Stochastic and differentiable simulators of drifting objects trajectories

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Drift reconstruction: not a trivial problem

Predicting the drift of objects on the ocean's surface is an important issue, both scientifically and operationally.
At first glance, reconstructing the successive displacements $d\mathbf{X}(t)$ of objects transported by Sea Surface Currents (SSC) $\mathbf{u}_C\big(\mathbf{X}(t), t\big)$, evaluated at their position $\mathbf{X}(t)$ at time t, knowing their initial position $\mathbf{X}_0$, may seem trivial and can be expressed by the Ordinary Differential Equation:

$$d\mathbf{X}(t) = \mathbf{u}_C\big(\mathbf{X}(t), t\big)\, dt$$

$$\mathbf{X}\big(t_0\big) = \mathbf{X}_0$$

However, this formulation of the drift function is naive and can lead to incoherent reconstructions for two main reasons:

1. Objects are not only displaced by SSC but also by the direct effect of Sea Surface Wind (SSW) $\mathbf{u}_W$, known as Leeway $\mathbf{u}_L \approx k\mathbf{u}_W$.
$$d\mathbf{X} = \big(\mathbf{u}_C + \mathbf{u}_L\big)\, dt$$
2. Our knowledge of SSC is imperfect as they are derived from the geostrophic balance and mapped Sea Surface Height (SSH) along-track altimetric observations.
$$d\mathbf{X} = \Big(\mathbf{u}_g + \mathbf{u}_L\Big)\, dt + \epsilon_{sub}$$

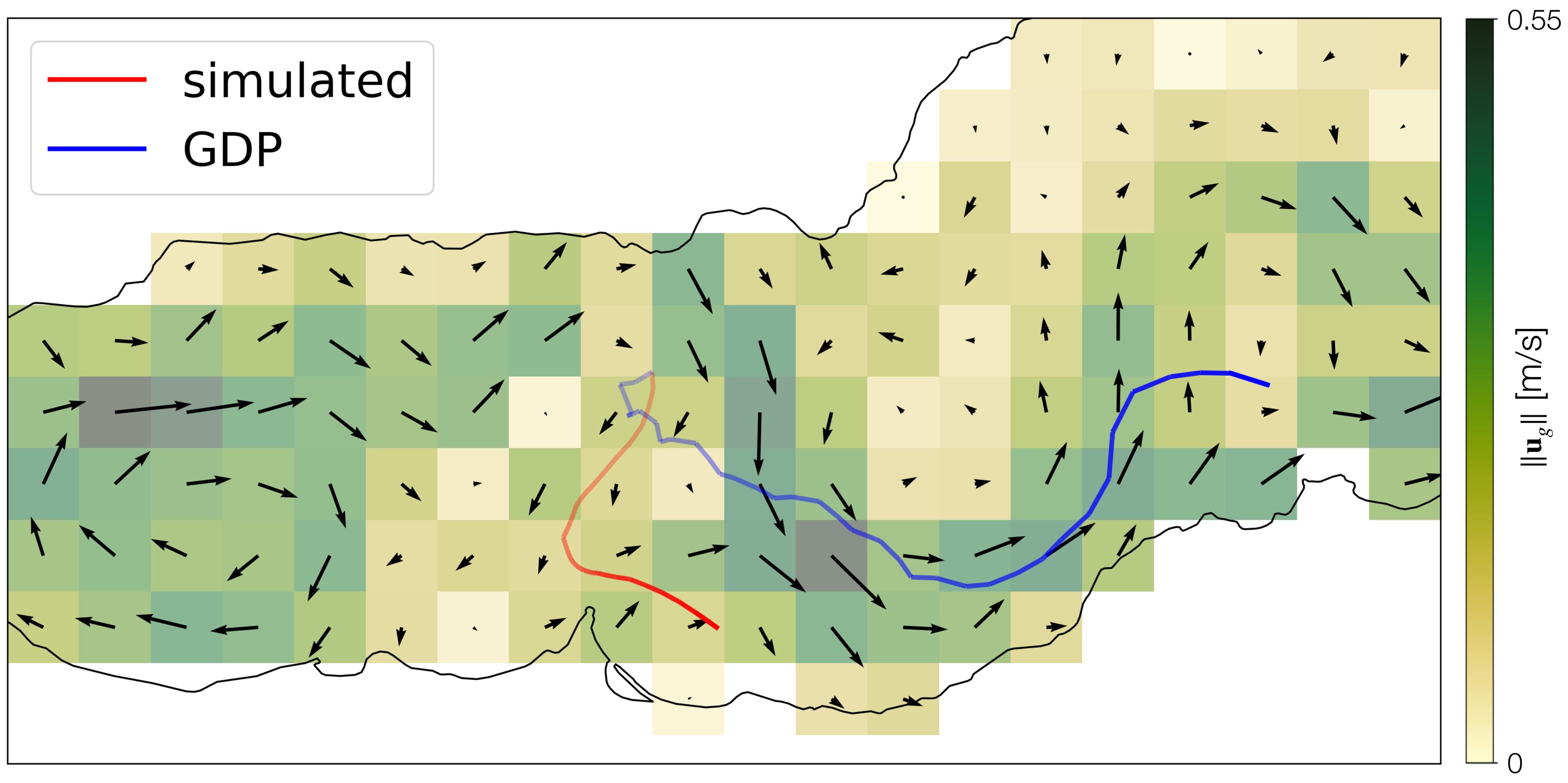

Fig 1. Example of a poor drifter trajectory reconstruction when using a naive simulator with DUACS geostrophic SSC as inputs.

The need to account for uncertainties and build well-calibrated simulators

The inability to resolve fine-scale physical processes in the available observations leads to uncertainties in the reconstruction of trajectories, even with a better representation of the physics of the drift.

Existing approaches address this by parameterising the terms of the drift function with random variables, which are often assumed to be independent and normally distributed, and then generating ensembles of trajectories using a Monte Carlo procedure.

Despite the increasing availability of real drifter trajectory data, through the Global Drifter Program (GDP) for instance, those models are still calibrated in an ad hoc manner, which limits their generalisation.

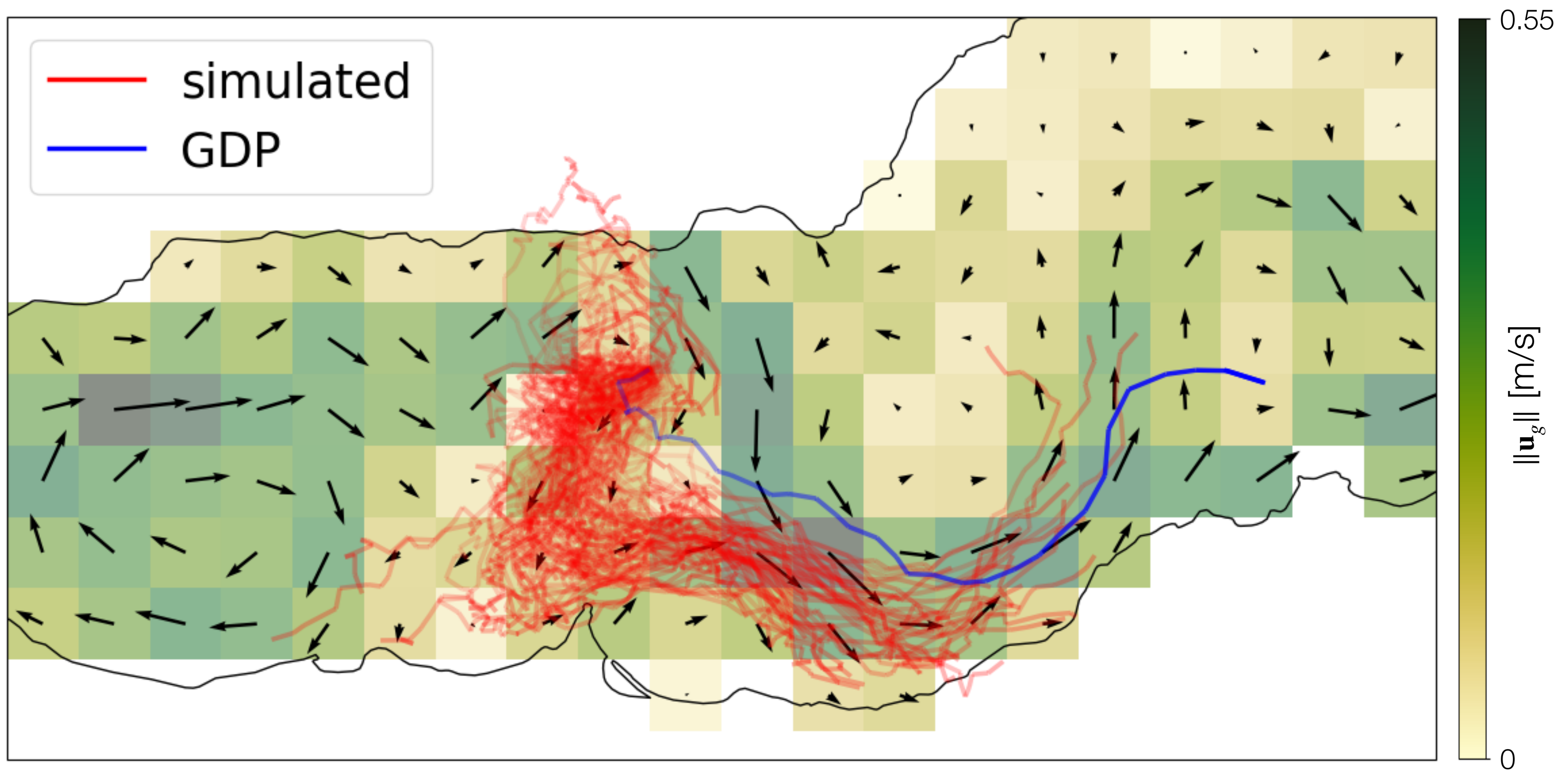

Fig 2. A slightly better reconstruction using an uncalibrated stochastic simulator.

Proposed approach

We propose to represent the drift process using Stochastic Differential Equations (SDE) of the general form:

$$d\mathbf{X}(t) = \mathscr{F}\,\big(\mathbf{X}(t), t\ ;\ \theta_{\mathscr{F}}, Y\big)\, dt + \mathscr{G}\,\big(\mathbf{X}(t), t\ ;\ \theta_{\mathscr{G}}, Y\big) \cdot d\mathbf{W}(t)$$

where $d\mathbf{W}(t)$ is a Wiener increment, Y represents a set of physical fields in the neighbourhood of $\mathbf{X}(t)$, $\mathscr{F}$ and $\mathscr{G}$ are arbitrary drift and diffusion functions parametrised by $\theta_{\mathscr{F}}$ and $\theta_{\mathscr{G}}$, themself possibly conditioned on the fields Y.

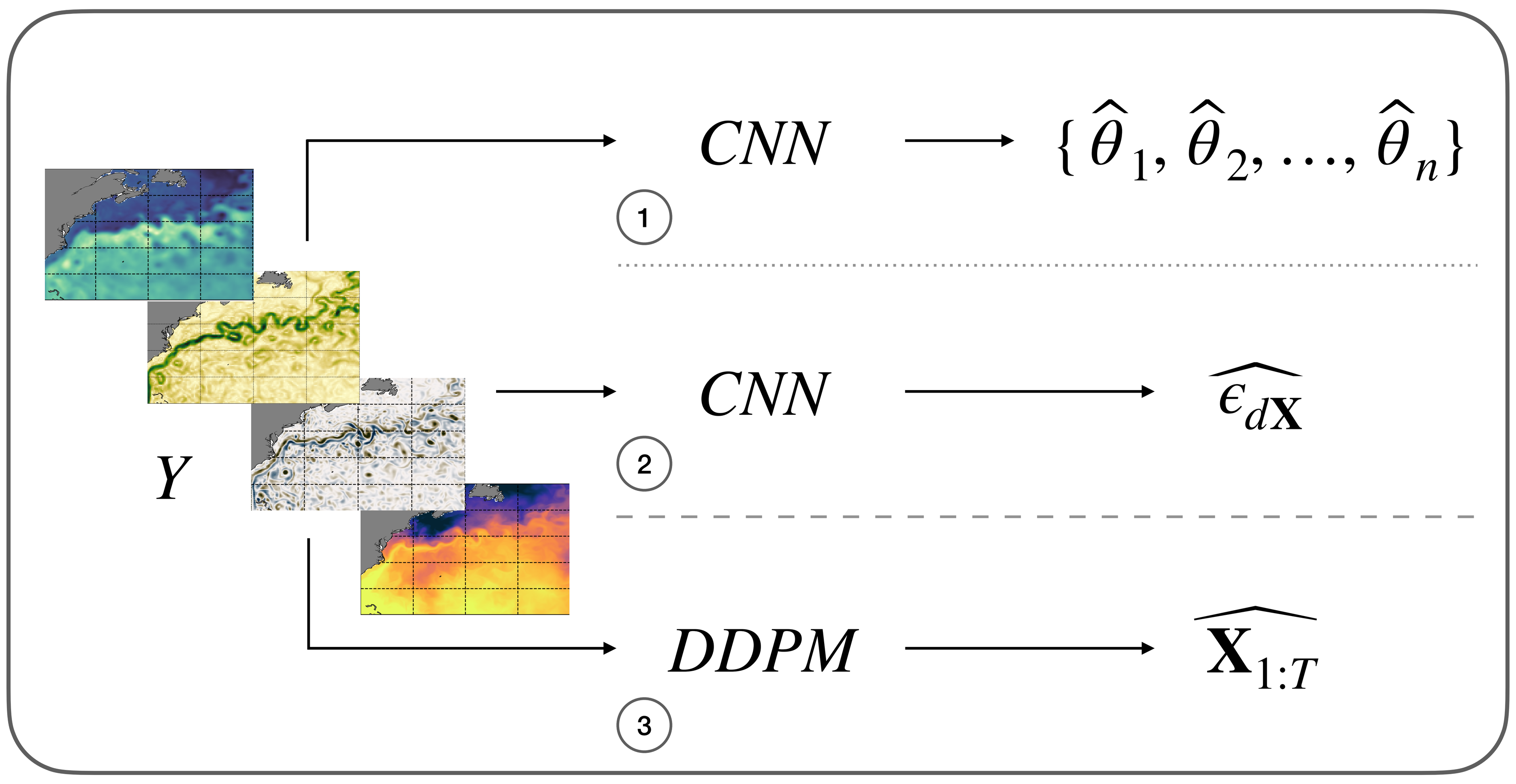

Fig 3. Possible stochastic simulator designs with different levels of abstraction.

Provided that the simulator $\mathscr{S}$ solving the previous SDE is automatically differentiable, $\theta_{\mathscr{F}}$ and $\theta_{\mathscr{G}}$ can then be optimised using gradient descent to minimise the cost function J, which aggregates a pairwise trajectory metric l using an aggregation operator $\langle \cdot \rangle$ —for example, the Continuous Ranked Probability Score (CRPS).

$$\mathscr{X}^{(i)} \sim \mathscr{S}\,\Big(\mathbf{X}_0^{(i)}; \theta_{\mathscr{F}}, \theta_{\mathscr{G}}, Y\Big)$$

$$J\,\big(\theta_{\mathscr{F}}, \theta_{\mathscr{G}}\big) = \Big\langle l\,\Big(\mathbf{X}_{1:T}^{(i)}, \mathscr{X}_{(j)}^{(i)}\Big)\Big\rangle$$

$$\theta_{\mathscr{F}}^{\star}, \theta_{\mathscr{G}}^{\star} = \arg\min_{\theta_{\mathscr{F}}, \theta_{\mathscr{G}}} J\,\big(\theta_{\mathscr{F}}, \theta_{\mathscr{G}}\big)$$