Assignment - 2

Quantum Computation and Machine Learning

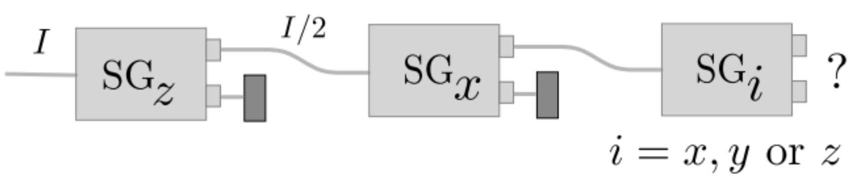
March 2023

Module 2: Postulates of Quantum Computing

1. Which of the following vectors represent valid quantum states? why or why not??

a) $\begin{bmatrix}0\\1\end{bmatrix}$ b) $\begin{bmatrix}0.5\\0.5\end{bmatrix}$ c) $\begin{bmatrix}0\\1\\1\\0\end{bmatrix}$ d) $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$ e) $\frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\end{bmatrix}$ f) $\frac{1}{\sqrt{2}}\begin{bmatrix}0.3\\0.7\end{bmatrix}$ g) $\begin{bmatrix}0\\0\\1\\0\end{bmatrix}$

2. What is the expected output from the series of Stern-Gerlach Machines shown Below:



3. Suppose A and B are commuting Hermitian operators. Prove that:

$$exp(A)\ exp(B) = exp(A + B)$$

4. In a three-dimensional vector space spanned by the *orthonormal* basis $\{|1\rangle, |2\rangle, |3\rangle\}$, the linear operators A and B has the following action on the basis vectors :

$$A|1\rangle = |1\rangle, \qquad A|2\rangle = \frac{1}{\sqrt{2}}|2\rangle - \frac{1}{\sqrt{2}}|3\rangle, \qquad A|3\rangle = \frac{1}{\sqrt{2}}|2\rangle + \frac{1}{\sqrt{2}}|3\rangle$$

$$B|1\rangle = |1\rangle, \qquad B|2\rangle = \frac{\sqrt{3}}{2}|2\rangle + \frac{1}{2}|3\rangle, \qquad B|3\rangle = -\frac{1}{2}|2\rangle + \frac{\sqrt{3}}{2}|3\rangle$$

Is it possible to find a common set of eigenvectors for A and B ? Justify your answer.

5. Show that the average value of the observable $X_1 Z_2$ for a two qubit system measured in the state $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$ is zero.

6. Show that the set of four Bell States

$$|\psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$$

$$|\phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$$

Forms an *orthonormal* basis for four dimensional Hilbert Space.

7. Consider a state $\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$. Suppose all the three qubits of this state are measured in the $\left\{\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right\}$ basis. What are the possible joint outcomes of these measurements? With what probabilities do they occur? Suppose the first qubit is measured in the $\left\{\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right\}$ basis and the second and third is measured in $\left\{\frac{1}{\sqrt{2}}(|0\rangle + i\,|1\rangle), \frac{1}{\sqrt{2}}(|0\rangle - i\,|1\rangle)\right\}$ basis. Again What are the possible joint outcomes of these measurements? With what probabilities do they occur? What is the expected value of the observable $\sigma_x(1) \otimes \sigma_x(2) \otimes \sigma_x(3)$? of the observable $\sigma_x(1) \otimes \sigma_x(2) \otimes \sigma_x(3)$?

8. Find out the transformation U that transform the ordered basis $B_1 \equiv \{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$ into another ordered basis $B_2 \equiv \{|\phi^+\rangle, |\phi^-\rangle, |\psi^+\rangle, |\psi^-\rangle\}$, where $|\psi^{\pm}\rangle$ and $|\phi^{\pm}\rangle$ are same as defined in ques. 6. let this unitary transformation be represented as $U = e^{iH}$ where H is Hermitian. Find out H.

9. Let $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ be a qubit state. The density matrix in this case is $\rho = |\psi\rangle\langle\psi|$, which can be determined using the vector representations of $|\psi\rangle$ and $\langle\psi|$. Calculate eigenvectors and eigenvalues of density matrix ρ.

10. Consider the operator (4 x 4 matrix) in the Hilbert space $\mathbb{C}^2$

$$\rho = \frac{1}{4}(1-\epsilon)\mathbb{I}_4 + \epsilon(|0\rangle \otimes |0\rangle)(\langle 0| \otimes \langle 0|)$$

where ϵ is a real parameter with $\epsilon \in [0, 1]$ and

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

Does ρ define a density matrix?