

# Causal inference in the latent spaces of Canonical Correlation Analysis

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# The principle of causal inference for latent spaces

Methods of causal inference and Structural Causal Models assume that there exist

- 1) explainable, labeled measurements represented as variables,
- 2) a computational graph in which each vertex corresponds to a single variable.

In AI, data have excessively high dimensionality. Causality can be re only in a low-dimensional latent space. The model that maps the data into this space is non-interpretable. The problem is therefore to identify causality in the latent space. This causality

- 1) learns the relationship between source and target variables in a reduced-dimensional space,
- 2) explains the observable variables by back-propagation for the source and by forward-propagation for the target.

*Significance of this principle:* it introduces a new method to dimensionality reduction.

## The problem of causal inference for latent spaces

A set of time series  $x_t$  and  $y_t$  is given, generated by a single observable dynamical system. They are represented as time-delay matrices  $X$  and  $Y$ . The problem is to forecast the vector  $y_{t+1}$ .

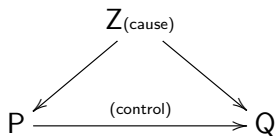
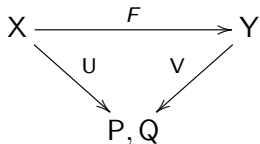
The forecasting model  $f : t \mapsto y \ni Y$  assumes that  $X$  and  $Y$  are mapped by orthogonal operators  $U$  and  $V$  into low-dimensional manifolds. The components of the resulting vectors are independent. The goal is to find the elements of the linear alignment  $\Lambda$  that express causality in some components of the time series  $x_t$  and  $y_t$ .

Denote by  $\hat{Y} = F(X)$  the forecast  $\hat{Y}$  obtained via the linear operator  $F$ . To approximate the target  $Y$ , make the singular value decomposition of the operator

$$F = U\Lambda V^T$$

and select the components of the diagonal matrix  $\Lambda$  that express the causality.

# Causal Inference for Canonical Correlation Analysis



The Canonical Correlation Analysis reduce the dimensionality of the source data. It projects observed data into low-dimensional space and uses it as new model features. It maps the source and the target  $X, Y$  onto joint latent space  $P, Q$  and maximizes the covariance between the projections

$$\text{cov}(P, Q) \rightarrow \max.$$

The latent variables  $P, Q$  approximate hidden dependencies in joint space using linear models  $U, V$ .

In the terms of Causality Inference, the linear operator  $\Lambda$ , the diagonal matrix, defines the *bilateral graph of causality*.

# The plan of action

## The background research

- 1) Write Bayesian Causality Inference for CCA
- 2) Generalize for CCM and Transformer
- 3) Propose Causality Propagation model
- 4) Propose Generative Causality Search

## The code

- 1) Make a simplest demo for latent spaces
- 2) Compare with Operator Learning for time series
- 3) Include Transformer and other models for text data
- 4) Make computational experiment for available models