Assignment - 4

Quantum Computation and Machine Learning

April 2023

Module 4: Introduction to quantum algorithm

1. Consider $y \in \{0,1\}$ to be unknown bit and U_f the unitary operator $|\phi\rangle \longrightarrow (-1)^y |\phi\rangle$. Design a network consisting of one conditional U_f gate and two Hadamard gates which can be used to determine y.

2. Consider the quantum state $\frac{|0\rangle+(-1)^x|1\rangle}{\sqrt{2}}$ where $x = 0 \; or \; 1.$. Which single operator application will yield value of x.

3. Prove that

$$H^{\otimes n} |x\rangle_n = \sum_y (-1)^{x \cdot y} |y\rangle_n$$

where x_n, y_n are n-bit strings and x.y is bit-wise multiplication, i.e., x.y $= x_1 y_1 \oplus x_2 y_2 \oplus ... x_n y_n$

4. Suppose an n–bit input string gets mapped by a balanced function and a total of 'r' queries are made on the output string one–by–one. Show that the probability of detection of balanced nature of function increases with the number of queries r.

5. In Berstein–Vazirani algorithm, suppose that f(x) = x·a, where the multiplication is bit–wise, and the final state is given by

$$\sum_{x,y \in \{0,1\}^n} (-1)^{f(x)+x \cdot y} |y\rangle$$

Show that the probability of getting a y $\neq$ a is zero.

6. Write down the complete circuit(including the oracle U_f in terms of CNOT operations) that Execute Bernstein-Vazirani for $f(x) = a.x$ with $a = 100101$

7. In the Bernstein-Vazirani algorithm, suppose a = 0110. Analyze each step of the protocol and write down the overall state after each step to show explicitly how the algorithm works to find a.

8.Apply QFT and Find the New Quantum State:

(i) $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$

(ii) $|\psi\rangle = |10\rangle$

(iii)$|\psi\rangle = \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$

9. Consider the task of constructing a quantum circuit to compute $|x\rangle \longrightarrow |x + y\, mod 2^n\rangle$, where y is a fixed constant, and $0 \leq x < 2^n$. Show that one efficient way to do this, for values of y such as 1, is to first perform a quantum Fourier transform, then to apply single qubit phase shifts, then an inverse Fourier transform. What values of y can be added easily this way, and how many operations are required?

10. Show by explicit calculation that the final state after the first stage of quantum phase estimation algorithm may be written as

$$\frac{1}{2^{n/2}}(|0\rangle + e^{2\pi i 0.\phi_n}|1\rangle)(|0\rangle + e^{2\pi i 0.\phi_{n-1}\phi_n}|1\rangle)...(|0\rangle + e^{2\pi i 0.\phi_1\phi_2....\phi_n}|1\rangle)|u\rangle$$