Kalman Filter summary.

Gaussian Assumption, Bayes Filter

1D Example:

higher center. smaller variance

1. initialize state x_0. Process covariance matrix
2. predict the state. P.
3. calculate the difference measurement − Prediction.
4. caculate the kalman Gain.
5. update the state and P

back 1.

non-lin process

predict state noise

x_{k-1}
P_{k-1}

1

$x = Ax + Bu + w_k$.
$P = APA^T + Q$

2

process noise

$K = \frac{PH}{HPH^T + R}$ sensor noise

$x_k = x + K[\boxed{Y} - \boxed{Hx}]$

3.4.

might from different space.

$P = P + [I - KH]P_K$
x_p

5

Y. x different space, Example $Y \to \begin{bmatrix} \rho \\ \phi \\ \dot{\rho} \end{bmatrix}$ $x \to \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}$ diff $= Y - h(x')$

non linear.

1. Extended kalman: linearization. Taylor Expansion.

$h(x)|_{\mu} = h(\mu) + h'(x)[x - \mu]$

Jacobian matrix.

$$H_j = \begin{bmatrix} \frac{\partial \rho}{\partial x} & \frac{\partial \rho}{\partial y} & \frac{\partial \rho}{\partial \dot{x}} & \frac{\partial \rho}{\partial \dot{y}} \\ \frac{\partial \phi}{\partial x} & - & - & - \\ - & - & - & \frac{\partial \rho}{\partial \dot{y}} \end{bmatrix}$$

2. Uncentered Kalman Filter: sample points. refit a Gaussian.

Generate sigma points (subsample points)

1. state of dimension
2. number of the sigma points

$x = [x_{center} \quad x_{center} \pm \sqrt{(\lambda + n_x) P}]$ with different weight.

design params, scalar, Process covariance

$\lambda = 3 - n_x$ (emperical).

$w_i = \begin{cases} \frac{\lambda}{\lambda + n_\sigma} & i = 0 \\ \frac{1}{2(\lambda + n_\sigma)} & i = 1 \cdots n_\sigma \end{cases}$

UKF Augmentation ← Process noise. $\nu_k = \begin{bmatrix} \nu_{k_1} \\ \nu_{k_2} \end{bmatrix}$

$x_a = \begin{bmatrix} \bar{x} \\ \bar{\nu} \end{bmatrix} \quad P = \begin{bmatrix} \bar{P} & 0 \\ 0 & \bar{Q} \end{bmatrix} \quad \bar{Q} = \nu_k \cdot \nu^T.$

prediction: x_k, $f(x)$, process.

reuse the sigma points for prediction / resample sigma points

skip augmentation in $H(x)$ process

markov localization:

observation list: $z_{1:t} = \{z_t, \cdots z_1\}$. $z_t = [z_t^1 \cdots z_t^k]$

$u_{1:t}$ control variable. $u_{1:t} = [u_t \cdots u_1]$

direct move between consecutive time stamps.

$bel(x_t) = [bel(x_t = 0), \cdots bel(x_t = 99)]$ x. pose.

$P(a|b, c, d) \cdot P(b|c, d) = P(b|a, c, d) \cdot P(a|c, d)$. Total probability

$P(a, b, c, d)$. $\quad P(a, b, c, d)$.

$$P(x_t|z_t, z_{1:t-1}, u_{1:t}, m) = \frac{P(z_t|x_t, z_{1:t-1}, u_{1:t}, m) \cdot P(x_t|z_{1:t-1}, u_{1:t}, m)}{P(z_t|z_{1:t-1}, u_{1:t}, m)}$$

observation model, motion model

$\eta = \frac{1}{P(z_t|z_{1:t-1}, u_{1:t}, m)} = \sum P(z_t|x_t, z_{1:t-1}, u_{1:t}, m) \cdot P(x_t|z_{1:t}, u_{1:t}, m)$

Prior $= \int P(x_t|x_{t-1}, z_{1:t-1}, u_{1:t}, m) P(x_{t-1}|z_{1:t-1}, u_{1:t}, m) \, dx_{t-1}$.

markov Assumption: $\boxed{x_0} \rightarrow \boxed{x_1} \rightarrow \boxed{x_2}$ only rely on the previous state $P(x_t|x_{1:t-1}) = P(x_t|x_1, x_2 \cdots x_{t-1})$

$\Rightarrow P(x_t|x_{1:t-1}) = P(x_t|x_{t-1})$

$\int P(x_t|x_{t-1}, z_{1:t-1}, u_{1:t}, m) P(x_{t-1}|z_{1:t-1}, u_{1:t}, m) \, dx_{t-1}$

$= \int P(x_t|x_{t-1}, u_t, m) P(x_{t-1}|z_{1:t-1}, u_{1:t-1}, m) \, dx_{t-1}$.

$\underline{P(x_t|z_{1:t-1}, u_{1:t}, m)} = \int P(x_t|x_{t-1}, u_t, m) \underline{P(x_{t-1}|z_{1:t-1}, u_{1:t-1}, m)} \, dx_{t-1}$

Recursive

observation model: $P(z_t \mid x_t, z_{1:t-1}, u_{1:t}, m)$

$x_t \rightarrow z_t \leftarrow z_{1:t-1}$, $u_{1:t} \rightarrow z_t \leftarrow m$ markov Assumption $x_t \rightarrow z_t \leftarrow m$.

no affect : $P(z_t \mid x_t, m)$.

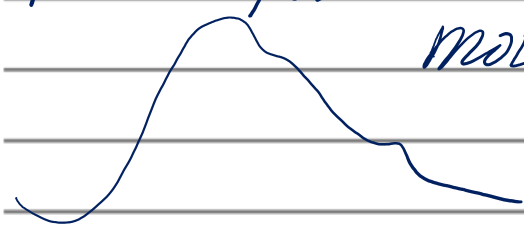
$= P(z_t^1 \cdots z_t^k \mid x_t, m)$

$= \prod_{k=1} P(z_t^k \mid x_t, m)$.

$bel(x) = P(x_t \mid z_t, z_{1:t-1}, u_{1:t}, m)$

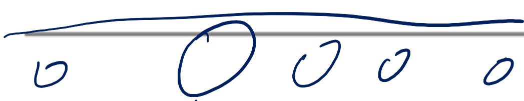
$= \eta P(z_t \mid x_t, m)\, \overline{bel}(x_t)$.

Particle filter.



more expensive than kalman.

$P(x) = \sum_{j=1}^{J} w^j \delta_{x[j]}(x)$.



more weight more samples.

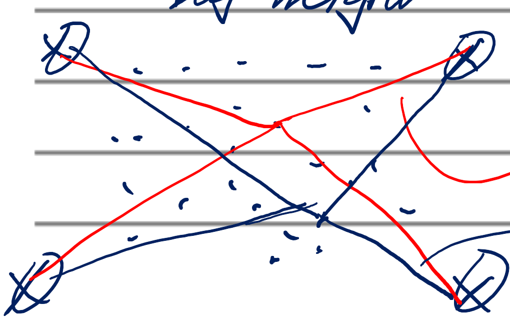
Importance sampling principle

1. proposal distribution.
2. correction. account for the difference between "q" and "f" using a weight. $w = f/q$.

Target f. Proposal q.

Pre condition $f(x)>0$. $q(x)>0$.

big difference big weight.



landmark.

predicted weight > importance weight

actual

resampling $\alpha_i = \frac{w_i}{W}$ normalized weight.

α_1 α_2
α_3 . - - -
α_n

particles with normalized weight as the probability.

$\Downarrow$

draw with replacement

$\Downarrow$

P_1 . - - -

P_n.