Causal inference in the latent spaces of Canonical Correlation Analysis

IA pour la Science

December 19th, 2025

The principle of causal inference for latent spaces

Methods of causal inference and Structural Causal Models assume that there exist

1) explainable, labeled measurements represented as variables,
2) a computational graph in which each vertex corresponds to a single variable.

In AI, data have excessively high dimensionality. Causality can be re only in a low-dimensional latent space. The model that maps the data into this space is non-interpretable. The problem is therefore to identify causality in the latent space. This causality

1) learns the relationship between source and target variables in a reduced-dimensional space,
2) explains the observable variables by back-propagation for the source and by forward-propagation for the target.

Significance of this principle: it introduces a new method to dimensionality reduction.

The problem of causal inference for latent spaces

A set of time series x_t and y_t is given, generated by a single observable dynamical system. They are represented as time-delay matrices X and Y. The problem is to forecast the vector y_{t+1}.

The forecasting model $f : t \mapsto y \ni Y$ assumes that X and Y are mapped by orthogonal operators U and V into low-dimensional manifolds. The components of the resulting vectors are independent. The goal is to find the elements of the linear alignment Λ that express causality in some components of the time series x_t and y_t.

Denote by $\hat{Y} = F(X)$ the forecast $\hat{Y}$ obtained via the linear operator F. To approximate the target Y, make the singular value decomposition of the operator

$$F = U \Lambda V^{\mathsf{T}}$$

and select the components of the diagonal matrix Λ that express the causality.

Causal Inference for Canonical Correlation Analysis

$$
\begin{tikzcd}
\mathrm{X} \arrow[rr, "F"] \arrow[dr, "\mathrm{U}"'] & & \mathrm{Y} \arrow[dl, "\mathrm{V}"] \\
& \mathrm{P}, \mathrm{Q} &
\end{tikzcd}
\qquad
\begin{tikzcd}
& \mathrm{Z}_{\text{(cause)}} \arrow[dl] \arrow[dr] & \\
\mathrm{P} \arrow[rr, "\text{(control)}"] & & \mathrm{Q}
\end{tikzcd}
$$

The Canonical Correlation Analysis reduce the dimensionality of the source data. It projects observed data into low-dimensional space and uses it as new model features. It maps the source and the target X, Y onto joint latent space P, Q and maximizes the covariance between the projections

$$\mathrm{cov}(P, Q) \to \max .$$

The latent variables P, Q approximate hidden dependencies in joint space using linear models U, V.

In the terms of Causality Inference, the linear operator Λ, the diagonal matrix, defines the *bilateral graph of causality*.

The plan of action

The background research

1) Write Bayesian Causality Inference for CCA
2) Generalize for CCM and Transformer
3) Propose Causality Propagation model
4) Propose Generative Causality Search

The code

1) Make a simplest demo for latent spaces
2) Compare with Operator Learning for time series
3) Include Transformer and other models for text data
4) Make computational experiment for available models