model description paper

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Abstract

This is my abstract.

1 Introduction

Bedrock river erosion is one of the principal ways in which perturbations to geomorphic equilibrium, such as shifts in climate or tectonic regimes, are propagated across the landscape (e.g., Sklar and Dietrich, 1998, Whipple, 2004). Bedrock rivers, then, play a vital role in setting landscape response times to such perturbations (e.g., Whipple and Tucker, 1999). It follows that in order to predict these large-scale landscape responses, we require a thorough understanding of the mechanics of bedrock river erosion.

Since the 1980s, much thought and attention has been given to developing a mathematical representation of erosional processes in bedrock rivers. Howard and Kerby (1983) studied both bedrock and alluvial channels developed over short timescales in a badland setting. They observed that alluvial rivers mainly transported upstream sediment supply, with only minor amounts of excess transport capacity contributing to bed erosion. The bedrock channels, however, exhibited erosion rates proportional to bed shear stress during high flow events. The recognition of this relationship led Howard and Kerby (1983) to define a power law function relating bedrock river erosion rate to drainage area and slope. This relationship emerges whether erosion is assumed to depend on either shear stress or stream power (Siedl and Dietrich, 1992, Whipple and Tucker, 1999). Equations of this form are commonly known as "stream power equations," (Lague, 2014) and are written as

$$E = KA^m S^n \tag{1}$$

where m and n are positive exponents, and K is an erodibility factor that encapsulates bedrock properties and climate variables. The righthand side of Equation 1 is equivalent in form to a geomorphic transport capacity (Howard, 1994 and references therein). If a river has bedrock exposed on its bed, it is reasonable to assume that the transport capacity of the river exceeds the rate of sediment supply (Howard and Kerby, 1983, Siedl and Dietrich, 1992, Howard, 1994b). These settings are termed "detachment-limited," implying that the erosion rate is limited not by the river's ability to transport alluvium downstream, but by the rate at which material can be detached and mobilized for transport from the riverbed itself (e.g., Howard, 1994b, Whipple and Tucker, 1999). If bedrock erodibility (K) is constant, then erosion rate will increase with shear stress/stream power.

In a steady state landscape, erosion rate everywhere along a river profile will be equal to the tectonic uplift rate. The stream power model captures this behavior when in predicts concave-upward steady state profiles, where slopes decrease in the downstream direction as contributing drainage area increases. If lithologic heterogeneities are present along the profile (resulting in different values of K), then the stream power law will predict that local slopes adjust to keep erosion rate the same everywhere despite differences in K.

The use of stream power models in the fluvial geomorphology and landscape evolution modeling communities has become widespread. There are two main reasons for their persistent popularity: (1) they are able to reproduce certain characteristic aspects of river profiles, such as smooth, concave-upward profiles at steady state and upstream-migrating knickpoints during transient response states, and (2) they are simple, as their derivation is based on reasonable assumptions about channel geometry and hydraulics and they are cast in terms of the physically observable metrics of slope and area (Lague, 2014). However, a recent review by Lague (2014) points out many of the shortcomings of the stream power model, including its inability to capture stochastic processes, the challenges it presents in terms of upscaling short-term discharge measurements into geologically meaningful flow characteristics, and its neglect of sediment abrasion as an important erosional process (Sklar and Dietrich, 2001, 2004). The stream power model's use of K, the erodibility factor, presents an additional challenge in that its value can span orders of magnitude, its units vary with the exponents m and n of Equation 1, and it is difficult to pinpoint the exact factors that go into determining its value (Barnhart et al., 2020). Even if one ignores these challenges, the stream power model was developed for bedrock river settings where erosion is "detachment-limited" (Howard and Kerby, 1983, Howard, 1994b, Whipple and Tucker, 1999). The stream power law is not applicable to lowland or "transport-limited" settings, where elevation changes are set by the divergence of sediment discharge rather than incision into bedrock. This represents a major challenge for the geomorphic modeling community: what to do when attempting to study large-scale river systems that transition from bedrock beds in their headwaters to alluvial beds in their downstream reaches?

Even in tectonically active, montane catchments, where the stream power model may be deemed most appropriate, recent work suggests that sediment transport is an important process that sets slopes and consumes a significant portion of the energy budget of "bedrock" rivers (Pfeiffer et al., 2017, Lai et al., 2021). Gravel-bed rivers, which are common around the world in montane settings, exhibit behavior characteristic of transport-limited, alluvial, "equilibrium channels" (Parker, 1978) – their widths are self-adjusted to transport a median grain size (Wickert and Schildgen, 2019). Additional work by Wickert and Schildgen (2019) and Johnson et al. (2009) has shown that rivers that may be described as "bedrock rivers" can exhibit transport-limited behavior with dynamics best captured by transport-limited models. In addition to setting slopes and controlling channel width, sediment also plays a key role in controlling channel incision. Work by Sklar and Dietrich (2001) demonstrated that saltating bedload sediment is capable of incising bedrock via abrasion, a process that involves bed material being dislodged as a result of particle impacts. The authors recognized that the effect is non-linear: at low sediment supply rates, the sediment mainly acts as abrasive "tools" that promote erosion of the bed; as supply increases, increasing "cover" on the bed inhibits further erosion. This work led to the development of mechanistic models that focus on erosion accomplished via sediment abrasion rather than through a stream power formulation (Sklar and Dietrich, 2004, Chatanantevet and Parker, 2009.)

These challenges and complexities summarize the need for models that explicitly account for sediment effects, both in terms of how sediment influences the erosional process (namely, the relative importance of abrasion) and how sediment supply controls (both spatially and temporally) the general erosional style: transport-limited versus detachment-limited. Efforts have been made to address both of these issues via the development of "erosion-deposition" models that are capable of capturing both the removal of material from the bed and the redeposition of that material at some point downstream. If the distance at which redeposition occurs is sufficiently short, then the model is effectively "transport-limited," while if the distance is long the model is "detachment-limited." These models are capable of capturing intermediate erosional behaviors, and some can explicitly capture dynamics such as the thinning and thickening of alluvial bed cover (Davy and Lague, 2009, Shobe et al., 2017). While transport-limited and detachment-limited erosional models can produce the same steady state river profiles, the choice of erosion law does result in markedly different transient responses to external forces (Tucker and Hancock, 2010). The study of transient river responses to climate or tectonic perturbations can therefore be very useful in determining the correct fluvial erosion law to apply to a given river system.

Recent advances in erosion-deposition models have provided nuance to our understanding of rivers that exhibit traits of both detachment- and transport-limited behavior, and these models seem promising in terms of their ability to improve our understanding of fluvial response to external forcing. However, these models often assume uniform lithology and sediment supply. Meanwhile, questions surrounding how rivers adjust to heterogeneous lithology and evolving sediment supply are active areas of research in their own right. Work by Gabet (2020a) found that in the Sierra Nevada (a highly heterogenous terrain), rivers adjust their profiles to accommodate differences in both the erodibility of the bedrock itself and in the lithologically controlled median grain size. Gabet (2020b) then showed that rivers with heterogenous lithology will generate different transient knickpoint behavior in response to two types of tectonic perturbation: uniform uplift and tilting. Work by Lai et al. (2021) in eastern Taiwan also demonstrated differences in channel steepness attributed to upstream controls on sediment supply where rivers cross a lithologic contact.

Other studies have focused more explicitly on how sediment load changes across lithologic boundaries; in these studies, the boundaries represent major shifts in physiographic province, such as crossing from a range composed of crystalline rocks into an adjacent sedimentary basin. Studies conducted in both the Himalaya (Dingle et al., 2017) and the Colorado Front Range (Menting et al., 2015) have demonstrated that crossing such a contact typically does not result in representation of downstream units in the sediment load. Rather, the sediment load continues to be dominated by sediment derived from the most resistant units found along profile. These units become enriched in the sediment load downstream, even as they represent a smaller and smaller fraction of the total drainage basin. In the Himalaya, all but the most resistant clasts experience rapid downstream fining which is attributed to sediment abrasion (Dingle et al., 2017). In the Colorado Front Range, downstream fining occurs on a spatial scale shorter than that predicted in abrasion mill experiments, which points to a selective transport control on streamwise grain size in addition to sediment abrasion (Menting et al., 2015).

This study draws from the areas of research outlined above. We are inspired by a case study in the Southern Rocky Mountains, where rivers traversing the crystalline Front Range exit into the sedimentary basin of the High Plains. In this setting we observe the sediment load being dominated by granitic clasts sourced from the mountains (Menting et al., 2015). We investigate what impact this coarse, erosion-resistant bedload has on the downstream profile of rivers that flow over sedimentary rocks. We implement an erosional scheme based on sediment cover that allows for both abrasion and stream-power erosion, and that honors the gradual transition from detachment-limited behavior in the mountains to transport-limited behavior on the plains. In this way, we can make predictions about river profiles produced when this lithologically heterogeneous landscape responds to changes in climate and tectonics. We pay special attention to how these changes may at times be compensated for through erosion of bedrock, and at other times through changes in the thickness of an alluvial mantle.

2 Methods

Here we present a numerical model that attempts to fill gaps in our current modeling capabilities. We envisage a river that traverses two distinct physiographic provinces: an upstream reach comprising erosion-resistant crystalline rocks, and a downstream reach composed of highly erodible, fine-grained sedimentary units. In both domains, total topography is the sum of bedrock elevation plus some thickness of alluvial sediment that mantles the riverbed.

In brief, our model erodes sediment from the riverbed, carries a fraction of that eroded material as bedload, and loses some fraction of bedload mass downstream to grain attrition. We assume that erosion occurs only through plucking and abrasion, as these are thought to be the dominant erosional processes in bedrock rivers (Whipple, 2004). The efficiency of bedrock erosional processes is modulated by the fraction of bedrock exposed and the sediment flux, honoring the observation that bedrock erosion rates initially increase under conditions of increasing sediment supply due to the presence of "tools" that promote erosion via abrasion; above a critical threshold, however, increasing sediment supply inhibits erosion by covering the bed and shielding it from abrasive impacts (Sklar and Dietrich, 1998, 2004). Finally, rather than assuming downstream increases in sediment load scale only with slope and discharge (Smith and Bretherton, 1972), we will honor the observation that coarse sediment decreases in size downstream due to particle attrition (Sklar and Dietrich, 2001, Attal and Lave, 2006, Menting et al., 2015). The attention paid in our model to both bedrock and sediment properties leads to a unique feature of our model: the relative importance of different erosional mechanisms (plucking and abrasion) varies depending on the properties of both the substrate and bedload material.

In the sections below we first outline the governing equations of our model, and then describe our methodology for testing the model.

FRANKENSTEIN "CAVEATS" SECTION INTO ABOVE:

Our model does not account for the differences in shear stress associated with different styles of plucking detachment, per Gabet 2020b. Instead, all plucking is accomplished through a simple stream power formulation (see below). We also do not account for large (boulder-sized) clasts in the system. We use a single discharge-area relationship, rather than a stochastic formulation. And finally, when calculating erosion via abrasion, we use generic abrasion coefficients that have been found for different rock types (Attal and Lave, 2006) in order to be broadly representative of "highly erodible" and "erosion-resistant" units; our study is not intended to be grounded in a specific field site, and therefore we have not conducted fieldwork to measure fracture density or other field-based observations that would inform a more realistic formulation of abrasion coefficients.

2.1 Governing Equations

The fundamental equation in our model that determines how river profiles change shape through time states that the rate of change of total elevation is equal to the rate of change of bedrock elevation, plus the rate of change of some thickness of sediment that sits atop the bedrock profile:

$$\frac{\partial \eta}{\partial t} = \frac{\partial \eta_b}{\partial t} + \frac{\partial H}{\partial t} \tag{2}$$

where η is the total topographic elevation, η_b is the bedrock elevation, and H is the sediment thickness. Each term on the righthand side can be broken down into its contributing pieces: bedrock elevation is controlled by uplift and bedrock erosion, and sediment thickness depends on the amount of coarse material present, the sediment flux, and any mass lost from the bedload due to grain attrition. The general forms of these equations are as follows:

$$\frac{\partial \eta_b}{\partial t} = \text{uplift rate} - \text{erosion rate} \tag{3}$$

$$\frac{\partial H}{\partial t} = \text{bedload} - \text{sediment flux} - \text{attrition} \tag{4}$$

Erosion and sedimentation rates in our model depend upon an interaction between bed cover, sediment flux, bedrock erodibility, and sediment hardness. These interactions are outlined in the following equations.

2.1.1 Bed Cover

We use an exponential function, formulated after Shobe et al. (2017), to calculate α, the effective bed exposure:

$$\alpha = e^{-H/H^*} \tag{5}$$

Here H represents the actual sediment thickness present on the riverbed, and H^* is a characteristic scale that approximates bed roughness height. When the actual sediment thickness, H, is extremely large relative to the roughness height H^*, bed exposure is minimized; when actual sediment is extremely thin relative to the roughness scale, bed exposure is maximized.

2.1.2 Sediment Flux

Our model uses a shear-stress dependent formulation for bedload transport derived by Wickert and Schildgen (2019). Their derivation results in a formulation in which sediment flux is dependent on slope, discharge, and flow intermittency; we modify their expression by adding in our bed cover term:

$$q_s = cIqS(1 - \alpha) \tag{6}$$

where c is a lumped, dimensionless coefficient whose value assumes a constant shear stress slightly above that needed to transport the median grain size at bankfull conditions (Parker, 1978), I is an intermittency factor that describes how often geomorphically effective flows occur within the channel, q is the water discharge per unit channel width, S is the channel slope, and the term $(1 - \alpha)$ describes the fraction of the riverbed covered by sediment. When α (bed exposure) is sufficiently small, the bed is effectively 100% covered by mobile sediment and sediment flux, q_s, will be maximized.

2.1.3 Bedrock Erosion

We allow for erosion to occur through two mechanisms: plucking and abrasion. The effectiveness of each of these mechanisms depends on the percentage of bedrock exposure on the riverbed. Dubinski and Wohl (2012) found a linear relationship between stream power and plucking rate in bedrock; we therefore use a version of the stream power model (Whipple and Tucker, 1999), modified to include our bedrock exposure term, to calculate the rate of erosion via plucking:

$$E_p = KqS\alpha \tag{7}$$

where K is the bedrock erodibility and α is the fraction of bedrock exposed.

Abrasion is accomplished through sediment impacts on the riverbed. We borrow our abrasion formulation from Chatanantavet and Parker (2009), where the amount of material removed for each sediment impact depends on the abrasion coefficient of the bedrock substrate, β, and the sediment flux. As before, we modify these authors' equation to include the percentage of bedrock exposed:

$$E_a = \beta q_s \alpha \tag{8}$$

While both E_p and E_a include factors related to rock strength, the erodibility factor used in Equation (4) depends more on large-scale features of the bedrock such as jointing, while the abrasion coefficient used in Equation (5) is related to strength at the grain scale. Erosion via abrasion is expected to dominate in rocks that are weak at the grain scale (Whipple, 2004). Since our model traverses multiple lithologies (multiple K values), both equations (7) and (8) will have as many forms as there are lithologies represented in the model (in our case, two). These forms may be denoted, for example, $E_{p,ig}$ and $E_{p,sed}$ to differentiate plucking of igneous and sedimentary lithologies.

2.1.4 Attrition

In addition to sediment abrading the riverbed, gravel clasts in transport in a real system will also abrade one another, which is one of the principal mechanisms at play in downstream fining. We refer to this grain-on-grain action as "attrition" in order to differentiate it from "abrasion," which will refer solely to vertical lowering pf the profile resulting from grain impacts on the riverbed. The rate of grain attrition, γ is a function of the abrasion coefficient of the sediment comprising the bedload and the amount of sediment in transport, i.e. the sediment flux:

$$\gamma = \beta q_s \tag{9}$$

2.1.5 Sedimentation

Sedimentation in our model is the sum of processes that contribute to an increase in sediment thickness on the riverbed, and those that degrade bed sediment thickness. Recalling Equation (4), the sedimentation rate depends on the presence of bedload material, the sediment flux, and grain attrition. As math, this is written:

$$\frac{\partial H}{\partial t} = E_{p,ig} - q_s - \gamma \tag{10}$$

When calculating the sedimentation rate, only coarse material plucked from highly erosion-resistant lithologies is considered significant to controlling the amount of bedload material present. Although plucking occurs in softer units, the most erodible clasts in a river system tend to experience rapid downstream attrition and thus they are not persistent in the bedload for long distances (Menting et al., 2015, Dingle et al., 2017).

2.2 Model Implementation

Our model is implemented on a 1D grid with a total domain length of 100 km and grid spacing of 1 km. A lithologic boundary is imposed at one quarter of the length of the domain (25 km). Between kilometers 0-24, the bedrock is assigned properties (such as erodibility and abrasion coefficient) representative of granitic rocks (Attal and Lave, 2006); from kilometers 25-100, rocks are assigned properties representative of course sandstones and conglomerates (Attal and Lave, 2006). Our model evolves through time in response to "uplift," which we simulate through the progressive baselevel lowering of an outlet node.

The model is designed to capture the changes in dominant erosional process that occur streamwise as sediment builds up and influences the system, much the way real rivers are often detachment-limited in their headwaters and become progressively more transport-limited downstream (CITATION; DAVY AND LAGUE 2009?)In our model, different fluvial conditions can be forced or suppressed with intentional parameter choices. For example, making abrasion coeffcients so small that abrasion is negligible, while also making the characteristic sediment thickness (roughness height) so large that it is challenging to completely cover the bed, results in a model that is effectively the stream-power model. In our implementation, we make parameters choices based on a combination of empirical data and BEST GUESSES.

2.3 Analytical Solution

2.4 Model Testing

Rivers exhibit certain characteristic forms in both steady and transient cases. At steady state, these forms include a concave-upward profile, as well as power law scaling between slope and area and an increase in channel steepness with uplift rate (Lague, 2014). In transient cases, rivers also exhibit upstream-migrating knickpoints in response to baselevel fall (Lague, 2014). These fluvial behaviors are drawn from both observation and theory [CITATIONS: TAKE FROM LAGUE 2014]. Several field studies have also contributed to our understanding of how rivers behave in lithologically heterogeneous environments: in settings in which a more erosion-resistant lithology occurs upstream and contributes coarse sediment into the system, downstream channel segments are steeper and more concave relative to those in which hard sediment does not occur (Duvall et al., 2004, Johnson et al., 2009, Lai et al., 2021).

In the section below, we review how our model responds to these tests.