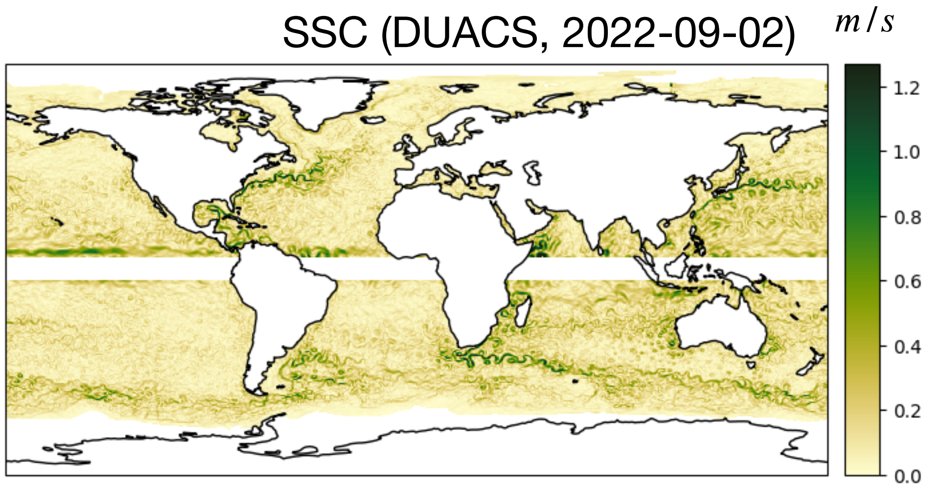
30 Years of Progress in Radar Altimetry Symposium
2-7 September 2024 | Montpellier, France

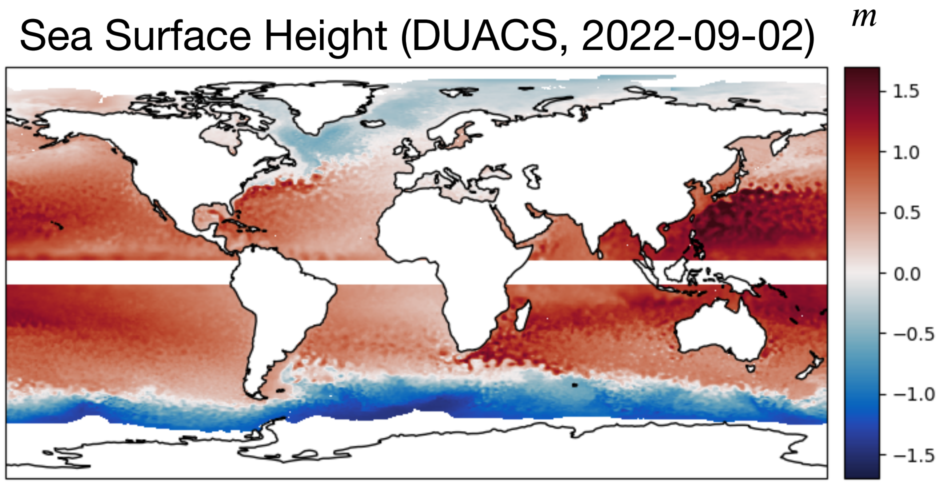
# Cyclogeostrophic inversion for estimating Sea Surface Currents from SWOT altimeter data

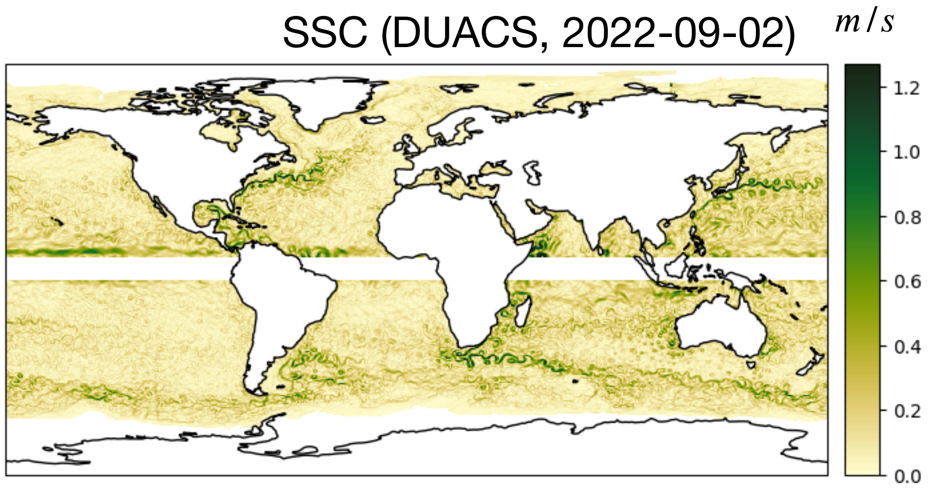
***<u>Vadim Bertrand</u>, Léo Boux de Casson, Victor EVZ De Almeida, Julien Le Sommer, Emmanuel Cosme***
***Univ. Grenoble Alpes, CNRS, INRAE, IRD, Grenoble INP, IGE, 38000 Grenoble, France***

# Estimating geostrophic Sea Surface Currents (SSC)

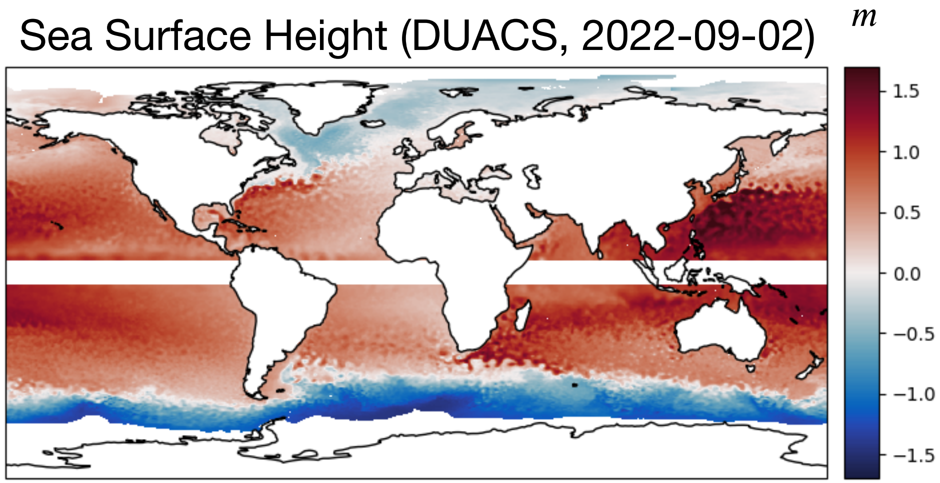


# Estimating geostrophic Sea Surface Currents (SSC)

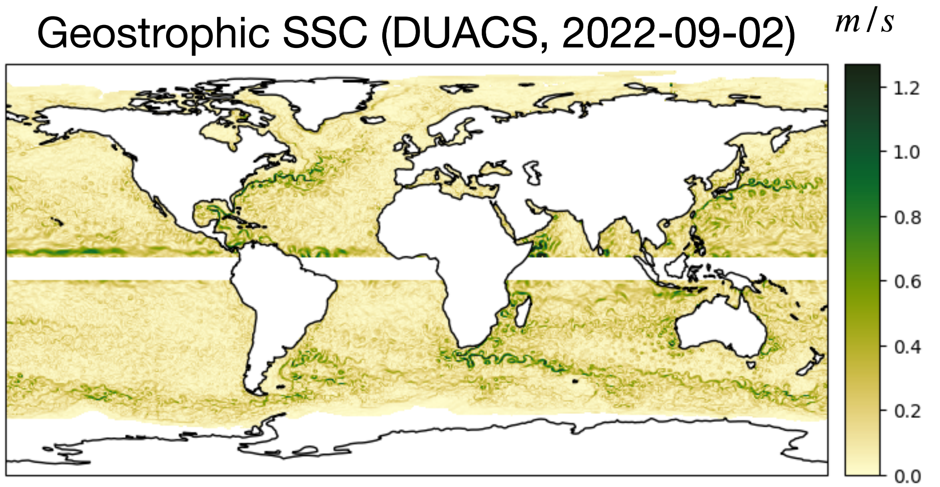


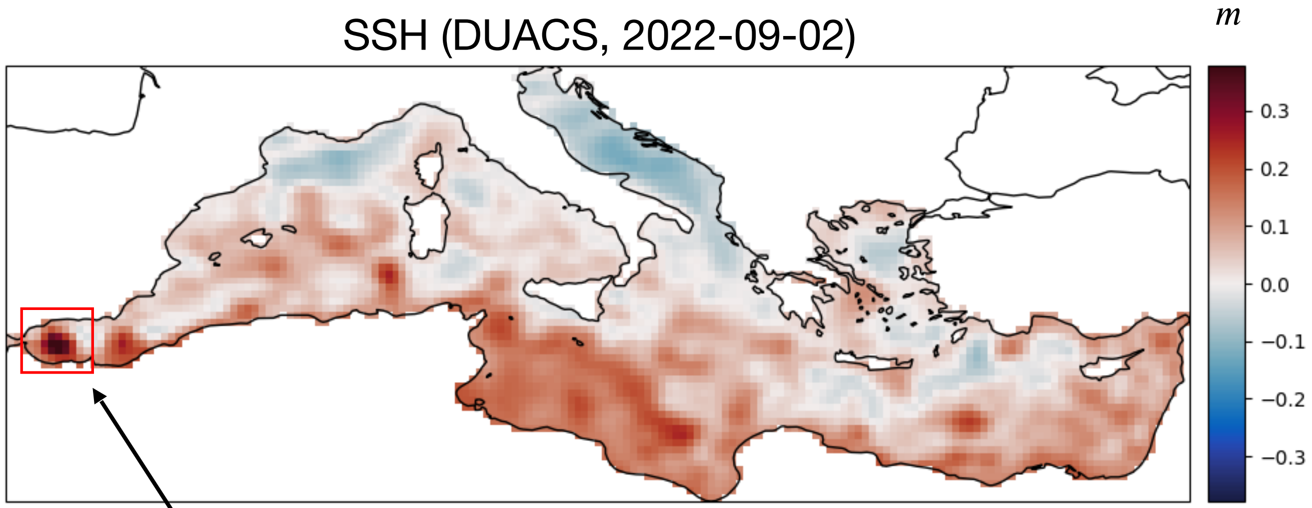
# Estimating geostrophic Sea Surface Currents (SSC)




$$f\vec{k} \times \vec{u}_g = -g\nabla\eta$$



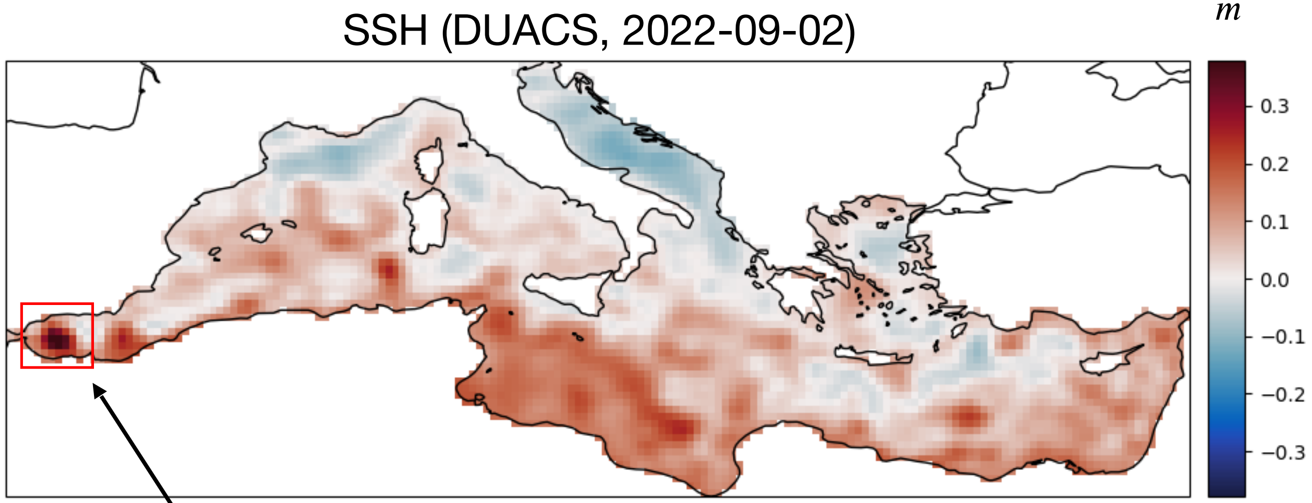
# What about cyclogeostrophic Sea Surface Currents?




$$\vec{u}_c \cdot \nabla \vec{u}_c + f\vec{k} \times \vec{u}_c = -\, g \nabla \eta$$

! In energetic conditions, the inertial acceleration is non-negligible

# What about cyclogeostrophic Sea Surface Currents?




$$\vec{u}_c \cdot \nabla \vec{u}_c + f\vec{k} \times \vec{u}_c = -g \nabla \eta$$

! In energetic conditions, the inertial acceleration is non-negligible

X Non-linear relation → no available operational products

# Cyclogeostrophic inverse problem

### Iterative method

*Penven et al. 2014*, *Cao et al. 2023*

- Not always guaranteed to converge

$$\vec{u}_c^{(n+1)} = \vec{u}_g + \frac{\vec{k}}{f}(\vec{u}_c^{(n)} \cdot \nabla \vec{u}_c^{(n)})$$

$$\vec{u}_c^{(0)} = \vec{u}_g$$

# Cyclogeostrophic inverse problem

## Iterative method

Penven et al. 2014, Cao et al. 2023

- Not always guaranteed to converge

$$\vec{u}_c^{(n+1)} = \vec{u}_g + \frac{\vec{k}}{f}(\vec{u}_c^{(n)} \cdot \nabla \vec{u}_c^{(n)})$$

$$\vec{u}_c^{(0)} = \vec{u}_g$$

---

## Variational formulation

$$J(\vec{u}_c) = \left\| \vec{u}_c - \frac{\vec{k}}{f} \times (\vec{u}_c \cdot \nabla \vec{u}_c) - \vec{u}_g \right\|^2$$

$$\vec{u}_c^{\star} = \underset{\vec{u}_c}{\arg\min}\; J(\vec{u}_c)$$

# Cyclogeostrophic inverse problem

## Iterative method

Penven et al. 2014, Cao et al. 2023

- Not always guaranteed to converge

$$\vec{u}_c^{(n+1)} = \vec{u}_g + \frac{\vec{k}}{f}(\vec{u}_c^{(n)} \cdot \nabla \vec{u}_c^{(n)})$$

$$\vec{u}_c^{(0)} = \vec{u}_g$$

## Variational formulation

Cyclogeostrophic imbalance ⟵

$$J(\vec{u}_c) = \left\| \vec{u}_c - \frac{\vec{k}}{f} \times (\vec{u}_c \cdot \nabla \vec{u}_c) - \vec{u}_g \right\|^2$$

$$\vec{u}_c^\star = \underset{\vec{u}_c}{\arg\min}\ J(\vec{u}_c)$$

# Cyclogeostrophic inverse problem

## Iterative method

[Penven et al. 2014](), [Cao et al. 2023]()

- Not always guaranteed to converge

$$\vec{u}_c^{(n+1)} = \vec{u}_g + \frac{\vec{k}}{f}(\vec{u}_c^{(n)} \cdot \nabla \vec{u}_c^{(n)})$$

$$\vec{u}_c^{(0)} = \vec{u}_g$$

---

## Variational formulation

Cyclogeostrophic imbalance ⟵

$$J(\vec{u}_c) = \left\| \vec{u}_c - \frac{\vec{k}}{f} \times (\vec{u}_c \cdot \nabla \vec{u}_c) - \vec{u}_g \right\|^2$$

$$\vec{u}_c^\star = \underset{\vec{u}_c}{\arg\min}\; J(\vec{u}_c)$$

- Gradient-based minimisation

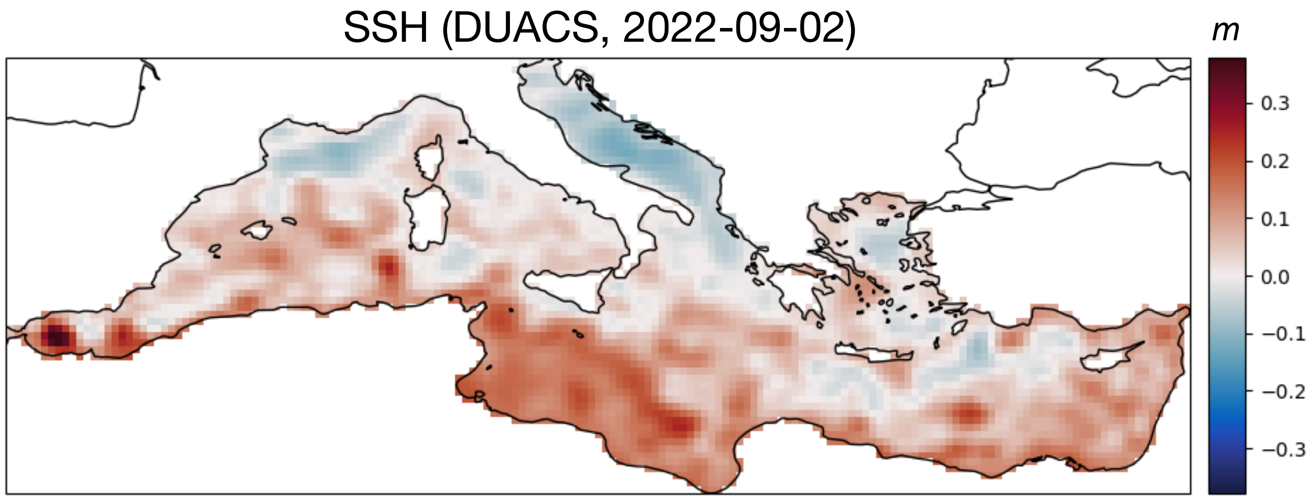
$$\vec{u}_c^{(n+1)} = \vec{u}_c^{(n)} - \gamma \nabla J(\vec{u}_c^{(n)})$$

$$\vec{u}_c^{(0)} = \vec{u}_g$$

# `jaxparrow` : Python / JAX-powered implementation

```
%pip install jaxparrow

import jax.numpy as jnp
from jaxparrow import cyclogeostrophy
from jaxparrow.tools import kinematics


ds = *** your favorite dataset ***

# some data manipulation...
lon, lat = jnp.meshgrid(ds.lon.values,
                        ds.lat.values)
adt = jnp.asarray(ds.adt.isel(time=0))
```

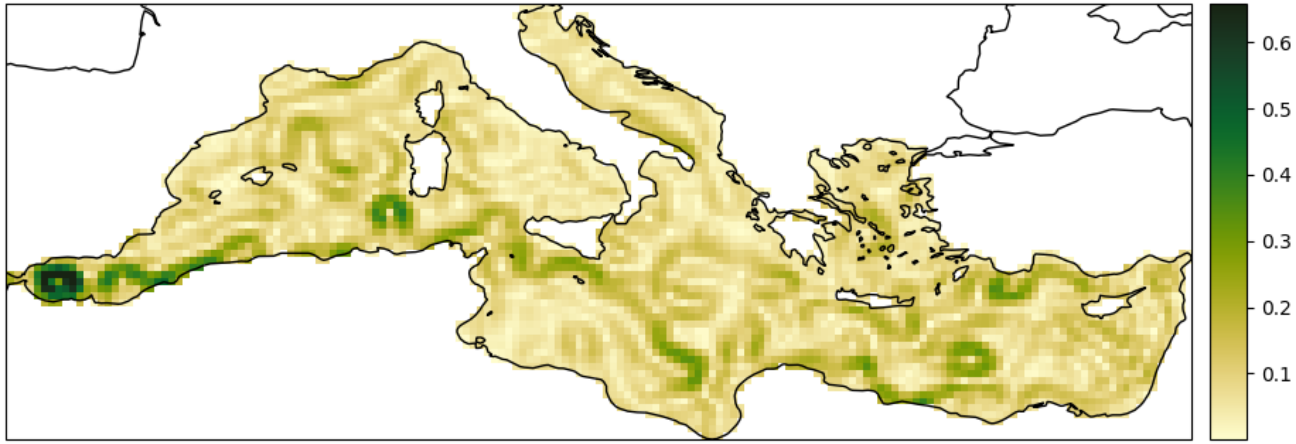
https://github.com/meom-group/jaxparrow



# jaxparrow : Python / JAX-powered implementation

```
%pip install jaxparrow

import jax.numpy as jnp
from jaxparrow import cyclogeostrophy
from jaxparrow.tools import kinematics


ds = *** your favorite dataset ***

# some data manipulation...
lon, lat = jnp.meshgrid(ds.lon.values,
                        ds.lat.values)
adt = jnp.asarray(ds.adt.isel(time=0))

# jaxparrow cyclogeostrophic inversion
u, v = cyclogeostrophy(adt, lat, lon)
ssc = kinematics.magnitude(u, v)
```

https://github.com/meom-group/jaxparrow

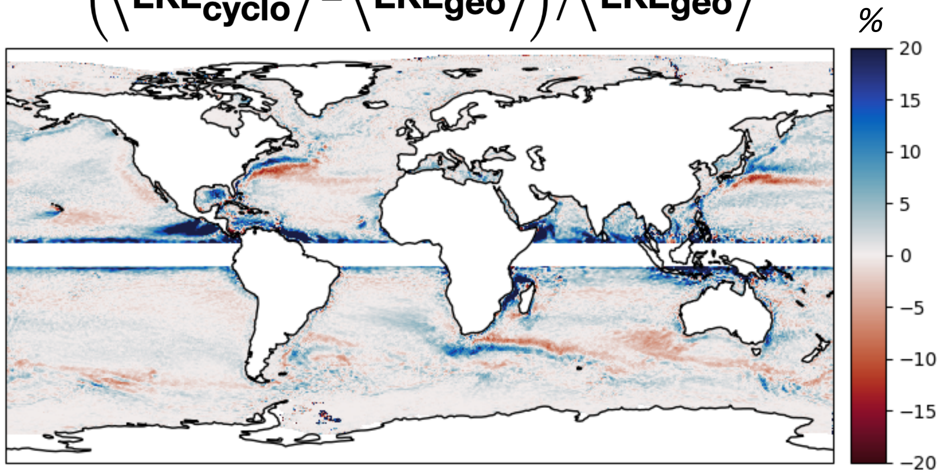


# Variational method validation

## **Variational method** *(Ours)*

- DUACS (*10.48670/moi-00148*)
  - from 2000 to 2022
- EKE computed from MDT + SLA
  - Arakawa C grid

$$\left(\left\langle \mathbf{EKE_{cyclo}} \right\rangle - \left\langle \mathbf{EKE_{geo}} \right\rangle\right) / \left\langle \mathbf{EKE_{geo}} \right\rangle$$



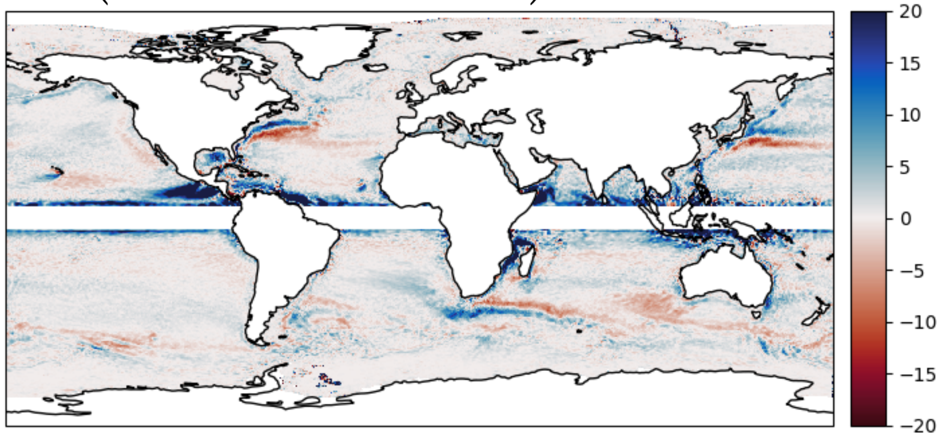

~10% EKE differences at mid-latitudes

# Variational method validation

## **Variational method** *(Ours)*

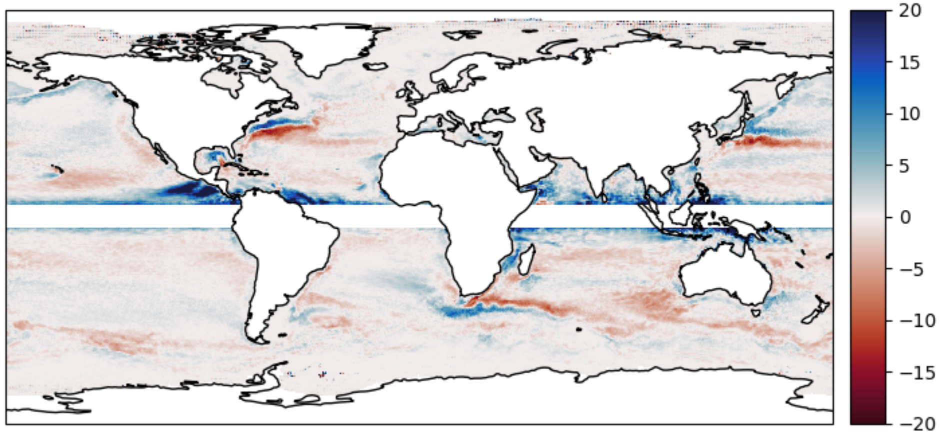
- DUACS (*10.48670/moi-00148*)
    - from 2000 to 2022
- EKE computed from MDT + SLA
    - Arakawa C grid

$$\left(\left\langle \mathbf{EKE_{cyclo}} \right\rangle - \left\langle \mathbf{EKE_{geo}} \right\rangle\right) / \left\langle \mathbf{EKE_{geo}} \right\rangle$$




## **Iterative method** (*Cao et al. 2023*)

- DUACS (*10.48670/moi-00148*)
    - from 1993 to 2018
- EKE computed from SLA
    - Arakawa A grid




~10% EKE differences at mid-latitudes

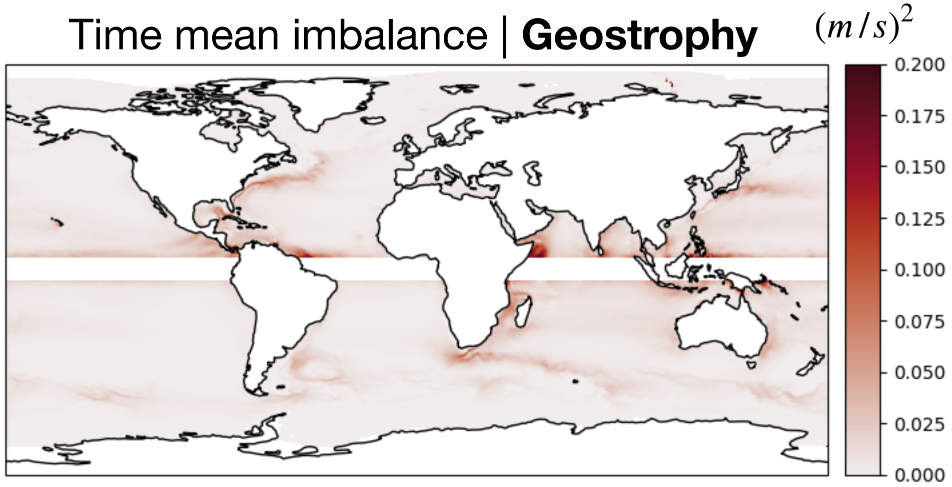
# Variational method validation

DUACS (*10.48670/moi-00148*)

- from 2000 to 2022

$$J(\vec{u}) = \left\| \vec{u} - \frac{\vec{k}}{f} \times (\vec{u} \cdot \nabla \vec{u}) - \vec{u}_g \right\|^2$$

Time mean imbalance | **Geostrophy**

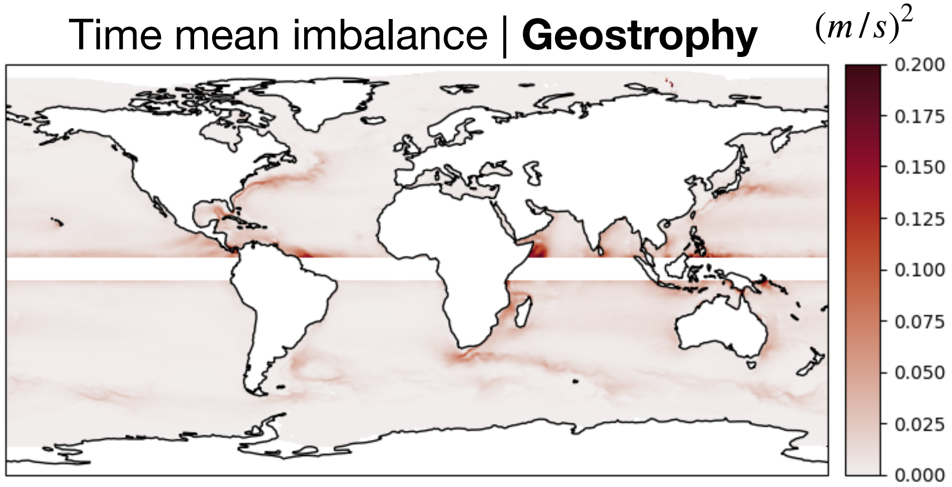


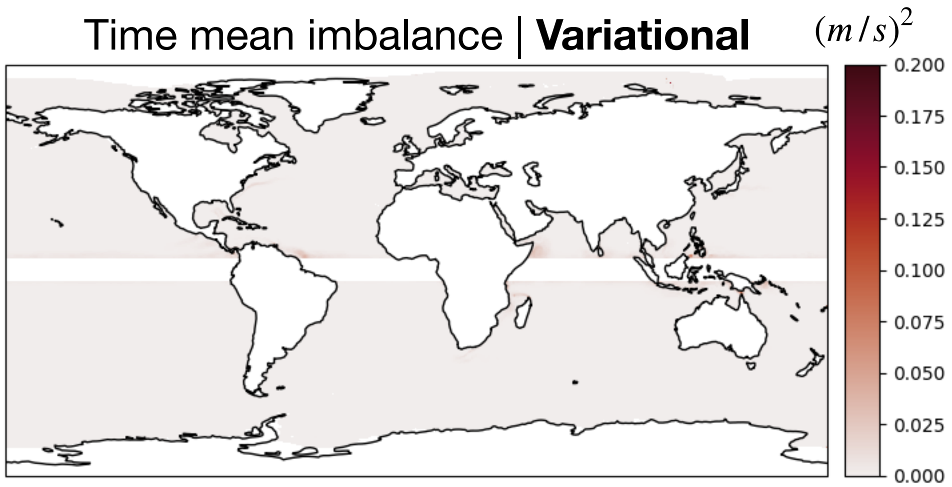
# Variational method validation

DUACS (*10.48670/moi-00148*)

- from 2000 to 2022

$$J(\vec{u}) = \left\| \vec{u} - \frac{\vec{k}}{f} \times (\vec{u} \cdot \nabla \vec{u}) - \vec{u}_g \right\|^2$$



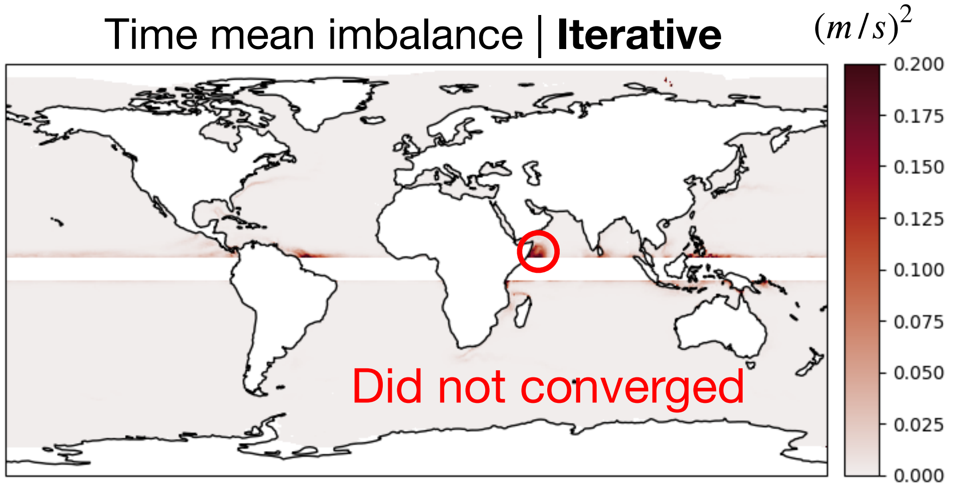



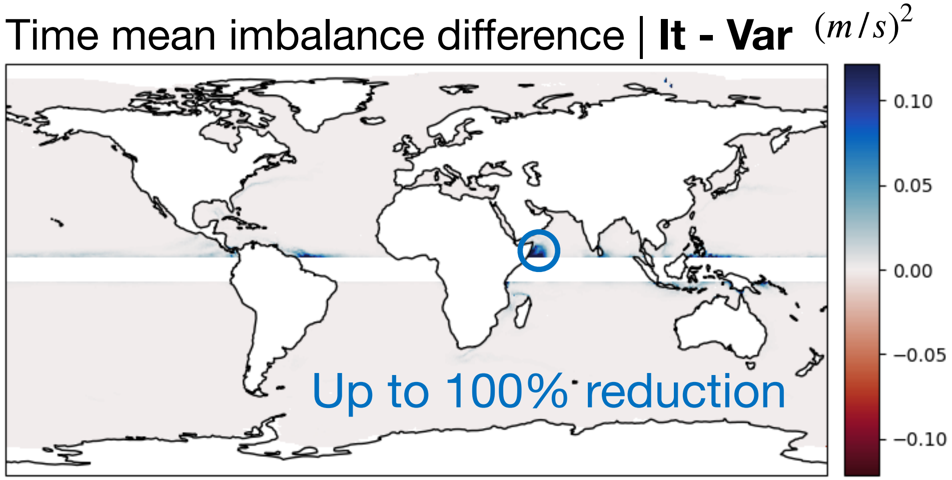
# Variational method validation

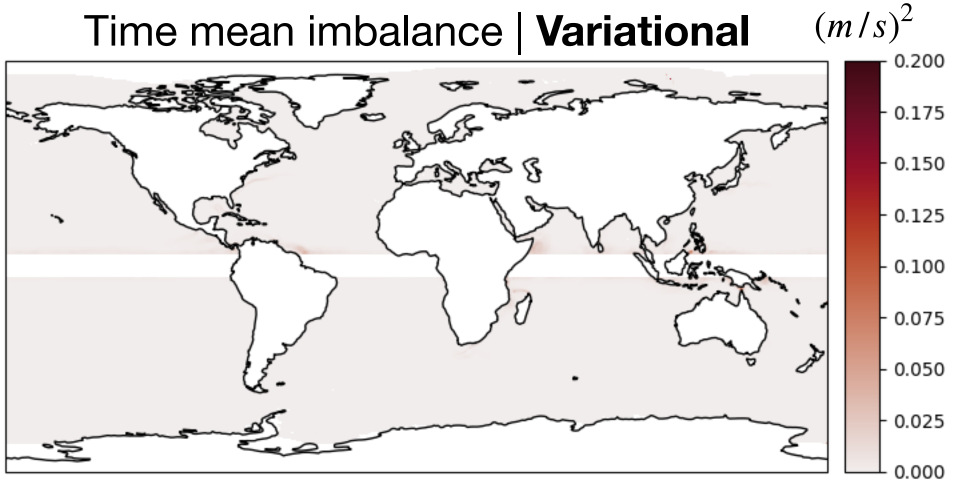
DUACS (10.48670/moi-00148)

- from 2000 to 2022

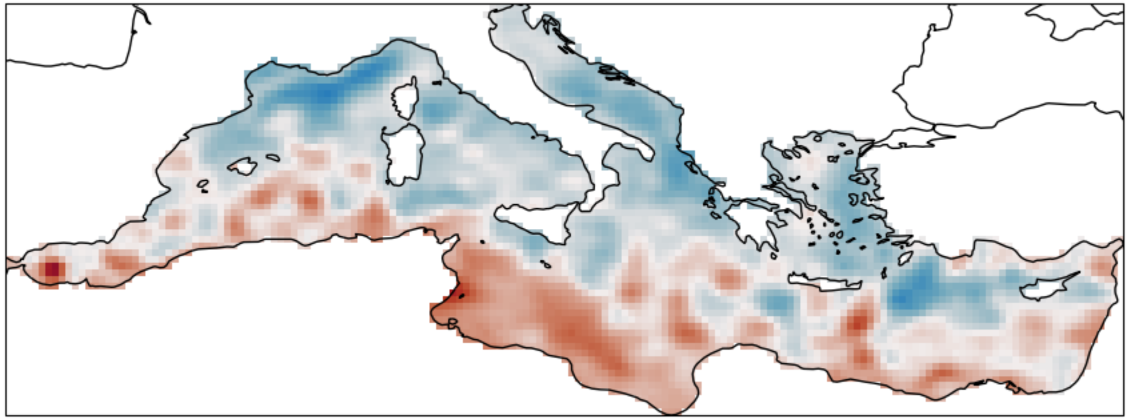
$$J(\vec{u}) = \left\| \vec{u} - \frac{\vec{k}}{f} \times (\vec{u} \cdot \nabla \vec{u}) - \vec{u}_g \right\|^2$$



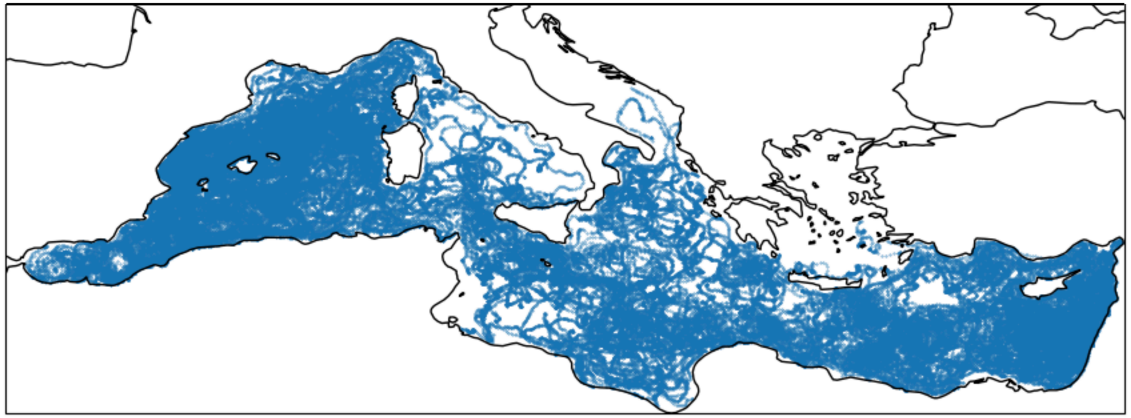



# Comparison of DUACS cyclogeostrophic SSC to drifter data



**Global Ocean Gridded L4 SSH (DUACS)**
*10.48670/moi-00148*

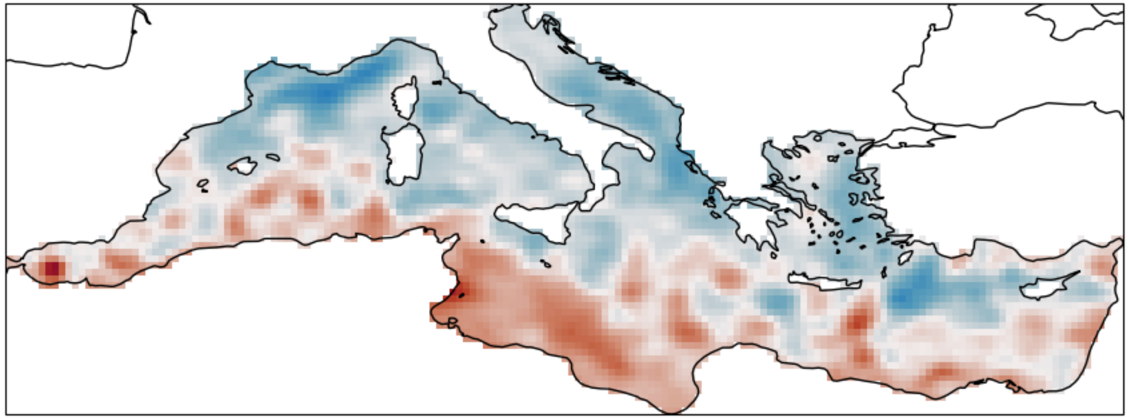
- SSH measurements
- 1/4° x 1/4° ; daily average
- From 2000-09 to 2022-11

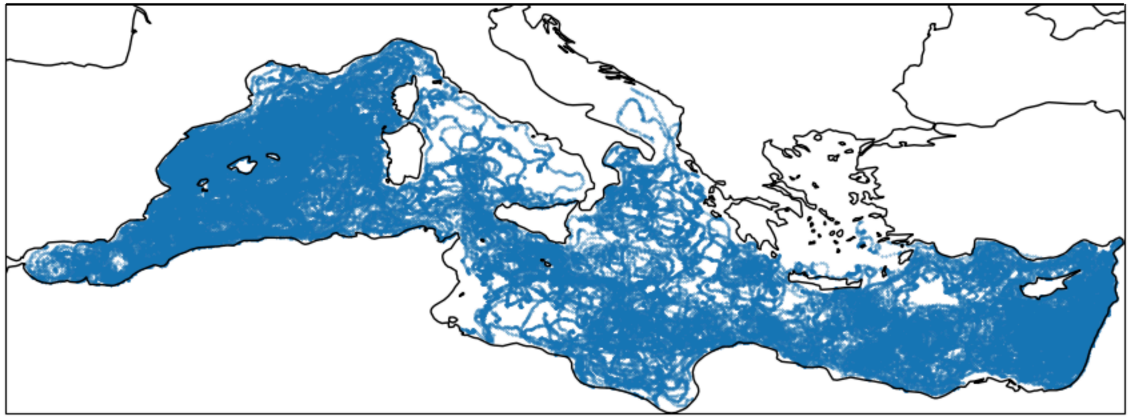


**Global Drifter Program (GDP) 6H Interpolated data**
*10.25921/7ntx-z961*

- Velocity measurements
- ~185k observations (~25 per day)
- From 2000-09 to 2022-11

# Comparison of DUACS cyclogeostrophic SSC to drifter data





SSH → `jaxparrow`

cyclogeostrophic field estimates | geostrophic field estimates

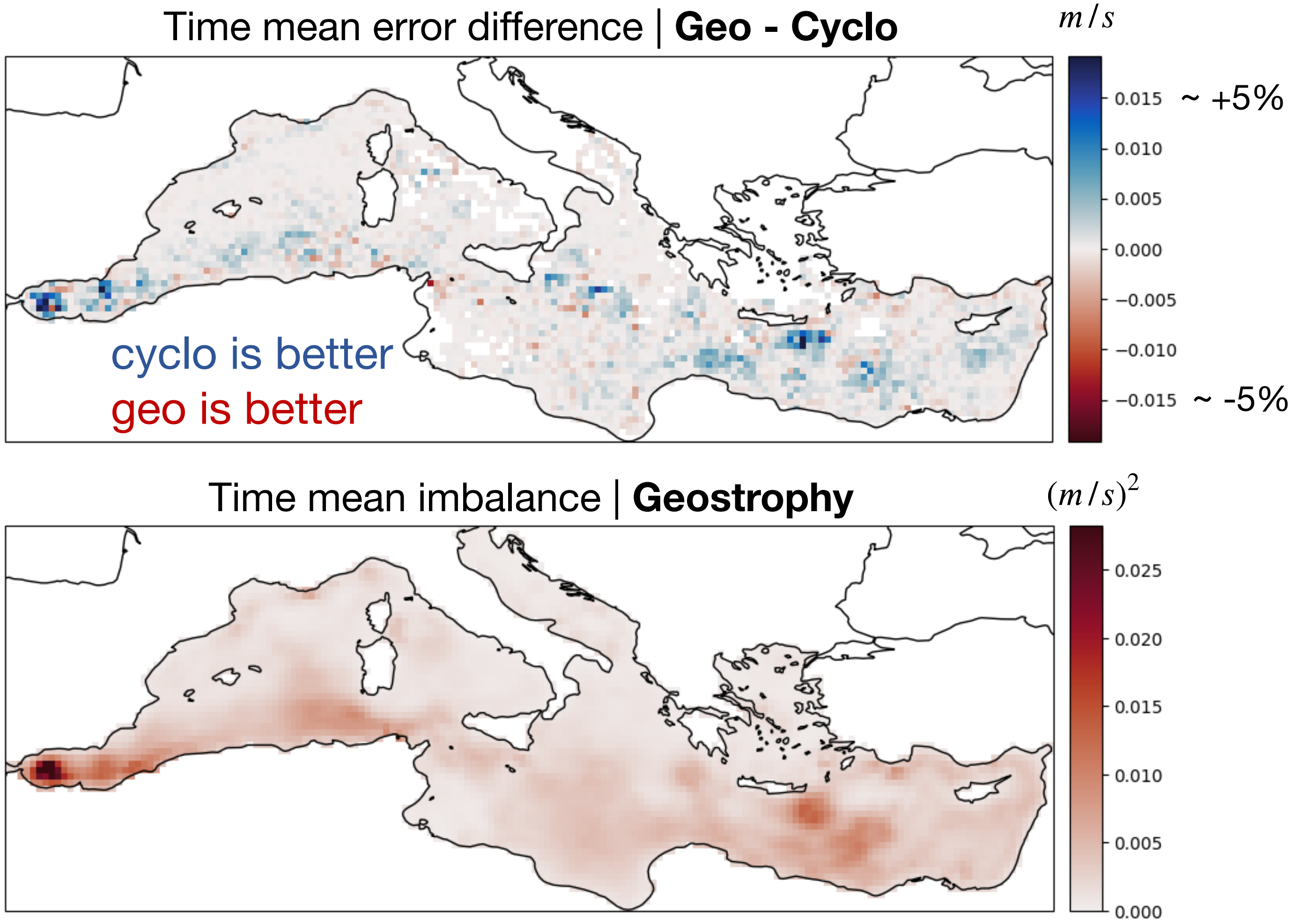
interpolation at drifters location

cyclogeostrophic point estimates | geostrophic point estimates

$(u_u, u_v)$ → $\epsilon = \sqrt{(u_u - \hat{u}_u)^2 + (u_v - \hat{u}_v)^2}$

cyclogeostrophic errors | geostrophic errors

# Comparison of DUACS cyclogeostrophic SSC to drifter data



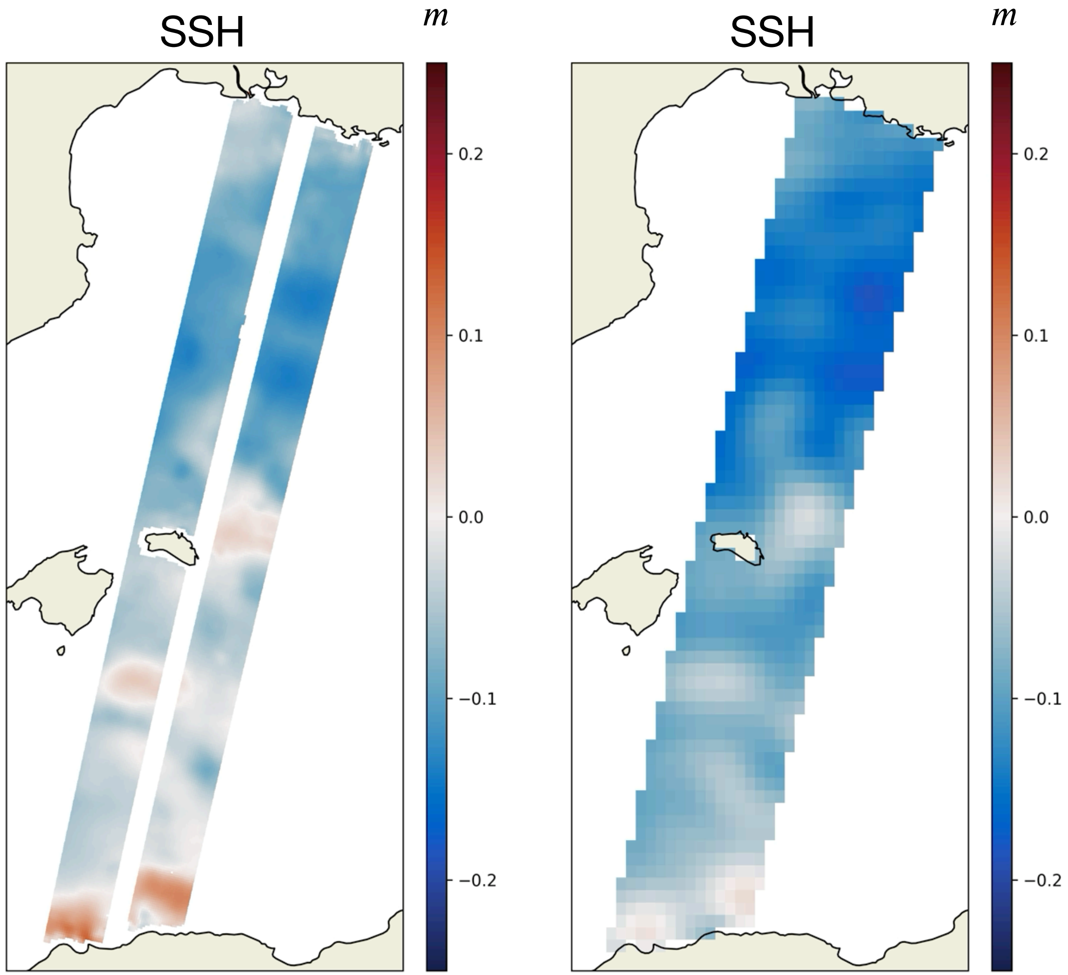
# Estimation of SWOT cyclogeostrophic SSC

**SWOT L3 Expert v1.0**
[10.24400/527896/A01-2023.018]

- 2km x 2km
- ~1 day

- Pass 003 (Mediterranea)
- From 2023-04 to 2023-07

- mdt + ssha_noiseless




**SWOT L4 MIOST**
[Ubelmann et al. 2021]

- 0.1° x 0.1°
- Daily

- Restricted to the swath
- From 2023-04 to 2023-07

- adt

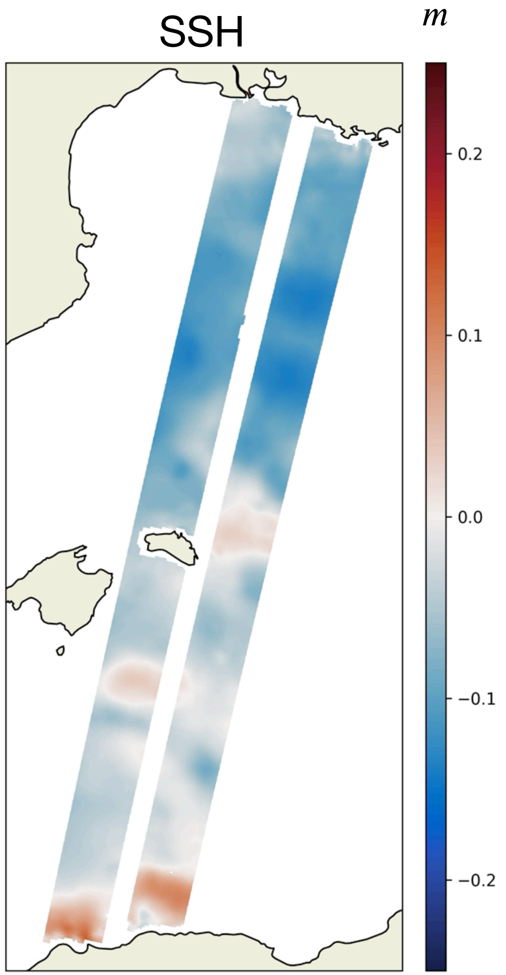
# Estimation of SWOT cyclogeostrophic SSC

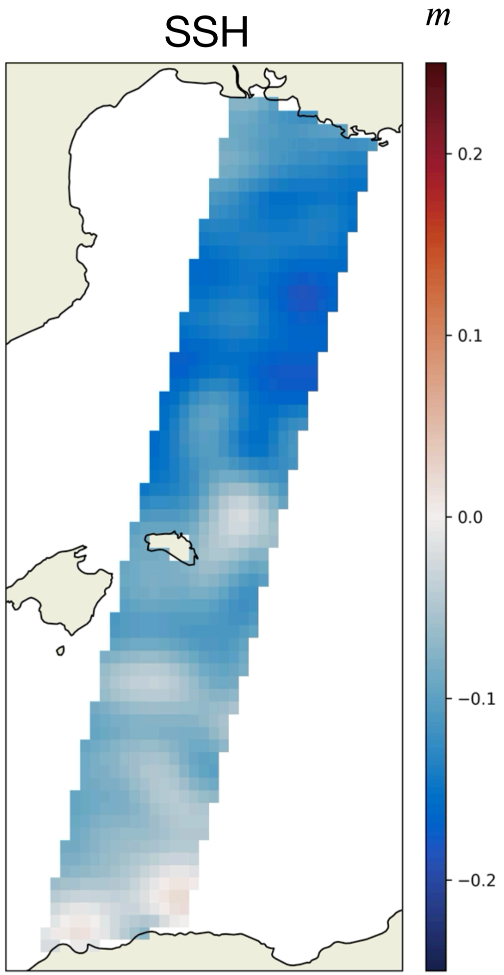
**SWOT L3 Expert v1.0**
[10.24400/527896/A01-2023.018](10.24400/527896/A01-2023.018)

- 2km x 2km
- ~1 day

- Pass 003 (Mediterranea)
- From 2023-04 to 2023-07

- mdt + ssha_noiseless



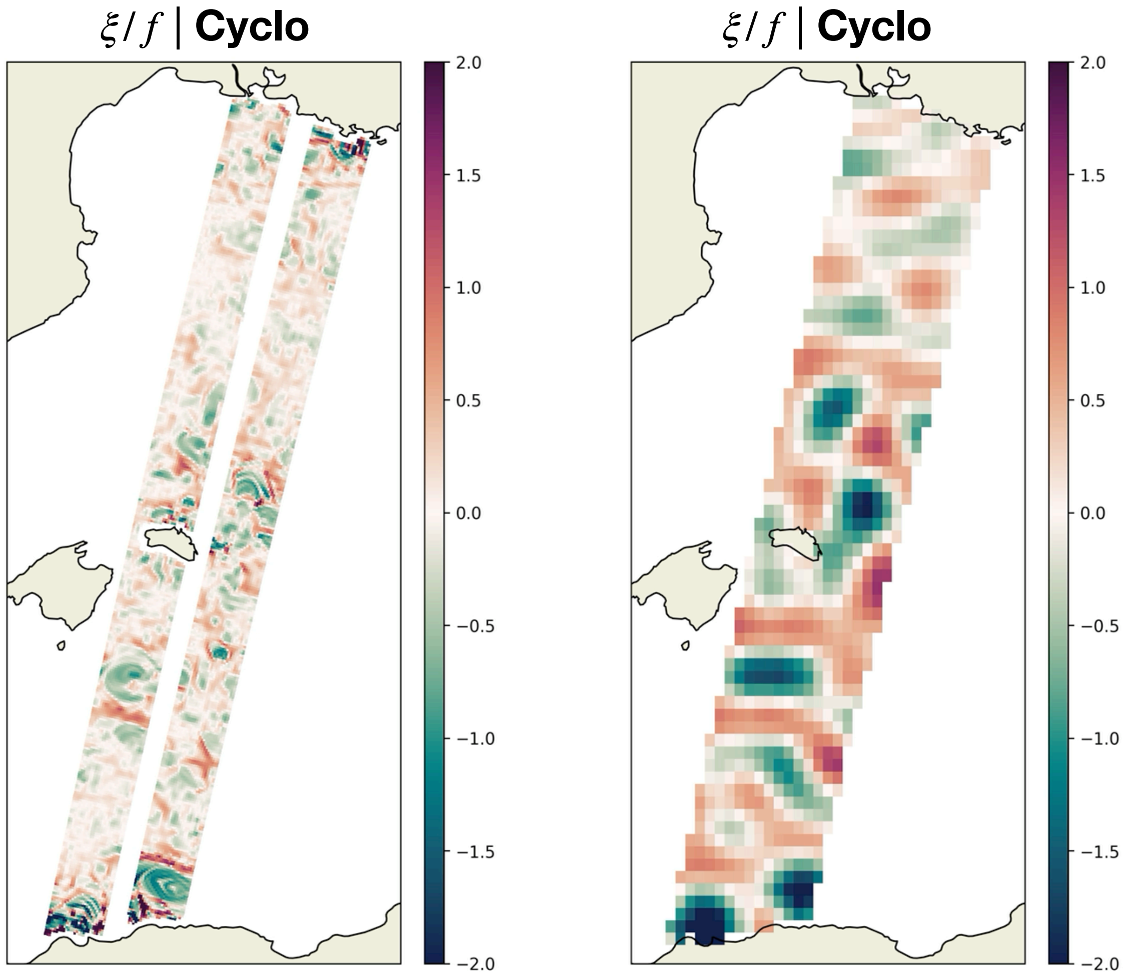

**SWOT L4 MIOST**
*Ubelmann et al. 2021*

- 0.1° x 0.1°
- Daily

- Restricted to the swath
- From 2023-04 to 2023-07

- adt

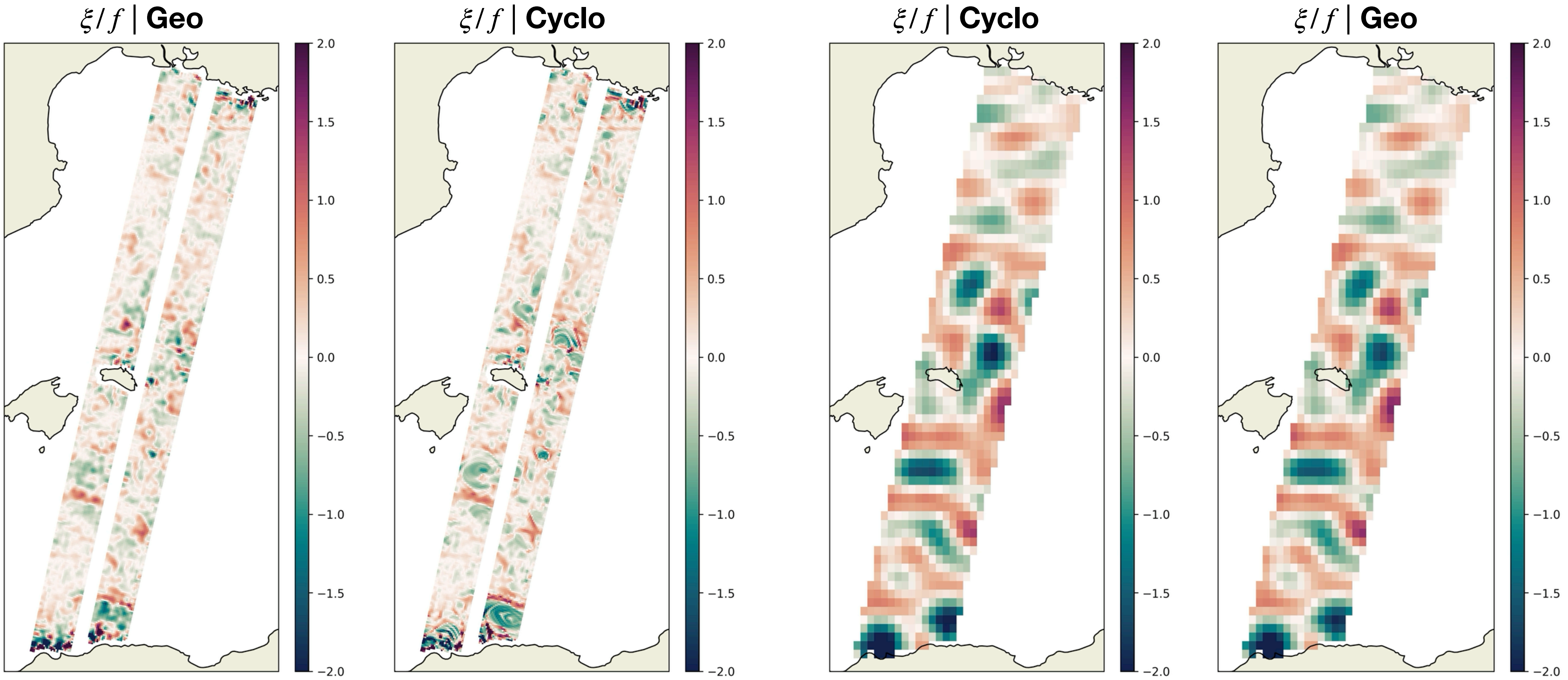
# Estimation of SWOT cyclogeostrophic SSC




L3 — ssha_noiseless

L4 — MIOST

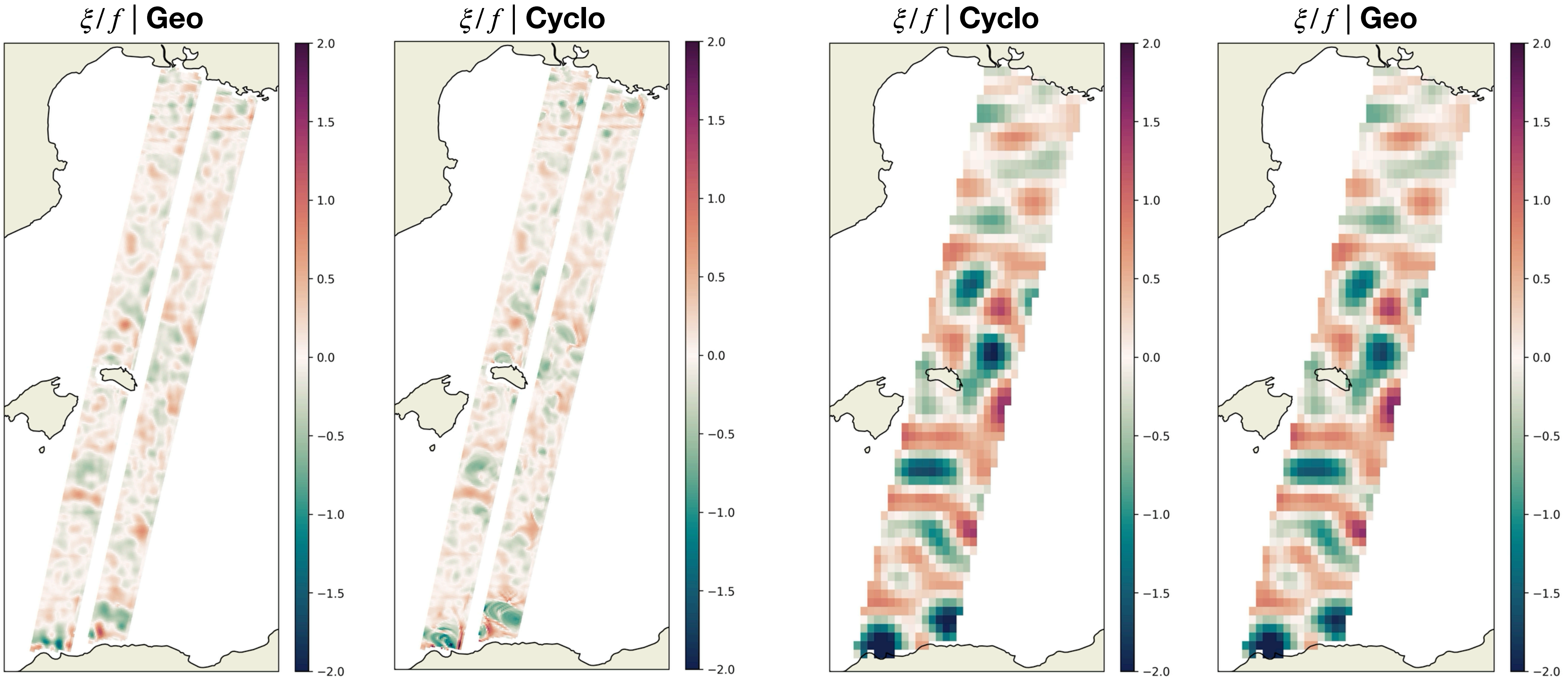
# Estimation of SWOT cyclogeostrophic SSC




L3 — ssha_noiseless

L4 — MIOST

# Estimation of SWOT cyclogeostrophic SSC




L3 — ssha + Gaussian filter ($\sigma_{time} = 24\ h,\ \sigma_{space} = 5\ km$)

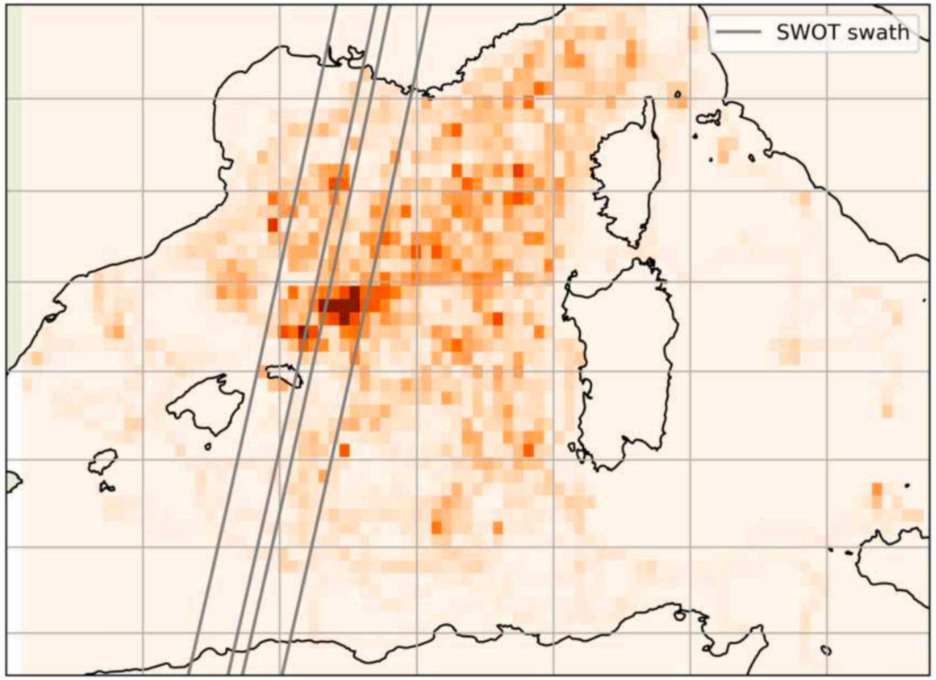
L4 — MIOST

# SWOT cyclogeostrophic SSC vs BioSWOT AdaC drifter data

## Impact of filtering

**C-SWOT** (10.17600/18002077392)
**BioSWOT** (10.17600/18002392)
(cf. Laura Gomez-Navarro & Louise Rousselet Keynote)

- L2 – 30 min interpolated data
- 65 SVP drifters ; ~27k obs. in the swath
- Derived velocities $\overrightarrow{u}_{drifter}$
- + low-pass velocity filtering



# SWOT cyclogeostrophic SSC vs BioSWOT AdaC drifter data

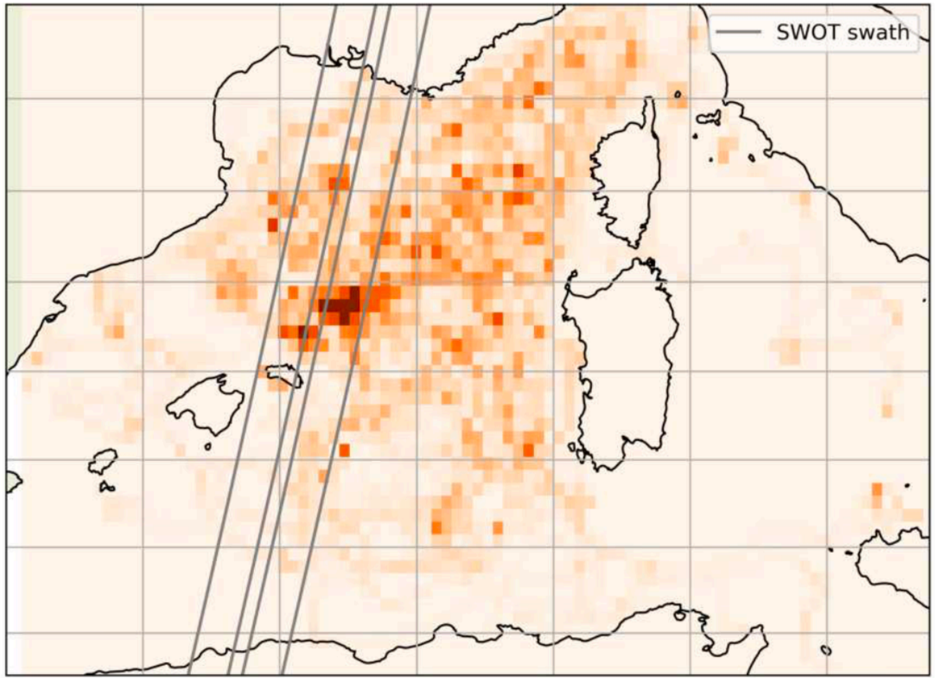
## Impact of filtering

**Many thanks to the MedSea Lagrangian working group**

**Margot Demol**, Pierre Garreau, Aurélien Ponte and Fabrice Ardhuin (LOPS)
Maristella Berta, Marco Bellacicco and Luca Centurioni (CNR-SCRIPPS)
Franck Dumas (SHOM) ; Alexey Mironov (Eodyn) ; Milenna Menna (OGS)

**C-SWOT** (10.17600/18002077392)
**BioSWOT** (10.17600/18002392)
(cf. Laura Gomez-Navarro & Louise Rousselet Keynote)

- L2 – 30 min interpolated data
- 65 SVP drifters ; ~27k obs. in the swath
- Derived velocities $\vec{u}_{drifter}$
- + low-pass velocity filtering



# SWOT cyclogeostrophic SSC vs BioSWOT AdaC drifter data

## Impact of filtering

$$\left\| \vec{u}_{drifter} - \vec{u}_{swath} \right\| \ (m/s)$$

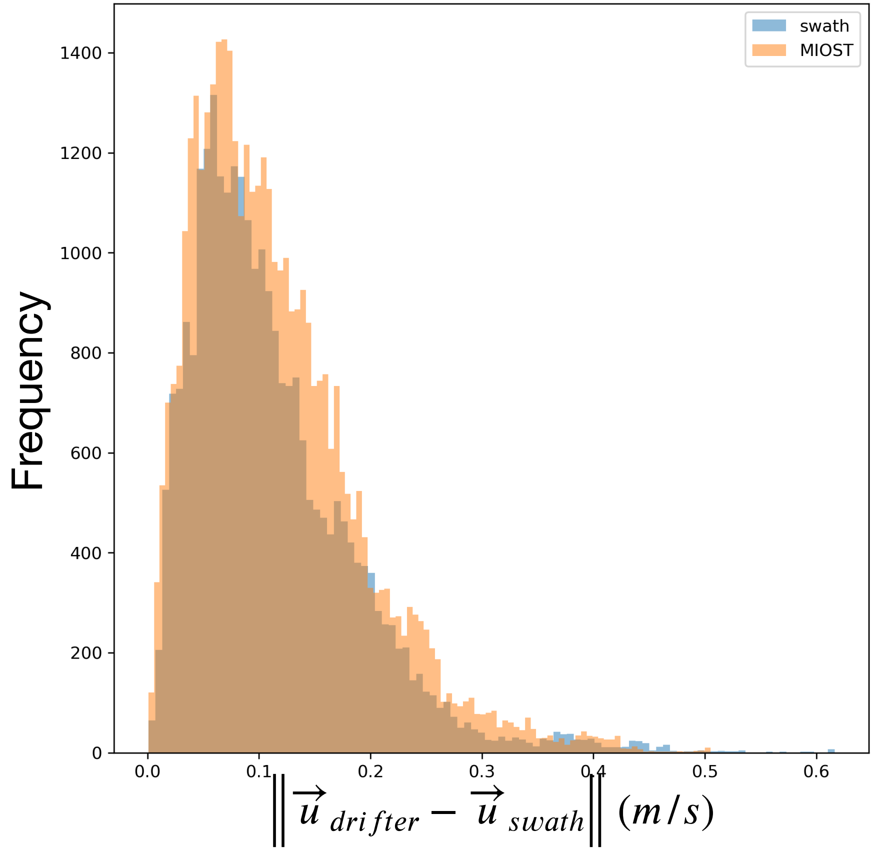
| $\sigma_{time}$ \ $\sigma_{space}$ | 2 km | 3.5 km | 5 km | 6.5 km |
|---|---|---|---|---|
| 0 h | 0.15 | 0.13 | 0.12 | 0.11 |
| 12 h | 0.15 | 0.13 | 0.12 | 0.11 |
| 24 h | 0.14 | 0.12 | 0.11 | 0.11 |
| 36 h | 0.14 | 0.12 | 0.11 | 0.11 |

# SWOT cyclogeostrophic SSC vs BioSWOT AdaC drifter data

## Impact of filtering

$\left\| \vec{u}_{drifter} - \vec{u}_{swath} \right\| \ (m/s)$

| $\sigma_{space}$ / $\sigma_{time}$ | 2 km | 3.5 km | 5 km | 6.5 km |
|---|---|---|---|---|
| 0 h | 0.15 | 0.13 | 0.12 | 0.11 |
| 12 h | 0.15 | 0.13 | 0.12 | 0.11 |
| 24 h | 0.14 | 0.12 | 0.11 | 0.11 |
| 36 h | 0.14 | 0.12 | 0.11 | 0.11 |



# Conclusion

- New variational method for reconstructing cyclogeostrophic SSC,
- Open-source implementation: https://github.com/meom-group/jaxparrow,
- Validation using DUACS SSH (~10% EKE differences at mid-latitudes).

- DUACS-derived cyclogeostrophic SSC are closer to GDP drifter velocities in energetic conditions.

Preliminary results with SWOT:

- Applicable directly to the swath,
- Better agreement with BioSWOT AdaC drifters-derived velocities.