## a) Construct a Block Diagram

The system involves both physical and cyber components. Here's how we can represent them:

1. **Physical Components:**

   - **Mass-Spring-Damper System**: This is represented by the state-space equation in Problem 1:
   $$\dot{z} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b}{m} \end{bmatrix} z + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u$$

   Where:

   - $z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$ is the state vector (with $z_1$ as position and $z_2$ as velocity),
   - $b = 2$, $m = 4$,
   - $u$ is the control input applied to the system.
2. **Cyber Components:**

   - **Controller**: The control input $u$ is generated based on the state $z$ using the control law $u = \begin{bmatrix} k_1 & k_2 \end{bmatrix} z$.
   - The controller processes the output $y = z$, which is the state (position and velocity), and applies feedback using the gains $k_1$ and $k_2$.

   **Block Diagram Explanation**:

   - The plant (mass-spring-damper system) takes the control input $u$ and produces the output $y = z$.
   - The controller uses $y = z$ as the feedback and computes $u = k_1 z_1 + k_2 z_2$ to control the system.

## b) Define a Discrete Algorithm for the Cyber Part

To model the cyber part (controller) in discrete time, we can discretize the control law $u = \begin{bmatrix} k_1 & k_2 \end{bmatrix} z$.

For a discrete-time control algorithm:

1. Sample the state $z[n]$ at each time step $n$.
2. Compute the control input at each time step:

   $$u[n] = k_1 z_1[n] + k_2 z_2[n]$$

3. Apply the control input $u[n]$ to the system for the next time step.

## c) Trial and Error: Find $k_1$ and $k_2$ to Drive the System to Zero

After simulating in MATLAB, appropriate values of $k_1$ and $k_2$ can be found by trial and error. Typical values are:

- $k_1 = -10$
- $k_2 = -4$

These values ensure that, regardless of initial conditions, both the position $z_1$ and velocity $z_2$ will approach zero.

## d) Model of the Closed-Loop System

The closed-loop system is obtained by substituting the control law $u = \begin{bmatrix} k_1 & k_2 \end{bmatrix} z$ into the original system equation:

$$\dot{z} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{b}{m} \end{bmatrix} z + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} z$$

This results in:

$$\dot{z} = \begin{bmatrix} 0 & 1 \\ \frac{k_1}{m} & \frac{k_2 - b}{m} \end{bmatrix} z$$

For $b = 2$ and $m = 4$, the closed-loop system is:

$$\dot{z} = \begin{bmatrix} 0 & 1 \\ \frac{k_1}{4} & \frac{k_2 - 2}{4} \end{bmatrix} z$$

Since this is a state-space equation with constant coefficients, the system is **linear time-invariant (LTI)**.

## e) Analytical Expression of the Trajectories

To find the trajectories, solve the differential equation for the closed-loop system. The general solution to a second-order linear system like this can be expressed in terms of the eigenvalues $\lambda_1$ and $\lambda_2$ of the system matrix $A = \begin{bmatrix} 0 & 1 \\ \frac{k_1}{4} & \frac{k_2 - 2}{4} \end{bmatrix}$.

The eigenvalues are the roots of the characteristic equation:

$$\det(A - \lambda I) = 0$$

This will give two eigenvalues, and the solution for $z_1(t)$ and $z_2(t)$ will be a combination of exponential terms involving these eigenvalues.

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