Análisis Dinámico Modal Espectral

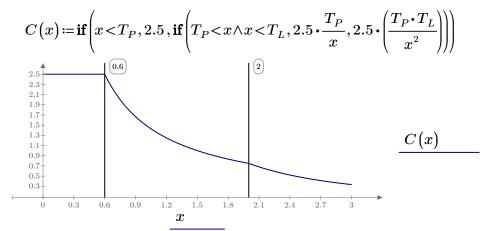
Ingeniería Sismorresistente y Prevención de Desastres ES831 H

R.C.

PARÁMETROS SÍSMICOS E030

Ubicación: Zona 4	$Z \coloneqq 0.45$
Uso: Centro de Salud	$U\!\coloneqq\!1.5$
Tipo de suelo: S2	$S \coloneqq 1.05$
Periodo corto	$T_P \coloneqq 0.6$
Periodo largo	$T_L \coloneqq 2.0$
Sistema Estructural: C.A. Pórticos	$R_0 \coloneqq 8$
Regular en planta	$I_p \coloneqq 1$
Regular en altura	$I_a \coloneqq 1$
Coeficiente de reducción	$R \coloneqq R_0 \cdot I_n \cdot I_a = 8$

COEFICIENTE DE AMPLIFICACIÓN SÍSMICA E030



DEFINICIÓN DE MASAS Y RIGIDECES

Peso sísmico

Peso sismico

$$m6 := \frac{w6}{g} = 0.908 \ tonnef \cdot \frac{s^2}{cm}$$
 $k6 := 906.449 \ \frac{tonnef}{cm}$
 $w5 := 1011.459 \ tonnef$
 $m5 := \frac{w5}{g} = 1.031 \ tonnef \cdot \frac{s^2}{cm}$
 $k5 := 906.449 \ \frac{tonnef}{cm}$
 $w4 := 1011.459 \ tonnef$
 $m4 := \frac{w4}{g} = 1.031 \ tonnef \cdot \frac{s^2}{cm}$
 $k4 := 906.449 \ \frac{tonnef}{cm}$
 $w3 := 1011.459 \ tonnef$
 $m3 := \frac{w3}{g} = 1.031 \ tonnef \cdot \frac{s^2}{cm}$
 $k3 := 906.449 \ \frac{tonnef}{cm}$
 $w2 := 1013.072 \ tonnef$
 $m2 := \frac{w2}{g} = 1.033 \ tonnef \cdot \frac{s^2}{cm}$
 $k2 := 922.508 \ \frac{tonnef}{cm}$
 $w1 := 993.843 \ tonnef$
 $m1 := \frac{w1}{g} = 1.013 \ tonnef \cdot \frac{s^2}{cm}$
 $k1 := 1797.101 \ \frac{tonnef}{cm}$

$$mM \coloneqq \begin{bmatrix} m6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & m5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m1 \end{bmatrix} \qquad mK \coloneqq \begin{bmatrix} k6 & -k6 & 0 & 0 & 0 & 0 \\ -k6 & k5 + k6 & -k5 & 0 & 0 & 0 & 0 \\ 0 & -k5 & k4 + k5 & -k4 & 0 & 0 \\ 0 & 0 & -k4 & k3 + k4 & -k3 & 0 \\ 0 & 0 & 0 & -k3 & k2 + k3 & -k2 \\ 0 & 0 & 0 & 0 & -k2 & k1 + k2 \end{bmatrix}$$

OBTENCIÓN DE FRECUENCIAS Y PERIODOS

$$\det(mK - a \cdot mM) = 0$$

$$A = \frac{mI}{a} = mK^{-1} \cdot mM$$

$$matrizA := mK^{-1} \cdot mM = \begin{bmatrix} 0.005 & 0.005 & 0.004 & 0.003 & 0.002 & 0.001 \\ 0.004 & 0.005 & 0.004 & 0.003 & 0.002 & 0.001 \\ 0.003 & 0.004 & 0.004 & 0.003 & 0.002 & 0.001 \\ 0.002 & 0.003 & 0.003 & 0.003 & 0.002 & 0.001 \\ 0.001 & 0.002 & 0.002 & 0.002 & 0.002 & 0.001 \\ 0.001 & 0.001 & 0.001 & 0.001 & 0.001 & 0.001 \end{bmatrix} s^2$$

Las frecuencias se denominan de menor a mayor

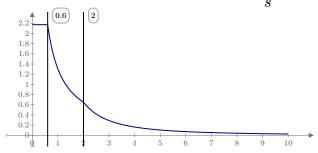
$$\omega_{\mathbf{i}} \coloneqq \operatorname{csort}\left(\frac{1}{\operatorname{eigenvals}\left(matrizA\right)}, 0\right) = \begin{vmatrix} 62.56878 \\ 534.30568 \\ 1339.69031 \\ 2256.4016 \\ 3040.339 \\ 3491.92354 \end{vmatrix} \frac{1}{\boldsymbol{s}^{2}}$$

Periodos

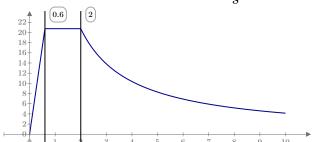
$$\begin{split} \omega_{1} &:= \left\| \sqrt{\omega_{i}^{\widehat{0}}} \right\| = 7.91 \, \frac{1}{s} & \omega_{2} := \left\| \sqrt{\omega_{i}^{\widehat{0}}} \right\| = 23.1151 \, \frac{1}{s} & \omega_{3} := \left\| \sqrt{\omega_{i}^{\widehat{0}}} \right\| = 36.6018 \, \frac{1}{s} \\ T_{1} &:= \frac{2 \cdot \pi}{\omega_{1}} = 0.794 \, s & T_{2} := \frac{2 \cdot \pi}{\omega_{2}} = 0.272 \, s & T_{3} := \frac{2 \cdot \pi}{\omega_{3}} = 0.172 \, s \\ \omega_{4} &:= \left\| \sqrt{\omega_{i}^{\widehat{0}}} \right\| = 47.5016 \, \frac{1}{s} & \omega_{5} := \left\| \sqrt{\omega_{i}^{\widehat{0}}} \right\| = 55.1393 \, \frac{1}{s} & \omega_{6} := \left\| \sqrt{\omega_{i}^{\widehat{0}}} \right\| = 59.0925 \, \frac{1}{s} \\ T_{4} &:= \frac{2 \cdot \pi}{\omega_{4}} = 0.132 \, s & T_{5} := \frac{2 \cdot \pi}{\omega_{5}} = 0.114 \, s & T_{6} := \frac{2 \cdot \pi}{\omega_{6}} = 0.106 \, s \end{split}$$

ESPECTROS DE DISEÑO E030

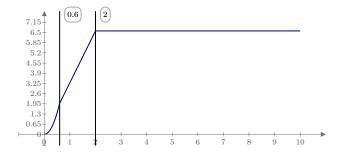
ESPECTRO DE ACELERACIONES $\frac{m}{c^2}$



ESPECTRO DE VELOCIDADES $\frac{cm}{s}$



ESPECTRO DE DESPLAZAMIENTOS cm

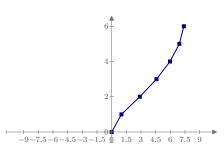


$$S_a = \begin{bmatrix} 1.641 \\ 2.172 \\ 2.172 \\ 2.172 \\ 2.172 \\ 2.172 \\ 2.172 \\ 2.172 \end{bmatrix} \frac{\textit{m}}{\textit{s}^2} \ S_v = \begin{bmatrix} 20.741 \\ 9.397 \\ 5.934 \\ 4.573 \\ 3.939 \\ 3.676 \end{bmatrix} \frac{\textit{cm}}{\textit{s}} \ S_d = \begin{bmatrix} 2.622 \\ 0.407 \\ 0.162 \\ 0.096 \\ 0.071 \\ 0.062 \end{bmatrix} \textit{cm}$$

OBTENCIÓN DE MODOS DE VIBRACIÓN

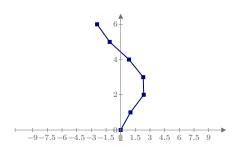
$$i \!\coloneqq\! \begin{bmatrix} 6 & 5 & 4 & 3 & 2 & 1 \end{bmatrix}^{^{\mathrm{T}}}$$

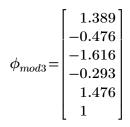
$$\phi_{mod1} = \begin{bmatrix} 7.386\\ 6.923\\ 5.967\\ 4.587\\ 2.879\\ 1 \end{bmatrix}$$

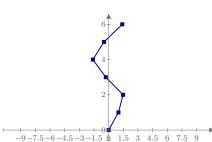


$$\underbrace{ \text{Mod2} \coloneqq mK - \omega_2^{\ 2} \cdot mM = \begin{bmatrix} 421.21 & -906.45 & 0 & 0 & 0 & 0 \\ -906.45 & 1261.81 & -906.45 & 0 & 0 & 0 \\ 0 & -906.45 & 1261.81 & -906.45 & 0 & 0 \\ 0 & 0 & -906.45 & 1261.81 & -906.45 & 0 \\ 0 & 0 & 0 & -906.45 & 1276.99 & -922.51 \\ 0 & 0 & 0 & 0 & -922.51 & 2178.12 \end{bmatrix}} \underbrace{ \begin{array}{c} \text{tonnef} \\ \text{cm} \\ \end{array} }$$

$$\phi_{mod2} = \begin{bmatrix} -2.414 \\ -1.122 \\ 0.853 \\ 2.309 \\ 2.361 \\ 1 \end{bmatrix}$$



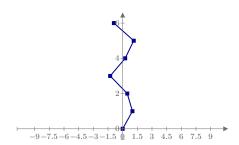




CUARTO MODO

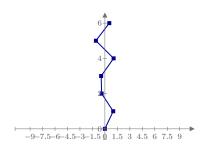
$$\underbrace{Mod4 \coloneqq mK - \omega_4^{\ 2} \cdot mM} = \begin{bmatrix} -1142.72 & -906.45 & 0 & 0 & 0 & 0 \\ -906.45 & -514.36 & -906.45 & 0 & 0 & 0 \\ 0 & -906.45 & -514.36 & -906.45 & 0 & 0 \\ 0 & 0 & -906.45 & -514.36 & -906.45 & 0 \\ 0 & 0 & 0 & -906.45 & -502.01 & -922.51 \\ 0 & 0 & 0 & 0 & -922.51 & 432.89 \end{bmatrix} \underbrace{\begin{array}{c} \textit{tonnef} \\ \textit{cm} \\ \end{aligned}}$$

$$\phi_{mod4}\!=\!\begin{bmatrix} -0.898\\ 1.132\\ 0.256\\ -1.278\\ 0.469\\ 1 \end{bmatrix}$$



QUINTO MODO

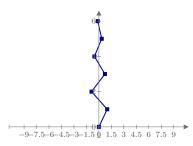
$$\phi_{mod5} = \begin{bmatrix} 0.528 \\ -1.081 \\ 1.049 \\ -0.45 \\ -0.392 \\ 1 \end{bmatrix}$$



SEXTO MODO

$$\underbrace{ \text{SEXTO MODO} }_{Mod6 := mK - \omega_6^2} \cdot mM = \begin{bmatrix} -2264.77 & -906.45 & 0 & 0 & 0 & 0 \\ -906.45 & -1788.68 & -906.45 & 0 & 0 & 0 \\ 0 & -906.45 & -1788.68 & -906.45 & 0 & 0 \\ 0 & 0 & -906.45 & -1788.68 & -906.45 & 0 \\ 0 & 0 & 0 & -906.45 & -1778.36 & -922.51 \\ 0 & 0 & 0 & 0 & -922.51 & -819.24 \end{bmatrix} \underbrace{ \begin{array}{c} \textit{tonnef} \\ \textit{cm} \\ \end{array} }_{}$$

$$\phi_{mod6}\!=\!\begin{bmatrix} -0.138\\ 0.344\\ -0.542\\ 0.725\\ -0.888\\ 1\end{bmatrix}$$



MATRIZ MODAL

$$\phi_{mod} \coloneqq \operatorname{augment} \left(\phi_{mod1}, \phi_{mod2}, \phi_{mod3}, \phi_{mod4}, \phi_{mod5}, \phi_{mod6} \right)$$

$$\phi_{mod} = \begin{bmatrix} 7.386 & -2.414 & 1.389 & -0.898 & 0.528 & -0.138 \\ 6.923 & -1.122 & -0.476 & 1.132 & -1.081 & 0.344 \\ 5.967 & 0.853 & -1.616 & 0.256 & 1.049 & -0.542 \\ 4.587 & 2.309 & -0.293 & -1.278 & -0.45 & 0.725 \\ 2.879 & 2.361 & 1.476 & 0.469 & -0.392 & -0.888 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

FACTORES DE PARTICIPACIÓN

PRECÁLCULOS

$$B \coloneqq \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}^{\mathrm{T}}$$

$$\phi_{mod}^{\mathrm{T}} \boldsymbol{\cdot} mM \boldsymbol{\cdot} B = \begin{bmatrix} 2872.201 \\ 336.343 \\ 134.143 \\ 79.645 \\ 59.109 \\ 51.465 \end{bmatrix} \boldsymbol{s}^{2} \boldsymbol{\cdot} \boldsymbol{tonnef} \qquad \phi_{mod}^{\mathrm{T}} \boldsymbol{\cdot} mM \boldsymbol{\cdot} \phi_{mod} \boldsymbol{\cdot} B = \begin{bmatrix} 16698.819 \\ 1960.957 \\ 803.241 \\ 504.757 \\ 397.607 \\ 281.183 \end{bmatrix} \boldsymbol{s}^{2} \boldsymbol{\cdot} \boldsymbol{tonnef}$$

FACTOR DE PARTICIPACIÓN DE MASA (FPM)

$$FPM \coloneqq \frac{\phi_{mod}^{\mathrm{T}} \cdot mM \cdot B}{\phi_{mod}^{\mathrm{T}} \cdot mM \cdot \phi_{mod} \cdot B} = \begin{bmatrix} 0.172 \\ 0.172 \\ 0.167 \\ 0.158 \\ 0.149 \\ 0.183 \end{bmatrix} \quad \begin{array}{l} \mathsf{Modo} \ 1 \\ \mathsf{Modo} \ 2 \\ \mathsf{Modo} \ 3 \\ \mathsf{Modo} \ 4 \\ \mathsf{Modo} \ 5 \\ \mathsf{Modo} \ 5 \\ \mathsf{Modo} \ 6 \\ \end{bmatrix}$$

MASA EFECTIVA MODAL - PORCENTAJE DE PARTICIPACIÓN DE MASA

$$\varphi \coloneqq \frac{\left(\phi_{mod}^{\mathsf{T}} \cdot mM \cdot B\right)^{2}}{\phi_{mod}^{\mathsf{T}} \cdot mM \cdot \phi_{mod} \cdot B} = \begin{bmatrix} 494.019 \\ 57.69 \\ 22.402 \\ 12.567 \\ 8.787 \\ 9.419 \end{bmatrix} \frac{\boldsymbol{s}^{2}}{\boldsymbol{m}} \cdot \boldsymbol{tonnef}$$

$$\sum \varphi = 604.885 \frac{s^2}{m} \cdot tonnef \qquad ppm \coloneqq \frac{\varphi}{\sum \varphi} = \begin{bmatrix} 0.817 \\ 0.095 \\ 0.037 \\ 0.021 \\ 0.015 \\ 0.016 \end{bmatrix} \quad \text{Mayor al 90% (E.030)} \\ ppm_0 + ppm_1 = 0.912$$

El 90% de participación de masa se alcanza con el segundo modo; de acuerdo con la norma E.030, se debe trabajar al menos con los tres primeros modos o hasta que se alcance el 90% de participación con modos superiores. Con fines académicos, se seguirá analizando considerando los 6 modos obtenidos de los cálculos matriciales.

DESPLAZAMIENTOS

DESPLAZAMIENTOS n MÁXIMOS

$$n_{max} \coloneqq \overrightarrow{FPM \cdot S_d} = \begin{bmatrix} 0.451 \\ 0.07 \\ 0.027 \\ 0.015 \\ 0.011 \\ 0.011 \end{bmatrix} \boldsymbol{cm}$$

MATRIZ R MODAL

$$R_{mod}\!\coloneqq\!\mathrm{diag}\left(n_{max}\right)\!=\!\begin{bmatrix} 0.451 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.07 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.027 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.015 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.011 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.011 \end{bmatrix} \boldsymbol{cm}$$

DESPLAZAMIENTOS MODALES

$$U_{mod} \coloneqq \phi_{mod} \cdot R_{mod} = \begin{bmatrix} 3.331 & -0.168 & 0.038 & -0.014 & 0.006 & -0.002 \\ 3.122 & -0.078 & -0.013 & 0.017 & -0.011 & 0.004 \\ 2.691 & 0.059 & -0.044 & 0.004 & 0.011 & -0.006 \\ 2.069 & 0.161 & -0.008 & -0.019 & -0.005 & 0.008 \\ 1.299 & 0.165 & 0.04 & 0.007 & -0.004 & -0.01 \\ 0.451 & 0.07 & 0.027 & 0.015 & 0.011 & 0.011 \end{bmatrix} \quad \begin{matrix} \text{Piso 6} \\ \text{Piso 5} \\ \text{Piso 3} \\ \text{Piso 2} \\ \text{Piso 1} \end{matrix}$$

COMBINACIÓN DE RESPUESTAS

De acuerdo con Art. 29.3.4. (E.030)

DESPLAZAMIENTOS

$$U_{E030_1} \coloneqq \left\| \begin{array}{l} \text{for } i \in 0 \dots 5 \\ \left\| U^{\widehat{i}} \leftarrow 0 \text{ } \mathbf{cm} \\ \text{for } j \in 0 \dots 5 \\ \left\| \left\| U^{\widehat{i}} \leftarrow U^{\widehat{i}} + \right\| \left(0.75 \cdot R \cdot U_{mod}{}^{(j)} \right)^{\widehat{i}} \right\| \right\| \\ U \end{array} \right\| \qquad \qquad U_{E030_1} = \begin{bmatrix} 21.3485 \\ 19.477 \\ 16.8944 \\ 13.6197 \\ 9.1476 \\ 3.51 \end{bmatrix} \mathbf{cm}$$

$$U_{E030_1} = egin{bmatrix} 21.3485 \ 19.477 \ 16.8944 \ 13.6197 \ 9.1476 \ 3.51 \end{bmatrix}$$
 cm

$$U_{E030_2} \coloneqq \left\| \begin{array}{l} \text{for } i \in 0 ...5 \\ \left\| U_{\widehat{\psi}}^{\widehat{i}} \leftarrow 0 \text{ } \boldsymbol{cm}^2 \\ \text{for } j \in 0 ...5 \\ \left\| U_{\widehat{\psi}}^{\widehat{i}} \leftarrow U_{\widehat{\psi}}^{\widehat{i}} + \left\| \left(0.75 \cdot R \cdot U_{mod}^{\langle j \rangle} \right)^2 \widehat{\psi} \right\| \right\| \\ \sqrt{U} \end{array} \right\| U_{E030_2} = \begin{bmatrix} 20.0148 \\ 18.7413 \\ 16.1544 \\ 12.4499 \\ 7.858 \\ 2.7461 \end{bmatrix} \boldsymbol{cm}$$

$$U_{E030_2} \! = \! egin{bmatrix} 20.0148 \\ 18.7413 \\ 16.1544 \\ 12.4499 \\ 7.858 \\ 2.7461 \end{bmatrix} {m cm}$$

$$U_{E030} \coloneqq 0.25 \boldsymbol{\cdot} U_{E030_1} + 0.75 \boldsymbol{\cdot} U_{E030_2} = \begin{bmatrix} 20.34822 \\ 18.92525 \\ 16.33936 \\ 12.74238 \\ 8.18042 \\ 2.9371 \end{bmatrix} \boldsymbol{cm}$$

DERIVAS

$$\Delta_{mod} \coloneqq \left\| \begin{array}{l} \text{for } i \in 0 \dots 5 \\ \left\| \text{for } j \in 0 \dots 5 \\ \left\| \begin{array}{l} \text{if } i < 5 \\ \left\| \Delta_{i,j} \leftarrow U_{mod_{i,j}} - U_{mod_{i+1,j}} \right\| \\ \text{else} \\ \left\| \Delta_{i,j} \leftarrow U_{mod_{i,j}} \end{array} \right\| \right\|$$

$$\delta_{mod} \! = \! \begin{bmatrix} 0.000674 & -0.000291 & 0.000163 & -0.0001 & 0.000055 & -0.000018 \\ 0.001391 & -0.000444 & 0.0001 & 0.000043 & -0.000073 & 0.000033 \\ 0.002009 & -0.000327 & -0.000116 & 0.000075 & 0.000051 & -0.000047 \\ 0.002484 & -0.000012 & -0.000155 & -0.000086 & -0.000002 & 0.000059 \\ 0.002734 & 0.000306 & 0.000042 & -0.000026 & -0.000048 & -0.000069 \\ 0.001455 & 0.000225 & 0.000087 & 0.000049 & 0.000034 & 0.000037 \end{bmatrix}$$

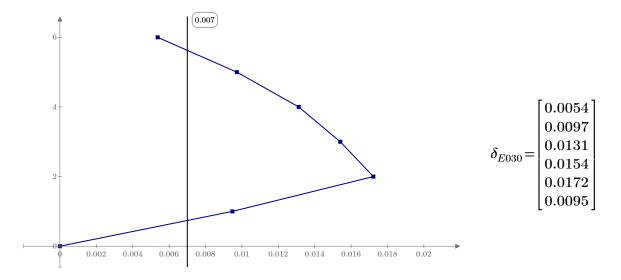
$$\delta_{E030_1} \coloneqq \begin{bmatrix} \text{for } i \in 0 \dots 5 \\ \delta^{\widehat{i}} \leftarrow 0 \\ \text{for } j \in 0 \dots 5 \\ \|\delta^{\widehat{i}} \leftarrow \delta^{\widehat{i}} + \| \left(0.75 \cdot R \cdot \delta_{mod}^{\langle j \rangle}\right)^{\widehat{i}} \| \end{bmatrix}$$

$$\delta_{E030_1} = \begin{bmatrix} 0.0078 \\ 0.0125 \\ 0.0157 \\ 0.0168 \\ 0.0193 \\ 0.0113 \end{bmatrix}$$

$$\delta_{E030_2} \coloneqq \left\| \begin{array}{l} \text{for } i \in 0 \dots 5 \\ \left\| \delta^{\widehat{i}} \leftarrow 0 \\ \text{for } j \in 0 \dots 5 \\ \left\| \delta^{\widehat{i}} \leftarrow \delta^{\widehat{i}} + \left\| \left(0.75 \cdot R \cdot \delta_{mod}^{\langle j \rangle} \right)^2 \widehat{i} \right\| \right\| \\ \sqrt{\delta} \end{array} \right\| \delta^{\widehat{i}} \leftarrow \delta^{\widehat{i}} + \left\| \left(0.75 \cdot R \cdot \delta_{mod}^{\langle j \rangle} \right)^2 \widehat{i} \right\| \right\| \delta^{\widehat{i}} \leftarrow \delta^{\widehat{i}} + \left\| \left(0.75 \cdot R \cdot \delta_{mod}^{\langle j \rangle} \right)^2 \widehat{i} \right\| \right\| \delta^{\widehat{i}} \leftarrow \delta^{\widehat{i}} + \left\| \left(0.75 \cdot R \cdot \delta_{mod}^{\langle j \rangle} \right)^2 \widehat{i} \right\| \delta^{\widehat{i}} = 0.0046 \\ 0.0088 \\ 0.0122 \\ 0.0149 \\ 0.0165 \\ 0.0089 \\ 0.$$

$$\delta_{E030} \coloneqq 0.25 \cdot \delta_{E030_1} + 0.75 \cdot \delta_{E030_2} = \begin{bmatrix} 0.00537 \\ 0.00972 \\ 0.01312 \\ 0.0154 \\ 0.01723 \\ 0.00947 \end{bmatrix}$$

GRÁFICA Y COMPARACIÓN CON LÍMITE No debe exceder 0.007 (E.030)



Como se observa en la comparación, las derivas superan los límites de la norma E.030 en todos los pisos excepto el último. Es necesario el reforzamiento y/o rediseño.