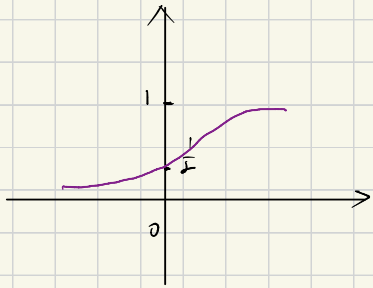
逻辑回归

线性回归 w^Tx $\xrightarrow[\text{sigmoid}]{\text{激活函数}}$ 线性分类 0, 1

Sigmoid 函数 $(R \to 0,1)$

$$\sigma(z) = \frac{1}{1+e^{-z}}$$



当 $z \to +\infty$ $\quad \lim \sigma(z) = 1$

当 $z = 0$ $\quad \lim \sigma(z) = \frac{1}{2}$

当 $z \to -\infty$ $\quad \lim \sigma(z) = 0$

$z = w^Tx \to p$

$$p_1 = P(y=1|x) = \sigma(w^Tx) = \frac{1}{1+e^{-w^Tx}}, \quad y=1 \Rightarrow \varphi(x;w)$$

$$p_0 = P(y=0|x) = 1 - P(y=1|x) = \frac{e^{-w^Tx}}{1+e^{-w^Tx}}, \quad y=0 \Rightarrow 1-\varphi(x;w)$$

$$\Rightarrow P(x|y) = p_1^{\,y} \, p_0^{\,1-y}$$

$$MLE = \hat{w} = \arg\max_w \log P(Y|X)$$

$$= \arg\max_w \log \prod_{i=1}^{N} P(y_i|x_i)$$

$$= \arg\max_w \sum_{i=1}^{N} [y_i \log p_1 + (1-y_i) \log p_0]$$

$$= \arg\max_w \sum_{i=1}^{N} y_i \log \varphi(x_i;w_i) + (1-y_i) \log (1-\varphi(x_i;w_i))$$ 交叉熵