Dimensionality reduction in the control problem

(Variants: The embedding of signals in dynamic systems, The signal representation in the dynamic control problem, Latent space reconstruction in brain-computer interfaces)

The goal is to propose a model that reconstructs the latent space.
To start with, construct the latent space with Riemannian and Graph diffusion models and compare these two models in the computational experiment.

There are given two signal sources: X, Y, and class labels C[1]:
1) EEG and IMU (table tennis),
2) a basic control problem (R1 to R1) signal (games),
3) TODO: list and discuss possible applications with datasets.
4) TODO: organize and republish the table tennis data.

The problem is to reconstruct a space of minimum dimensionality that 1) reconstructs both observable spaces and maps the source to the target, given the class label.

The models to analyze:
1. Canonical correlation analysis from Denis or Roman
2. Pytorch transformer solution, see variants from Eduard or Jeremy
3. Use the Pyriemann dimensionality reduction to construct the latent space
4. WaveNet arXiv:1906.00121 (or variants with diffusion convolution)
5. Use the GRAND arXiv:2106.10934 software if it is possible
6. Research the advances in the topics: Temporal Graph Transformers to combine diffusion with attention, Contrastive Learning for Graph Time Series, and Physics-informed Graph Diffusion Models

The next short step:
Select a simple-to-visualize dataset that contains low-dimensional (1–3 signals) X and Y and an assumed 2–3 dimensional latent space U, V with class labels C.

Further steps:
1. Visualize this simplest dataset with CCA and other models.
2. Select a proper theoretical configuration.
3. Compare and combine Riemannian and Graph-Diffusion models.

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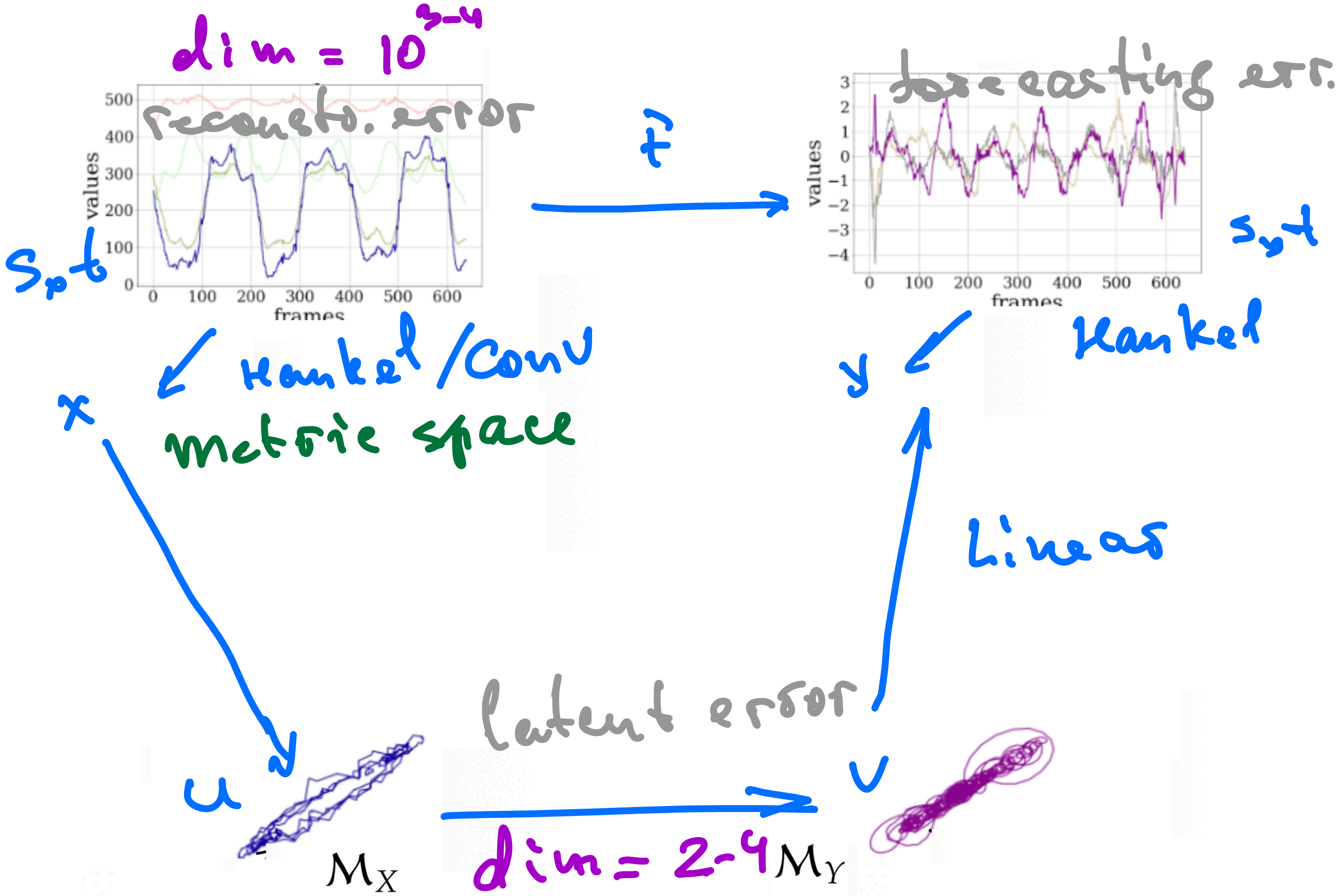
[1] The class labers is needed to ensure that the source X and the target Y are generated by the same model. It guarantees some causality between X and Y. The main hypothesis is: there is some manifold X_U in X that is represented by embedding U. This embedding corresponds to the trajectory V. The last one maps to Y.

The last step and **ultimate goal**: generalize both methods and propose a combined variant that constructs an optimal latent space by adequate topological mapping. Analyze theoretical properties.
The simplest statement to follow the ultimate goal: the change of a system state over time dx/dt=Lx(t) is described by the graph Laplacian (or an equivalent distance matrix in the Riemannian state) and the system state representation.



The latent space

class ⊂ z-generator

Metric 1) over distance between signals
2) over physical space S_{t-1}

Does the source X need low-dim state of y in the past?