Assignment 5

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In [1]:
```
from qiskit import *
import numpy as np
from qiskit.visualization import plot_state_city, plot_bloch_multivector, plot_histogram
from qiskit.quantum_info.operators import Operator, Pauli
from qiskit import IBMQ, Aer, transpile, execute
from scipy.fft import fft
from math import pi
from math import gcd
from qiskit.quantum_info import Statevector
from IPython.display import display, Latex
from qiskit.circuit.library import QFT
```

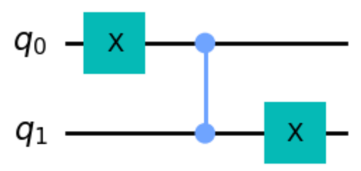
1. In Grover's algorithm, how do we flip the sign of the marked element without knowing which element is marked?

The marked element is flipped in sign by the Oracle function. It is also known as phase oracle. The oracle is designed to be specific for phase being flipped.

Example #1 In order to flip the state of 01 state of the complete state of 2 qubits.

In [2]:
```
no_of_qubits = 2
no_of_search_space = 2**no_of_qubits

oracle = QuantumCircuit(no_of_qubits, name="oracle")
oracle.x(0)
oracle.cz(0,1)
oracle.x(1)
oracle.to_gate()

oracle.draw(output="mpl")
```

Out[2]:



In [3]:
```
OracleTest = QuantumCircuit(2, name="OracleTest")
OracleTest.h([0, 1])
OracleTest.append(oracle, [0,1])
state = Statevector(OracleTest)
state.draw(output="latex")
```

Out[3]:

$$\frac{1}{2}|00\rangle - \frac{1}{2}|01\rangle + \frac{1}{2}|10\rangle + \frac{1}{2}|11\rangle$$

2. Suppose that there are 32 elements in the search space.

How many qubits do you need to represent 32 elements? (Don't count any extra qubits, only the ones required to represent the elements).

$2^n = 32$

$n = 5$

3. Compute the DFT of (3,4)

DFT transformation matrix for 2x2 is as below.

Transformation matrix = $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & e^{2.\pi.i/2} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$

$$\begin{bmatrix} 3 \\ 4 \end{bmatrix} \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

4. What happens if we apply DFT/QFT twice.

To explain in terms of DFT, apply DFT would return the signal to time domain but scaled by a factor. Observe the values below for two inputs at even iterations.

In [4]:
```
tinput = [3,4]

print('DFT input =', tinput )

for i in range(10):
    tinput = fft(tinput)
    print('DFT iter', i+1,' =', tinput )
```

```
DFT input = [3, 4]
DFT iter 1  = [ 7.-0.j -1.-0.j]
DFT iter 2  = [6.-0.j 8.+0.j]
DFT iter 3  = [14.+0.j -2.-0.j]
DFT iter 4  = [12.+0.j 16.+0.j]
DFT iter 5  = [28.+0.j -4.+0.j]
DFT iter 6  = [24.+0.j 32.+0.j]
DFT iter 7  = [56.+0.j -8.+0.j]
DFT iter 8  = [48.+0.j 64.+0.j]
DFT iter 9  = [112.+0.j -16.+0.j]
DFT iter 10  = [ 96.+0.j 128.+0.j]
```

In [5]:
```
tinput = [4,5]

print('DFT input =', tinput )

for i in range(10):
    tinput = fft(tinput)
    print('DFT iter', i+1,' =', tinput )
```

```
DFT input = [4, 5]
DFT iter 1  = [ 9.-0.j -1.-0.j]
DFT iter 2  = [ 8.-0.j 10.+0.j]
DFT iter 3  = [18.+0.j -2.-0.j]
DFT iter 4  = [16.+0.j 20.+0.j]
DFT iter 5  = [36.+0.j -4.+0.j]
DFT iter 6  = [32.+0.j 40.+0.j]
DFT iter 7  = [72.+0.j -8.+0.j]
DFT iter 8  = [64.+0.j 80.+0.j]
DFT iter 9  = [144.+0.j -16.+0.j]
DFT iter 10  = [128.+0.j 160.+0.j]
```

In case of quantum, after QFT twice the state returns back to compuational basis from entangled stated but will be scaled (or in this case bitwise operated upon.) For example, the following is for a 4 bit QFT transformation.

QFT iteration 0

$$|1000\rangle$$

QFT iteration 1

$$\frac{1}{4}|0000\rangle - \frac{1}{4}|0001\rangle + \frac{i}{4}|0010\rangle - \frac{i}{4}|0011\rangle + (0.176776695297 + 0.176776695297i)|0100\rangle + (-0.176776695297 - 0.1767766952$$
$$+ (0.095670858091 - 0.230969883128i)|1011\rangle + (0.095670858091 + 0.230969883128i)|1100\rangle + (-0.095670858091 - 0.230969$$
$$+ (-0.230969883128 + 0.095670858091i)|1110\rangle + (0.230969883128 - 0.095670858091i)|1111\rangle$$

QFT iteration 2

$$|1111\rangle$$

QFT iteration 3

$$\frac{1}{4}|0000\rangle - \frac{1}{4}|0001\rangle - \frac{i}{4}|0010\rangle + \frac{i}{4}|0011\rangle + (0.176776695297 - 0.176776695297i)|0100\rangle + (-0.176776695297 + 0.1767766952$$
$$+ (0.095670858091 + 0.230969883128i)|1011\rangle + (0.095670858091 - 0.230969883128i)|1100\rangle + (-0.095670858091 + 0.230969$$
$$+ (-0.230969883128 - 0.095670858091i)|1110\rangle + (0.230969883128 + 0.095670858091i)|1111\rangle$$

QFT iteration 4

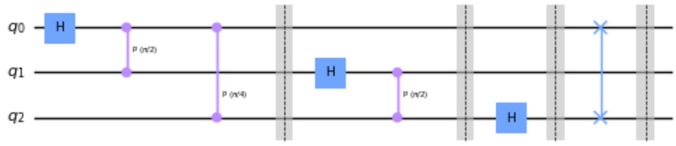
$$|1000\rangle$$

5. Draw the QFT circuit for three, four and five qubits.

Note that when we have an odd number, there is always one qubit in the middle that does not need to be swapped.

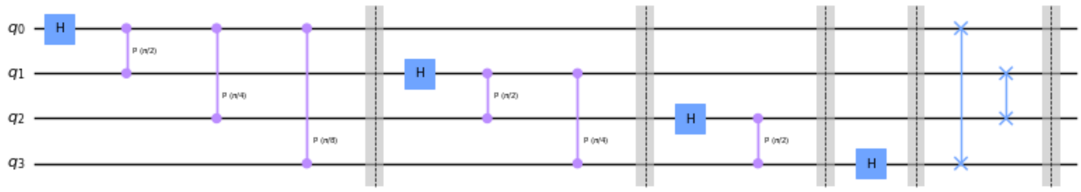
In [6]:
```python
NO_QUBITS = 3

QC = QuantumCircuit(NO_QUBITS)

def myQFT(qc, no_of_Qubits):
    for q in range(0, no_of_Qubits):
        qc.h(q)
        for i in range(q+1, no_of_Qubits):
            qc.cp(pi/(2**(i-q)), i, q)
        qc.barrier()
    for q in range(no_of_Qubits//2):
        qc.swap(q, no_of_Qubits-q-1)
    qc.barrier()
myQFT(QC, NO_QUBITS)
QC.draw(output="mpl", fold= 200, scale = 0.5)
```

Out[6]:



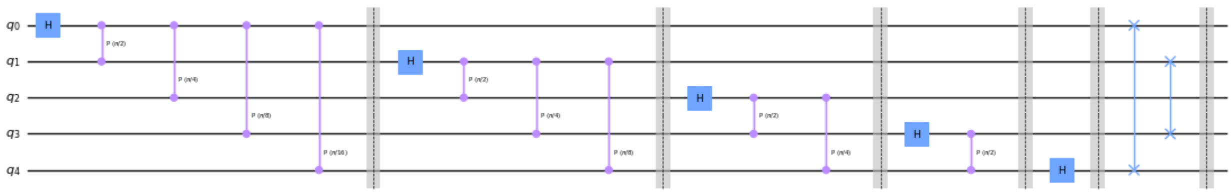
In [7]:
```python
NO_QUBITS = 4

QC = QuantumCircuit(NO_QUBITS)

def myQFT(qc, no_of_Qubits):
    for q in range(0, no_of_Qubits):
        qc.h(q)
        for i in range(q+1, no_of_Qubits):
            qc.cp(pi/(2**(i-q)), i, q)
        qc.barrier()
    for q in range(no_of_Qubits//2):
        qc.swap(q, no_of_Qubits-q-1)
    qc.barrier()
myQFT(QC, NO_QUBITS)
QC.draw(output="mpl", fold= 200, scale = 0.5)
```

Out[7]:



In [8]:
```python
NO_QUBITS = 5

QC = QuantumCircuit(NO_QUBITS)

def myQFT(qc, no_of_Qubits):
    for q in range(0, no_of_Qubits):
        qc.h(q)
        for i in range(q+1, no_of_Qubits):
            qc.cp(pi/(2**(i-q)), i, q)
        qc.barrier()
    for q in range(no_of_Qubits//2):
        qc.swap(q, no_of_Qubits-q-1)
    qc.barrier()
myQFT(QC, NO_QUBITS)
QC.draw(output="mpl", fold= 200, scale = 0.5)
```

Out[8]:



6. which states do you expect to observe more frequently?

Assuming that the question relates to QPE, none of the answers are correct. Please see the reasoning below.

if $\phi = \frac{3}{16}$, then the decimal notation of that expected is 0.0011 $\frac{1}{8} + \frac{1}{16} = \frac{3}{16}$

For phase rotation, the angle should be inserted into Unitary definition = $e^{2\pi i \phi}$

Inserting phi, angle rotation = $e^{2\pi i \frac{3}{16}} = e^{\frac{3}{8}\pi i}$

The following code demonstrates the usage of the angle $3 * pi/8$

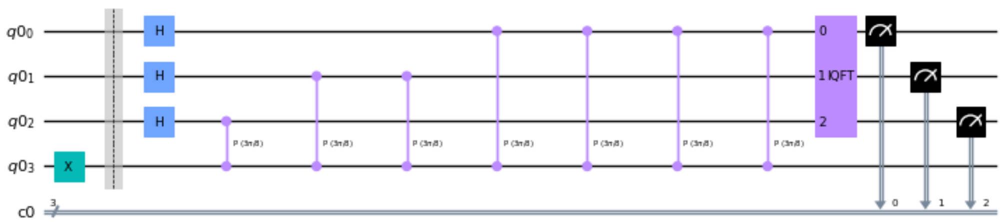
In [9]:
```
Precision = 3

QREG = QuantumRegister(Precision+1)
CREG = ClassicalRegister(Precision)

QC = QuantumCircuit(QREG, CREG)
QC.x(Precision)
QC.barrier()

angle = 3*pi/8

for i in range(Precision):
    QC.h(i)

for i in range(Precision):
    for t in range(2**(i)):
        QC.cp(angle, Precision-i-1, Precision)

QC.append(QFT(Precision, 0, inverse=True, do_swaps=False), range(Precision) )
state = Statevector(QC)
state.draw(output="latex")
```

Out[9]:

$$(0.125 + 0.187075720333i)|1000\rangle + (0.125 + 0.628417436516i)|1001\rangle + (0.125 - 0.628417436516i)|1010\rangle + (0.125 - 0.1870757$$
$$+ (0.125 - 0.08352232974i)|1100\rangle + (0.125 - 0.024864045922i)|1101\rangle + (0.125 + 0.024864045922i)|1110\rangle + (0.125 + 0.083522$$

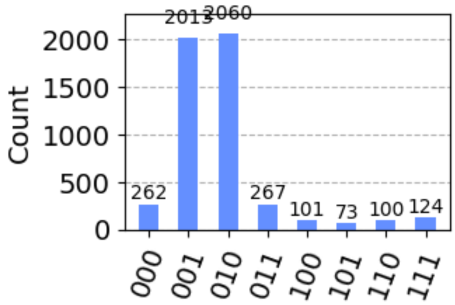
In [10]:
```
QC.measure(range(Precision),range(Precision))
QC.draw(output="mpl", scale=0.5, fold=1000)
```

Out[10]:



In [11]:
```
backend = Aer.get_backend('aer_simulator_statevector')
QC.save_statevector()
result = backend.run(transpile(QC, backend), shots=5000).result()
out_state = result.get_statevector()
plot_histogram(result.get_counts(), figsize=(3,2))
```

Out[11]:



Normally the circuit simulation would require 4 Qubits for precision. However since we only gave 3 Qubits, we would not be able to get the right precision, however it would be an approximation close the the real value. The small method below would convert a binary representation to decimal. As one can derive, the state1 and state 2 are equidistant and 33% approximate from the original value.

The rest of the values are further away in the options provided.

In [12]:
```
def convertBinaryToDecimal(str):
    str = list(str)
    o = 0
    multiplication_factor = 2
    while str:
        if int(str.pop(0)):
            o += 1/multiplication_factor
        multiplication_factor *= 2
    return o

orig = convertBinaryToDecimal("0011")
state1 = convertBinaryToDecimal("001")
state2 = convertBinaryToDecimal("010")

print ("0011 =",orig )
```

```
print ("001 =", convertBinaryToDecimal("001"), abs(state1-orig), state1/orig)
print ("010 =", convertBinaryToDecimal("010"), abs(state2-orig), state2/orig)
```

```
0011 = 0.1875
001 = 0.125 0.0625 0.6666666666666666
010 = 0.25 0.0625 1.3333333333333333
```

7. Implement shors algorithm for N = 15 for factorization

Step 1 : Initialize the circuit

In [13]:

```
first_register = 4
second_register = 4
a = 11

QC = QuantumCircuit(first_register+second_register, first_register)
QC.h(range(first_register))
QC.x(first_register+second_register-1)
```

Out[13]:

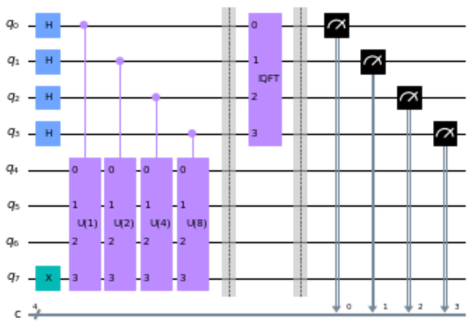
```
<qiskit.circuit.instructionset.InstructionSet at 0x7fb01af0a670>
```

Step 2: Create an x mod 15 gate.

In [14]:

```
def a_x_mod15(a, x):
    U = QuantumCircuit(4)
    for iteration in range(x):
        U.swap(1,3)
        U.swap(0,2)
        for q in range(4):
            U.x(q)
    U = U.to_gate()
    U.name = f"U({x})"
    c_U = U.control()
    return c_U

for x in range(first_register):
    exponent = 2**x
    QC.append(a_x_mod15(a, exponent), [x] + list(range(first_register, first_register+second_register)))

QC.barrier()
QC.append(QFT(first_register,do_swaps=False).inverse(),range(first_register))
QC.barrier()
for i in range(first_register):
    QC.measure(i, i)

QC.draw(output="mpl", scale=0.4)
```

Out[14]:

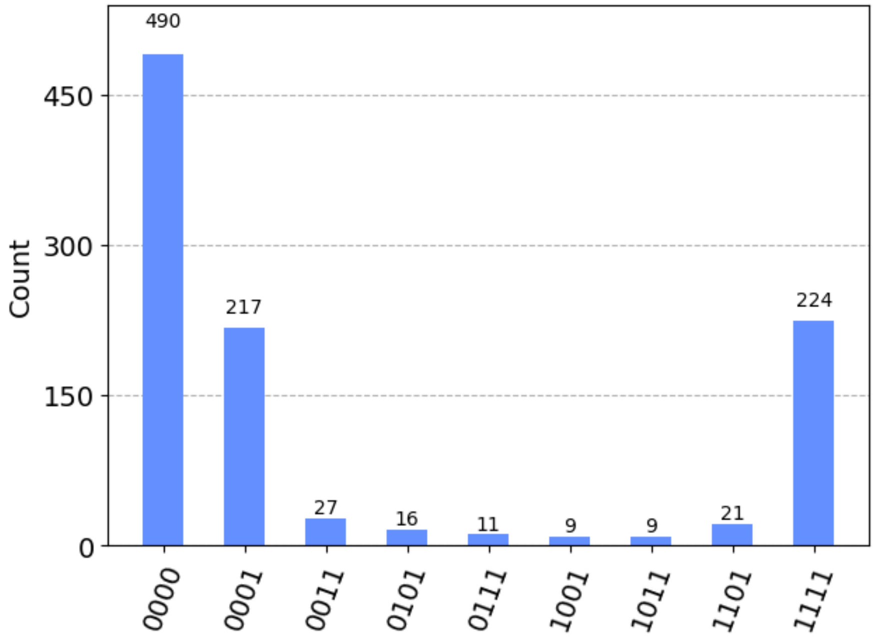


Step 3 Execute and get counts

In [15]:

```
simulator = Aer.get_backend('qasm_simulator')
counts = execute(QC, backend=simulator).result().get_counts(QC)

plot_histogram(counts)
```

Out[15]:



Step 4 Calcuate factos from the values measured.

In [16]:

```
for i in counts:
    measured_value = int(i[::-1], 2)
    if measured_value % 2 != 0:
        continue
    x = int((a ** (measured_value/2)) % 15)
    if (x + 1) % 15 == 0:
        continue
    factors = gcd(x + 1, 15), gcd(x - 1, 15)
    print(factors)
```

```
(1, 15)
(3, 5)
(3, 5)
(1, 15)
(1, 15)
```

8. How to implement Grover's algorithm?

The following steps can help to implement the grovers algorithm.

Step 1

Find the right number of Qubits required from the search space (S).

$N = math.ceil(log_2 S)$

Step 2

Design the right oracle required to phase flip the marked states (M). An example is mentioned in the #1.

Step 3

Design the diffusion circuit to replicate the formula $H^{\otimes n}\left[2|\psi\rangle\langle\psi|-I\right]H^{\otimes n}$ The goal of this step is for amplitude amplification.

Step 4

Repeat step 2 and 3 as grover iteration atleast $O(sqrt(\frac{S}{M}))$

9. What is an oracle in quantum computing ?

In quantum computing, an oracle is a black box function that takes a quantum state as input and performs a specific computation on it, returning the result as an output quantum state.

Oracles are used in a variety of quantum algorithms, including the Grover's algorithm for search and Shor's algorithm for factoring large numbers.

10. Compute numerically the speed of QFT and FFT for n= 10, 20, 50, 100

QFT would require $O(n^2)$ steps to compute. FFT uses $O(n.2^n)$ steps to compute

N	QFT	FFT
10	100	10.2^10
20	400	20971520
50	2500	56294995342131200
100	10000	126765060022822940149670320537600

In [17]:
```
qft_gates = lambda x: x**2
fft_gates = lambda x: x*(2**x)

for n in [10,20,50,100]:
    print(qft_gates(n), fft_gates(n))
```

```
100 10240
400 20971520
2500 56294995342131200
10000 126765060022822940149670320537600
```