Combinatorics Fundamentals

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1 Introduction

$$C_n^k = \frac{n!}{(n-k)! * k!} \tag{1}$$

$$\overline{C}_n^k = C_{n-k+1}^k \tag{2}$$

$$A_n^k = \frac{n!}{(k)!} \tag{3}$$

$$\overline{A}_n^k = n^k \tag{4}$$

$$P(n_1, ..., n_k) = \frac{n!}{n_1!...n_k!} \tag{5}$$

2 Pascal's rule

Pascal's triangle:
1
1 1 (first line)
1 2 1
1 3 3 1
1 4 6 4 1

3 Dirichlet Principle

4 box, 5 rabbits

4 Binom Newton

$$(x+y)^n = \sum_{k=0}^{n} C_n^k * x^k * y^{n-k} \tag{6}$$

5 Properties

$$C_n^k = C_n^{n-k} \tag{7}$$

$$C_n^k = C_{n-1}^k + C_{n-1}^{k-1} \tag{8}$$

$$\sum_{k=0}^{n} C_n^k = 2^n \tag{9}$$

$$\sum_{k=0}^{n} (C_n^k)^2 = C_{2n}^k \tag{10}$$

$$C_{n+m}^n = \sum_{k=0}^{m} C_{n+k-1}^{n-1} \tag{11}$$

$$\sum_{k=0}^{m} m^2 = \frac{m(m+1)(2m+1)}{6} \tag{12}$$

$$C_n^0 - C_n^1 + C_n^2 + ... + (-1)^n C_n^n = \begin{cases} 1 & ifn = 0, \\ 0 & ifn \geq 0. \end{cases} \tag{13}$$

6 Useful Tasks

Count of subsets from set (n items) if a power of each subset is odd (even): 2^{n-1}

7 Polynomial Coefficients