1

$$Y_i = f_\varphi(X_i, \xi_i) + \varepsilon_i \qquad \varepsilon_i \underset{iid}{\sim} \mathcal{N}(0, \omega^2)$$

$$\xi_i \mid \xi_{i-1} \sim \mathcal{N}\left(\xi_{i-1} e^{-\beta\Delta}, \frac{\sigma^2}{2\beta}\left(1 - e^{-2\beta\Delta}\right)\right)$$

$$\varphi = (A, B, a, b)$$

$$\psi = e^{-\beta\Delta}$$

$$\gamma^2 = \frac{\sigma^2}{\beta}.$$

$$\xi_i \mid \xi_{j-1} \sim \mathcal{N}\left(\xi_{i-1}\psi, \frac{\gamma^2}{2}(1-\psi^2)\right).$$

$\log p(Y, \varphi, \psi, \gamma^2, \omega^2)$ = pas explicite.

$$p(Y, \xi_i \mid \theta) = \prod_{i=1}^{n} p(Y_i \mid \xi_i) \prod_{i=1}^{n} p(\xi_i \mid \xi_{i-1}).$$

$$p(Y_i \mid \xi_i) = \mathcal{N}(f_\varphi(X_i, \xi_i), \omega^2).$$

$$\log p(Y, \xi_i \mid \theta) \propto -\frac{1}{2\omega^2}\sum_{i=1}^{n}(Y_i - f_\varphi(\xi_i))^2 - \frac{1}{2\frac{\gamma^2}{2}(1-\psi^2)} \sum_{i=1}^{n} (\xi_i - \xi_{i-1}\psi)^2$$

Expectation.

$$Q(\theta, \theta^k) =$$

$$\mathbb{E}\left(\log p(y, \xi_i \mid \theta) \mid y_i, \theta^k \right).$$

$$p(\xi_i \mid y_i, \theta^k).$$

Maximisation

$$\theta^k = \arg\max_{\theta} Q(\theta, \theta^k).$$

Simulat.

$$\xi_i^{(k)} \sim p(\xi_i \mid y_i, \theta^k) \quad \text{MCMC}$$

Approx. stoc.

$$Q^{(k)} = Q^{(k-1)} + (1-\alpha_k)\left(Q(\xi_i^{(k)}) - Q^{(k-1)}\right).$$

$$= \alpha_k Q^{(k-1)} + (1-\alpha_k)\, Q(\xi_i^{(k)}).$$

Maximisat

$$\theta^k = \arg\max Q(\theta, \theta^k).$$

mcmc

$\xi_i = (\xi_{i1}, \ldots, \xi_{in})$

for $j = 1 : n$

$\xi_{ij}^c \sim \xi_{ij} + \mathcal{N}(0, \delta_j)$ marche aléatoire

$$\text{proba} = \frac{L(Y, \xi_{ij}^c)}{L(Y, \xi_{ij})}$$

$\log U < \log \text{proba}$