Back propagation Algorithm Calculation.

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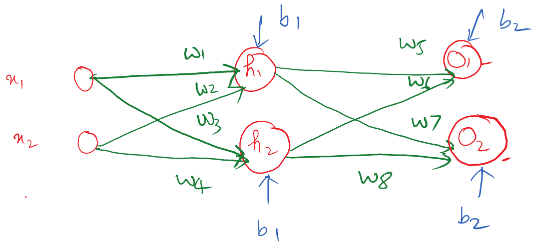

Input $x_1 = 0.05$ $x_2 = 0.10$

Target output $O_1 = 0.01$ Target output $O_2 = 0.99$

$w_1 = 0.15$ $w_5 = 0.40$ $b_1 = 0.35$

$w_2 = 0.20$ $w_6 = 0.45$ $b_2 = 0.60$

$w_3 = 0.25$ $w_7 = 0.50$

$w_4 = 0.30$ $w_8 = 0.55$

Forward Propagation

Activation function $= f(x) = \dfrac{1}{1+e^{-x}}$

$$net(h_1) = x_1 w_1 + x_2 w_2 + b_1$$
$$= 0.05 \times 0.15 + 0.10 \times 0.20 + 0.35 = 0.3775$$

$$out(h_1) = \frac{1}{1+e^{-net(h_1)}} = \frac{1}{1+e^{-.3775}} = \boxed{0.5932}$$

$$net(h_2) = x_1 w_3 + x_2 w_4 + b_1 = 0.05 \times 0.25 + 0.10 \times 0.30 + 0.35$$
$$= 0.3925 \quad ; \quad out(h_2) = \frac{1}{1+e^{-net(h_2)}} = \frac{1}{1+e^{-.3925}} = \boxed{0.5968}$$

$$net(O_1) = out(h_1) \cdot w_5 + out(h_2) \cdot w_6 + b_2$$
$$= 0.5932 \cdot 0.40 + 0.5968 \times 0.45 + 0.60 = 1.10584$$

$$out(O_1) = \frac{1}{1+e^{-net(O_1)}} = \frac{1}{1+e^{-1.10584}} = \boxed{0.7513}$$

$$net(O_2) = out(h_1) \cdot w_7 + out(h_2) \cdot w_8 + b_2$$
$$= 0.5932 \cdot 0.50 + 0.5968 \cdot 0.55 + 0.60$$
$$= 1.22484$$

$$out(O_2) = \frac{1}{1+e^{-net(O_2)}}$$
$$= \frac{1}{1+e^{-1.22484}} = \boxed{0.7729}$$

$$Error = \frac{1}{2}(\text{Target Value} - \text{Output})^2$$

$$Target(O_1) = 0.01$$
$$Target(O_2) = 0.99$$

$$E_{Total} = E_1 + E_2$$
$$= \frac{1}{2}\left(Target(O_1) - Out(O_1)\right)^2 + \frac{1}{2}\left(Target(O_2) - out(O_2)\right)^2$$
$$= \frac{1}{2}[0.01 - 0.7513]^2 + \frac{1}{2}[0.99 - 0.7729]^2$$
$$= \boxed{0.2983} \leftarrow$$

1^{st} ~~error~~ iteration error.

b

Backward Propagation

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Weight update b/w hidden layer and out put layer

$$\frac{\partial E_{total}}{\partial w_5} = \frac{\partial E_{total}}{\partial out(o_1)} \cdot \frac{\partial out(o_1)}{\partial net(o_1)} \cdot \frac{\partial net(o_1)}{\partial w_5}$$

$\cdot 7413$ $\quad 0.1868$

$$\frac{\partial E_{total}}{\partial out(o_1)} = \frac{\partial}{\partial out(o_1)} \left(\frac{1}{2}(target(o_1) - out(o_1))^2 + \frac{1}{2}(target(o_2) - out(o_2))^2 \right)$$

$= 0$

$$= \frac{1}{2} \cdot 2 \, (target(o_1) - out(o_1)) \times (-1)$$

$$= -(target(o_1) - out(o_1)) = -(0.01 - 0.7513) = \boxed{0.7413}$$

$$\frac{\partial out(o_1)}{\partial net(o_1)} :- \quad out(o_1) = \frac{1}{1+e^{-net(o_1)}}$$

$$out(o_1) \cdot (1 - out(o_1))$$

$$= 0.7513 \, (1 - 0.7513)$$

$$= 0.1868.$$

$$f(x) = \frac{1}{1+e^{-x}} \qquad \frac{d}{dx} f(x) = \frac{d}{dx} (1+e^{-x})^{-1}$$

$$-1 \, (1+e^{-x})^{-1-1} \times e^{-x} (-1)$$

$$= \frac{e^{-x}}{(1+e^{-x})^2} = \frac{1+e^{-x}-1}{(1+e^{-x})^2} =$$

$$\frac{1+e^{-x}}{(1+e^{-x})^2} - \frac{1}{(1+e^{-x})^2}$$

$$= \frac{1}{(1+e^{-x})} - \frac{1}{(1+e^{-x})^2}$$

$$= f(x) - (f(x))^2$$

$$= f(x) \, (1 - f(x))$$

$$\frac{\partial net(o_1)}{\partial w_5} = \frac{\partial}{\partial w_5} (out(h_1) \cdot w_5 + out(h_2) \cdot w_6 + b_2)$$

$$= out(h_1)$$

$$= 0.5932$$

$$\frac{\partial E_{total}}{\partial w_5} = 0.7413 \, (.1868) \, (0.5932)$$

$$= 0.082$$

$$w_5 = w_5 - \eta \left(\frac{\partial E_{total}}{\partial w_5} \right)$$

$\eta \rightarrow$ Learning Rate $(0, 1)$

$\eta = 0.5$

$$w_5 = 0.40 - 0.50 \times 0.082$$

$$= 0.40 - 0.041$$

$$w_5' = 0.359$$

$$\frac{\partial E_{total}}{\partial w_6} = \frac{\partial E_{total}}{\partial out(o_1)} \cdot \frac{\partial out(o_1)}{\partial net(o_1)} \cdot \frac{\partial net(o_1)}{\partial w_6}$$

$$\frac{\quad}{\partial w_6} \quad \partial out(o_1) \quad \partial net(o_1) \quad w_6$$

$$\frac{\partial E_{total}}{\partial w_7} = \frac{\partial E_{total}}{\partial out(o_2)} \cdot \frac{\partial out(o_2)}{\partial net(o_2)} \cdot \frac{\partial net(o_2)}{\partial w_7}$$

$$\frac{\partial E_{total}}{\partial w_8} = \frac{\partial E_{total}}{\partial out(o_2)} \cdot \frac{\partial out(o_2)}{\partial net(o_2)} \cdot \frac{\partial net(o_2)}{\partial w_8}$$

$w_5' = 0.359$

$w_6' = 0.40867$

$w_7' = 0.5133$

$w_8' = 0.56135$

Back propagation hidden and input layer

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$$\frac{\partial E_{total}}{\partial w_1} = \frac{\partial E_{total}}{\partial out(h_1)} \cdot \frac{\partial out(h_1)}{\partial net(h_1)} \cdot \frac{\partial net(h_1)}{\partial w_1}$$

$$= 0.03633 \quad out(h_1)(1-out(h_1)) \cdot x_1$$

$$\frac{\partial E_{total}}{\partial out(h_1)} = \frac{\partial (E_1+E_2)}{\partial out(h_1)} = \frac{\partial E_1}{\partial out(h_1)} + \frac{\partial E_2}{\partial out(h_1)}$$

$$= 0.05538 \qquad =$$

$$\frac{\partial E_1}{\partial out(h_1)} = \frac{\partial E_1}{\partial out(o_1)} \cdot \frac{\partial out(o_1)}{\partial net(o_1)} \cdot \frac{\partial net(o_1)}{\partial out(h_1)}$$

$$= -(target(o_1) - out(o_1)) \cdot out(o_1) \cdot (1-out(o_1)) \cdot w_5$$

$$= 0.7413 \times (.1868) \times (0.40)$$

$$= 0.05538$$

$$\frac{\partial E_2}{\partial out(h_1)} = \frac{\partial E_2}{\partial out(o_2)} \cdot \frac{\partial out(o_2)}{\partial net(o_2)} \cdot \frac{\partial net(o_2)}{\partial out(h_1)}$$

$$= -(target(o_2) - out(o_2)) \cdot out(o_2)(1-out(o_2)) \cdot w_7$$

$$= -(0.99 - 0.7729)(0.7729)(1-0.7729) \cdot 0.50$$

$$= -0.01905$$

$$\frac{\partial E_1}{\partial out(h_1)} + \frac{\partial E_2}{\partial out(h_1)} = 0.05538 + (-0.01905) = 0.03633$$

$$\frac{\partial E_{total}}{\partial w_1} = 0.03633 \cdot out(h_1)(1-out(h_1)) \cdot x_1$$

$$= 0.03633 (0.5932)(1-0.5932) \cdot 0.05$$

$$= 2.19 \times 10^{-4}$$

$$\eta \left(\partial E_{total} \right)$$

$= 2 \cdots$

$$w_1' = w_1 - \eta \left(\frac{\partial E_{total}}{\partial w_1} \right)$$

$$= 0.15 - 0.5 \times 2.19 \times 10^{-4}$$

$$w_1' = \boxed{0.14978}$$

$$\frac{\partial E_{total}}{\partial w_2} = \frac{\partial E_{total}}{\partial out(h_1)} \cdot \frac{\partial out(h_1)}{\partial net(h_1)} \cdot \frac{\partial net(h_1)}{\partial w_2}$$

$$\frac{\partial E_{total}}{\partial w_3} = \frac{\partial E_{total}}{\partial out(h_2)} \cdot \frac{\partial out(h_2)}{\partial net(h_2)} \cdot \frac{\partial net(h_2)}{\partial w_3}$$

$$\frac{\partial E_{total}}{\partial w_4} = \frac{\partial E_{total}}{\partial out(h_2)} \cdot \frac{\partial out(h_2)}{\partial net(h_2)} \cdot \frac{\partial net(h_2)}{\partial w_4}$$

$\cancel{w_2^2}$ $w_2' = 0.19956$

$w_3' = 0.24975$

$w_4' = 0.2995$

$w_5' = 0.3589$

$w_6' = 0.4086$

$w_7' = 0.511$

$w_8' = 0.5613$

$\text{Error}^2 =$