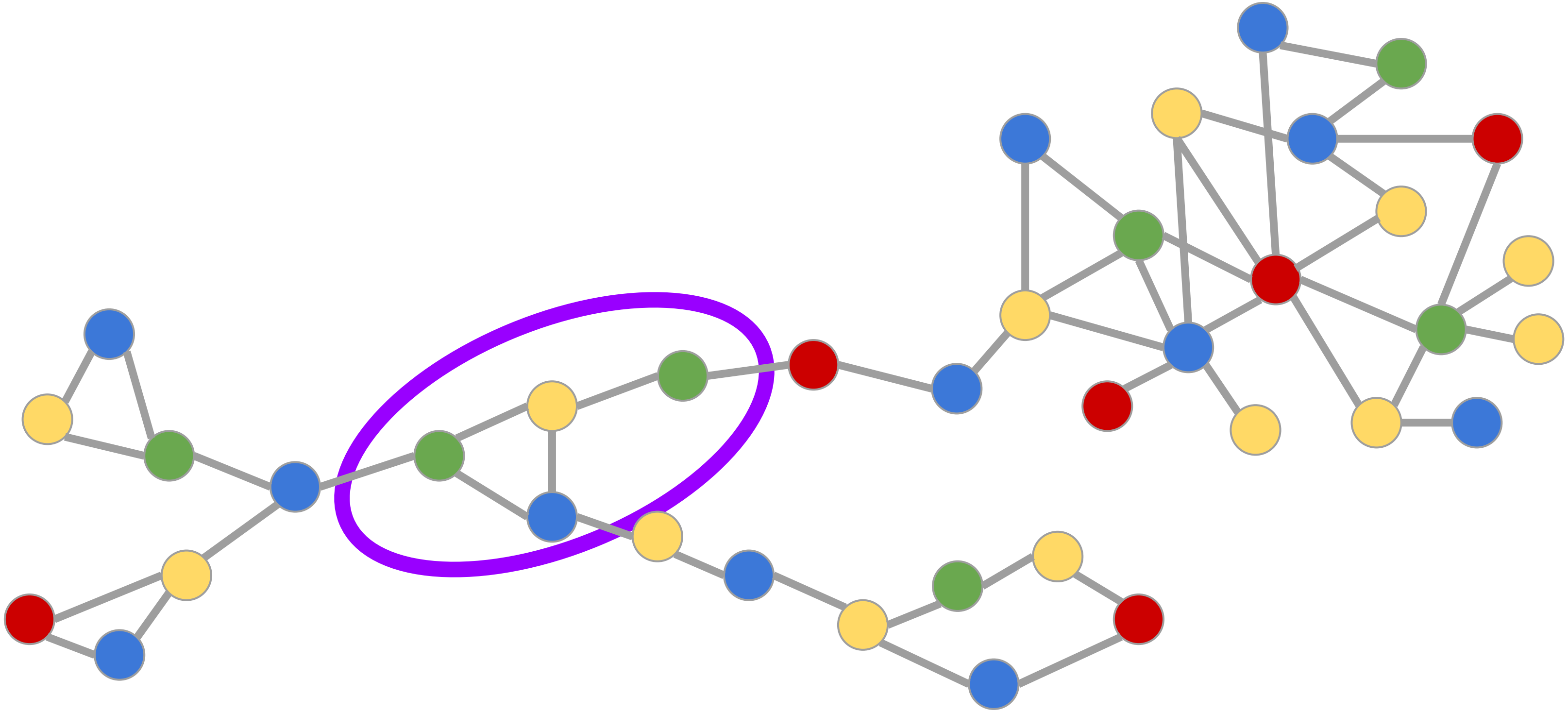
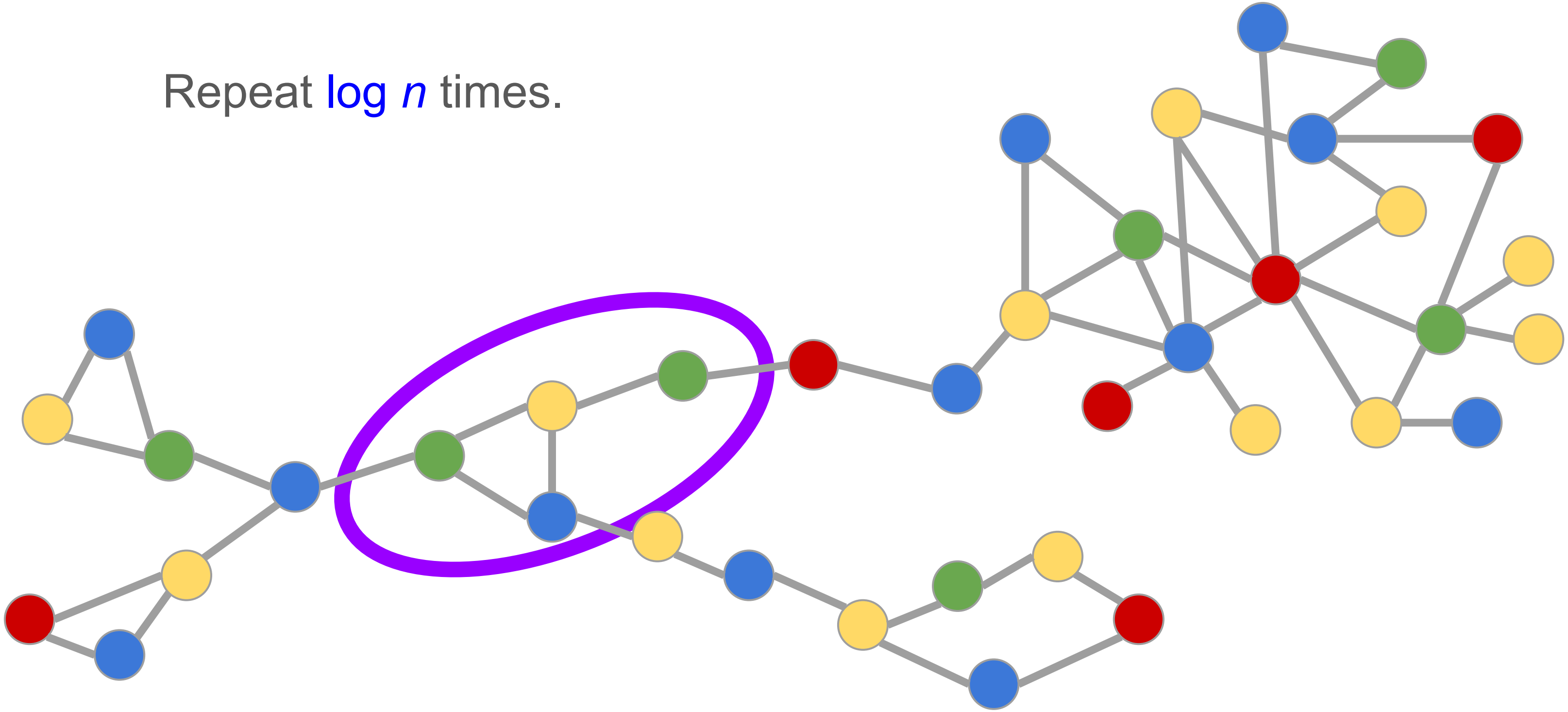


Principled approach

Repeat log *n* times.



Making things formal

Dictionary

Network decomposition with **C** colors and diameter **D**:

Coloring of the vertices with **C** colors, such that each component induced by a particular color has diameter at most **D**.

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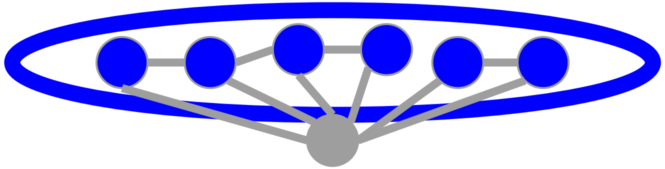

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Weak-diameter network decomposition

...any two vertices of a cluster are at most **D** hops apart in the original graph.

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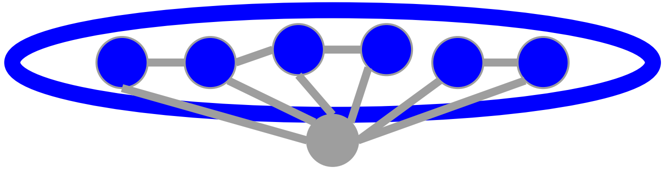

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Algorithm (that I show next) that finds independent clusters of diameter $O(\log n)$ while leaving at most ½ vertices unclustered.

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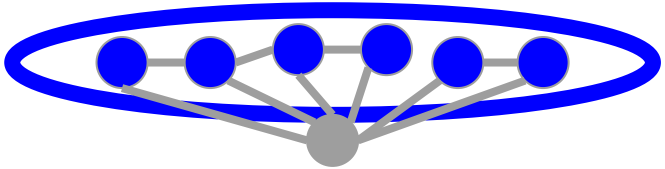

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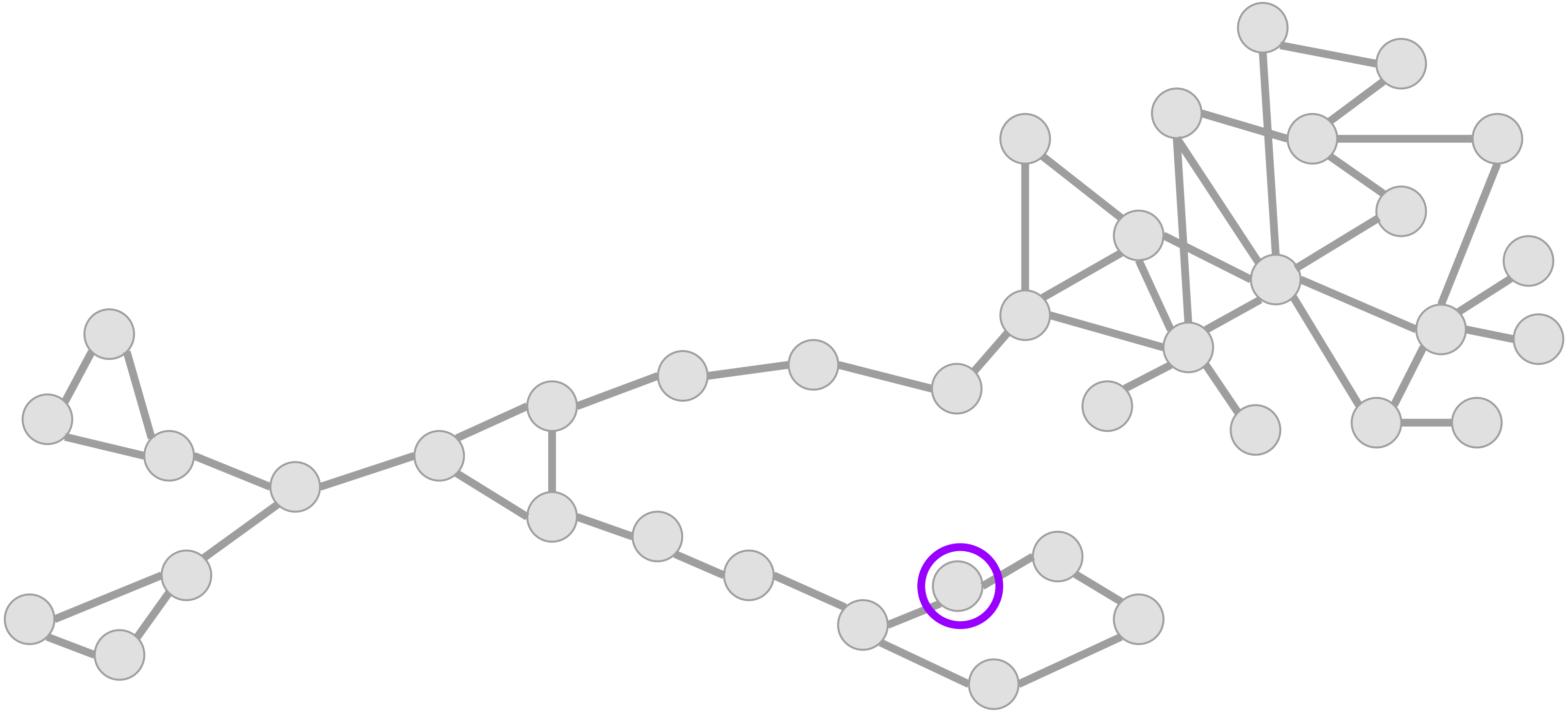
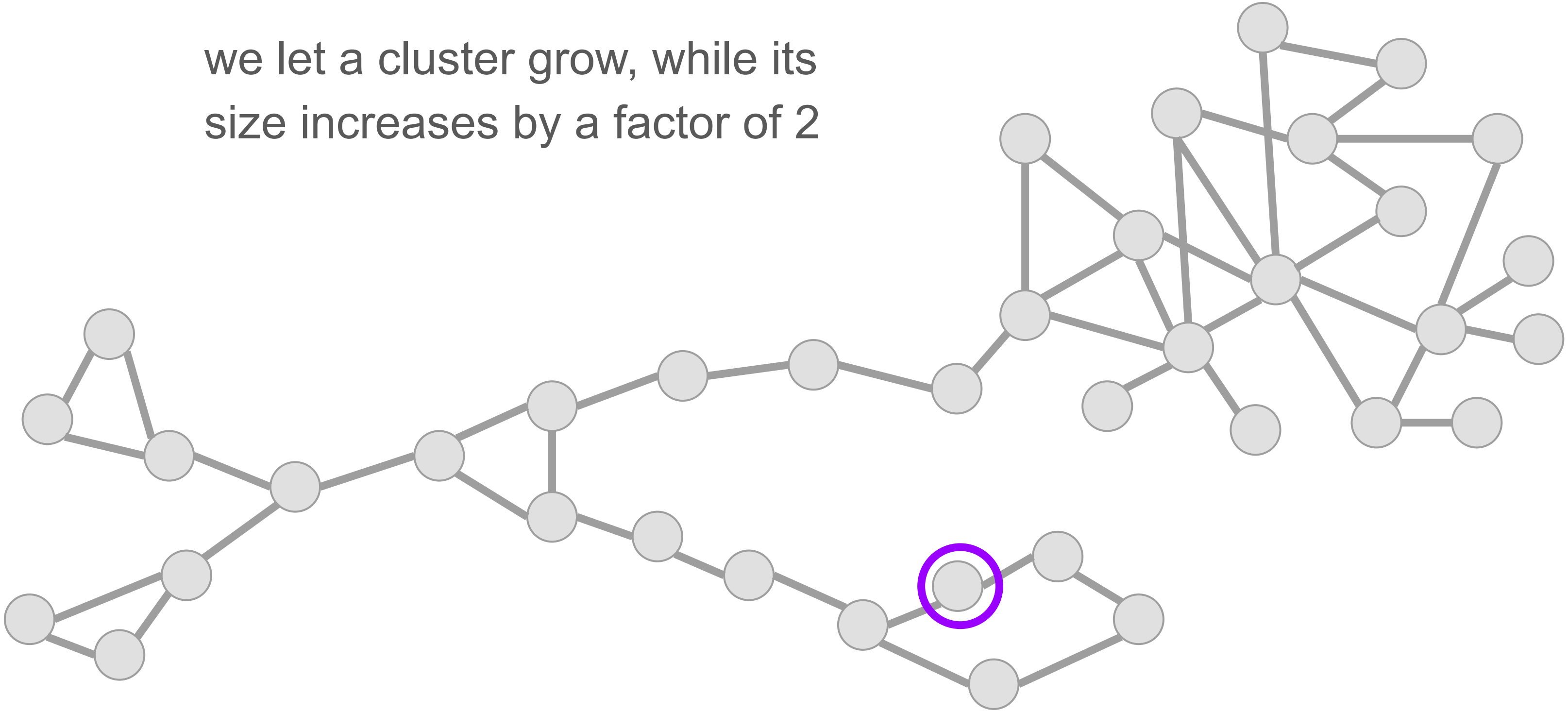
Algorithm (that I show next) that finds independent clusters of diameter $O(\log n)$ while leaving at most ½ vertices unclustered.
This implies existence of decomposition with $\mathbf{C}=O(\log n)$ and $\mathbf{D}=O(\log n)$.

Sequential ball carving



Sequential ball carving

we let a cluster grow, while its size increases by a factor of 2



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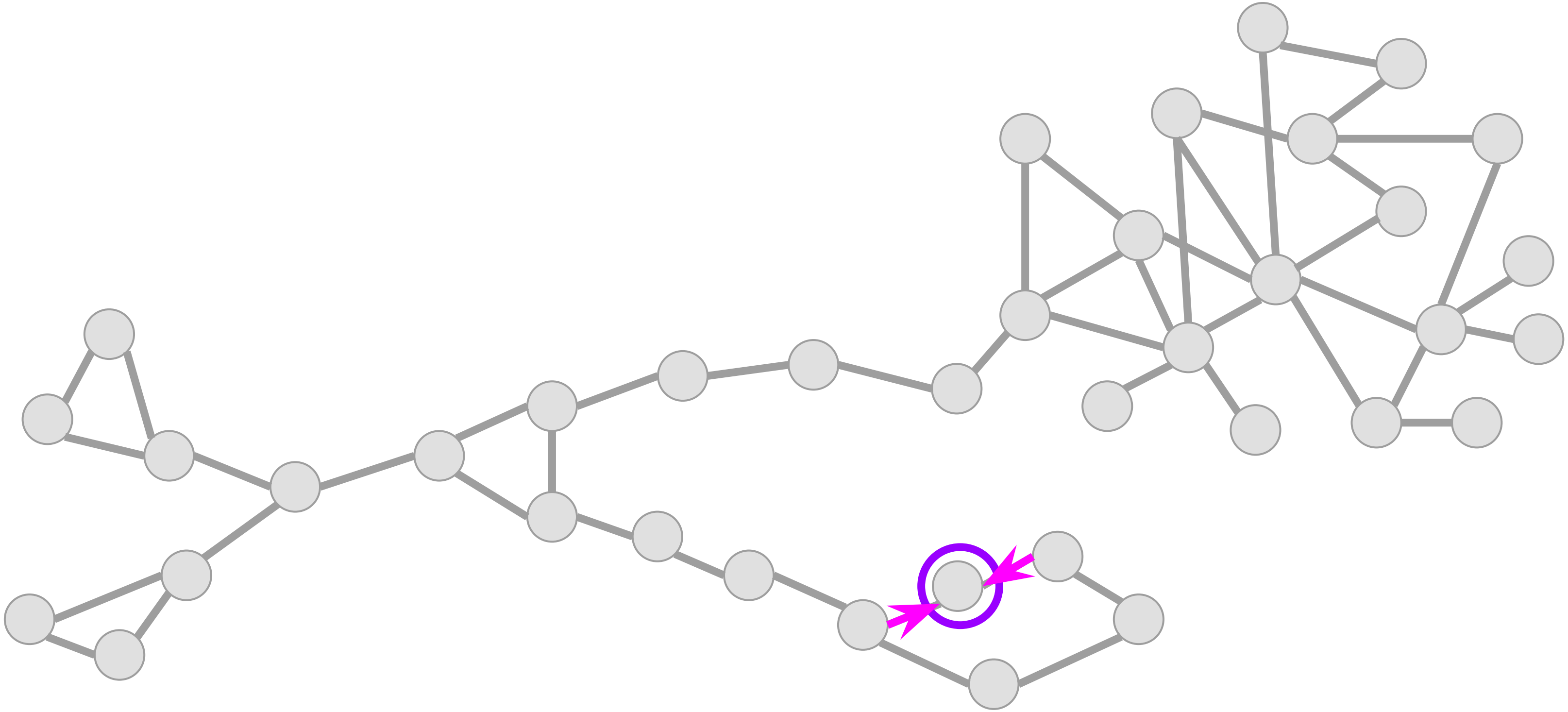

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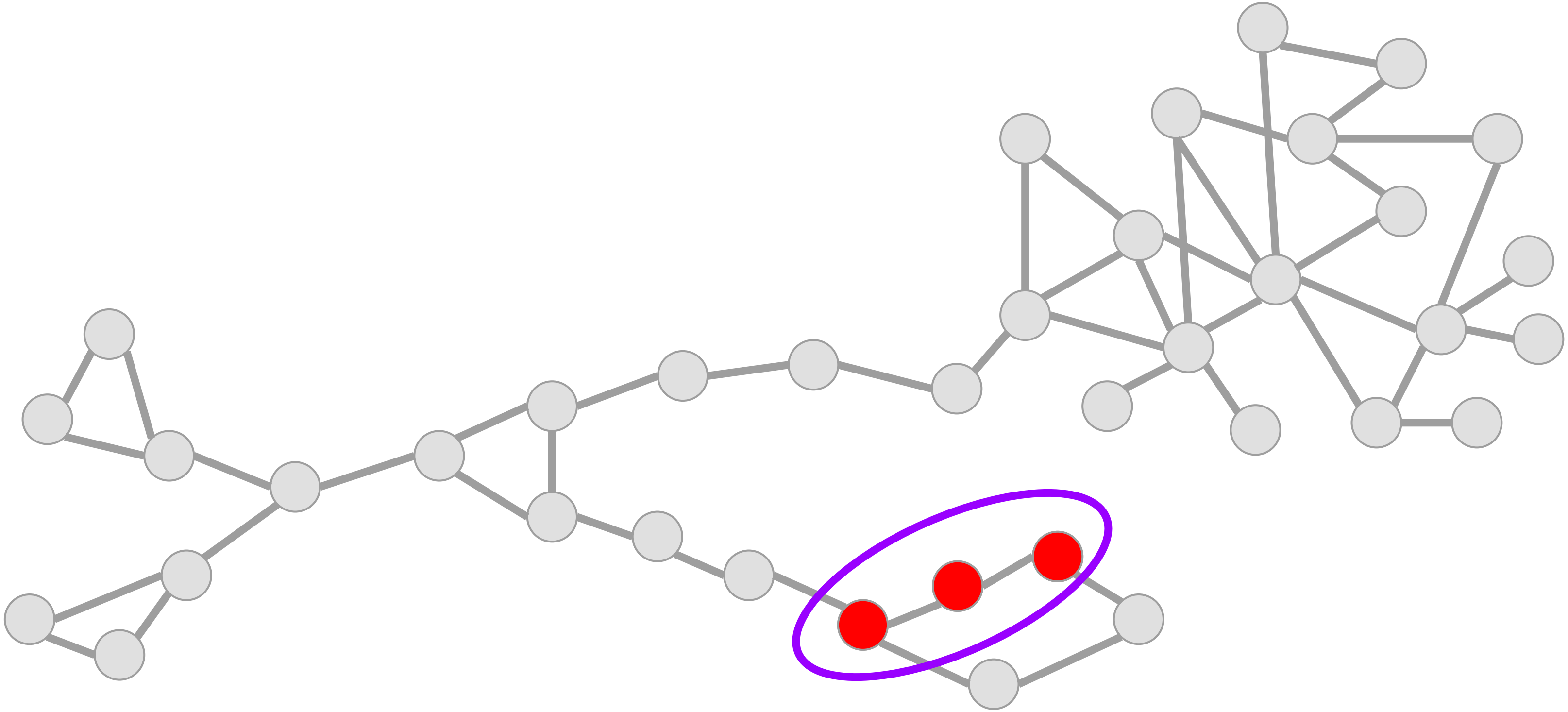

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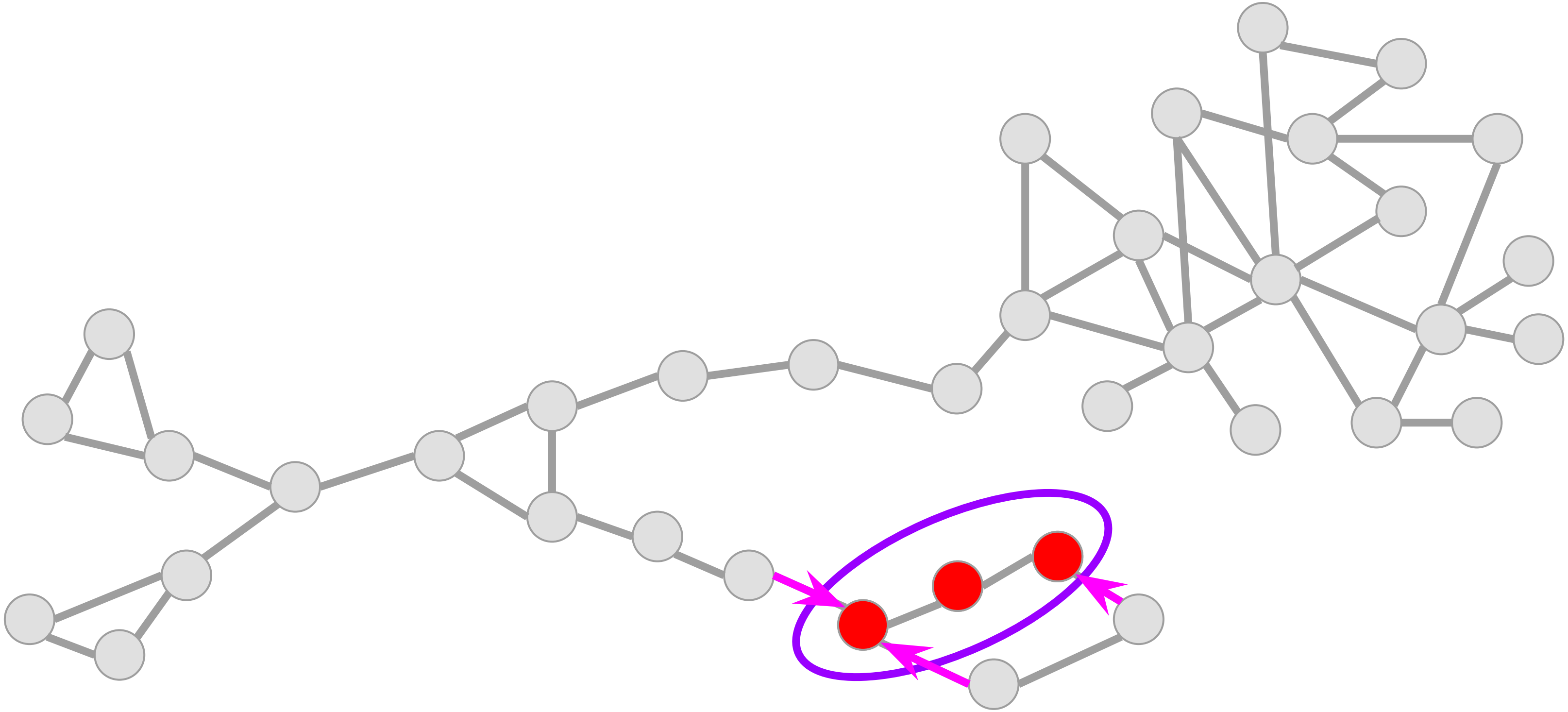

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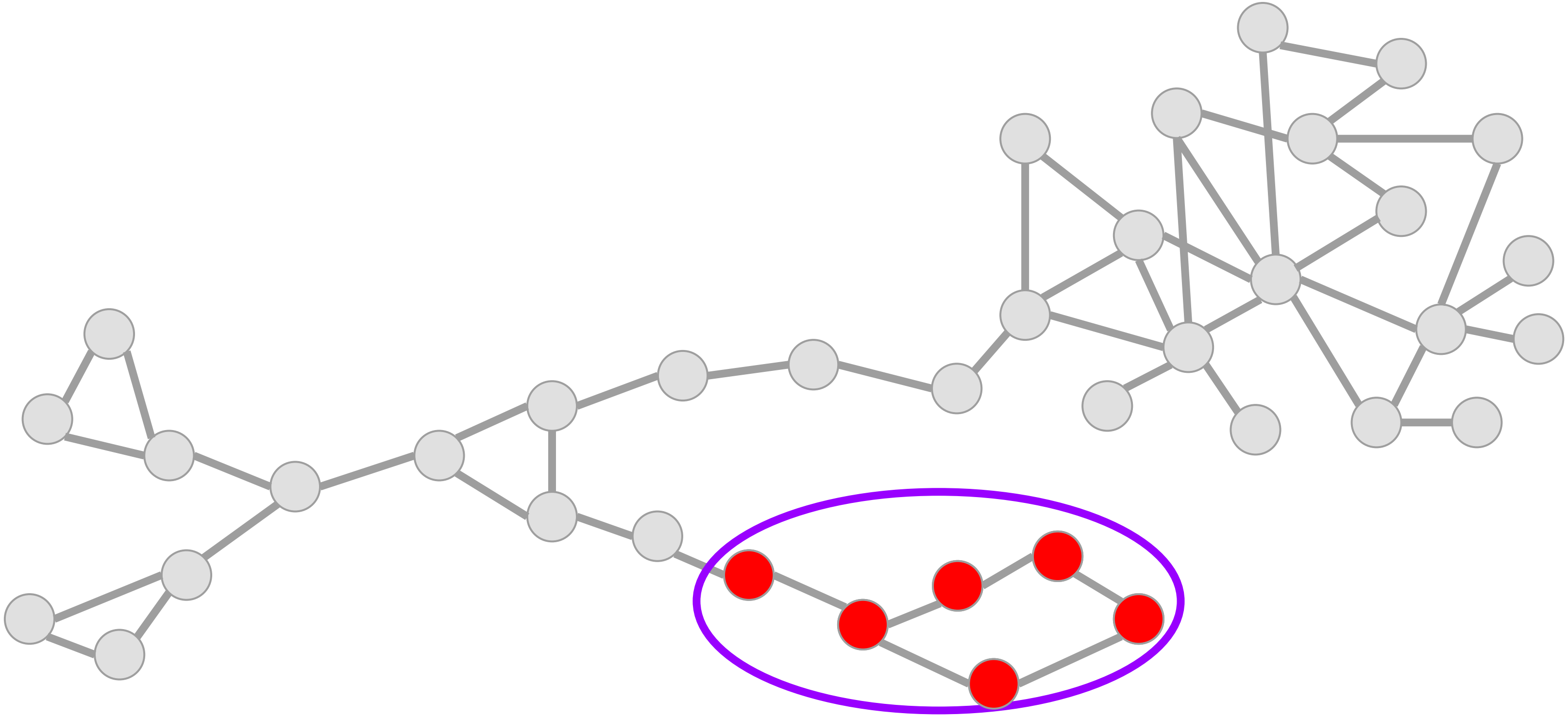

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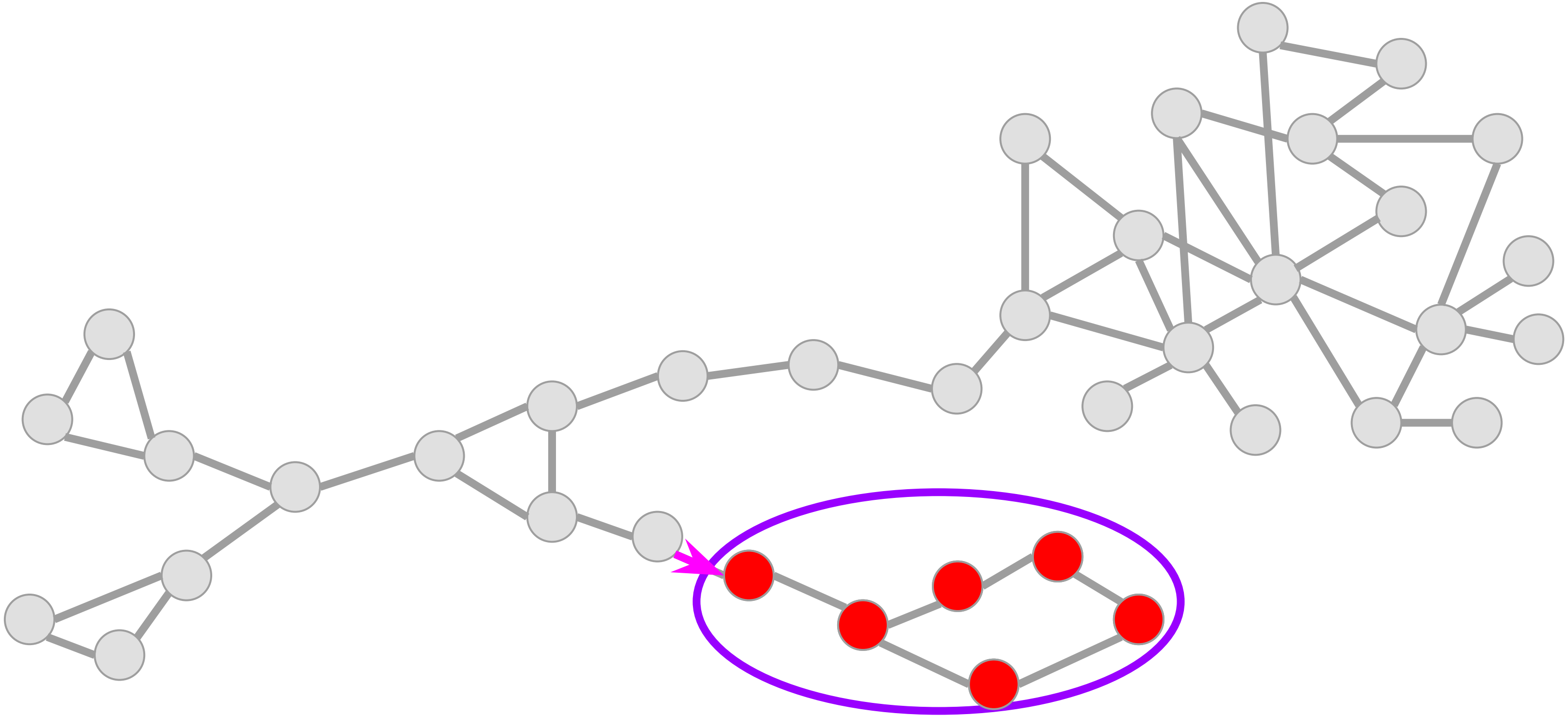

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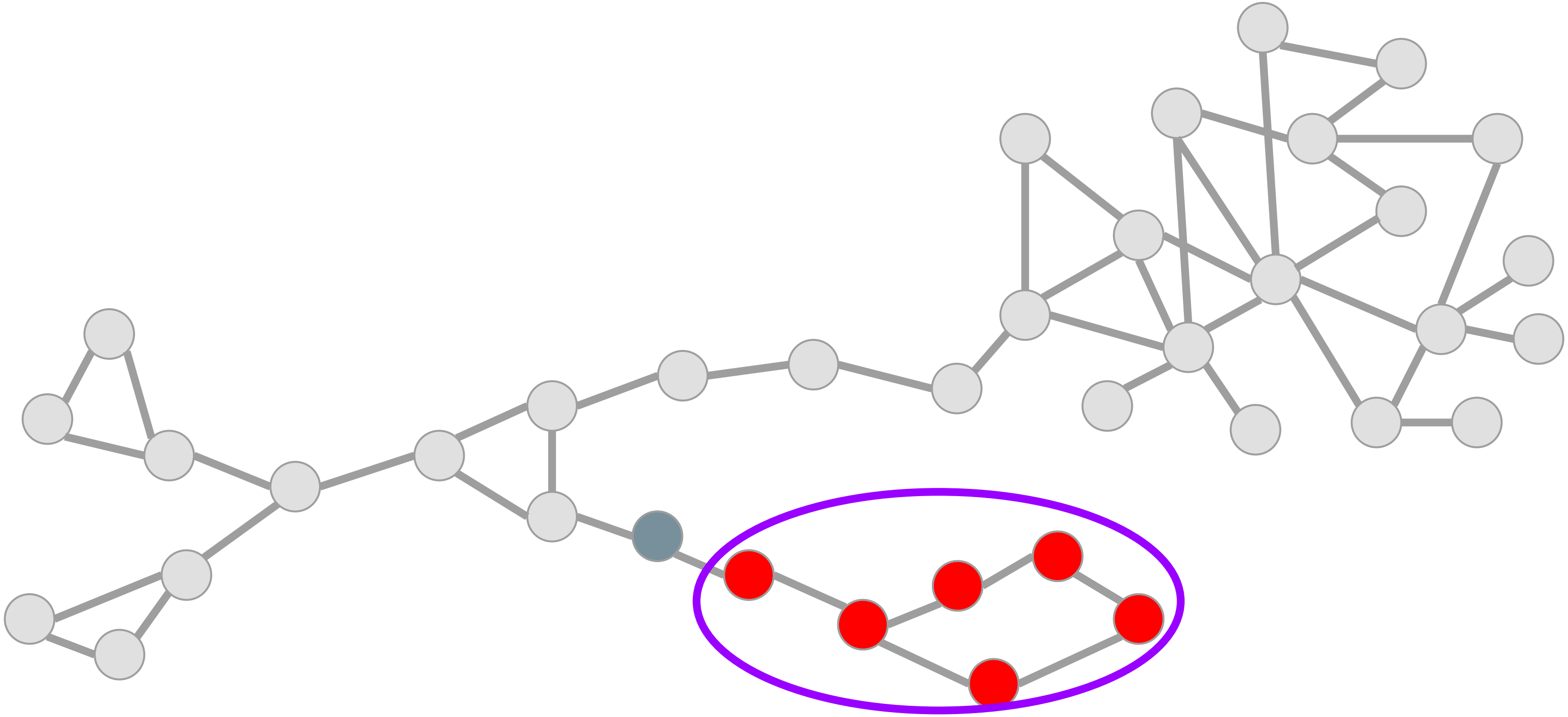

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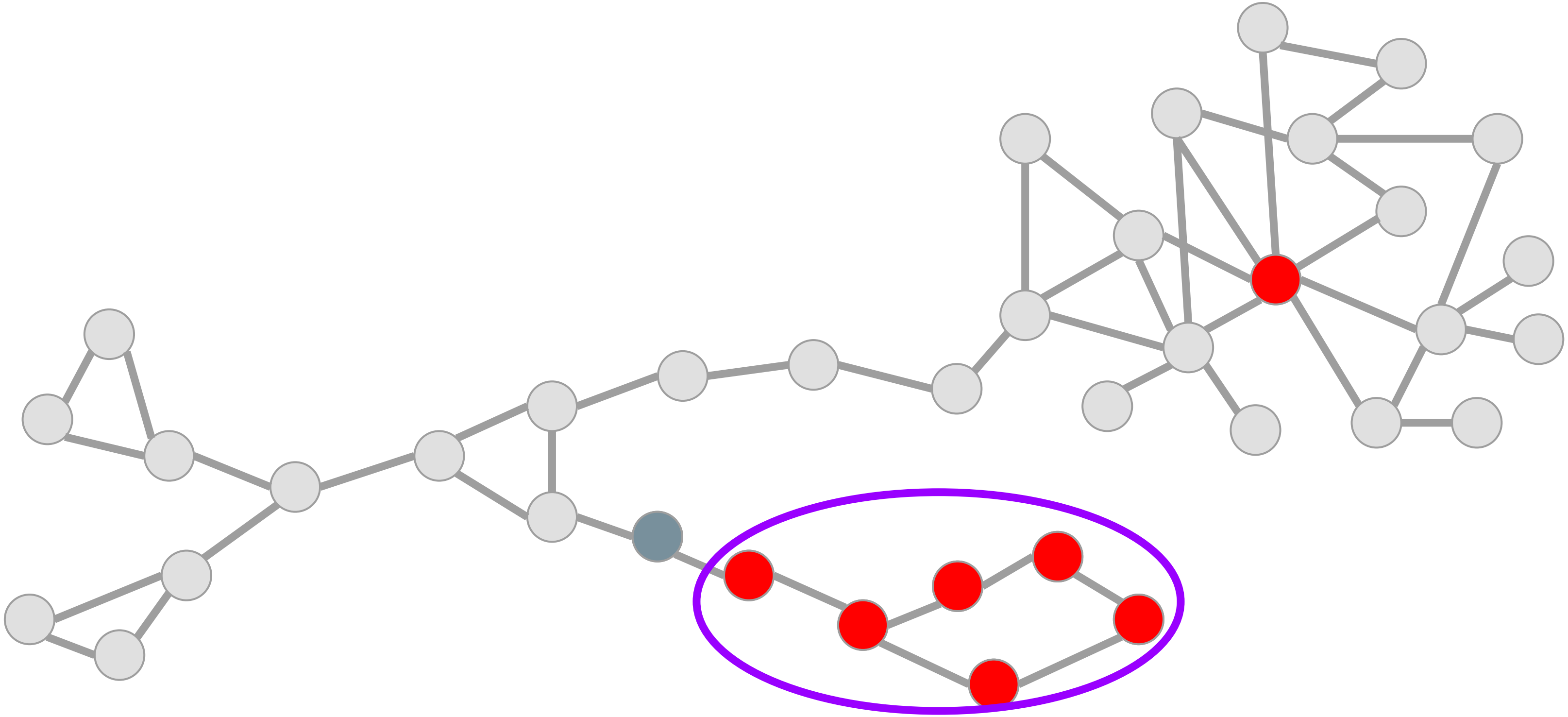

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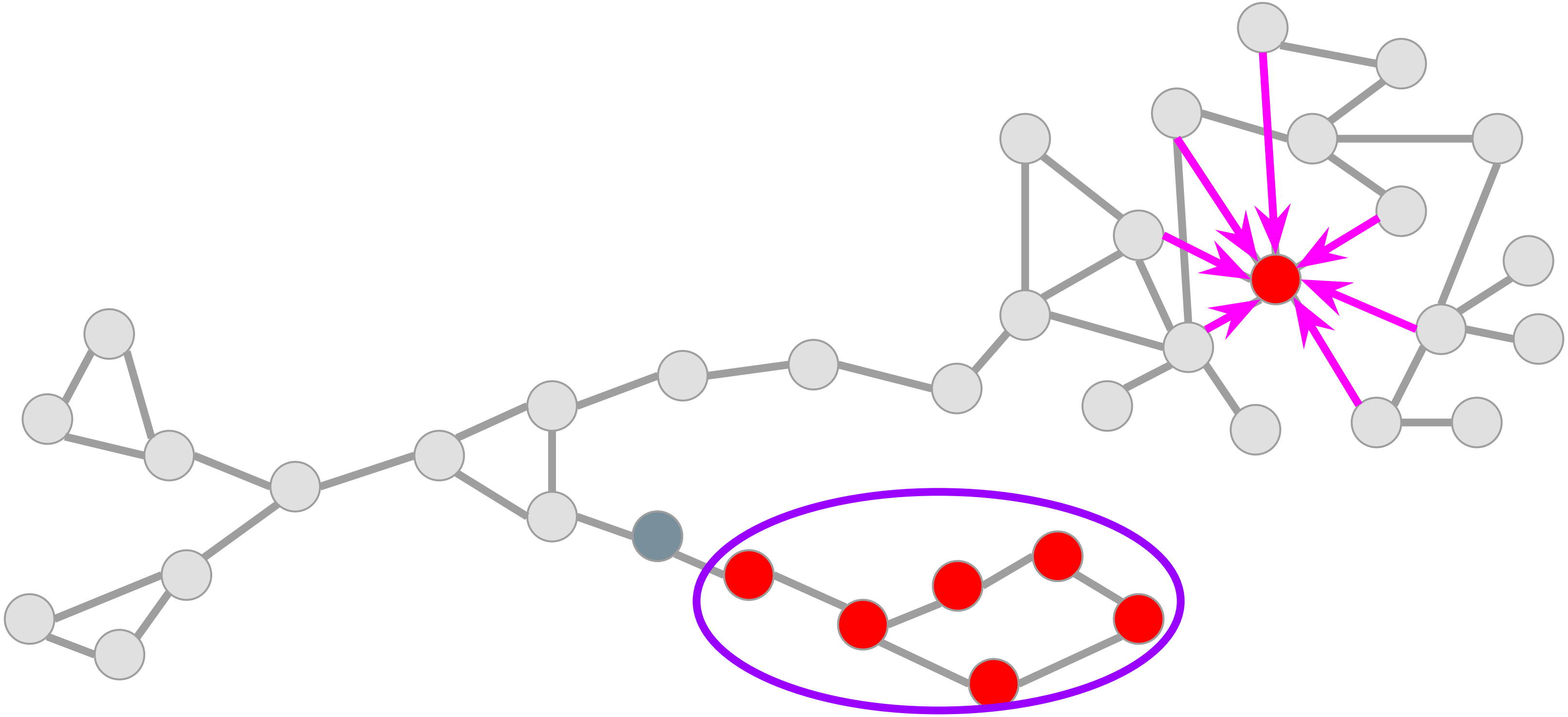

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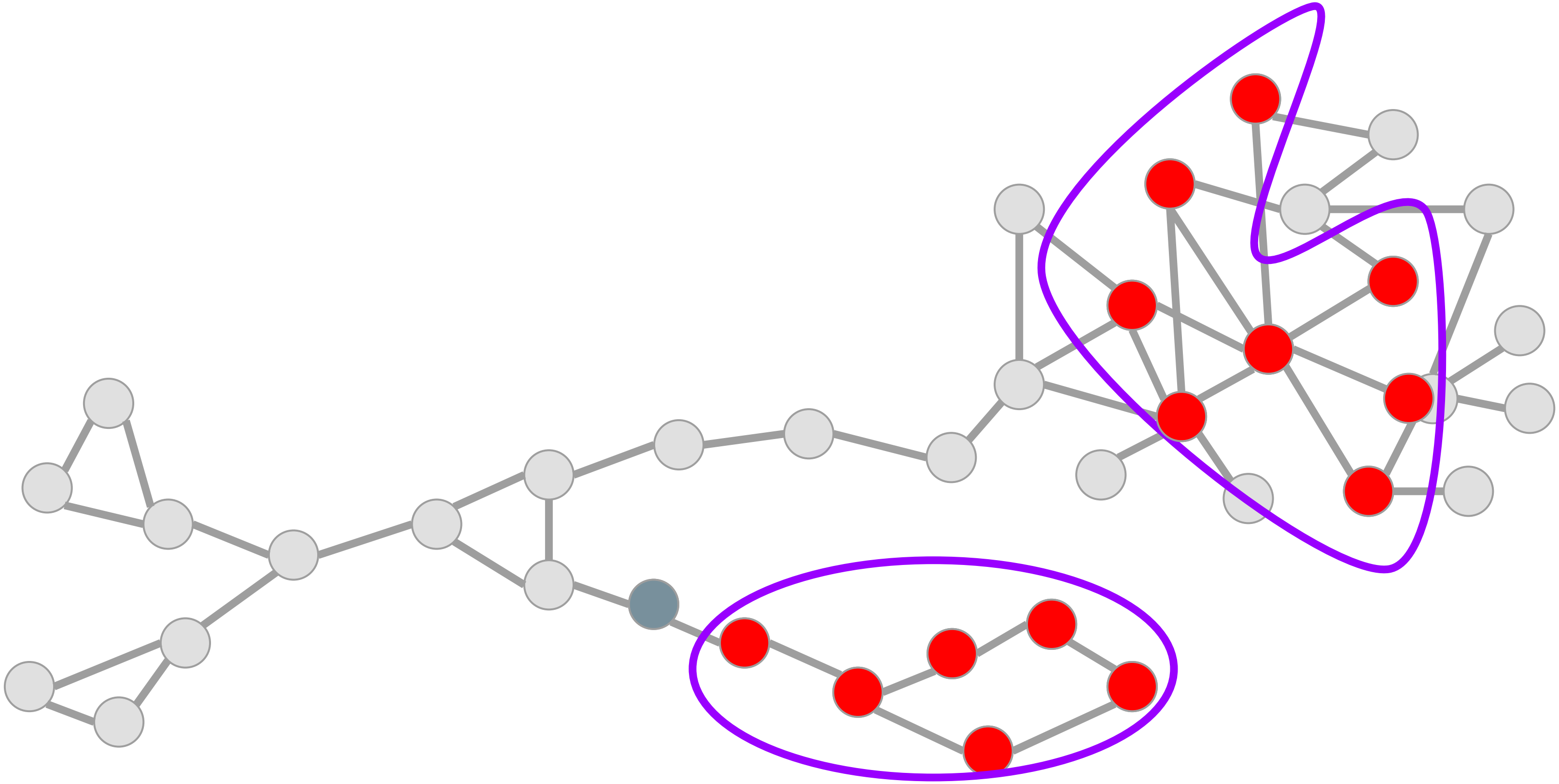

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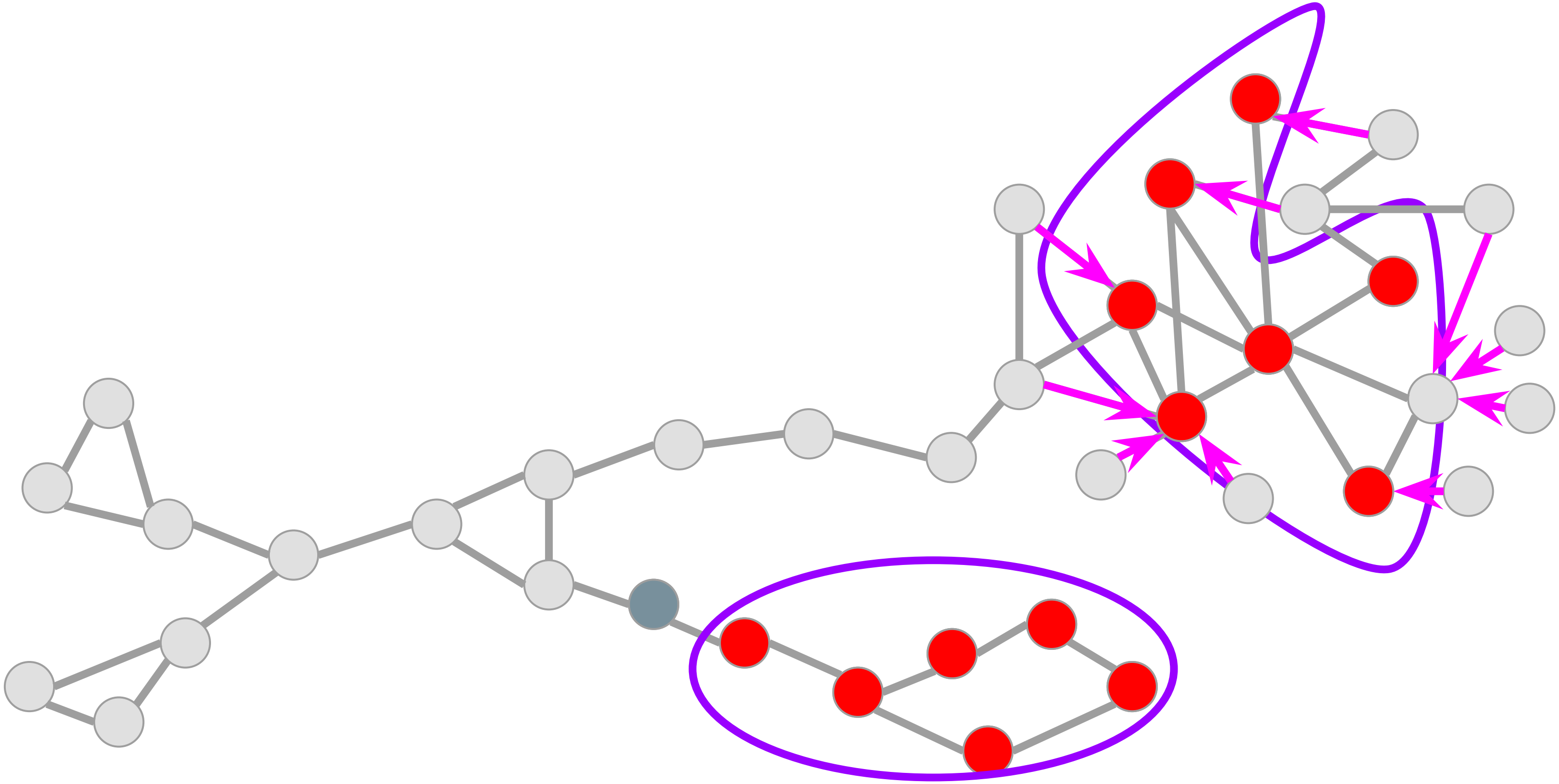

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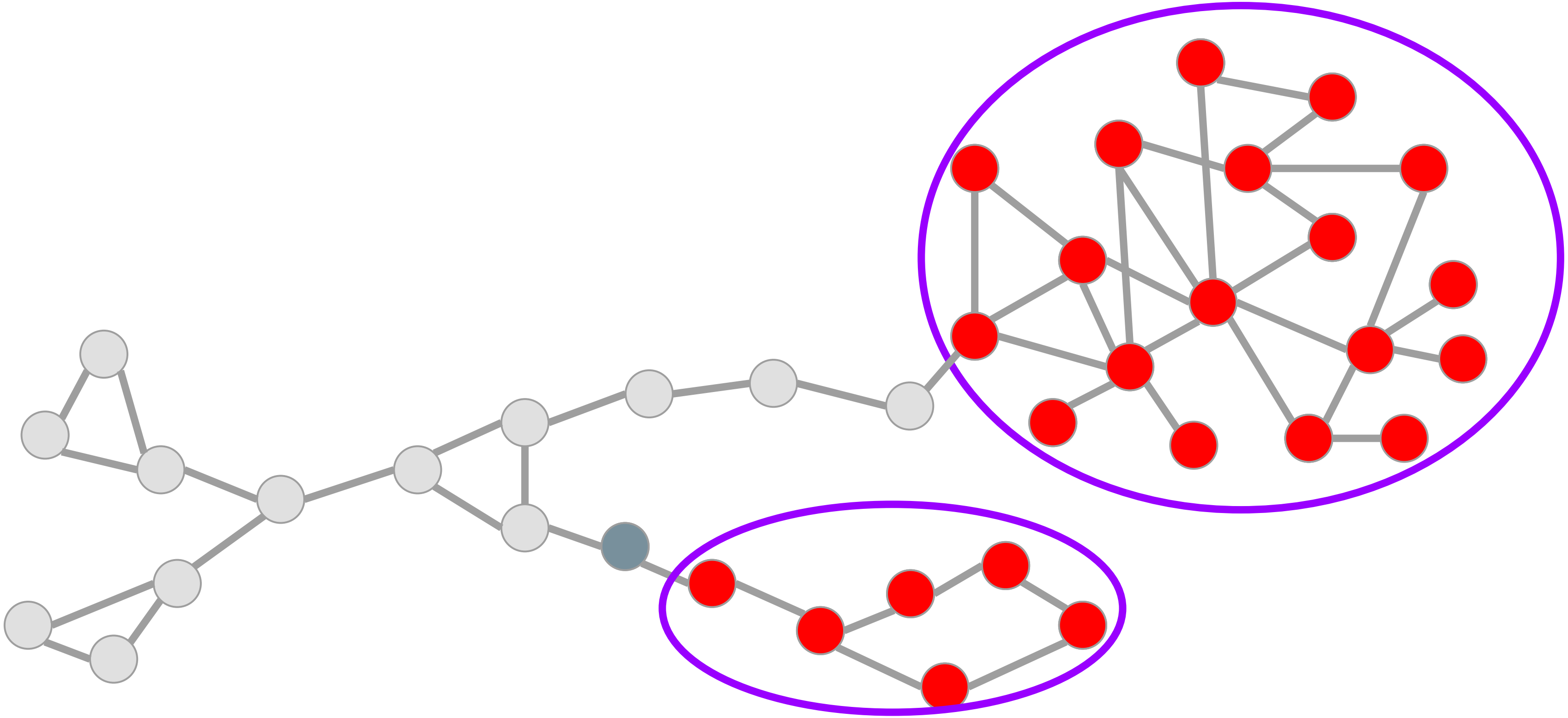

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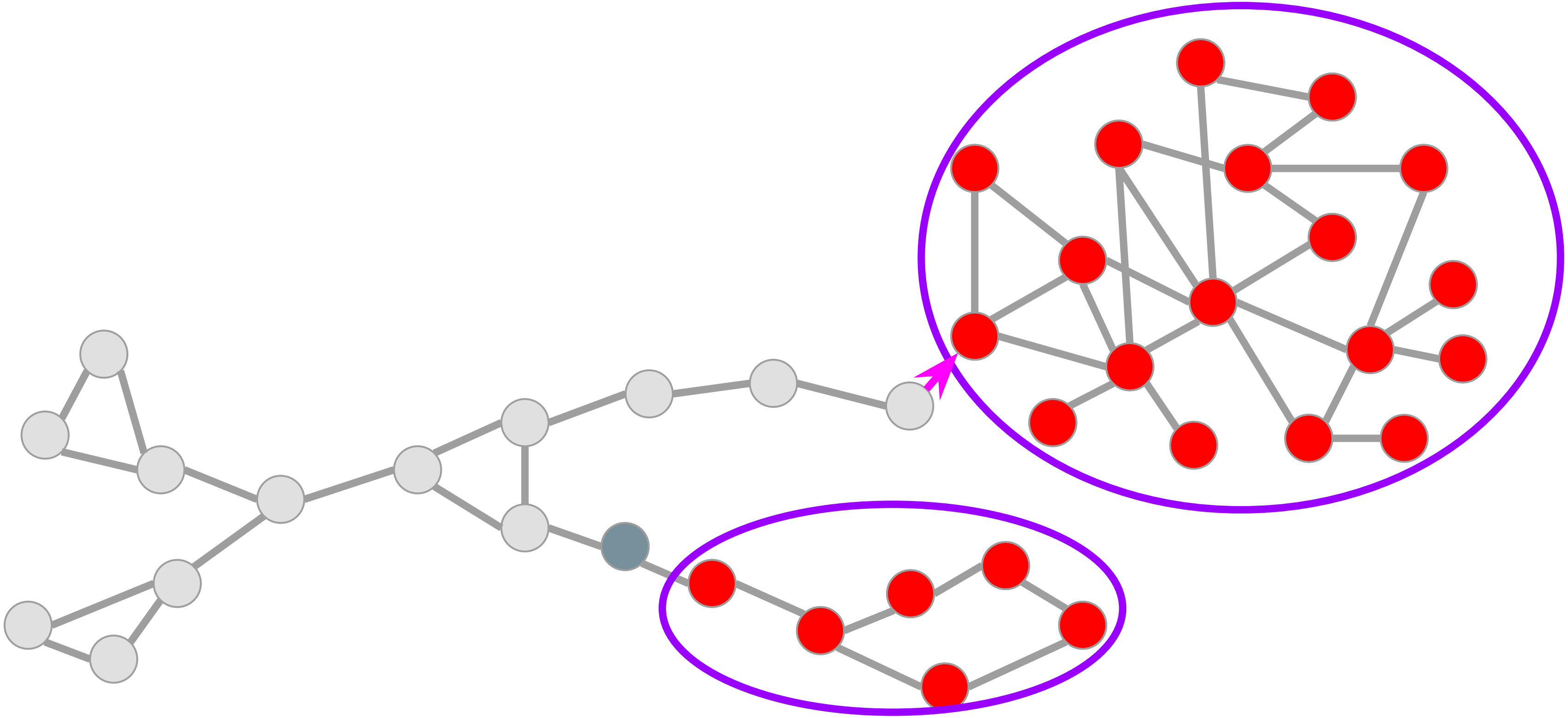

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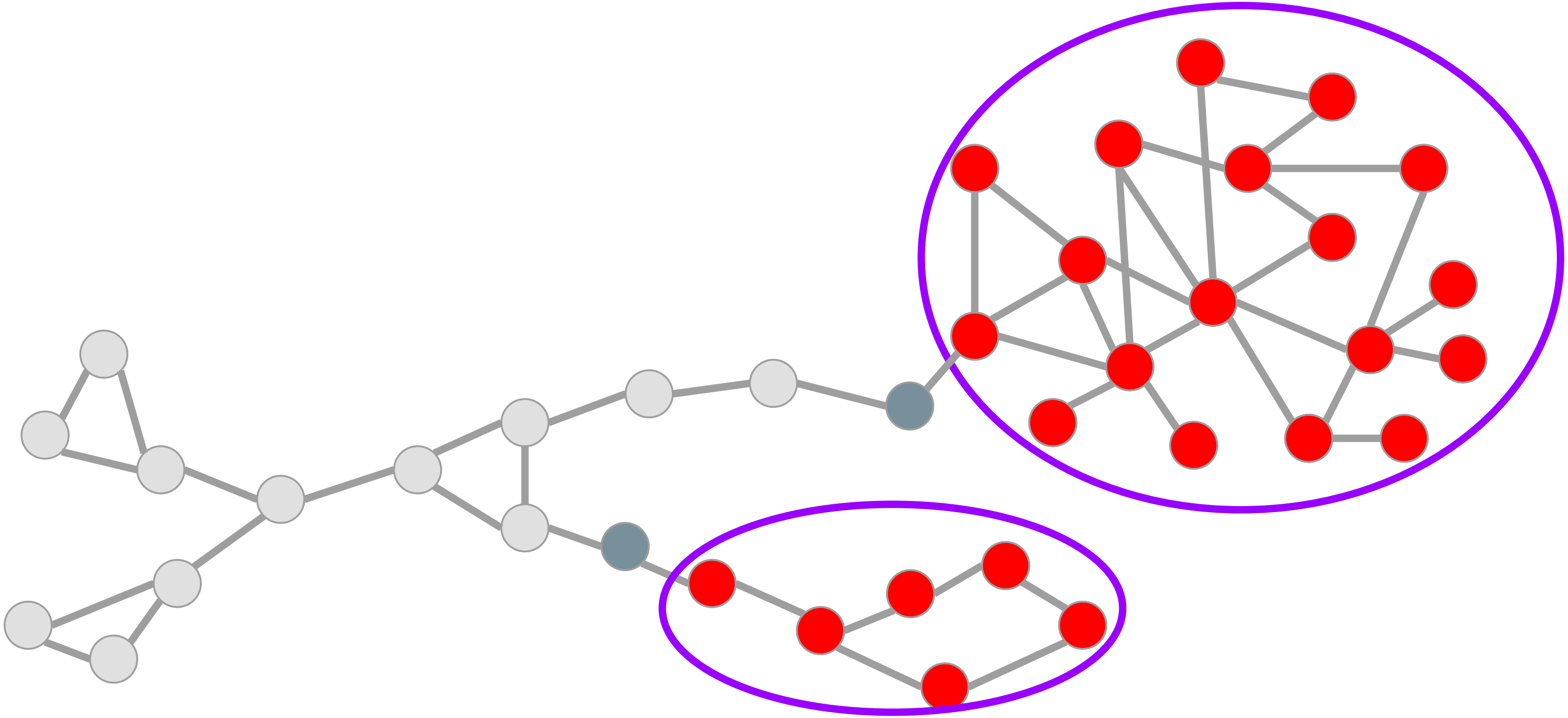

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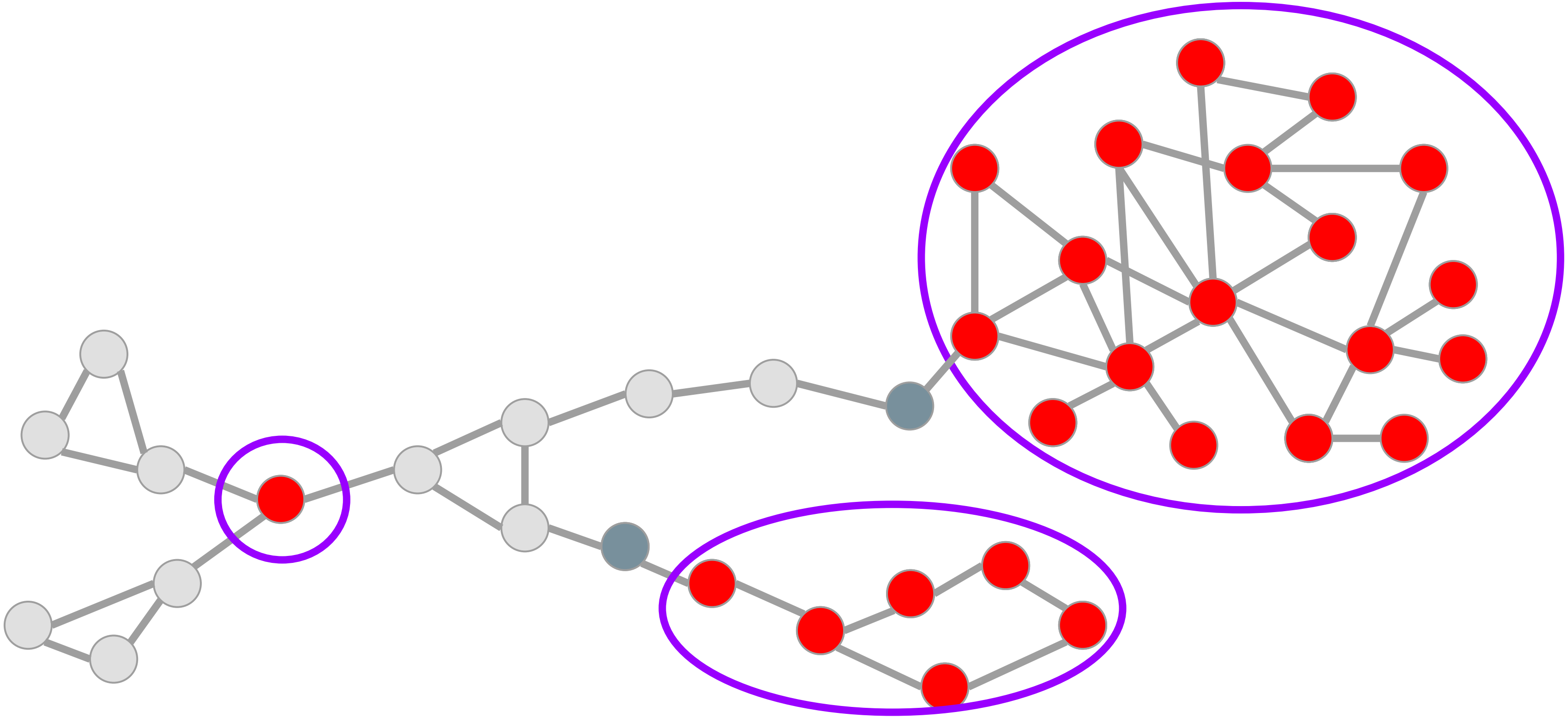

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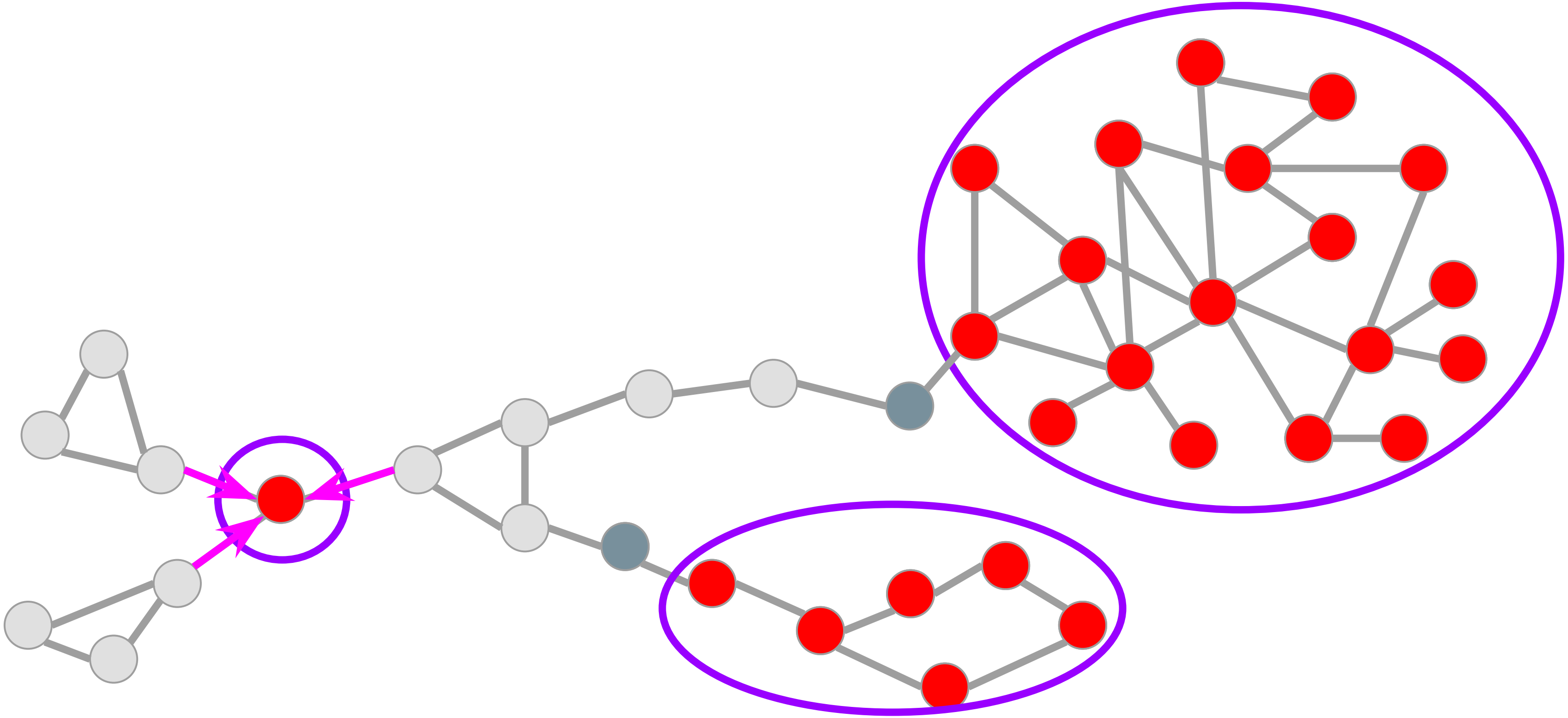

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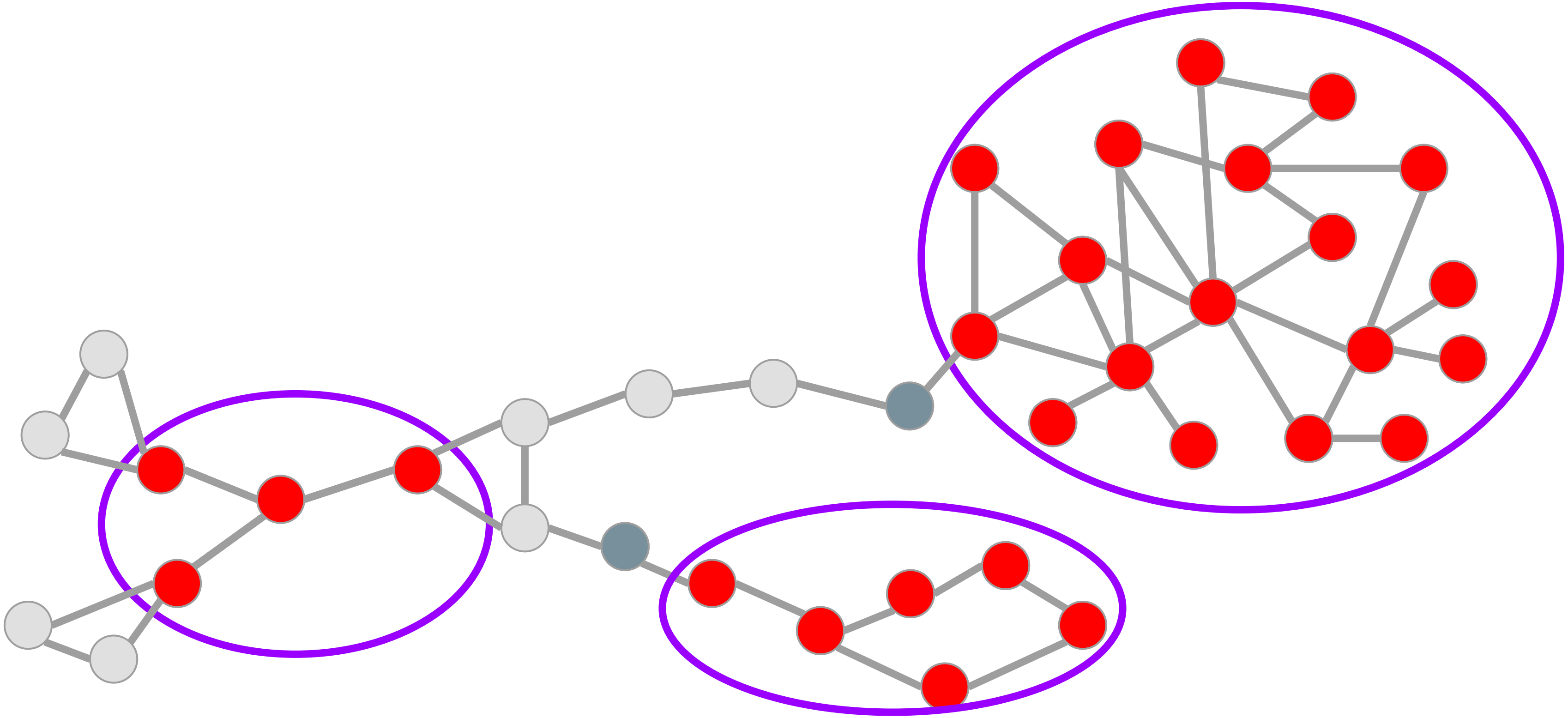

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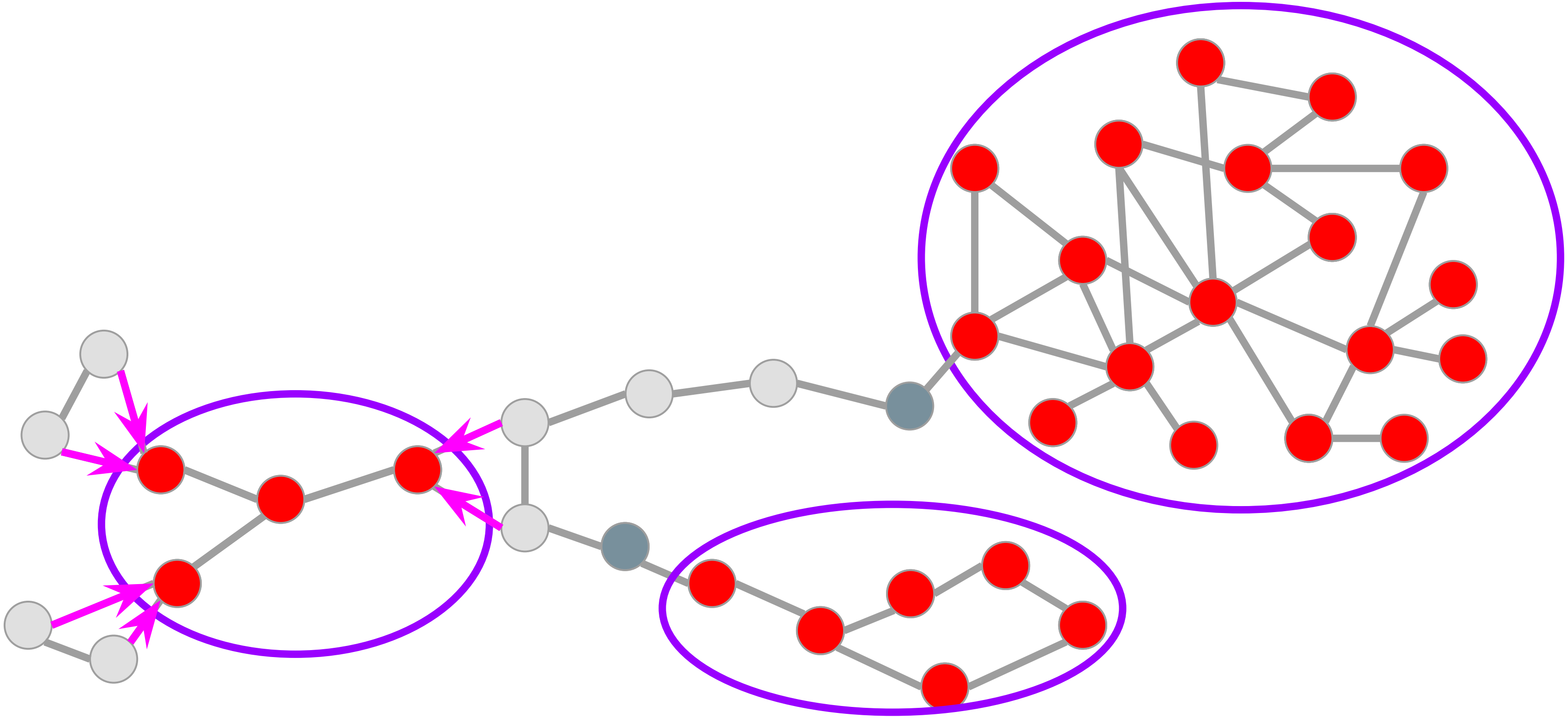

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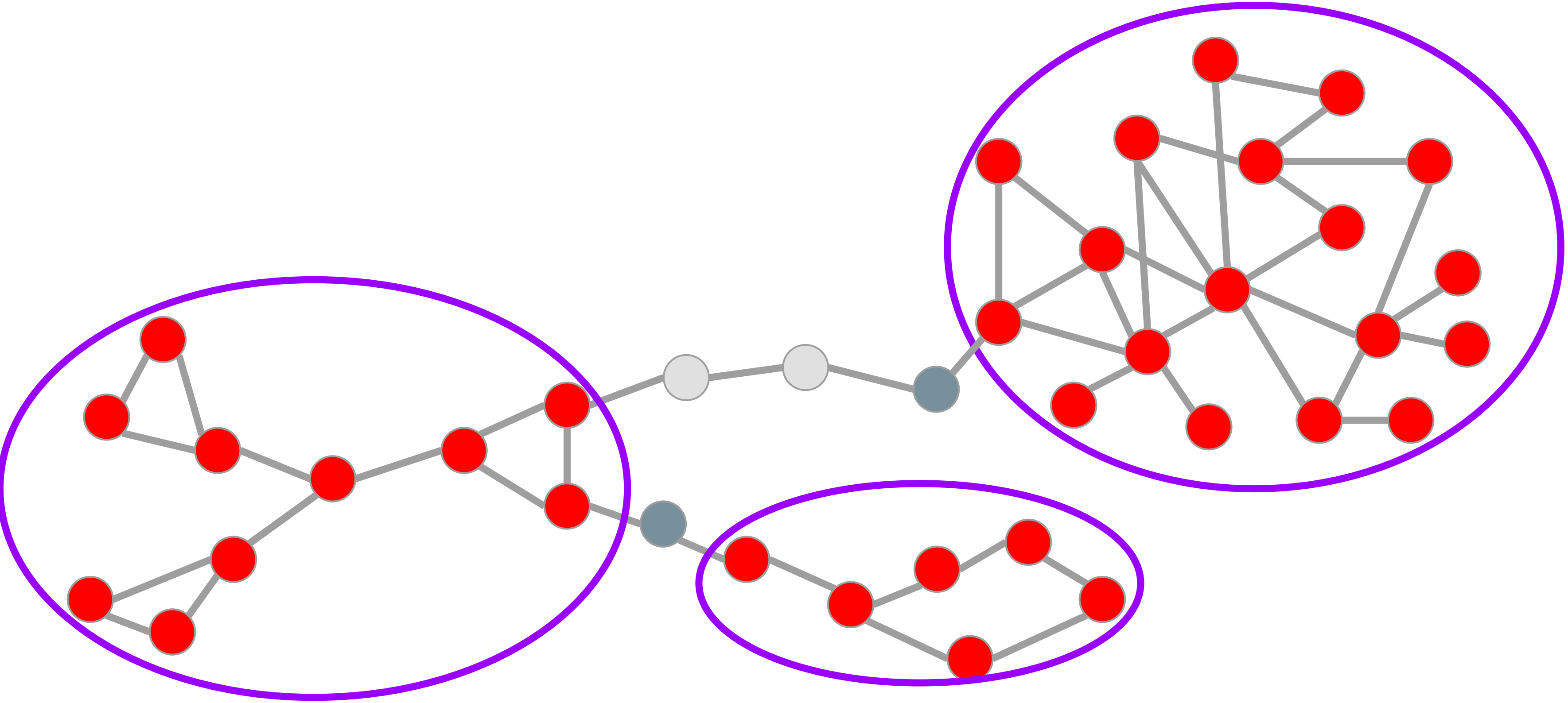

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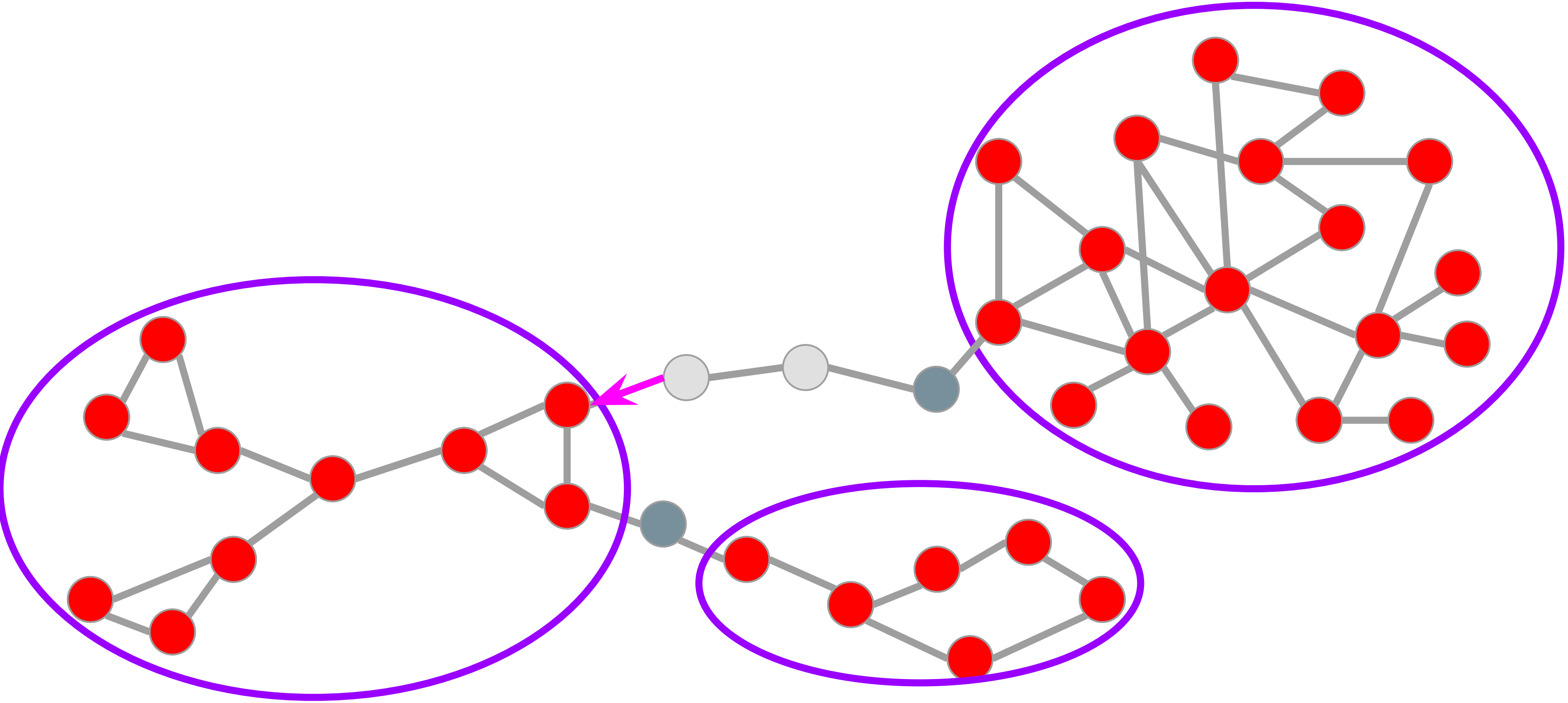

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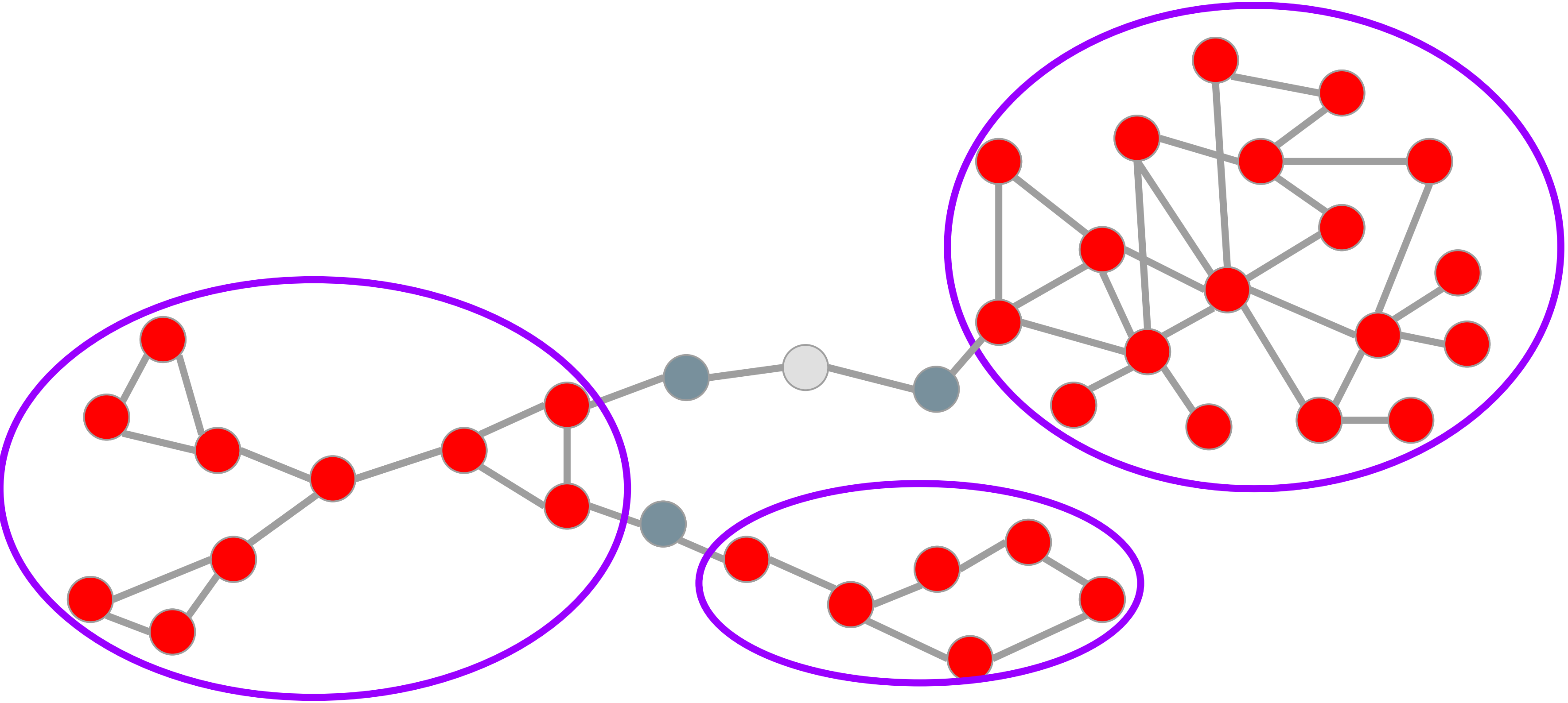
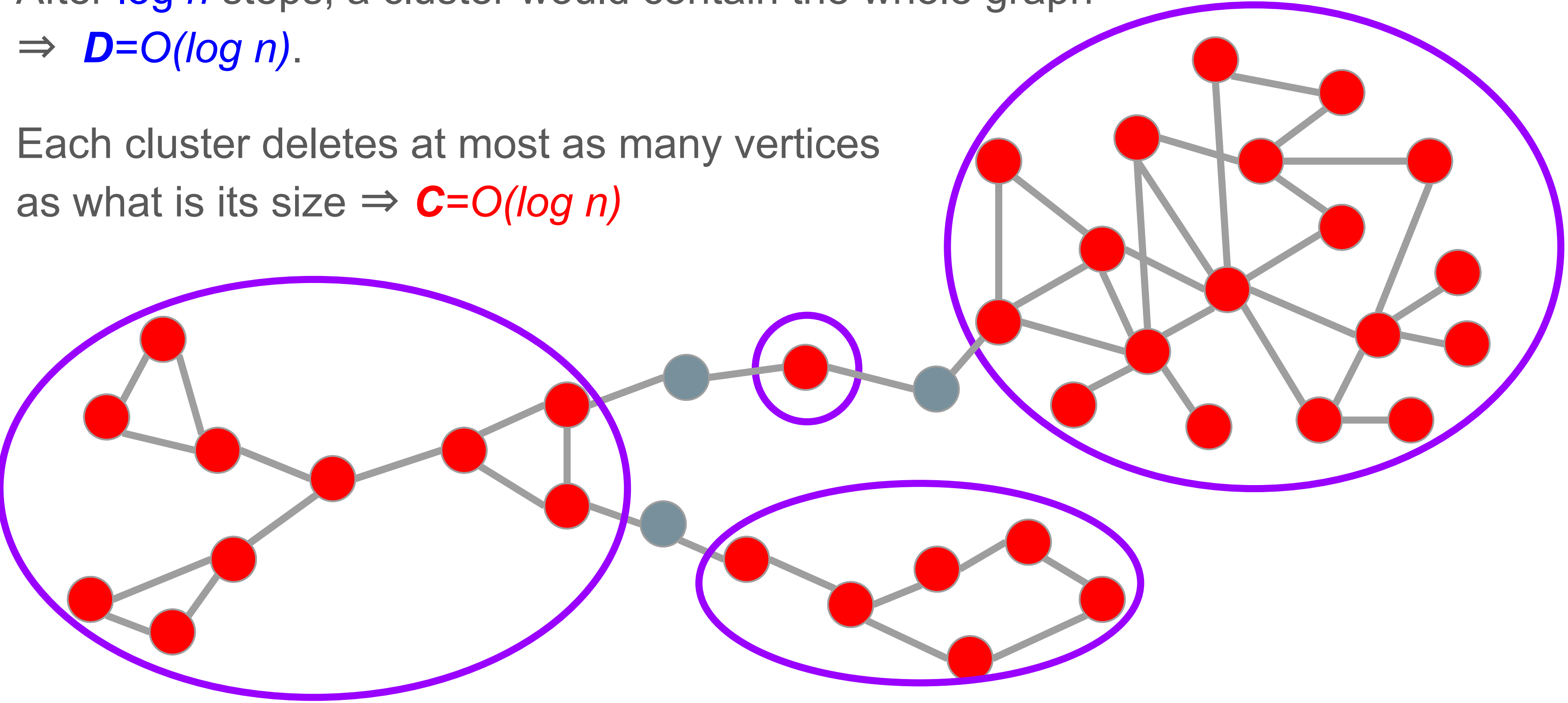


Sequential ball carving

After log n steps, a cluster would contain the whole graph
$\Rightarrow$ $\boldsymbol{D}=O(\log n)$.

Each cluster deletes at most as many vertices as what is its size $\Rightarrow$ $\boldsymbol{C}=O(\log n)$



In general

- This works generally for *Δ+1* coloring, maximal independent set, maximal matching, …
- If the problem has locality k, work in G^k.
- The right level of generality: sequential greedy algorithms [Ghaffari, Kuhn, Maus STOC'17]
- Think of the sequential algorithm for *Δ+1* coloring, maximal independent set, or even *ball carving*!

SLOCAL - sequential variant of the **LOCAL** model

Iterate over nodes in adversarial order.

Decide their label based only on their r-neighbourhood.

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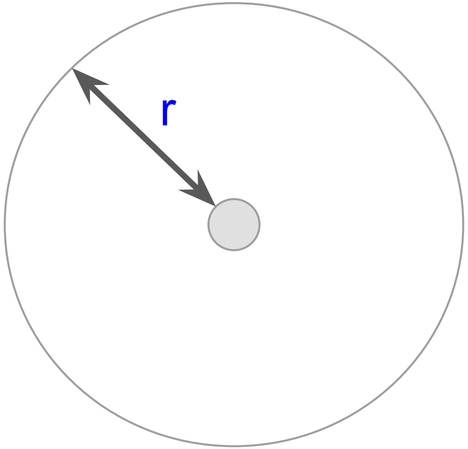

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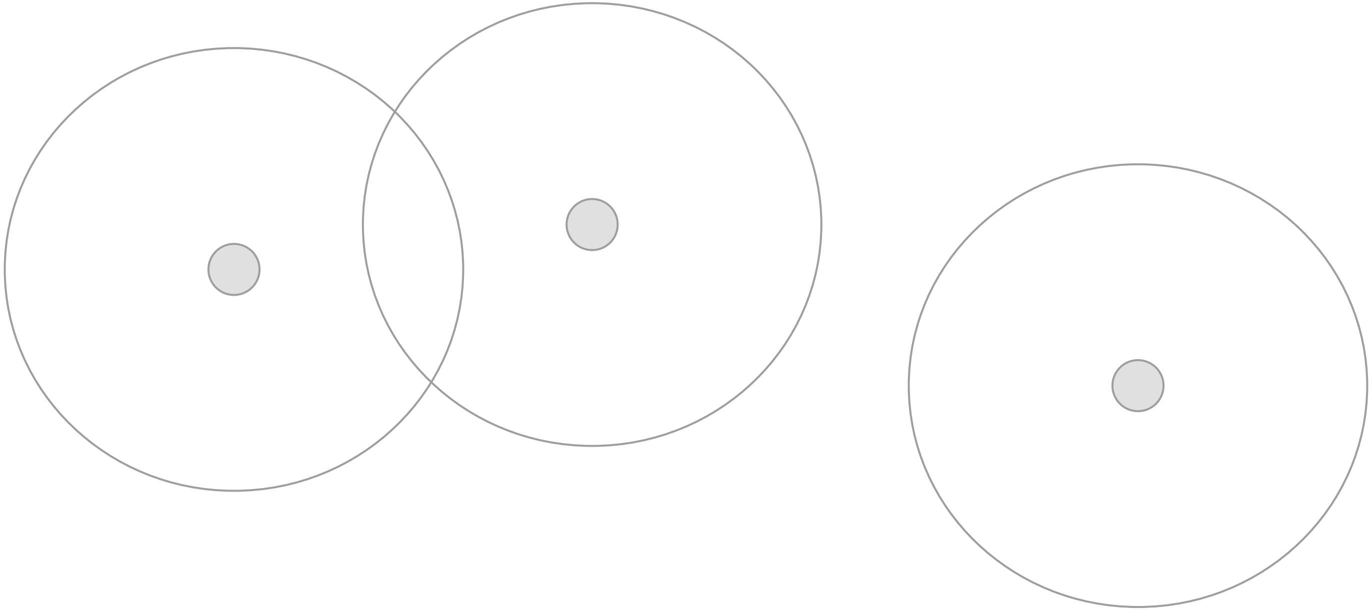

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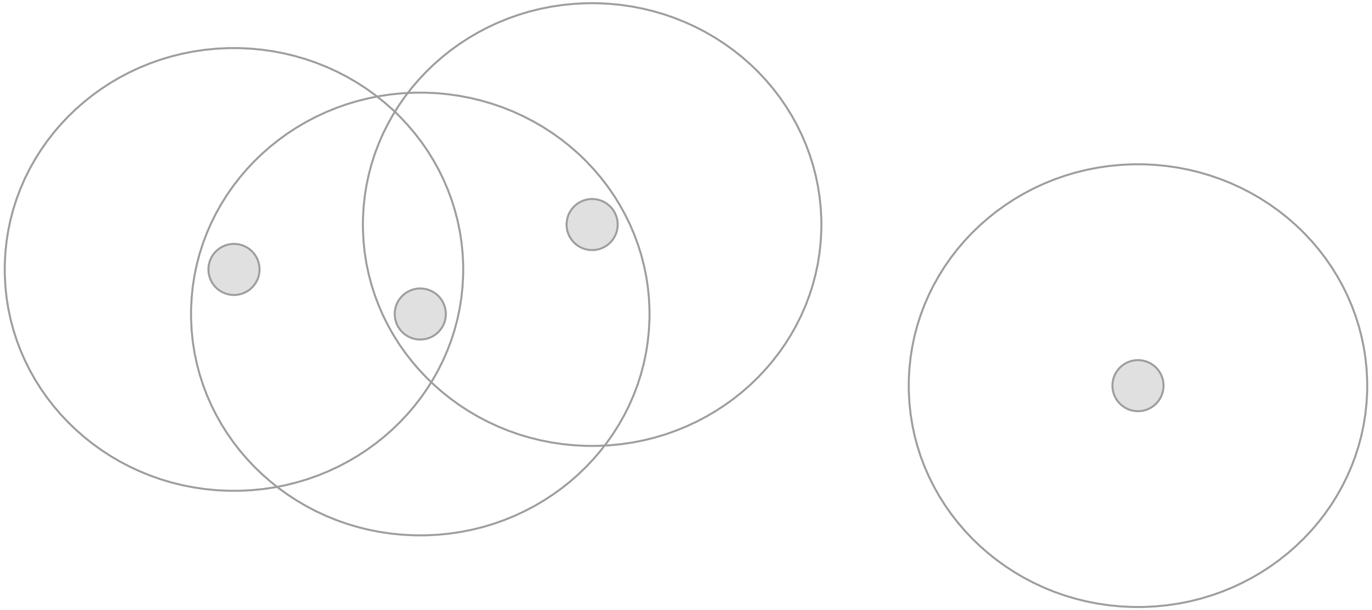

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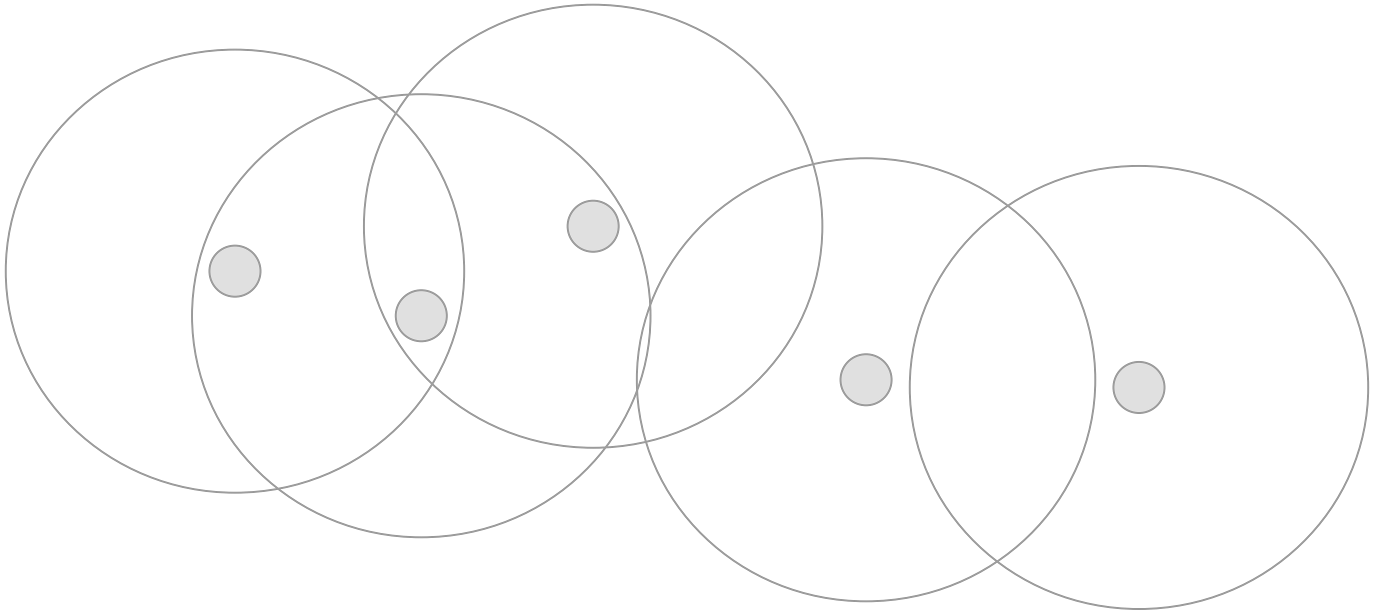

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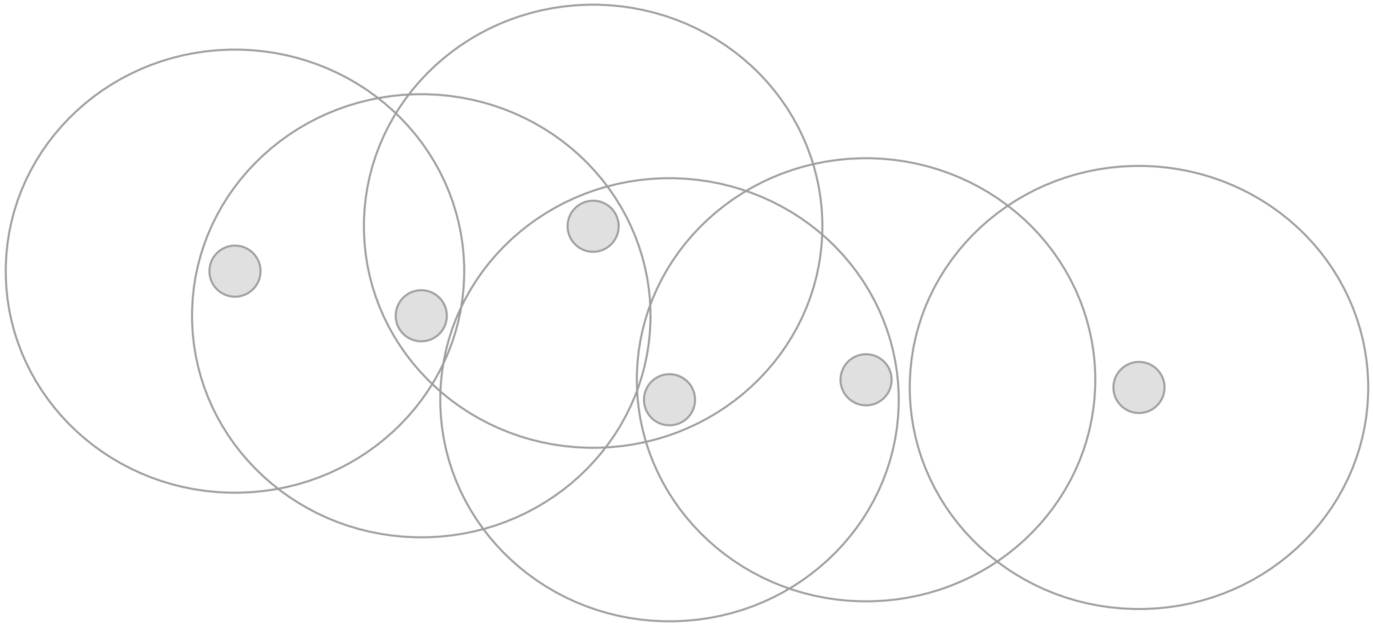

P-SLOCAL

"deterministic sequential"

deterministic network decomposition [R., Ghaffari 19+]

direct

P-LOCAL

"deterministic distributed"

P-SLOCAL

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P-LOCAL

"deterministic distributed"

Corollary [R., Ghaffari 19+] There is an efficient deterministic algorithm for $\Delta+1$ coloring, maximal independent set, ...

P-SLOCAL

“deterministic sequential”

deterministic network decomposition [R., Ghaffari 19+]

direct

P-LOCAL

“deterministic distributed”

P-RSLOCAL

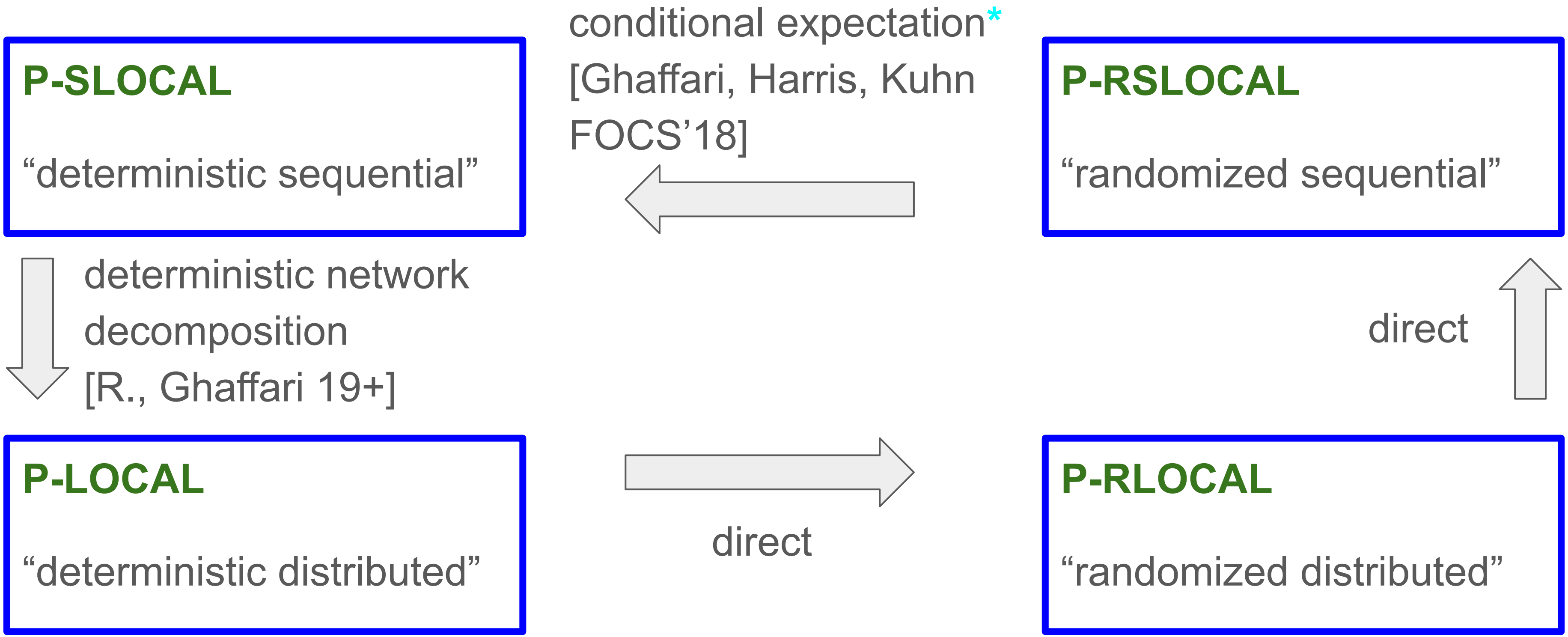
“randomized sequential”

randomized network decomposition [Linial, Saks SODA ‘91]

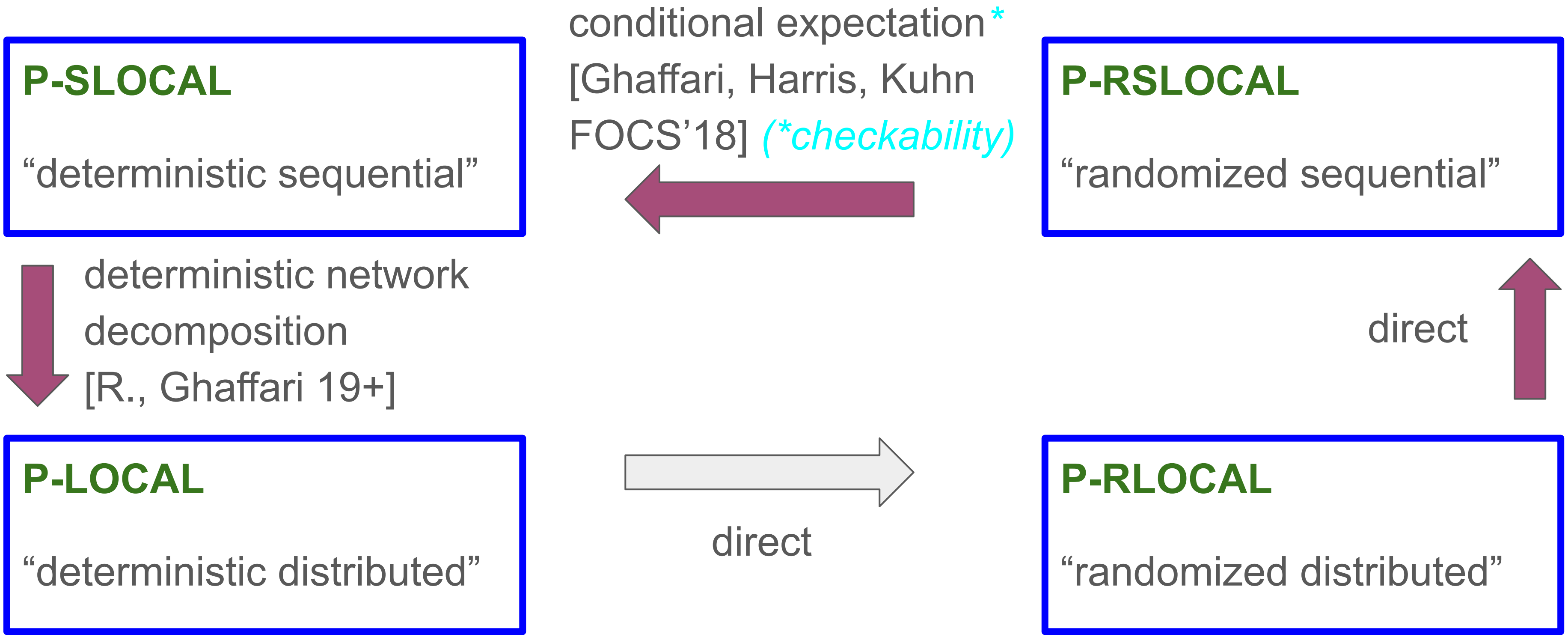
direct

P-RLOCAL

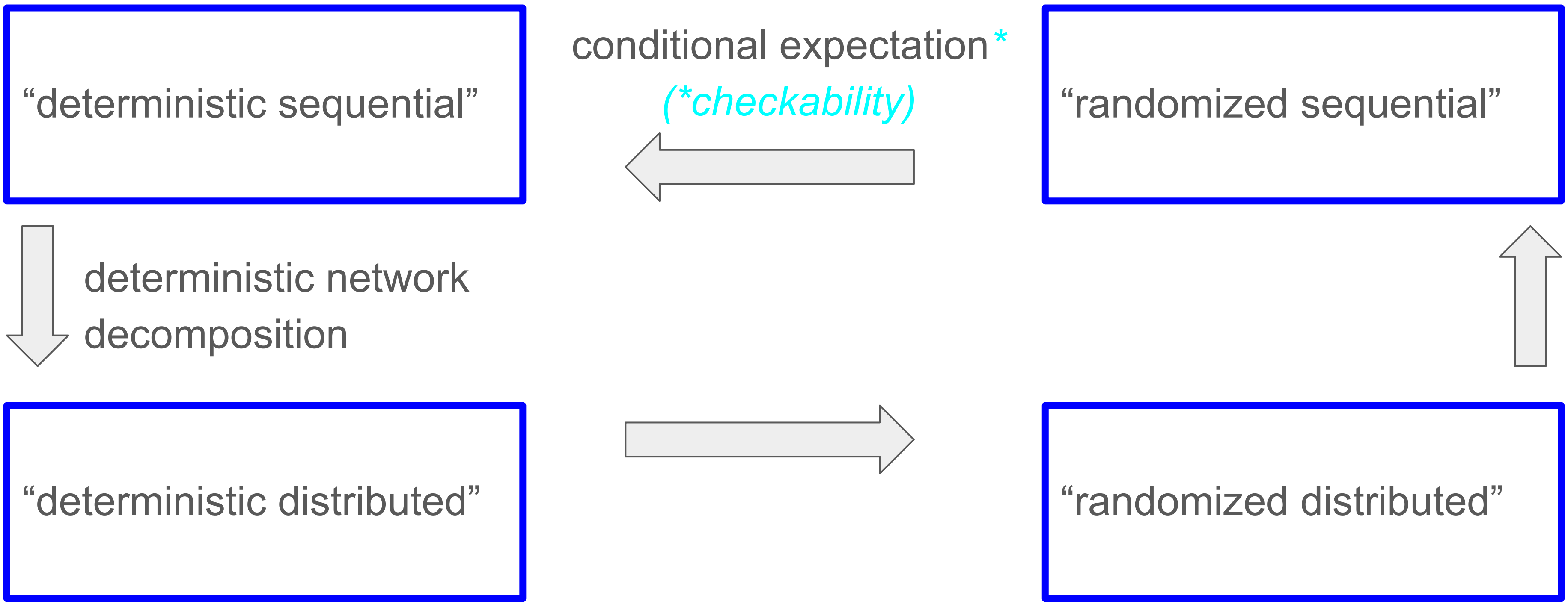
“randomized distributed”



*for problems checkable in poly(log n) rounds



Corollary [R., Ghaffari 19+] There is an efficient deterministic algorithm for hypergraph splitting, Lovász local lemma, ...

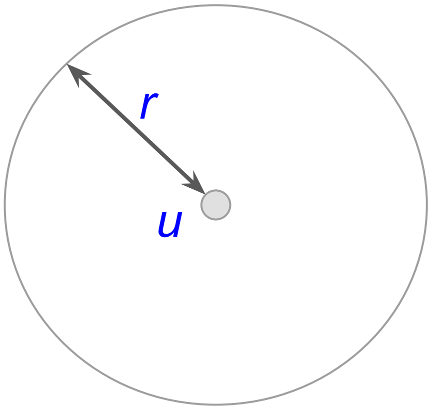


We see a clean first-order theory of the LOCAL model.

Moreover, techniques are simple and principled.

Techniques

Conditional expectation in the **SLOCAL** model



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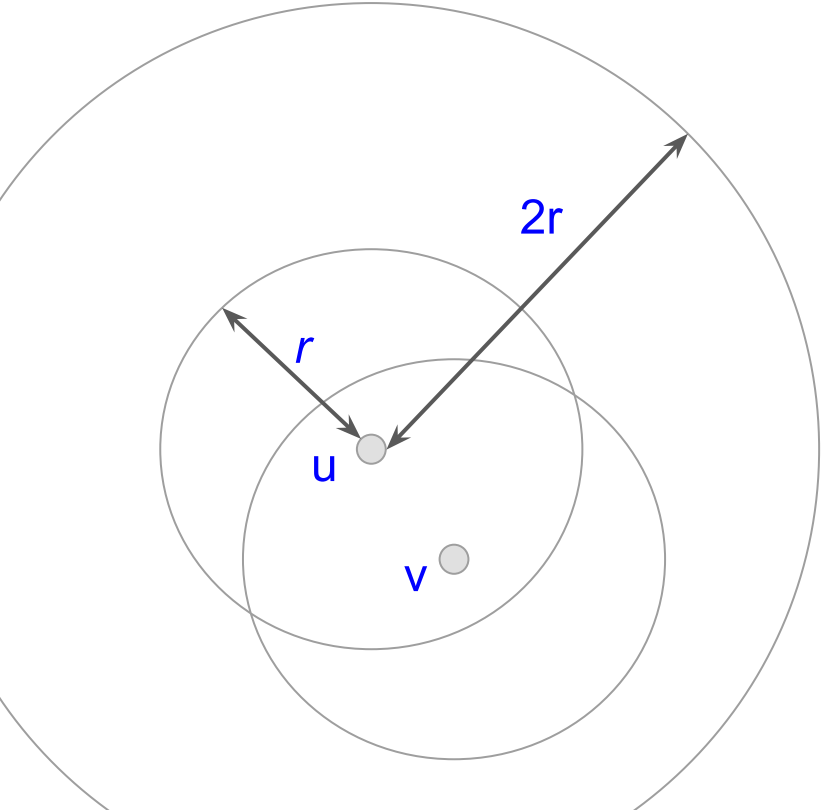

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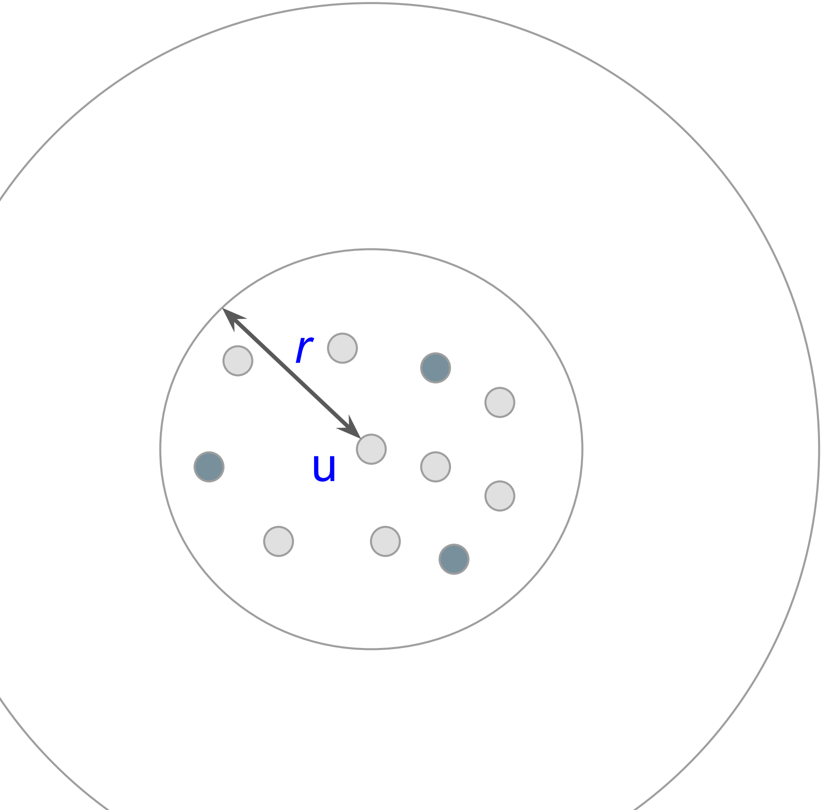

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- For any choice of u's randomness, compute sum of failure probabilities of all these vertices.

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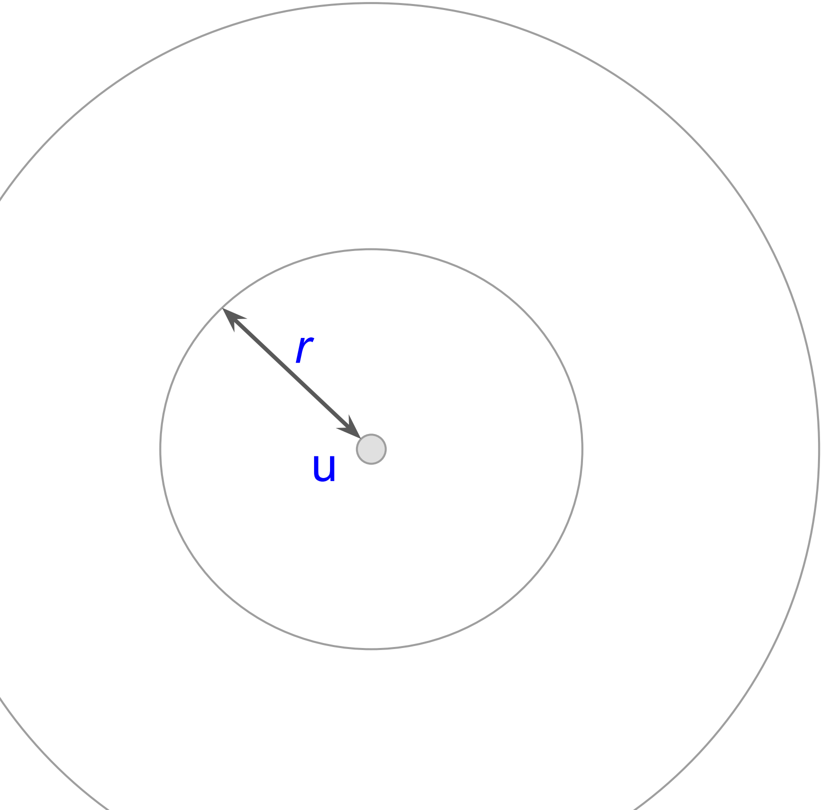

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- Looking at distance 2r from u, we can compute the probability of failure for any vertex that depends on u.
- For any choice of u's randomness, compute sum of failure probabilities of all these vertices.
- Fix the randomness of u so as to minimize expected sum of failure probabilities; it was $\ll 1$ at the beginning, hence no failure occurs.

Deterministic algorithms for network decomposition

Previous work:

$\boxed{2^{O(\sqrt{\log n \log \log n})} \text{[Awerbuch, Goldberg, Luby, Plotkin FOCS'89]}}$

$2^{O(\sqrt{\log n})}$ [Panconesi, Srinivasan STOC'92]

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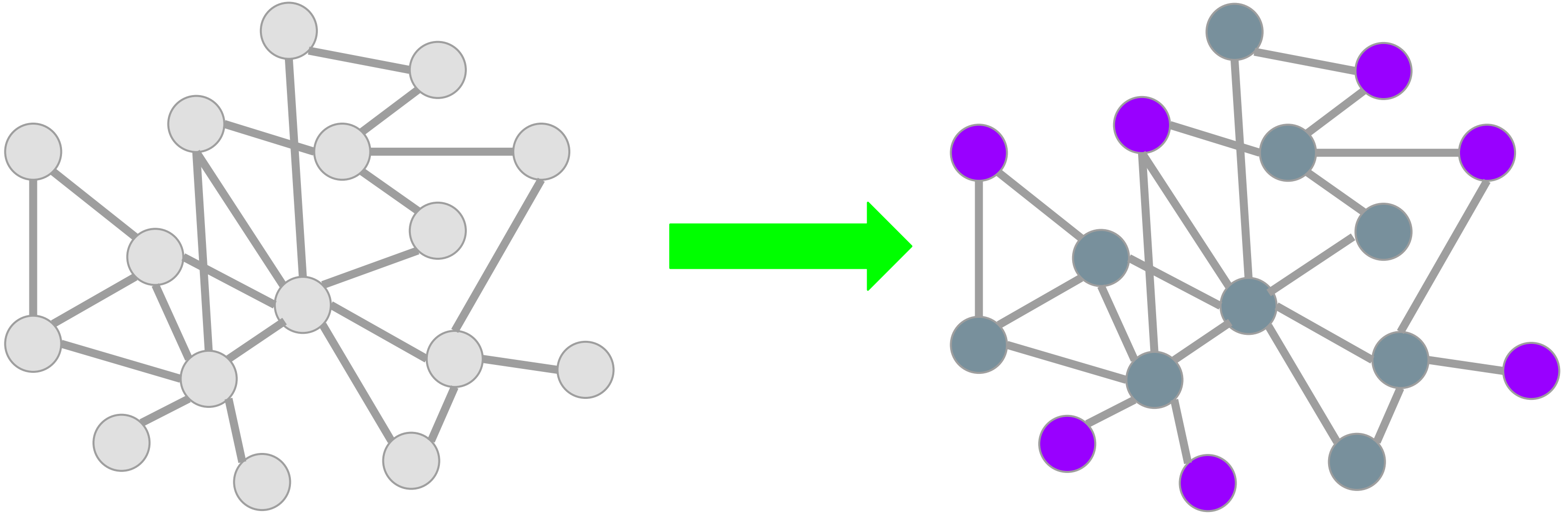
[Ghaffari, Grunau, R. '19+]: $O(\log^6 n)$ algorithm with $O(\log^2 n)$ weak diameter, $O(\log n)$ colors

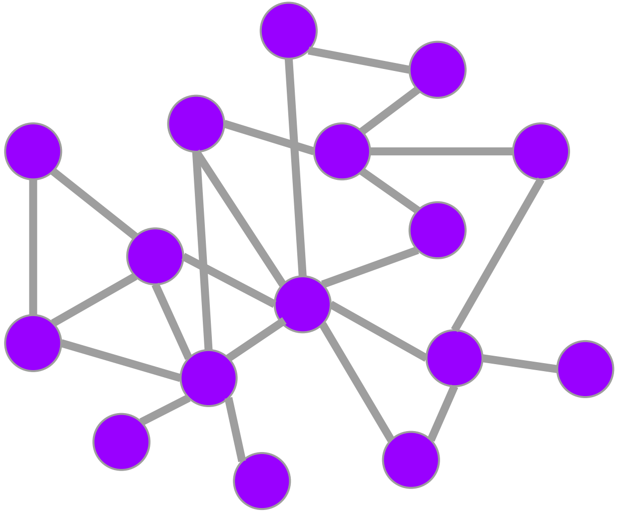
Ruling sets

Goal: find a maximal independent set S such that any vertex of G is of distance at most $O(\log n)$ from S.



Ruling sets

Think about the problem decrementally.



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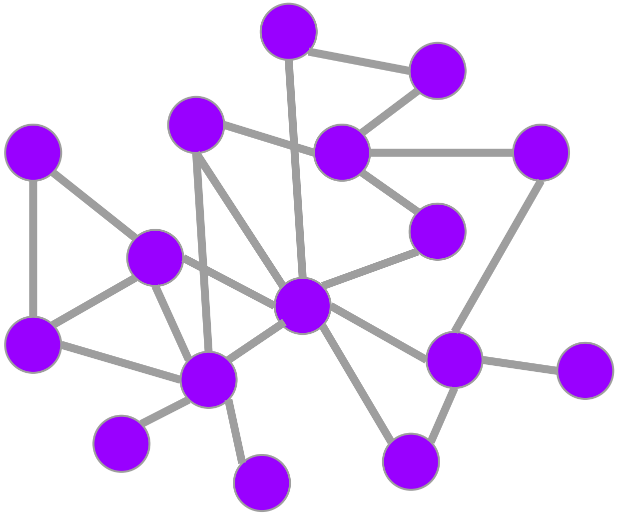

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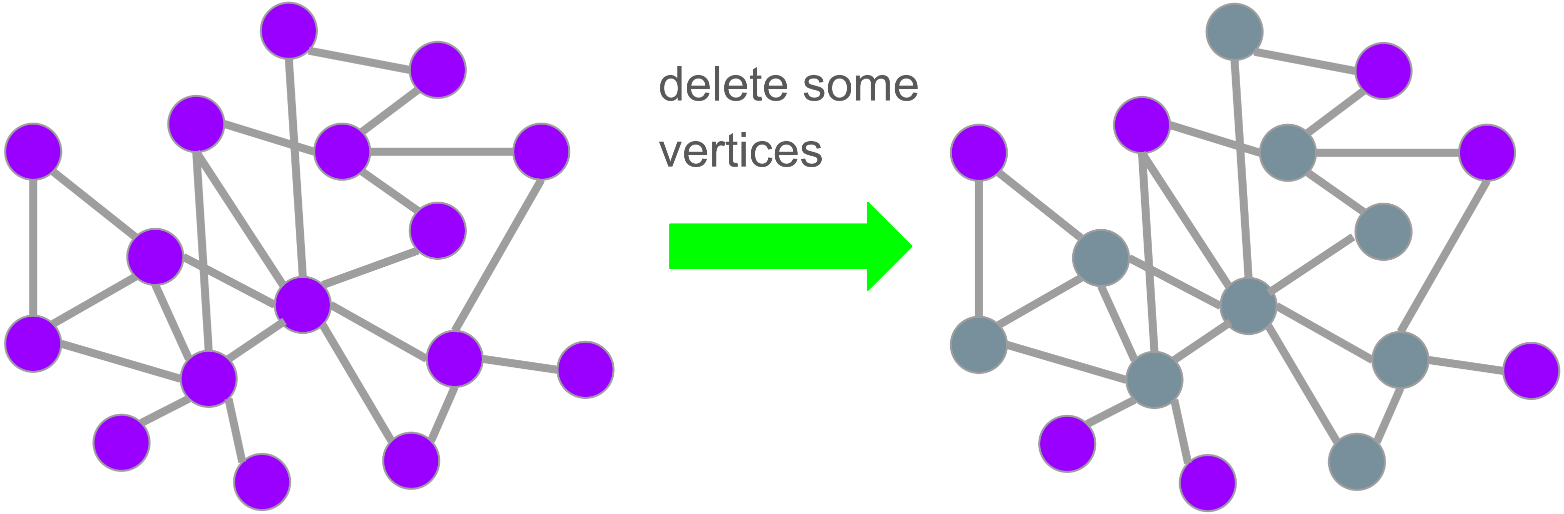
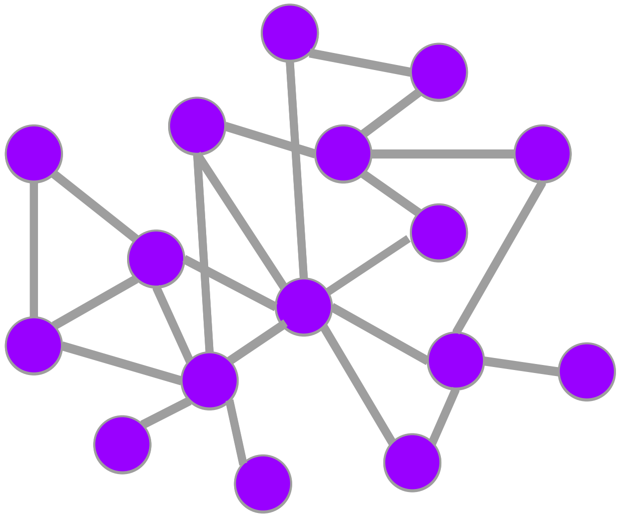


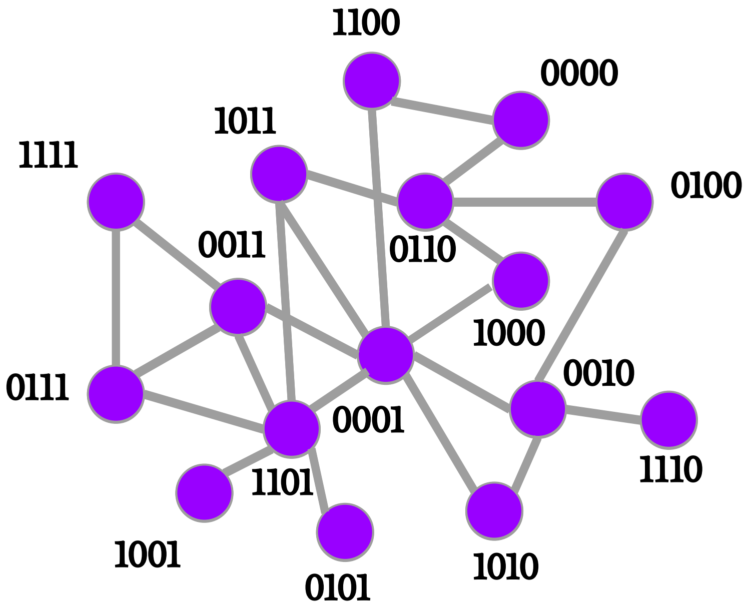
Ruling sets

$O(\log n)$ round algorithm

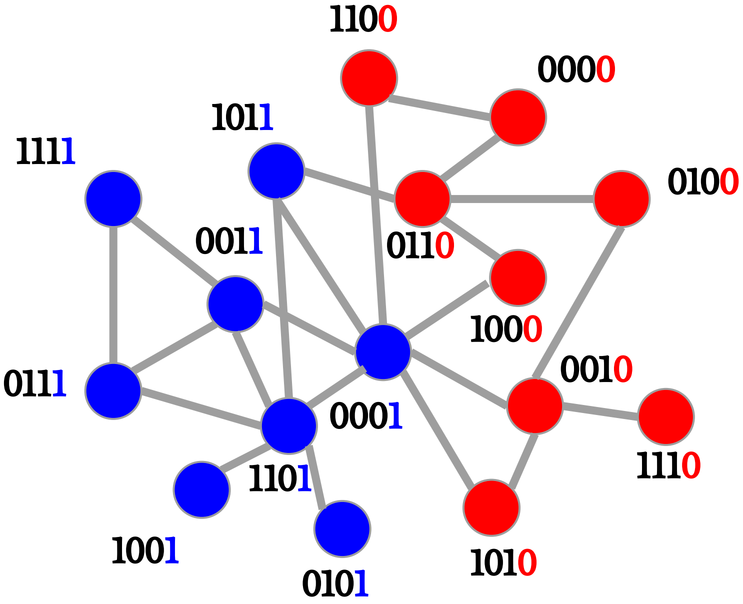


Ruling sets

O(log n) bit labels



Ruling sets


1100
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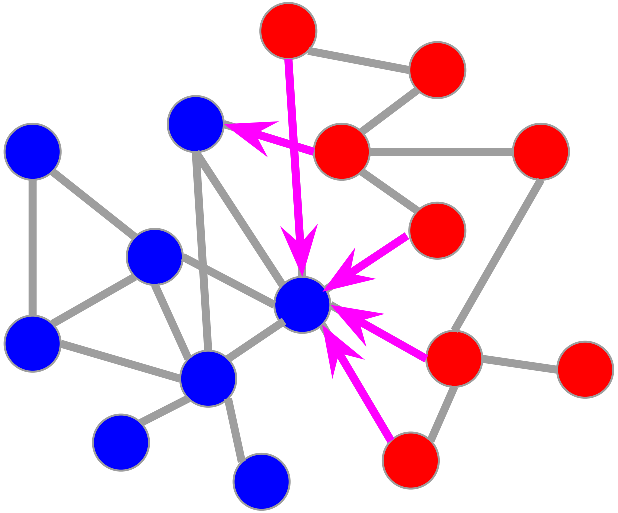
Ruling sets



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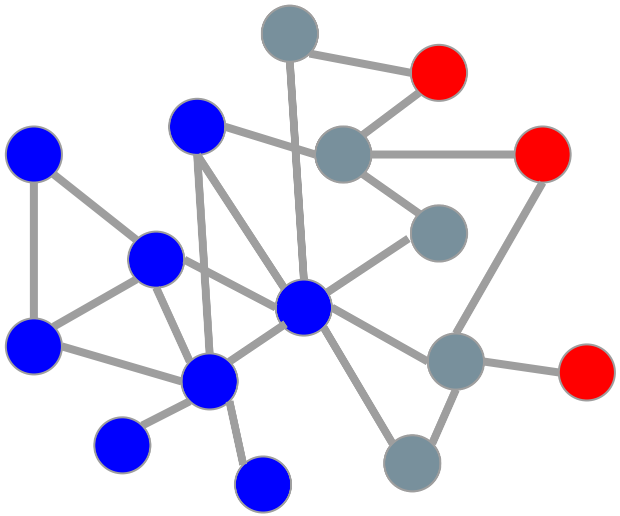

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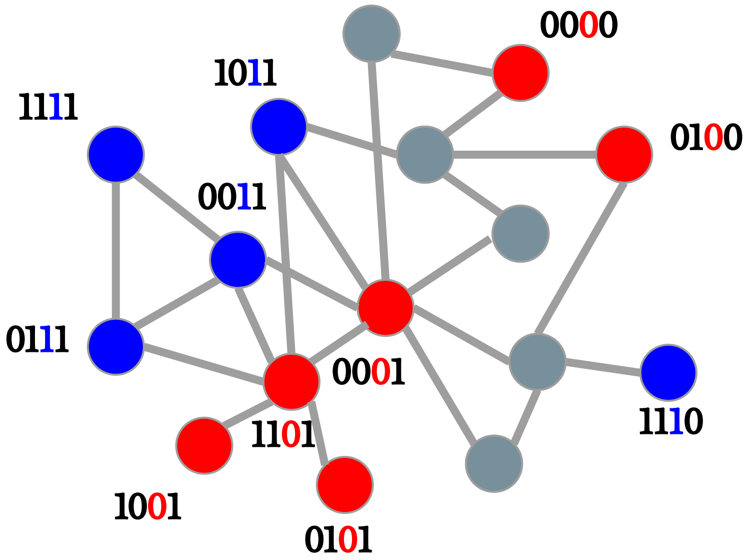

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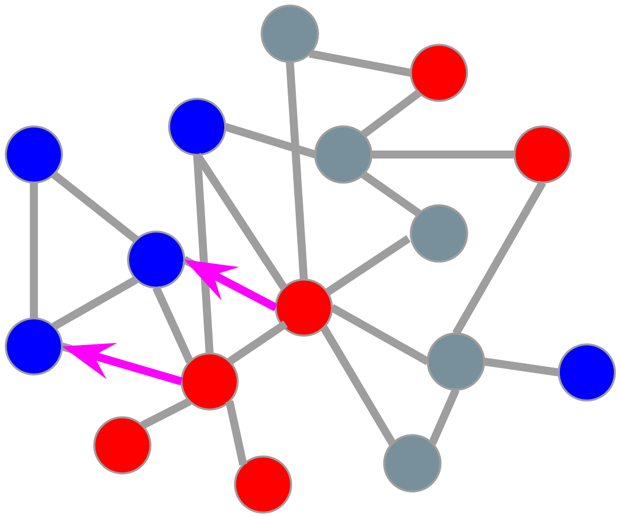

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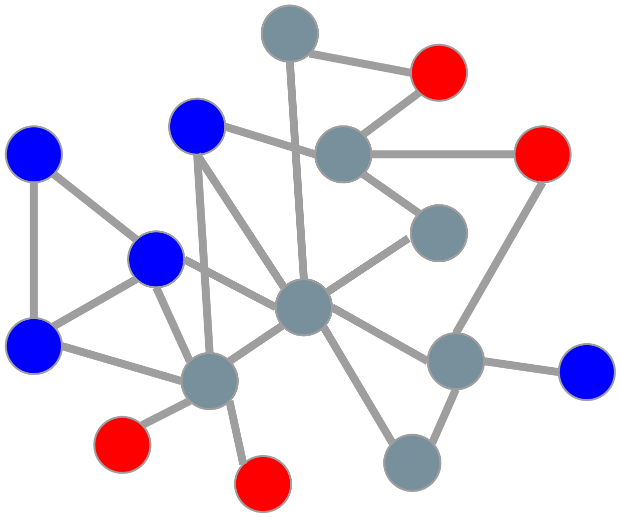

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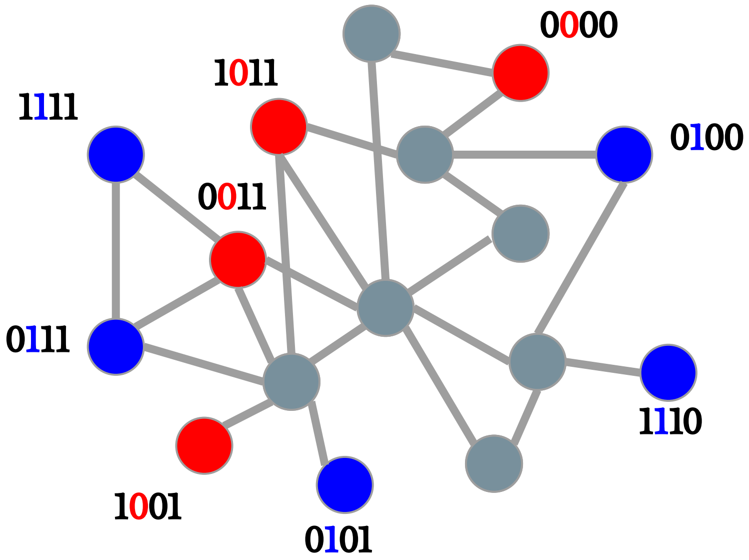

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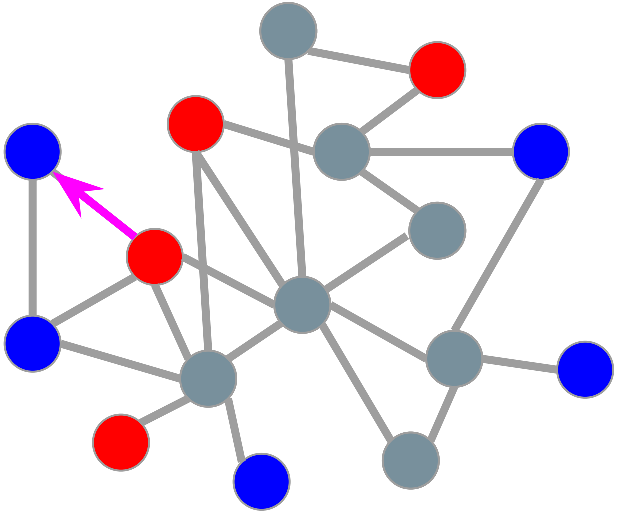

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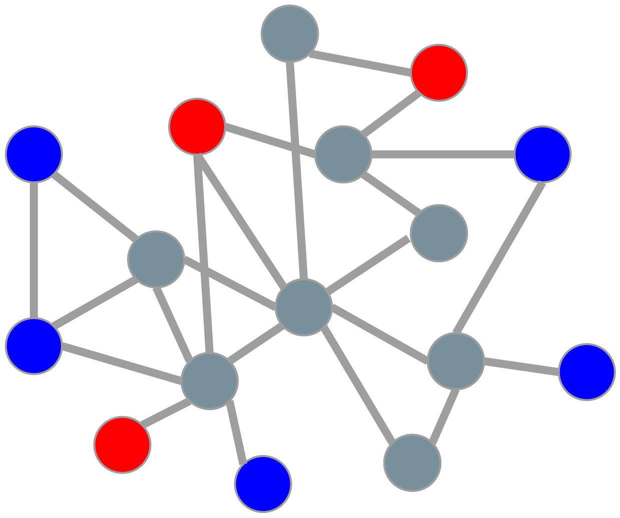

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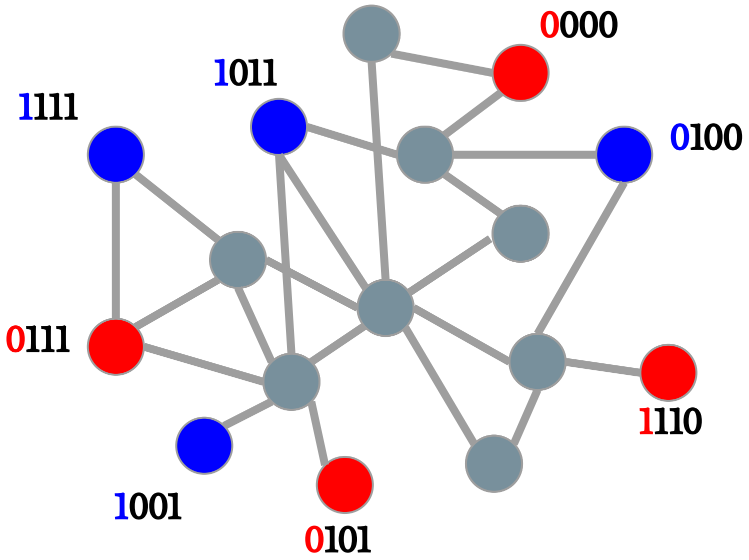

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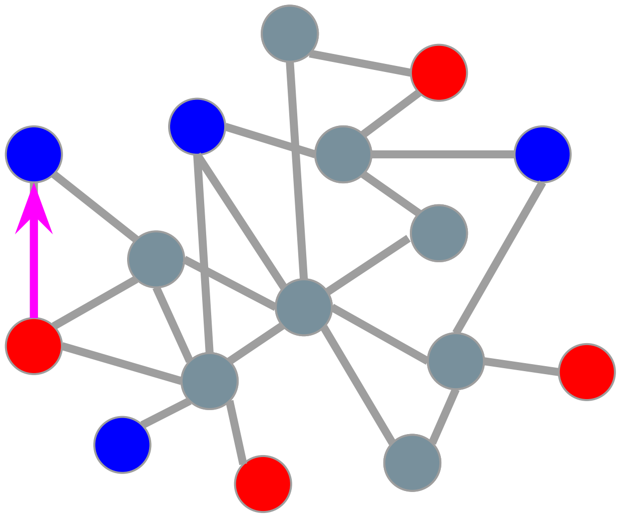

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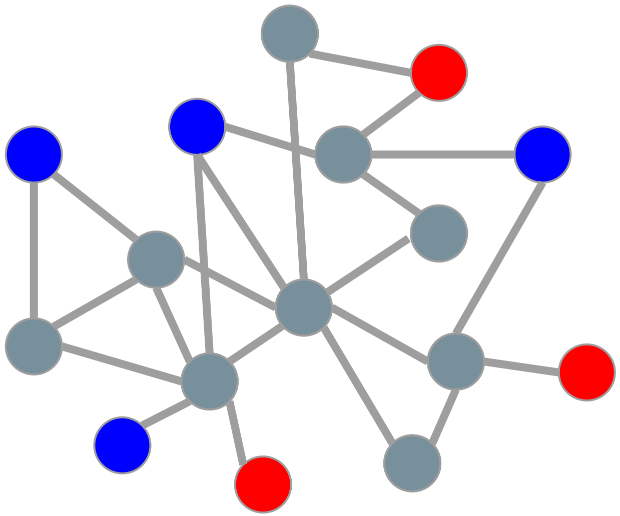
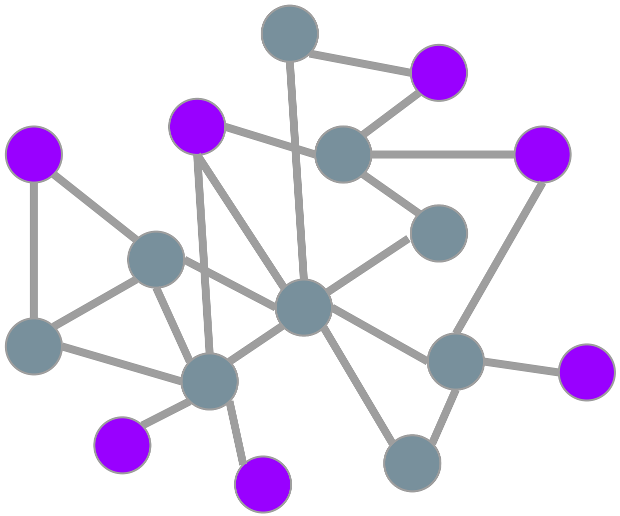


Ruling sets

We got independent set.

But why are distances to nearest vertex in S of order $O(\log n)$?



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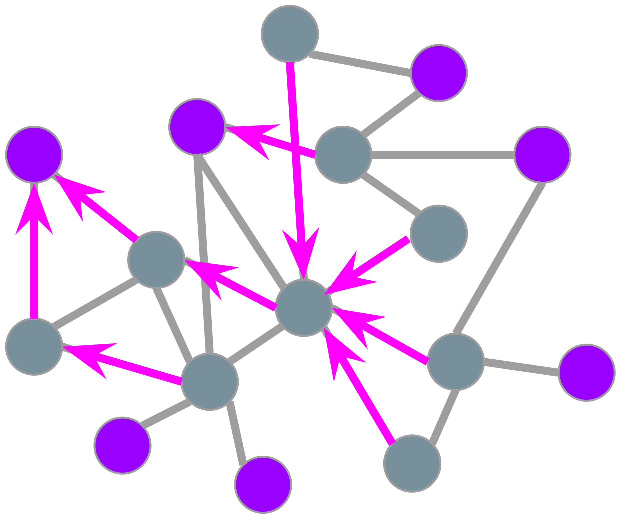
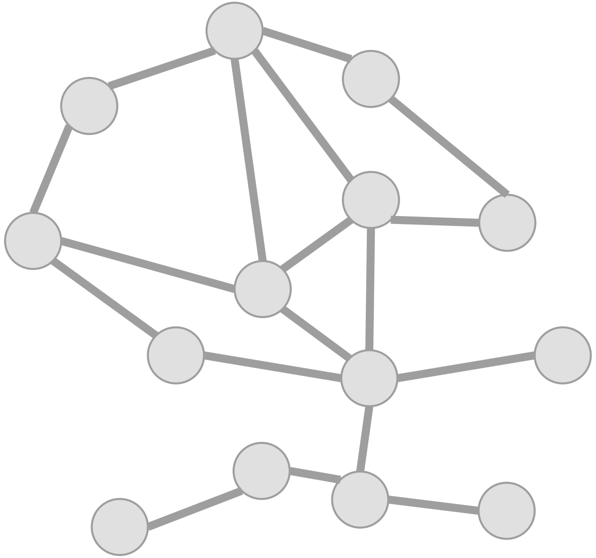


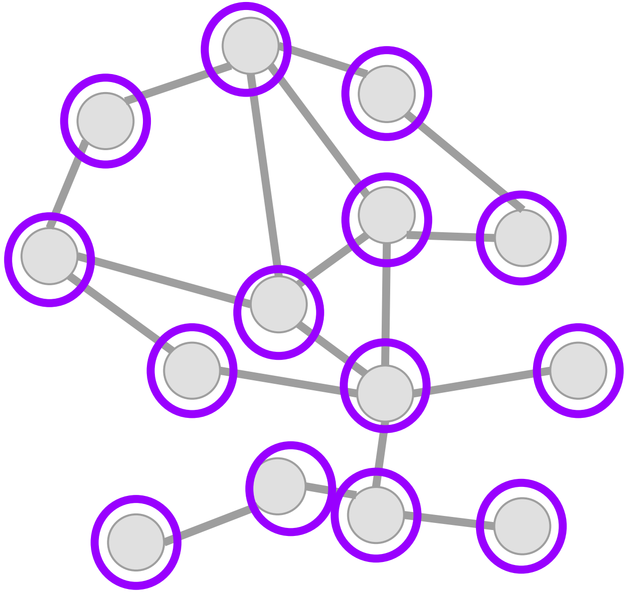
Distributed ball carving



Recall:

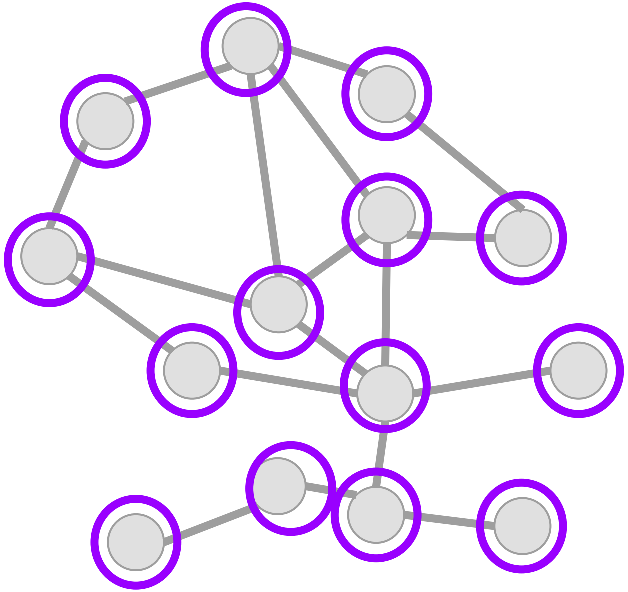
- *poly(log n)* weak-diameter non-neighbouring clusters
- at most ½ fraction of vertices deleted

Distributed ball carving



Each vertex thinks of itself as a root of a cluster

Distributed ball carving



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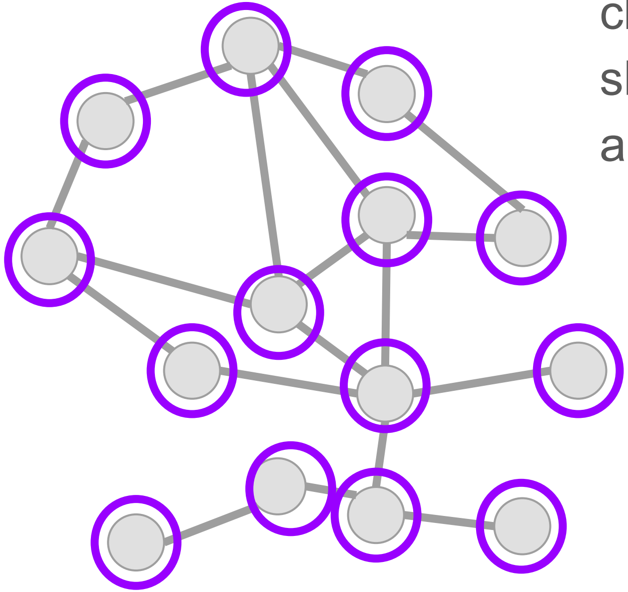
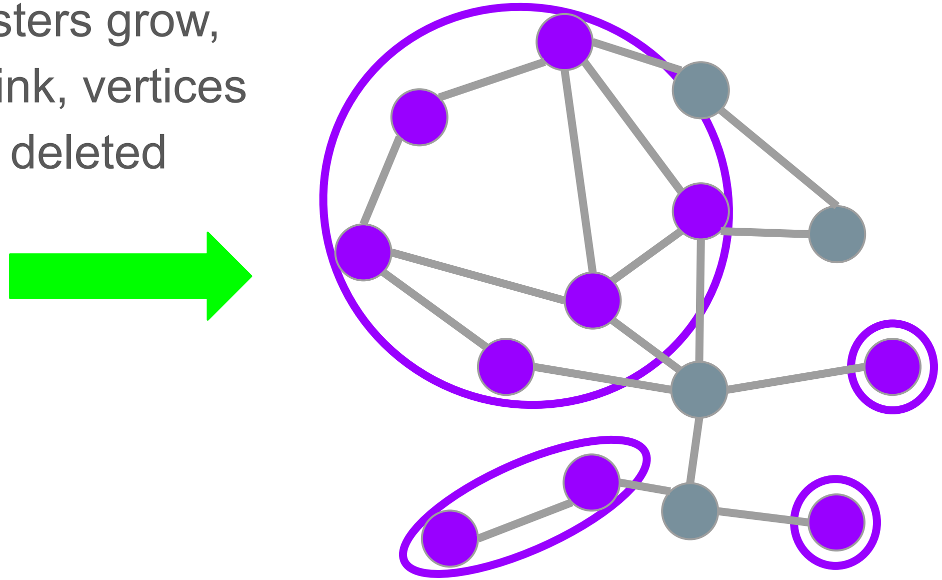


clusters grow,
shrink, vertices
are deleted



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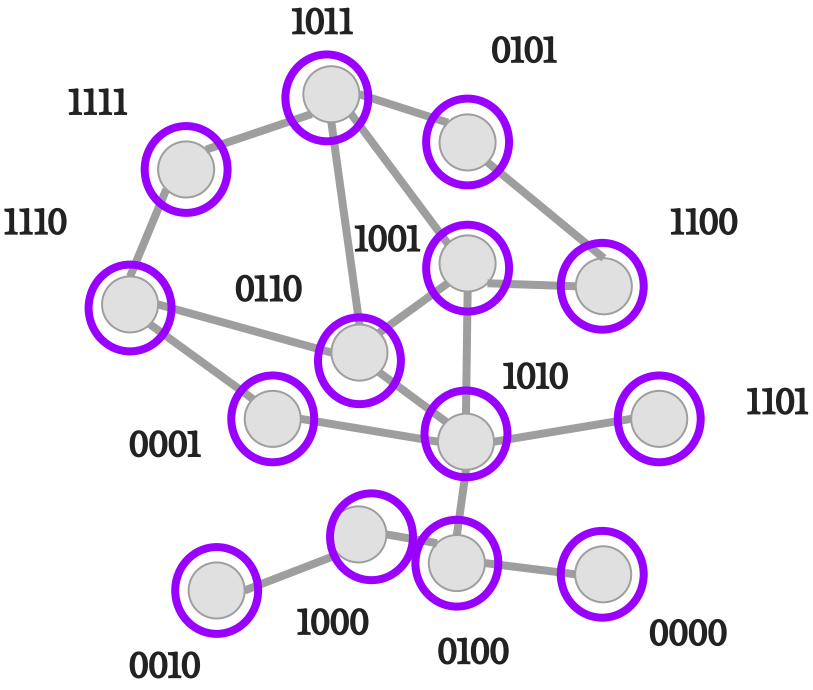

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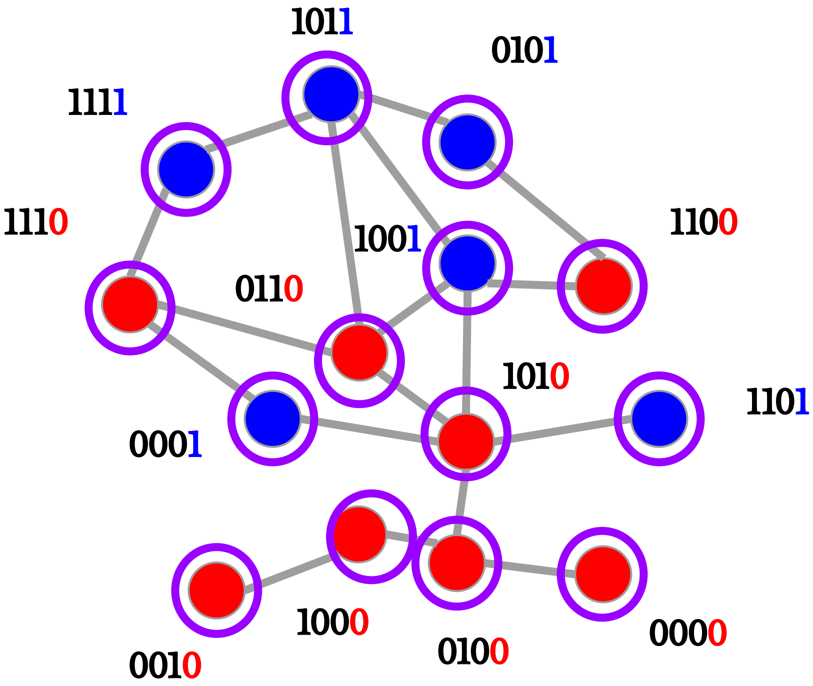

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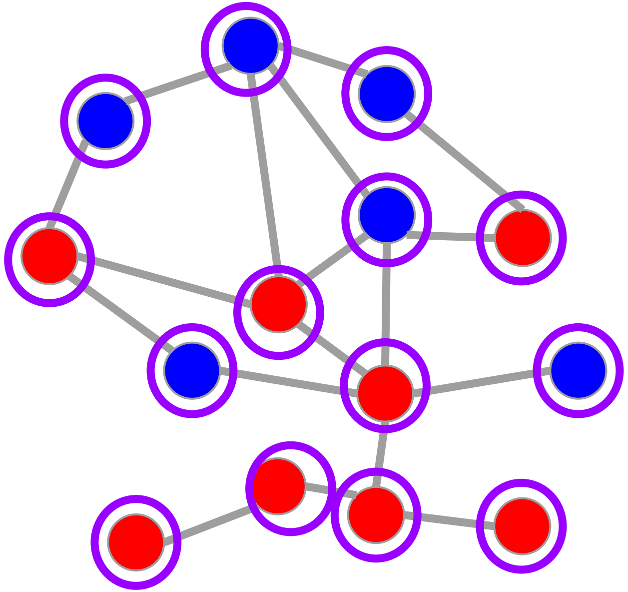

for ruling sets we just delete the boundary, now we have to be more clever

Distributed ball carving

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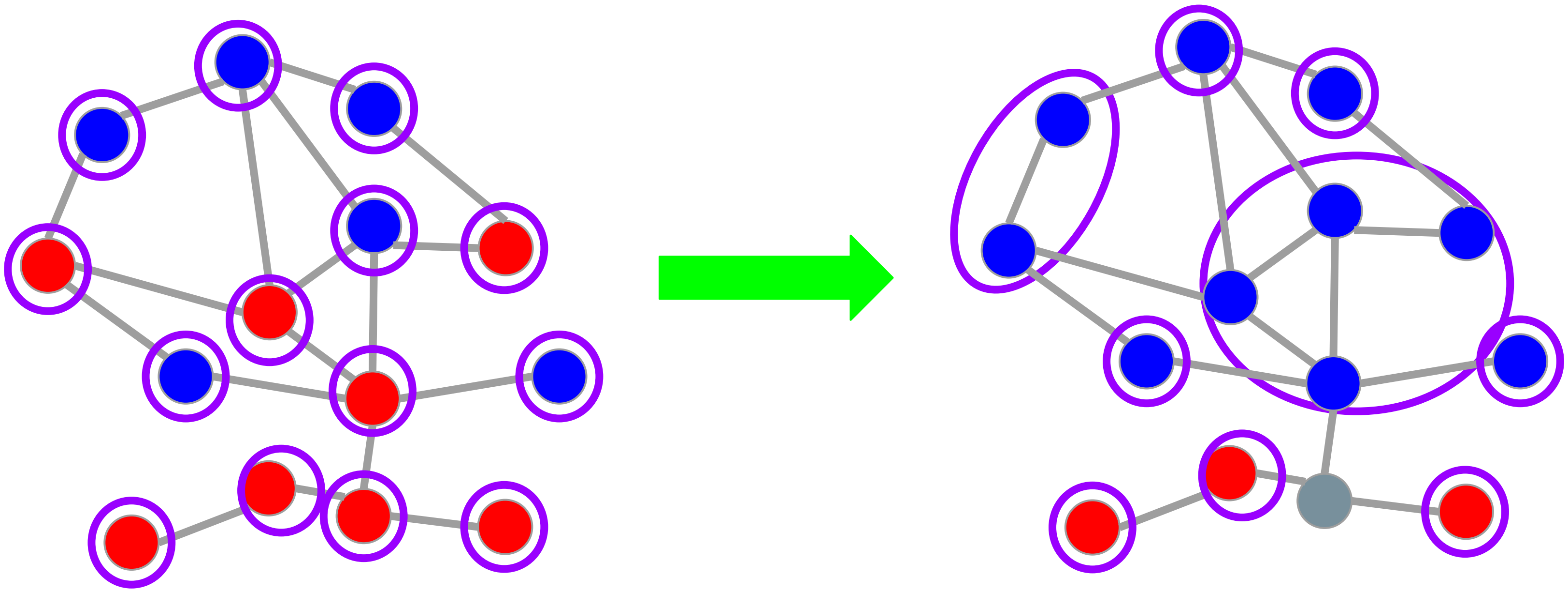

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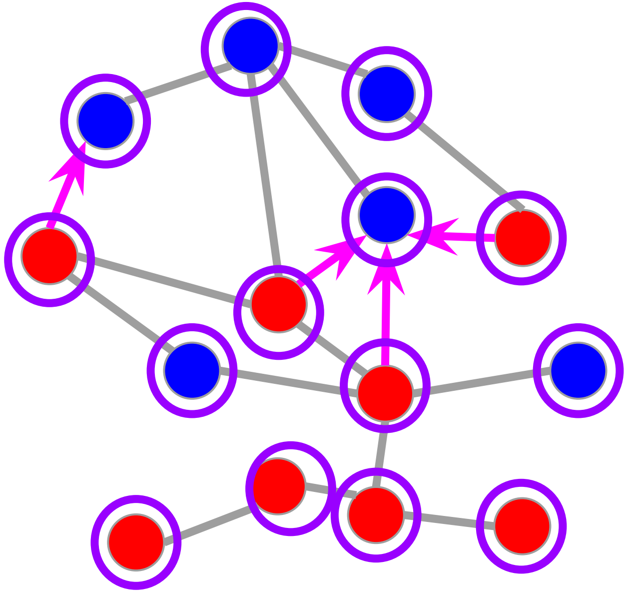

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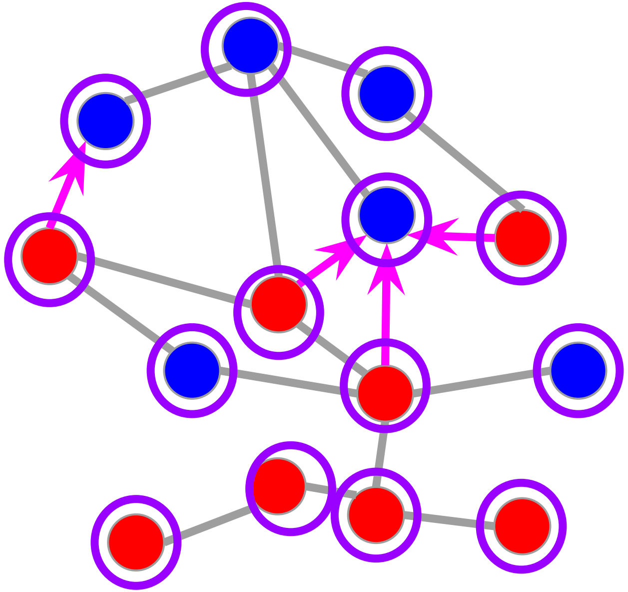
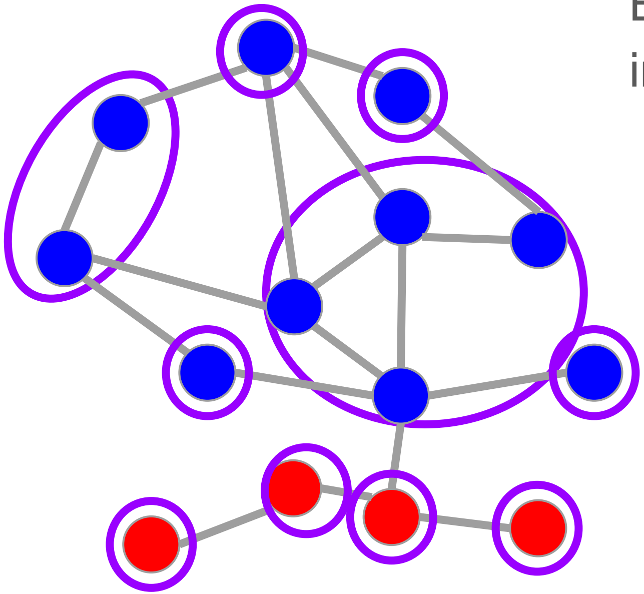


It is time to go back to the sequential algorithm.

Distributed ball carving



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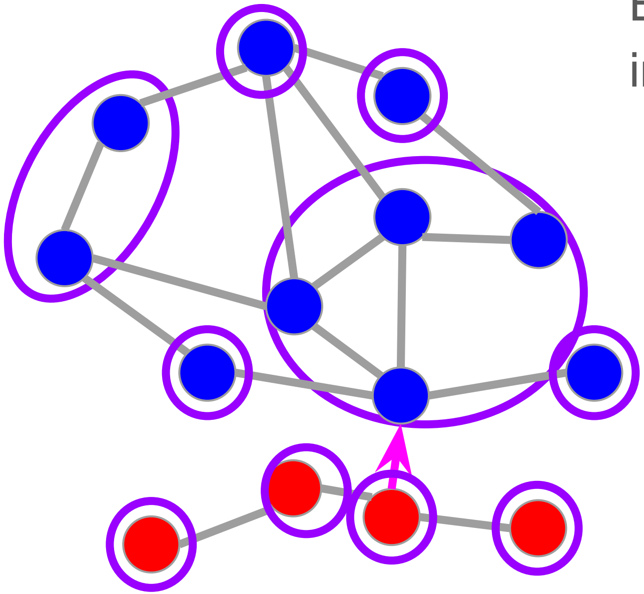
Blue clusters grow, if it means multiplicative increase by factor of $1+1/O(\log n)$.

Distributed ball carving



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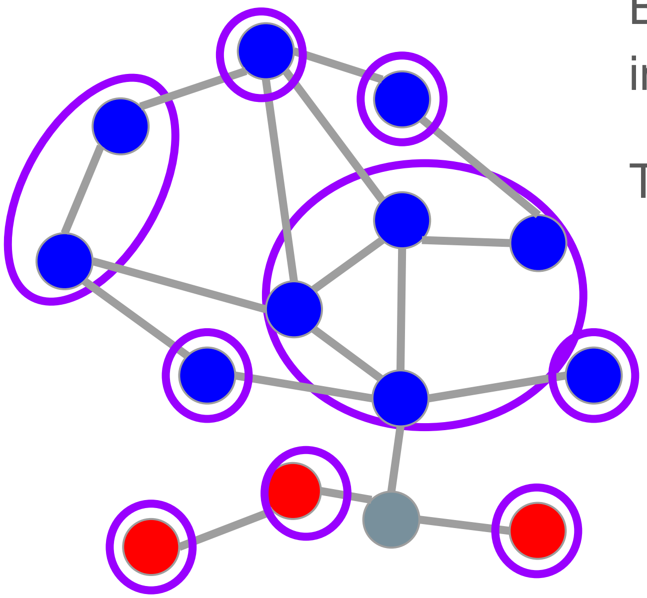

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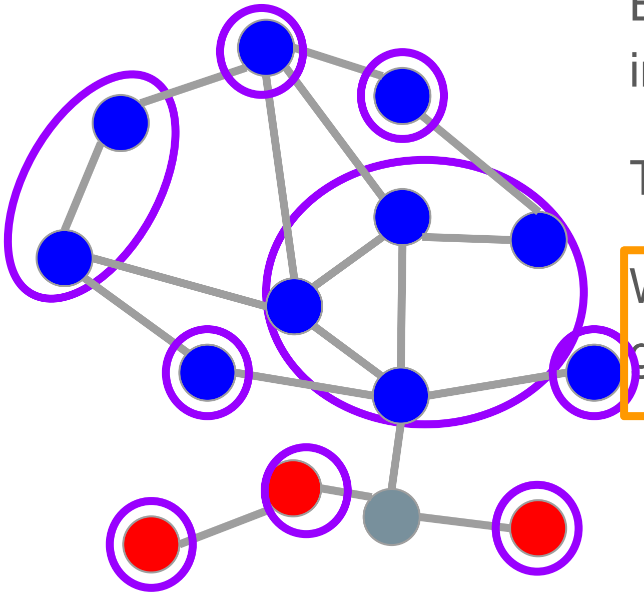

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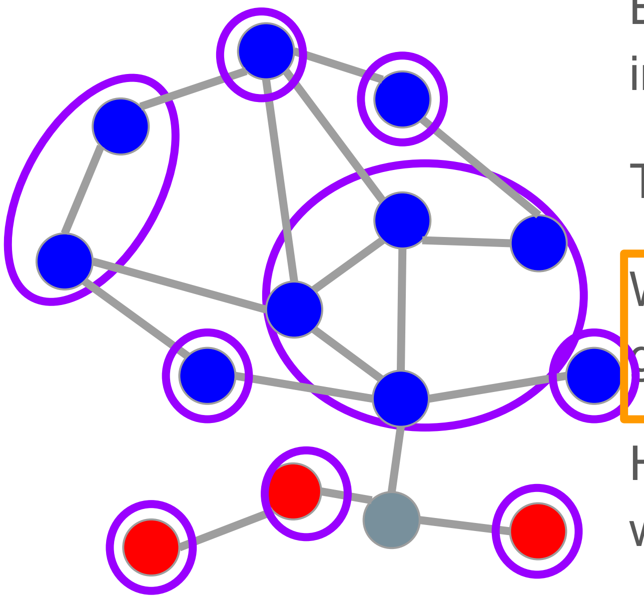

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Hence, if a cluster grows after $\Omega(\log^2 n)$ steps, it was growing for the whole time ⇒ Its size is $>n$.

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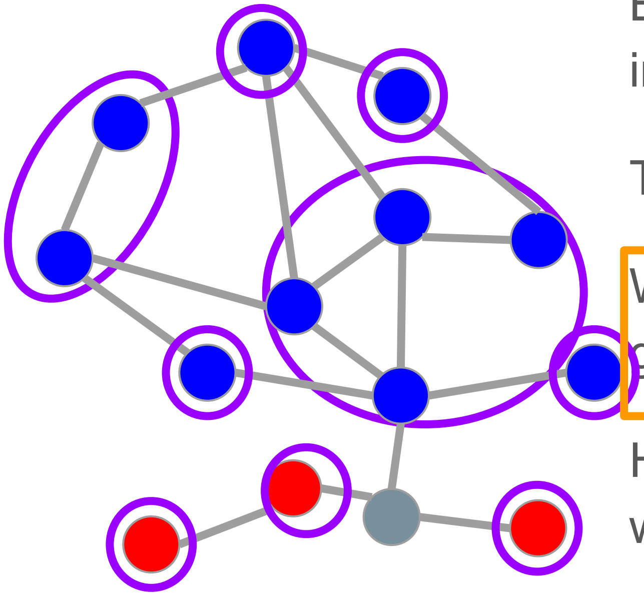

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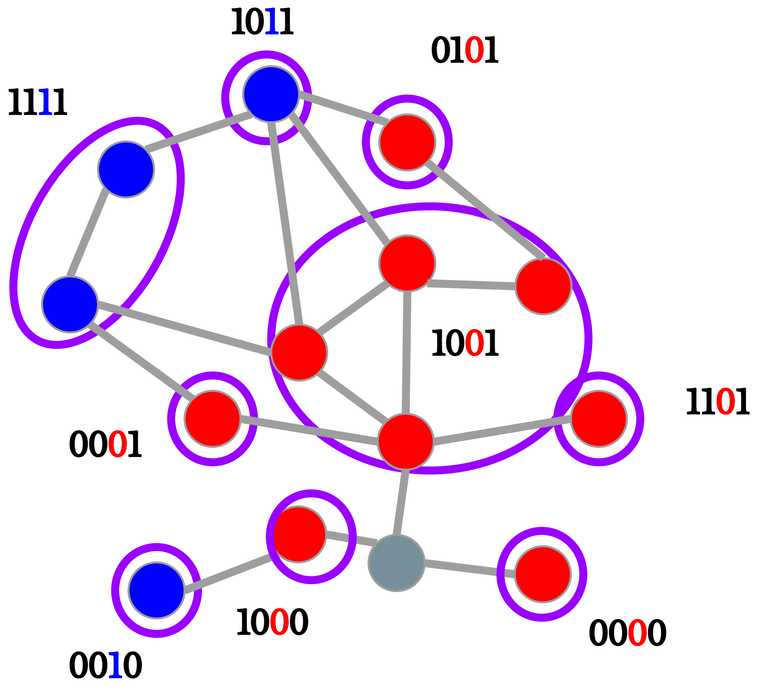
We finish after $O(\log^2 n)$ steps.

Distributed ball carving

Id of a cluster is the id of the original vertex.



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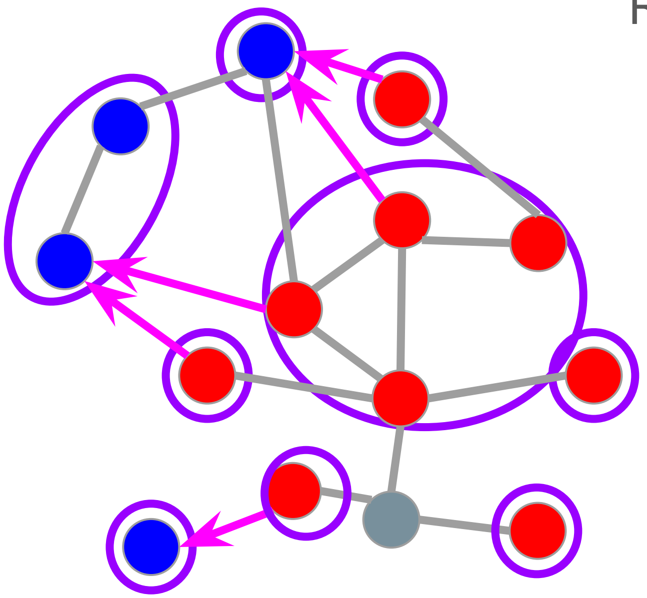

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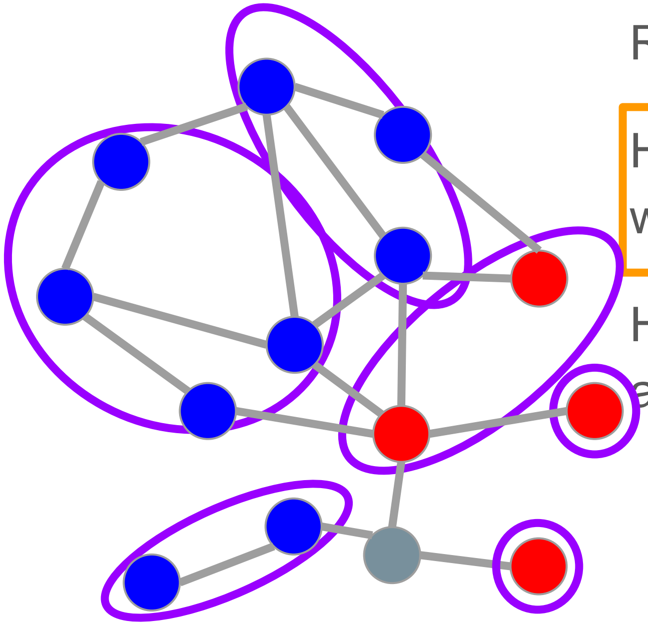
Distributed ball carving



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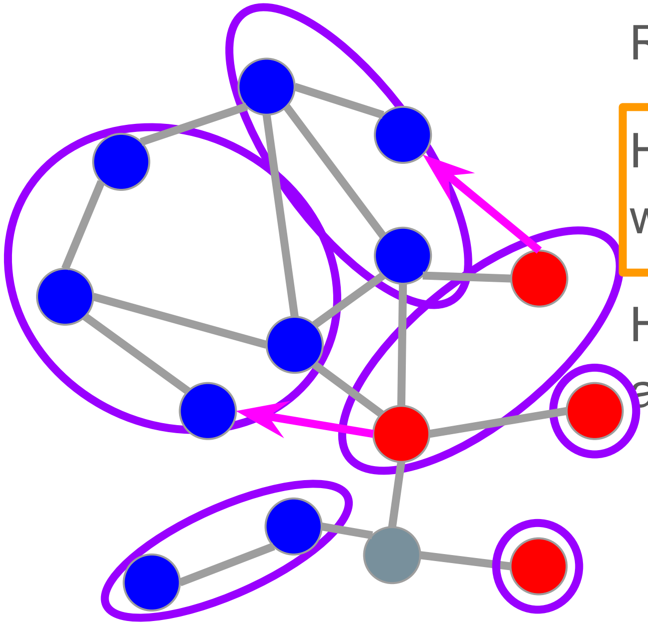
Hence, clusters may even disconnect, but weak-diameter increases by at most 2

Hence, the diameter of each cluster grows additively by $O(\log^2 n)$ per phase.

Distributed ball carving



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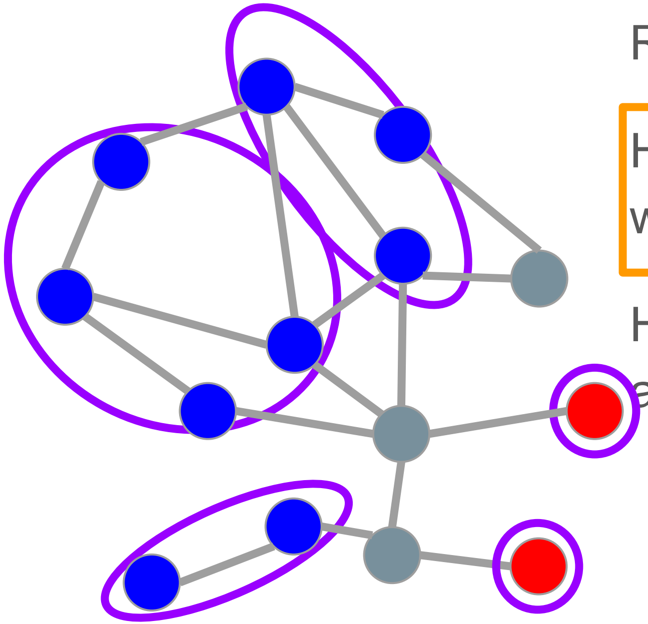

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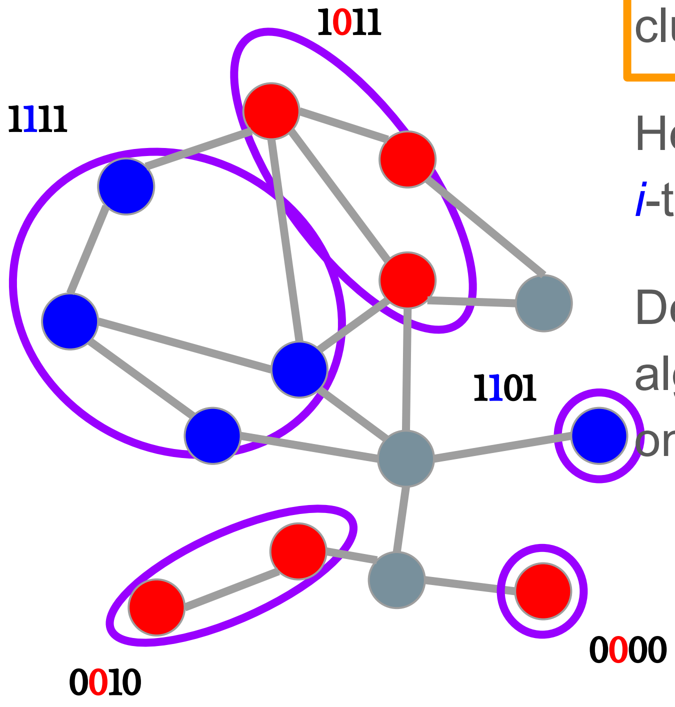
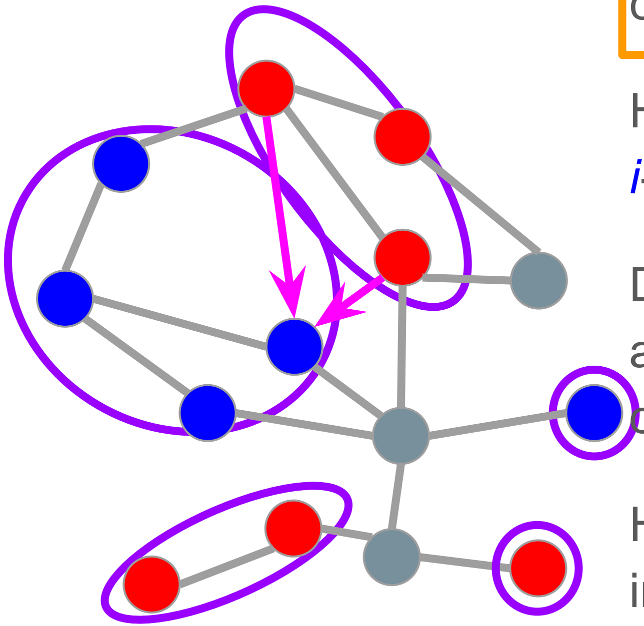


After *i*-th phase, there is no edge between two clusters with different *i*-th bit in their label.

Hence, all connected components agree on the *i*-th bit.

Deleted vertices stay delete for the whole algorithm; hence, all connected component agree on *i* rightmost bits after *i* phases.

Distributed ball carving



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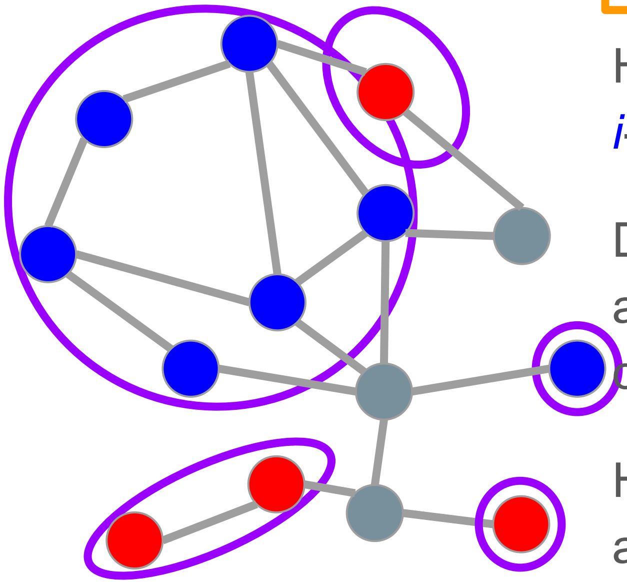
Hence, after we finish all remaining clusters are independent.

Distributed ball carving



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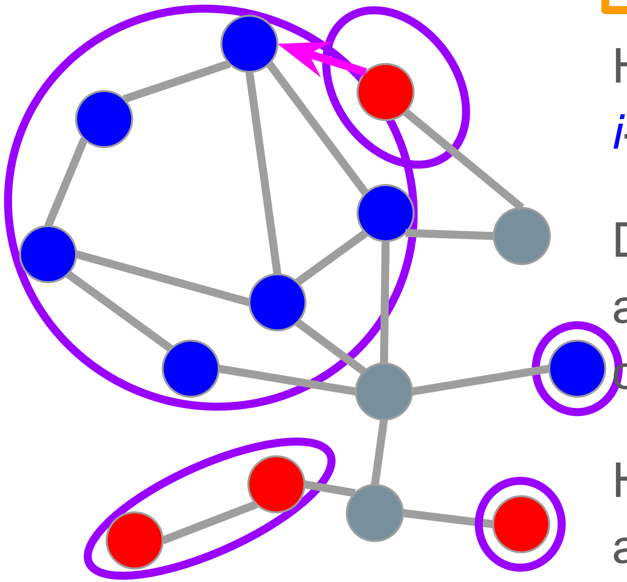
Hence, after $O(\log n)$ phases remaining clusters are independent.

Distributed ball carving



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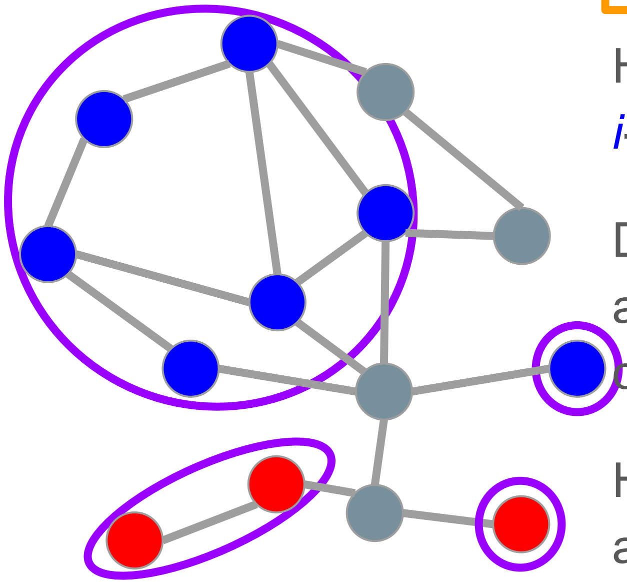

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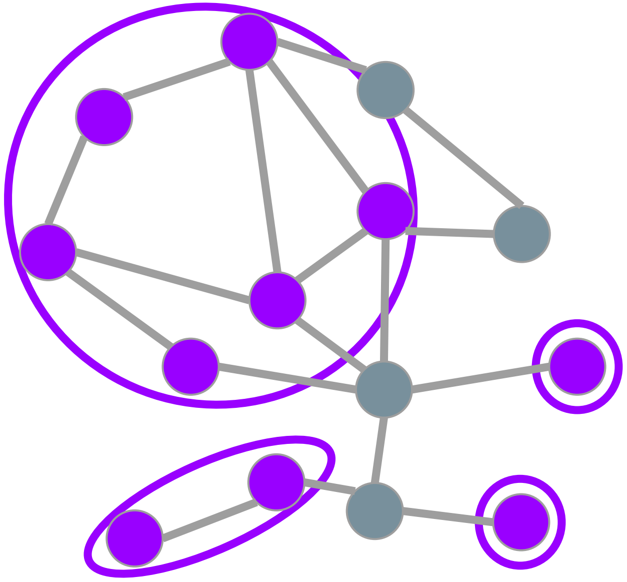
Hence, after *O(*log *n)* phases remaining clusters are independent.

Distributed ball carving



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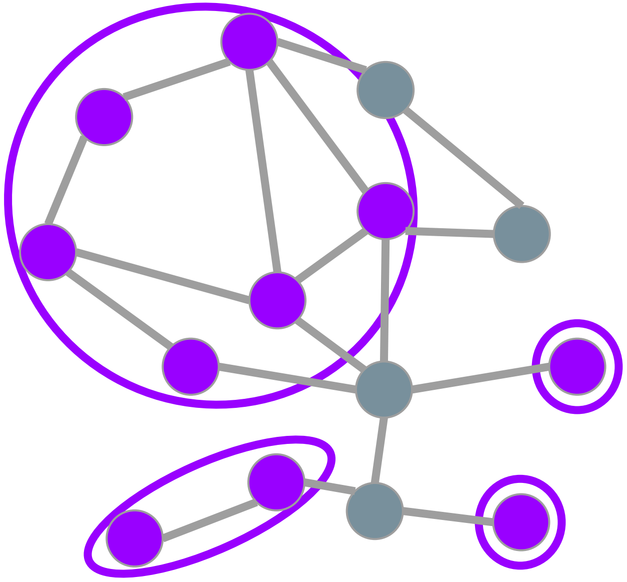
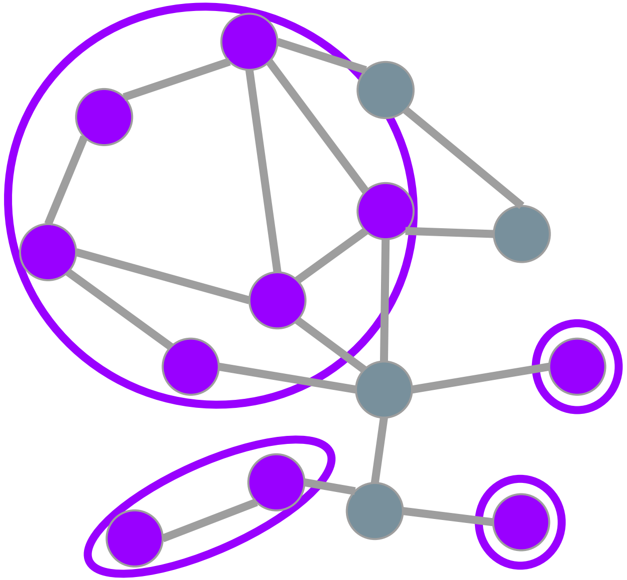


In each of $O(\log n)$ phases, we deleted $1/O(\log n)$ fraction of vertices.

Distributed ball carving



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Hence, in total we deleted at most ½ of the vertices (if we set up right constant).

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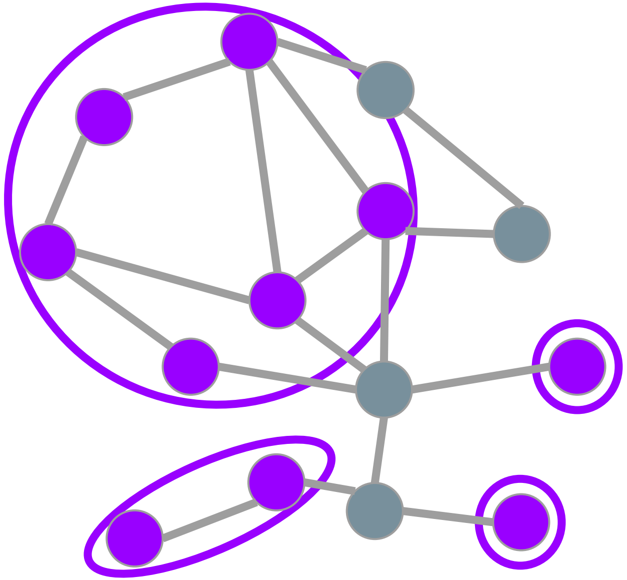

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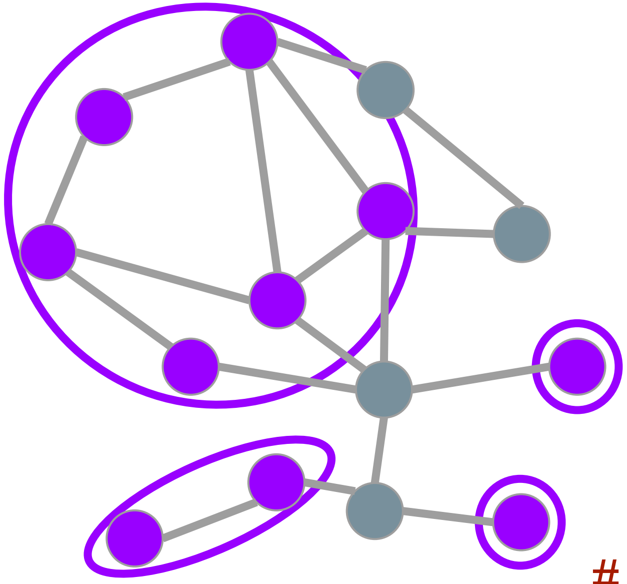

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Running time is $O(\log^7 n) =$

$\underbrace{O(\log n)}_{\text{\# of colors}} \cdot \underbrace{O(\log n)}_{\text{\# of phases}} \cdot \underbrace{O(\log^2 n)}_{\text{steps per phase}} \cdot \underbrace{O(\log^3 n)}_{\text{complexity of one step}}$

Outlook to **CONGEST** and randomized **LOCAL**/**MPC**

Our algorithm in **CONGEST**

Since our algorithm works also in the **CONGEST** model, we get some more results:

[Censor-Hillel, Parter, Schwartzman DISC‘17]:

“There is deterministic poly(log n)-round algorithm for maximal independent set in the **CONGEST** model. “

[Bamberger, Kuhn, Maus ‘19+]

“The same holds for Δ+1 coloring. “

Important: the algorithm also gives you underlying broadcast trees.

Our algorithm in **CONGEST**

The algorithm generates broadcast trees.

Each edge is in $O(\log n)$ of them.

Our algorithm in **CONGEST**

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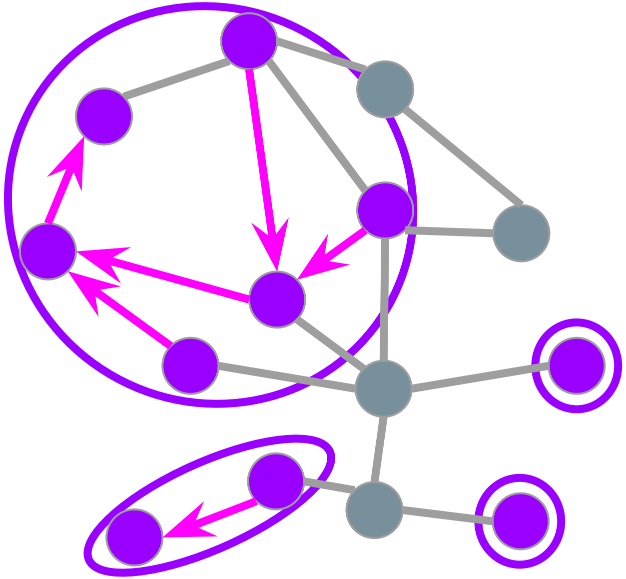

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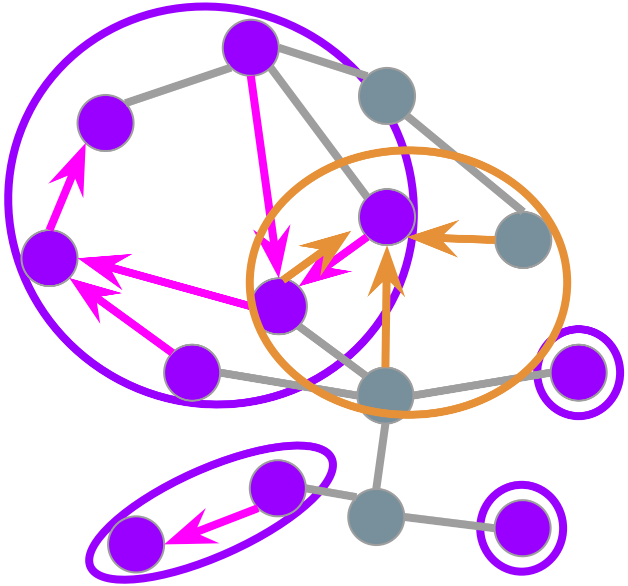

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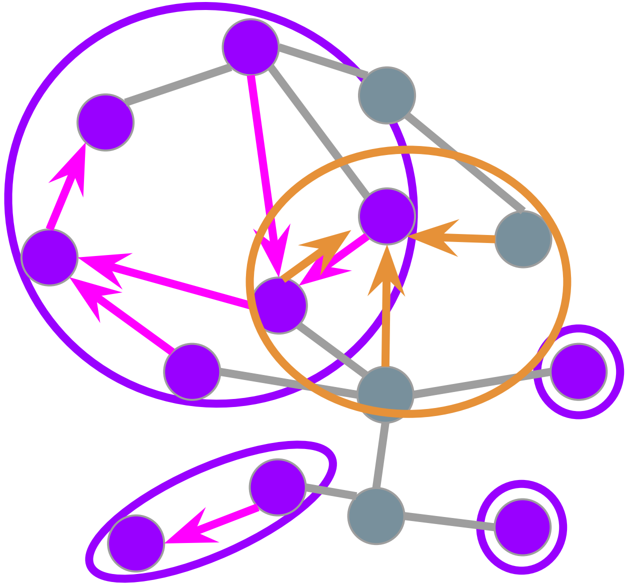
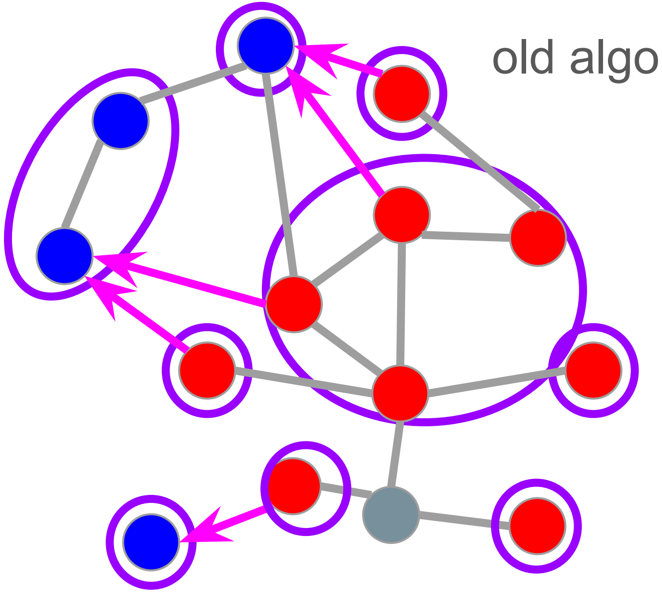


Natural extension of the algorithm gives decomposition of G^k.

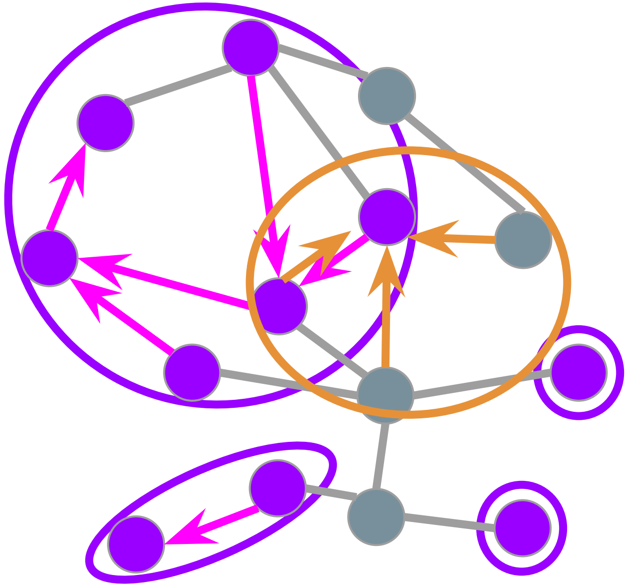


old algorithm

Our algorithm in **CONGEST**

The algorithm generates broadcast trees.

Each edge is in $O(\log n)$ of them.



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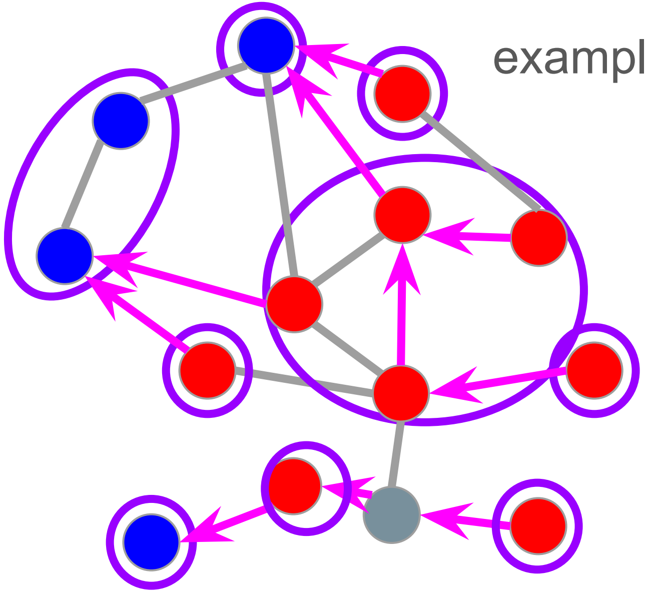

example for G^3

Outlook: Beyond deterministic & local

Δ+1 coloring in different models

LOCAL, deterministic	LOCAL, randomized	MPC, randomized
poly(log n)		

Outlook: Beyond deterministic & local

Δ+1 coloring in different models

LOCAL, deterministic		LOCAL, randomized	MPC, randomized
poly(log n)	→	poly(log log n)	

shattering [Chang, Li, Pettie STOC'18]
+network decomposition [R., Ghaffari 19+]

Outlook: Beyond deterministic & local

Δ+1 coloring in different models

amplification of success probability
[Chang, Kopelowitz, and Pettie FOCS'16]

LOCAL, deterministic	LOCAL, randomized	MPC, randomized
poly(log n)	poly(log log n)	

shattering [Chang, Li, Pettie STOC'18]
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Outlook: Beyond deterministic & local

Δ+1 coloring in different models

amplification of success probability
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LOCAL, deterministic

poly(log n)

LOCAL, randomized

poly(log log n)

MPC, randomized

O(log log log n)

shattering [Chang, Li, Pettie STOC'18]
+network decomposition [R., Ghaffari 19+]

graph exponentiation [Chang, Fischer, Ghaffari, Uitto, Zheng PODC'19]

Outlook: Beyond deterministic & local

Δ+1 coloring in different models

amplification of success probability
[Chang, Kopelowitz, and Pettie FOCS'16]

conditioned on hardness of connectivity
in MPC [Ghaffari, Kuhn, Uitto '19+]

LOCAL, deterministic

poly(log n)

LOCAL, randomized

poly(log log n)

MPC, randomized

O(log log log n)

shattering [Chang, Li, Pettie STOC'18]
+network decomposition [R., Ghaffari 19+]

graph exponentiation [Chang, Fischer,
Ghaffari, Uitto, Zheng PODC'19]

Open problems

Give $o(\log^6 n)$ **LOCAL** algorithm for net. decomposition, or even, say, $\Delta+1$ coloring.

Give poly(log n) **CONGEST** algorithm for strong diameter decomposition.

Find a *natural* deterministic poly(log n) algorithm for MIS/coloring in the **CONGEST** model.