Assignment 6

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In [1]:
```
from qiskit import *
from qiskit.quantum_info import *
from qiskit.extensions import *
from random import randrange
import numpy
from qiskit.circuit.library import CPhaseGate
import pennylane as qml
import numpy as np

from qiskit.extensions import HamiltonianGate
from qiskit.quantum_info import Statevector

np.set_printoptions(precision=3)

import math
```

1. Fixed point Binary notation of decimal numbers.

Number	Fixed point binary
53	0 110101
26.5	0 11010.1
-43.625	1 101011.101
0.6875	0 0.1011
55.66	0 110111.101010001111010111

2. no of qubits required for basis encoding

F1	F2	F3	F4	F5	F6	F7	F8	F9	F10	F11	F12	F13	F14	F15	F16	F17	F18	F19	F20
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6

1200 (6 *10* 20) qubits for 10 features, 20 data points 6 bits each (assumig that 6 bits contain the full representation.)

3. Superposition can improve the number of qubits required.

One training iteration data would be loaded at one shot which is one row of the table in problem #2.

That is 6*20 = 120 qubits.

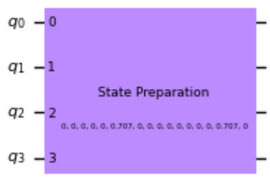
Loading and storage two branches - 2*120 = 240. Two aniclla bits, 242 Qubits.

4. Produce |0101> + |1110> using basis encoding technique.

Method 1 : Initialize

In [2]:
```
from qiskit import *
from qiskit.quantum_info import Statevector
import math
import random

data = [0,0,0,0,0, 1/math.sqrt(2), 0,0,0,0,0,0,0,0, 1/math.sqrt(2), 0]
total = math.sqrt(sum([i*i for i in data]))
normalized_data = [i/total for i in data]

num_qubits = 4
circ = QuantumCircuit(num_qubits)
circ.prepare_state(normalized_data, [0, 1, 2, 3])
circ.draw(output="mpl", scale=0.5)
```

Out[2]:



In [3]:
```
state = Statevector(circ)
state.draw(output="Latex")
```

Out[3]:

$$\frac{\sqrt{2}}{2}|0101\rangle + \frac{\sqrt{2}}{2}|1110\rangle$$

Method 2 : Superposition

A. If the number of bits of training data is 6 bits of a feature, then Processing branch + Storage branch + Ancilla Qubits = 6 + 6 + 2 = 14

B. Split the branches by using H gate on A2.

C. Load the training vector (all features of one training vector) into the processing branch.

D. Move the data to storage branch.

E. Split the processing branch with Unitary gate which calculates $U(\mu) = \begin{bmatrix} \sqrt{\frac{\mu-1}{\mu}} & \frac{1}{\sqrt{\mu}} \\ \frac{-1}{\sqrt{\mu}} & \sqrt{\frac{\mu-1}{\mu}} \end{bmatrix}$

F. Flip A1 for branch where loading register == storage register.

G. Reset the storage register and loading register of both both branches.

H. Repeat from C, until all vectors are loaded.

In [4]:
```
## Load the first training vector
from qiskit.circuit.library import MCXGate
from qiskit import *
from qiskit.quantum_info import *
from qiskit.extensions import *
from random import randrange
import numpy
from qiskit.circuit.library import CPhaseGate

L = QuantumRegister(4, name="L")
A = QuantumRegister(2, name="A")
S = QuantumRegister(4, name="S")

C = ClassicalRegister(4)
QC = QuantumCircuit(S,A,L,C)

QC.barrier()

# data = [format(randrange(1,100), "#010b")[6:] for _ in range(2)]
data = ['0101', '1110']
QC.h(A[0])
m = 1
M = 2**len(data[0])

for d in data:
    print (d[::-1])
    QC.barrier()
```

```
    for i, v in enumerate(d):
        if v == '1':
            QC.x(L[i])

    for i in range(4):
        if d[i] == '1':
            QC.ccx(A[0], L[i], S[i])
    QC.barrier()

    QC.cx(A[0], A[1])
    Mu = M + 1 - m
    UMu = UnitaryGate([[math.sqrt((Mu-1)/(Mu)),1/math.sqrt(Mu)],[-1/math.sqrt(Mu),math.sqrt((Mu-1)/(Mu))]], label="
    QC.append(UMu, [A[1], A[0]])
    QC.barrier()

    gate = MCXGate(8)

    # ## reset the first training vector
    for i, v in enumerate(d):
        if v == '0':
            QC.x(L[i])
            QC.x(S[i])

    QC.barrier()
    QC.append(gate, [0,1,2,3,6,7,8,9,5])
    QC.barrier()

    # ## reset the first training vector
    for i, v in enumerate(d):
        if v == '0':
            QC.x(L[i])
            QC.x(S[i])

    for i, v in enumerate(d):
        if v == '1':
            QC.x(L[i])


    for i, v in enumerate(d):
        if v == '1':
            QC.cx(A[0], S[i])
    m += 1
state = Statevector(QC)
state.draw(output="Latex")
```

```
1010
0111
```

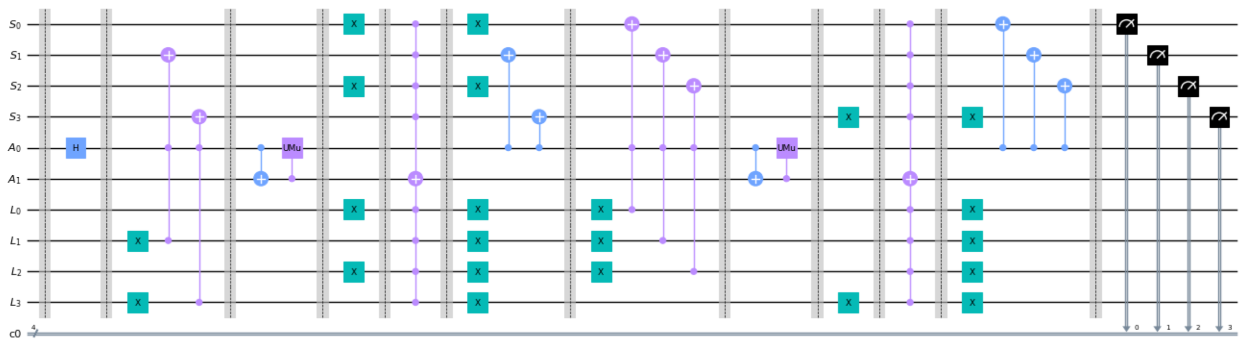
Out[4]:

$$\frac{\sqrt{2}}{2}|0000000000\rangle + 0.176776695297|0000000111\rangle + 0.176776695297|0000001010\rangle + \frac{\sqrt{7}}{4}|0000010000\rangle$$

In [5]:

```
QC.barrier()
for i in range(4):
    QC.measure(S[i], C[i])

QC.draw(output="mpl", scale=0.5, fold=1000)
```

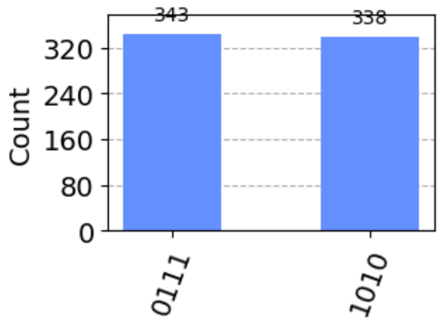
Out[5]:



In [6]:

```
backend = Aer.get_backend("aer_simulator")

result = execute(QC, backend=backend, shots=10000).result()

from IPython.display import Latex
from qiskit.visualization import *

state_to_latex = result.get_counts()

# removing for better visualization
del state_to_latex['0000']

plot_histogram(state_to_latex, figsize=(3,2))
```

Out[6]:



5.How many Qubits may be required for the state preperation in amplitude encoding ?

Total number of data points for all rows of data = 10*20 = 200

Number of qubits required = math.ceil(math.log(200,2)) = 8 qubits

6. Product the state below in amplitude encoding.

{0.2, 0.5}, {0.2, 0.1}

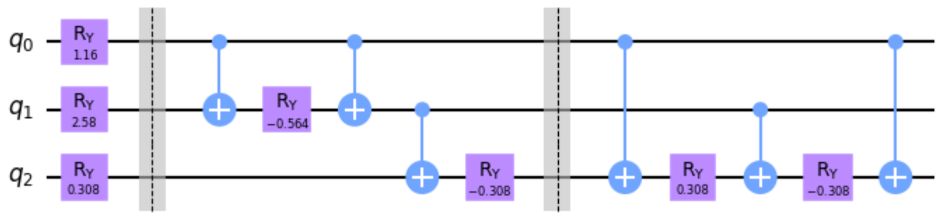
The following is an example of mottgen state preparration circuit with uniform rotation.

3 Qubit implementation

Qiskit

In [7]:
```
QC = QuantumCircuit(3)
QC.ry(1.15928, 0)
QC.ry(2.57765, 1)
QC.ry(0.30774, 2)
QC.barrier()
QC.cx(0,1)

QC.ry(-0.56394, 1)
QC.cnot(0,1)
QC.cnot(1,2)
QC.ry(-0.30774, 2)
QC.barrier()
QC.cnot(0,2)
QC.ry(0.30774, 2)
QC.cnot(1,2)
QC.ry(-0.30774, 2)
QC.cnot(0,2)

QC.draw(output="mpl", scale=0.75)
```

Out[7]:



In [8]:
```
S = Statevector(QC)
S.draw(output="Latex")
```

Out[8]:

$$0.447212589948|000\rangle + 7.26716 \cdot 10^{-7}|001\rangle + 0.707107248889|010\rangle + 0.447213680738|011\rangle + 0.316228021719|111\rangle$$

Pennylane

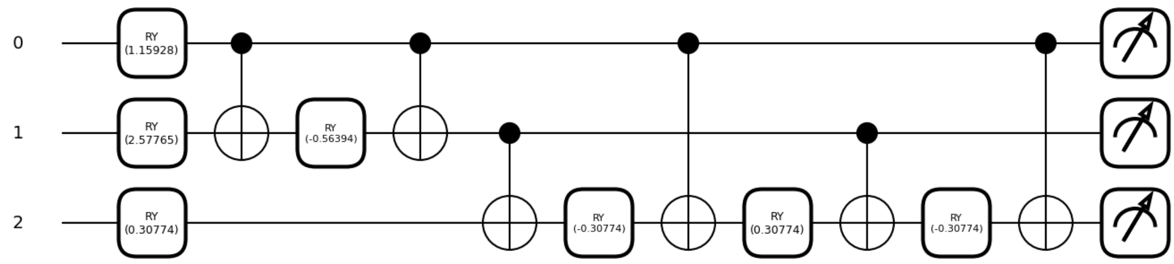
In [9]:
```
dev = qml.device('default.qubit', wires=3)

@qml.qnode(dev)
def circuit(state):
    qml.MottonenStatePreparation(state_vector=state, wires=range(3))
    return qml.state()

state = np.array([0.2, 0, 0.5, 0, 0, 0, 0.2, 0.1])
```

```
state = [math.sqrt(i) for i in state]
print (state)
```

[0.4472135954999579, 0.0, 0.7071067811865476, 0.0, 0.0, 0.0, 0.4472135954999579, 0.31622776601683794]

In [10]:
```
print(qml.draw_mpl(circuit, decimals=5, expansion_strategy="device", max_length=80)(state))
```

(<Figure size 1400x400 with 1 Axes>, <Axes: >)



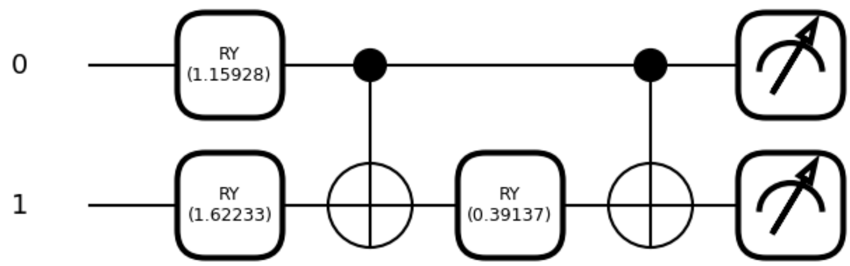
2 Qubit implementation

In [11]:
```
dev = qml.device('default.qubit', wires=3)

@qml.qnode(dev)
def circuit(state):
    qml.MottonenStatePreparation(state_vector=state, wires=range(2))
    return qml.state()

state = np.array([0.2, 0.5, 0.2, 0.1])
state = [math.sqrt(i) for i in state]
print (state)
```

[0.4472135954999579, 0.7071067811865476, 0.4472135954999579, 0.31622776601683794]

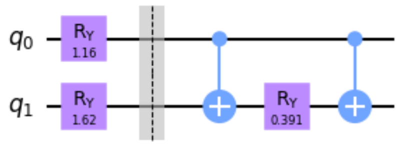
In [12]:
```
print(qml.draw_mpl(circuit, decimals=5, expansion_strategy="device", max_length=80)(state))
```

(<Figure size 700x300 with 1 Axes>, <Axes: >)



In [13]:
```
QC = QuantumCircuit(2)
QC.ry(1.15928, 0)
QC.ry(1.62233, 1)
QC.barrier()
QC.cx(0,1)

QC.ry(0.39137, 1)
QC.cnot(0,1)

QC.draw(output="mpl", scale=0.75)
```

Out[13]:



In [14]:
```
S = Statevector(QC)
S.draw(output="Latex")
```

Out[14]:

$$0.447216125478|00\rangle + 0.447213680739|01\rangle + 0.707105012817|10\rangle + 0.31622802172|11\rangle$$

7. For a given matrix A, Do hamiltonian encoding.

Provided matrix A is not hermitian, hence we need to convert matrix as below,

$$data = \begin{bmatrix} 0 & 0 & 0.073 & -0.438 \\ 0 & 0 & 0.73 & 0 \\ 0.073 & 0.73 & 0 & 0 \\ -0.438 & 0 & 0 & 0 \end{bmatrix}$$

Hamiltonian encoding of above is to convert the data to the form e^{iHt}

$e^{iHt} = \sum_i \lambda_i |i\rangle\langle i|$

Note : The example below uses the time evolution of 0.05

Method 1 : Eigen values and vectors

In [15]:

```
data = [[0,0,0.073,0.-0.438],
        [0,0,0.730,0],
        [0.073,0.730,0,0],
        [-0.438,0,0,0]]

eigval, eigvec = np.linalg.eig(data)

A = np.array(eigvec[0])
AT = np.atleast_2d(A).transpose()
l1 = numpy.kron(A, AT)

A = np.array(eigvec[1])
AT = np.atleast_2d(A).transpose()
l2 = numpy.kron(A, AT)

A = np.array(eigvec[2])
AT = np.atleast_2d(A).transpose()
l3 = numpy.kron(A, AT)

A = np.array(eigvec[3])
AT = np.atleast_2d(A).transpose()
l4 = numpy.kron(A, AT)

Z = np.exp(1j*eigval[0]*0.05)*l1 + np.exp(1j*eigval[1]*0.05)*l2 + np.exp(1j*eigval[2]*0.05)*l3 + np.exp(1j*eigval[3
print(numpy.matrix.round(Z, 3))
```

```
[[ 1.    -0.j      0.    +0.j      0.    +0.002j  0.    +0.022j]
 [ 0.    +0.j      0.999+0.j      0.    -0.037j -0.    -0.002j]
 [ 0.    +0.002j  0.    -0.037j  0.999+0.j     -0.    -0.j    ]
 [ 0.    +0.022j -0.    -0.002j -0.    -0.j      1.    -0.j    ]]
```

Method 2: Use internal qiskit support for HamiltonianGate.

In [16]:

```
gate = HamiltonianGate(data, 0.05, label="H")
Unitary = gate.to_matrix()
print(numpy.matrix.round(Unitary, 3))
QC = QuantumCircuit(2)
QC.append(gate, [0,1])

QC.draw()
decom = QC.decompose()
decom.draw()
```

```
[[ 1.    +0.j     -0.    +0.j      0.    -0.004j  0.    +0.022j]
 [-0.    +0.j      0.999+0.j      0.    -0.036j  0.    -0.j    ]
 [ 0.    -0.004j  0.    -0.036j  0.999+0.j      0.    +0.j    ]
 [ 0.    +0.022j  0.    -0.j      0.    +0.j      1.    +0.j    ]]
```

Out[16]:

```
     ┌──────────┐
q_0: ┤0         ├
     │  Unitary │
q_1: ┤1         ├
     └──────────┘
```

8. Construct a swap gate using CNOT gates.

The method below is from Nielsen Fig 1.7 and also in the following research paper.

https://www.researchgate.net/publication/216778423_The_cost_of_quantum_gate_primitives

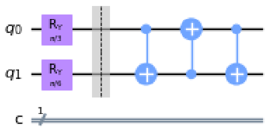
Swap gate with 3 CNOT gates

In [17]:

```
from numpy import pi

qreg_q = QuantumRegister(2, 'q')
creg_c = ClassicalRegister(1, 'c')
circuit = QuantumCircuit(qreg_q, creg_c)
```

```
circuit.ry(pi / 3, qreg_q[0])
circuit.ry(pi / 6, qreg_q[1])
circuit.barrier(qreg_q[0], qreg_q[1])
circuit.cx(qreg_q[0], qreg_q[1])
circuit.cx(qreg_q[1], qreg_q[0])
circuit.cx(qreg_q[0], qreg_q[1])
circuit.draw(output="mpl", scale=0.5)
```

Out[17]:



In [18]:
```
state = Statevector(circuit)
state.draw(output="Latex")
```

Out[18]:

$$0.836516303738|00\rangle + 0.224143868042|01\rangle + 0.482962913145|10\rangle + 0.129409522551|11\rangle$$

Swap gate primitive

In [19]:
```
qreg_q = QuantumRegister(2, 'q')
creg_c = ClassicalRegister(1, 'c')
circuit = QuantumCircuit(qreg_q, creg_c)

circuit.ry(pi / 3, qreg_q[0])
circuit.ry(pi / 6, qreg_q[1])
circuit.barrier(qreg_q[0], qreg_q[1])
circuit.swap(qreg_q[0], qreg_q[1])

state = Statevector(circuit)
state.draw(output="Latex")
```

Out[19]:

$$0.836516303738|00\rangle + 0.224143868042|01\rangle + 0.482962913145|10\rangle + 0.129409522551|11\rangle$$

9. Construct the control swap gate matrix using bra-ket notation.

A general swap gate, is like the one below.

I	O
00	00
01	10
10	01
11	11

SWAP = $|00\rangle\langle 00| + |01\rangle\langle 10| + |10\rangle\langle 01| + |11\rangle\langle 11|$

CSWAP = $(|0\rangle\langle 0| \otimes I \otimes I) + (|1\rangle\langle 1| \otimes SWAP)$

CSWAP matix representation is as below.

In [20]:
```
ZB = np.array([[1],[0]])
ZK = np.atleast_2d(ZB).transpose()

OB = np.array([[0],[1]])
OK = np.atleast_2d(OB).transpose()

S00 = np.kron(ZB, ZB)
S01 = np.kron(ZB, OB)
S10 = np.kron(OB, ZB)
S11 = np.kron(OB, OB)

S00x00 = np.kron(S00, S00.T)
S01x10 = np.kron(S01, S10.T)
S10x01 = np.kron(S10, S01.T)
S11x11 = np.kron(S11, S11.T)

SWAP = S00x00 + S01x10 + S10x01 + S11x11
Z = np.kron(np.kron(np.kron(ZB,ZK), np.eye(2)), np.eye(2))
O = np.kron(np.kron(OB, OK), SWAP)

Z + O
```

Out[20]:

```
array([[1., 0., 0., 0., 0., 0., 0., 0.],
       [0., 1., 0., 0., 0., 0., 0., 0.],
       [0., 0., 1., 0., 0., 0., 0., 0.],
       [0., 0., 0., 1., 0., 0., 0., 0.],
       [0., 0., 0., 0., 1., 0., 0., 0.],
       [0., 0., 0., 0., 0., 1., 0., 0.],
       [0., 0., 0., 0., 0., 1., 0., 0.],
       [0., 0., 0., 0., 0., 0., 0., 1.]])
```

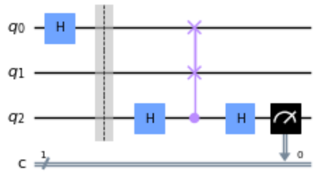
10. SWAP test

+, 0

In [21]:

```
qreg_q = QuantumRegister(3, 'q')
creg_c = ClassicalRegister(1, 'c')
circuit = QuantumCircuit(qreg_q, creg_c)

circuit.h(qreg_q[0])

circuit.barrier(qreg_q[0], qreg_q[1], qreg_q[2])
circuit.h(qreg_q[2])
circuit.cswap(qreg_q[2], qreg_q[0], qreg_q[1])
circuit.h(qreg_q[2])
circuit.measure(qreg_q[2], creg_c[0])
circuit.draw(output="mpl", scale=0.5)
```

Out[21]:



In [22]:

```
nShots = 5000
backend = Aer.get_backend('aer_simulator_statevector')
result = backend.run(transpile(circuit, backend), shots=nShots).result()
counts = result.get_counts()
if '0' in counts:
    b = counts['0']
else:
    b = 0

s = round(abs(1 - (2*(b/nShots))),1)

print("Squared Inner Product:",str(s), 'Inner product :', math.sqrt(s))
```

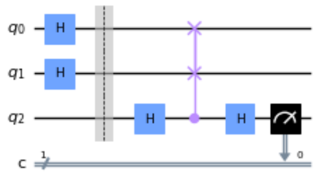
```
Squared Inner Product: 0.5 Inner product : 0.7071067811865476
```

+, +

In [23]:

```
qreg_q = QuantumRegister(3, 'q')
creg_c = ClassicalRegister(1, 'c')
circuit = QuantumCircuit(qreg_q, creg_c)

circuit.h(qreg_q[0])
circuit.h(qreg_q[1])

circuit.barrier(qreg_q[0], qreg_q[1], qreg_q[2])
circuit.h(qreg_q[2])
circuit.cswap(qreg_q[2], qreg_q[0], qreg_q[1])
circuit.h(qreg_q[2])
circuit.measure(qreg_q[2], creg_c[0])
circuit.draw(output="mpl", scale=0.5)
```

Out[23]:



In [24]:

```
nShots = 5000
backend = Aer.get_backend('aer_simulator_statevector')
result = backend.run(transpile(circuit, backend), shots=nShots).result()
counts = result.get_counts()
if '0' in counts:
    b = counts['0']
else:
    b = 0

s = round(abs(1 - (2*(b/nShots))),3)
```

```
print("Squared Inner Product:",str(s), 'Inner product :', math.sqrt(s))
```

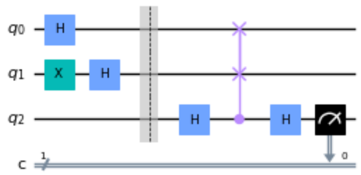
```
Squared Inner Product: 1.0 Inner product : 1.0
```

+, -

In [25]:

```
qreg_q = QuantumRegister(3, 'q')
creg_c = ClassicalRegister(1, 'c')
circuit = QuantumCircuit(qreg_q, creg_c)

circuit.h(qreg_q[0])
circuit.x(qreg_q[1])
circuit.h(qreg_q[1])

circuit.barrier(qreg_q[0], qreg_q[1], qreg_q[2])
circuit.h(qreg_q[2])
circuit.cswap(qreg_q[2], qreg_q[0], qreg_q[1])
circuit.h(qreg_q[2])
circuit.measure(qreg_q[2], creg_c[0])
circuit.draw(output="mpl", scale=0.5)
```

Out[25]:



In [26]:

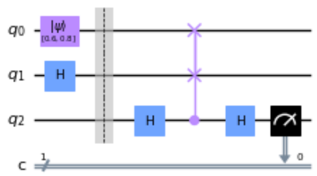
```
nShots = 5000
backend = Aer.get_backend('aer_simulator_statevector')
result = backend.run(transpile(circuit, backend), shots=nShots).result()
counts = result.get_counts()
if '0' in counts:
    b = counts['0']
else:
    b = 0

s = round(abs(1 - (2*(b/nShots))),1)

print("Squared Inner Product:",str(s), 'Inner product :', math.sqrt(s))
```

```
Squared Inner Product: 0.0 Inner product : 0.0
```

In [27]:

```
qreg_q = QuantumRegister(3, 'q')
creg_c = ClassicalRegister(1, 'c')
circuit = QuantumCircuit(qreg_q, creg_c)

circuit.initialize([3/5, 4/5], 0)
circuit.h(qreg_q[1])

circuit.barrier(qreg_q[0], qreg_q[1], qreg_q[2])
circuit.h(qreg_q[2])
circuit.cswap(qreg_q[2], qreg_q[0], qreg_q[1])
circuit.h(qreg_q[2])
circuit.measure(qreg_q[2], creg_c[0])
circuit.draw(output="mpl", scale=0.5)
```

Out[27]:



In [28]:

```
nShots = 5000
backend = Aer.get_backend('aer_simulator_statevector')
result = backend.run(transpile(circuit, backend), shots=nShots).result()
counts = result.get_counts()
if '0' in counts:
    b = counts['0']
else:
    b = 0

s = round(abs(1 - (2*(b/nShots))),10)

print("Squared Inner Product:",str(s), 'Inner product :', math.sqrt(s))
```

```
Squared Inner Product: 0.9796 Inner product : 0.9897474425326898
```