Assignment - 05

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1) a) Looking at the bode plot

→ there is a pole at origin
zero at $s = -\omega_1$
pole at $s = -\omega_2$ & $s = -10$ } minimum phase.

→ so TF is

$$T(s) = \frac{k\left(1+\frac{s}{\omega_1}\right)}{s\left(1+\frac{s}{\omega_2}\right)\left(1+\frac{s}{10}\right)}$$

at low ω,

$$|T(j\omega)| \to \left|\frac{k}{j\omega}\right|$$

$$20\log_{10}|T(j\omega)| = 20\log_{10}k - 20\log_{10}(0.1) = 32\text{dB}$$

$$20\log_{10}k = 12\text{dB}$$

$$k = 3.98 \approx 4.$$

$\underline{\omega_1}$

$$\frac{32-6}{\log(0.1)-\log(\omega_1)} = -20\text{ dB/dec} \Rightarrow \omega_1 = 1.995 \approx 2$$

$\underline{\omega_2}$

$$\frac{6-0}{\log(\omega_2)-\log_{10}(10)} = -20 \Rightarrow \omega_2 = 5.011 \approx 5$$

so TF $\Rightarrow$

$$T(s) = \frac{4\left(1+\frac{s}{2}\right)}{s\left(1+\frac{s}{5}\right)\left(1+\frac{s}{10}\right)}$$

(b) looking at graph

→ 2 poles at origin (type 2) system

→ 2 zeroes at $s=-2$

2 poles at $s=-10$

$$T(s) = \frac{k\left(1+\frac{s}{2}\right)^2}{s^2\left(1+\frac{s}{10}\right)^2}$$

$\lim_{\omega\to 0} |T(j\omega)| = \left(\frac{k}{\omega^2}\right)$ → low freq asymptote intersects ω-axis @ ($\omega_0 = \sqrt{k}$)

⇒ $s = \sqrt{k}$

⇒ $k = 25$

$$\therefore T(s) = \frac{25\left(1+\frac{s}{2}\right)^2}{s^2\left(1+\frac{s}{10}\right)^2}$$

(c) looking at graph

→ pole at origin

→ pole at $s = -\omega_1$

zero @ $s = -\omega_2$

pole @ $s = -\omega_3$

so $T(s) = \dfrac{k\left(1+\frac{s}{\omega_2}\right)}{s\left(1+\frac{s}{\omega_1}\right)\left(1+\frac{s}{\omega_3}\right)}$

$\underline{\omega_1}$

$$\frac{(24.1-0)}{\log(\omega_1)-\log(8)} = -40 \Rightarrow \omega_1 = 1.99 \approx 2$$

$\underline{\omega_2}$

$$\frac{24.1-(-12.05)}{\log(\omega_1)-\log(\omega_2)} = -40 \rightarrow \omega_2 = 16$$

$\underline{W_3}$

$$\frac{-12.05 - (20.05)}{\log \omega_2 - \log \omega_3} = -20 \Rightarrow \omega_3 = 40.19 \approx 40.$$

$$\frac{N - 24.1}{\log_{10} 1 - \log_{10} \omega_1} = -20 \rightarrow N = 30.12$$

$$20 \log_{10} k = 30.12$$

$$\Rightarrow k = 32.06$$

$$\rightarrow T(s) = \frac{32.06\left(1 + \frac{s}{46}\right)}{s\left(1 + \frac{s}{2}\right)\left(1 + \frac{s}{40}\right)}$$

3) <u>Bode plots</u>

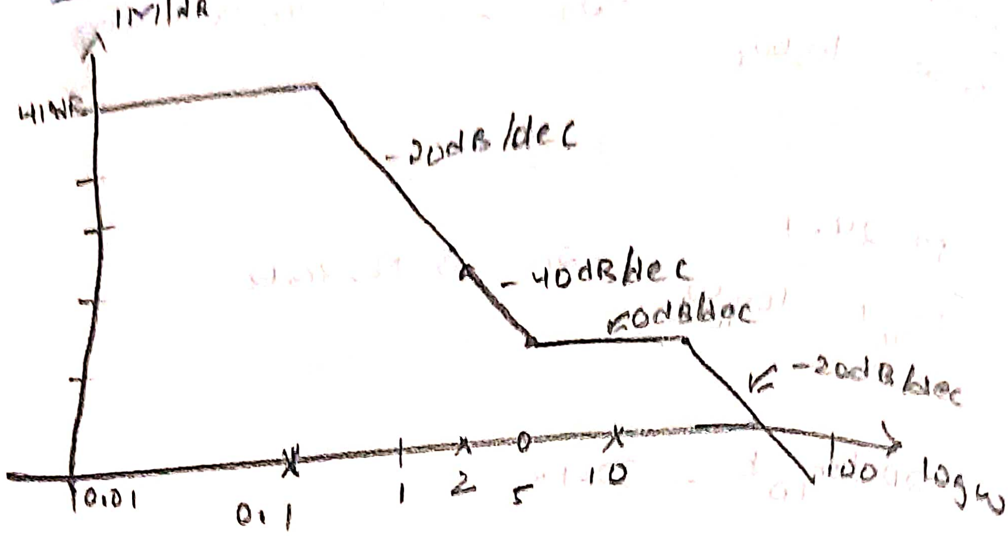
$$(a)\ G(s) = \frac{10(s+5)^2}{(s+0.1)(s+2)(s+10)}$$

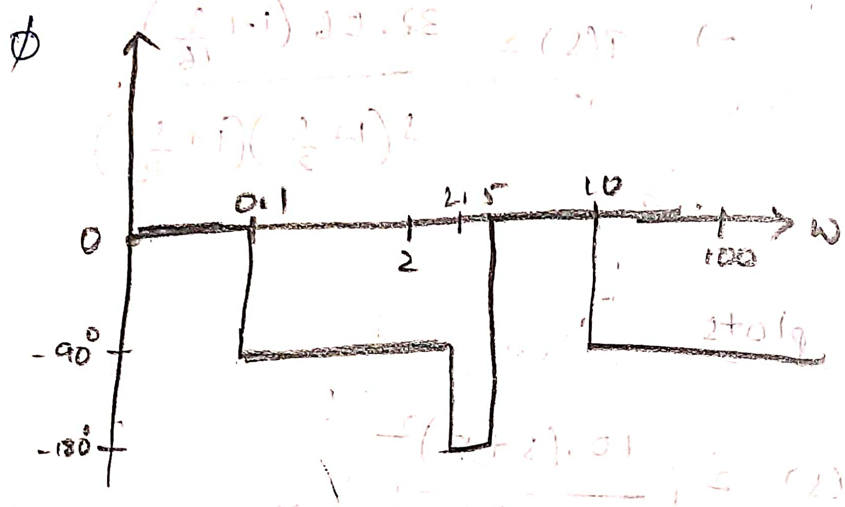
$$= \frac{10(25)\left(1 + \frac{s}{5}\right)^2}{10 \times 2 \times \left(1 + \frac{s}{0.1}\right)\left(1 + \frac{s}{2}\right)\left(1 + \frac{s}{10}\right)}$$

$$= \frac{125\left(1 + s/5\right)^2}{\left(1 + \frac{s}{0.1}\right)\left(1 + \frac{s}{2}\right)\left(1 + \frac{s}{10}\right)}.$$

$$G(j\omega) = \frac{125\left(\frac{j\omega}{5} + 1\right)^2}{\left(\frac{j\omega}{0.1} + 1\right)\left(\frac{j\omega}{2} + 1\right)\left(\frac{j\omega}{10} + 1\right)}$$

Magnitude Plot

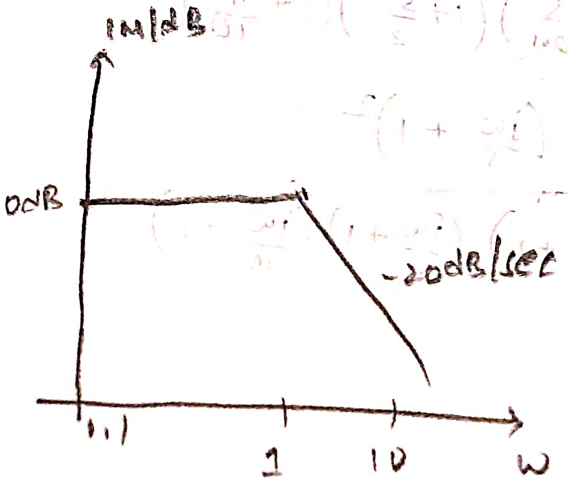


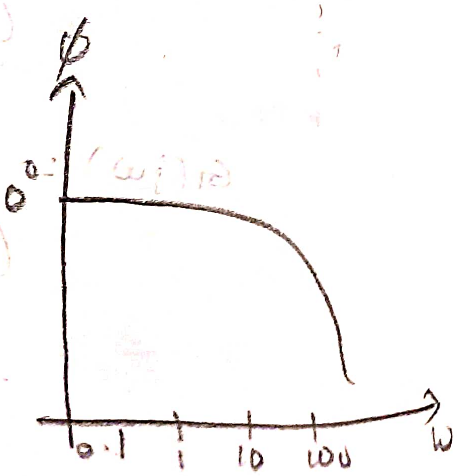


(b) $\dfrac{e^{-s}}{1+s}$

$$G(j\omega) = \frac{e^{-j\omega}}{1+j\omega}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1+\omega^2}} \qquad \phi(\omega) = \underbrace{-\omega * 57.29^\circ}_{\text{rad} \to \text{degree factor}} - \tan^{-1}\omega$$





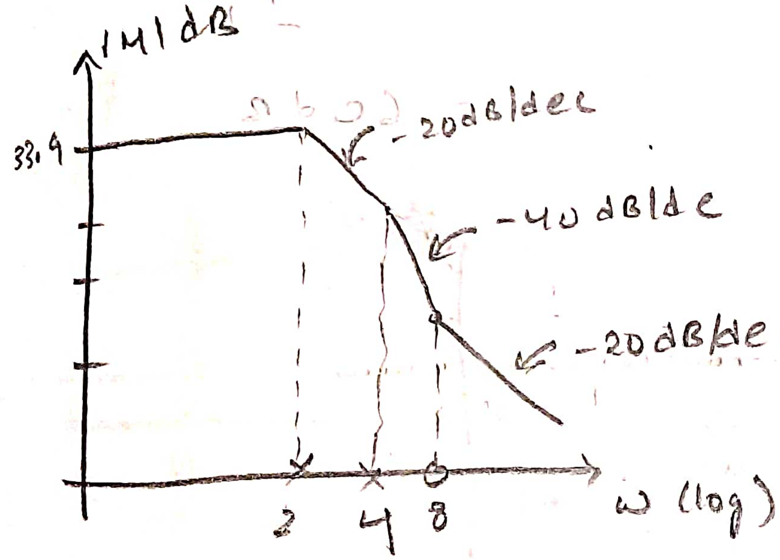
c) $G(s) = \dfrac{50(s-8)}{s^2+6s+8} = \dfrac{50(s-8)}{(s+4)(s+2)} = \dfrac{50\times 8(s/8-1)}{4\times 2(1+\frac{s}{4})(1+\frac{s}{2})}$

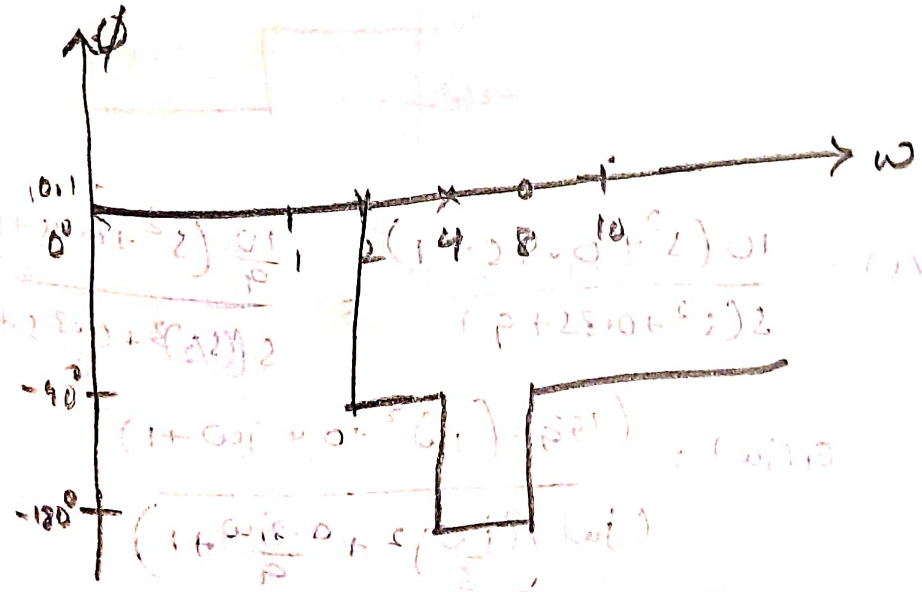
$= \dfrac{50(s/8-1)}{(1+\frac{s}{4})(1+\frac{s}{2})}$

$G(j\omega) = \dfrac{50(\frac{j\omega}{8}-1)}{(1+\frac{j\omega}{4})(1+\frac{j\omega}{2})}$

starts @ $20\log_{10}50$

$= 33.979$ dB





d) $G(s) = \dfrac{1}{s^3(s+1)}$

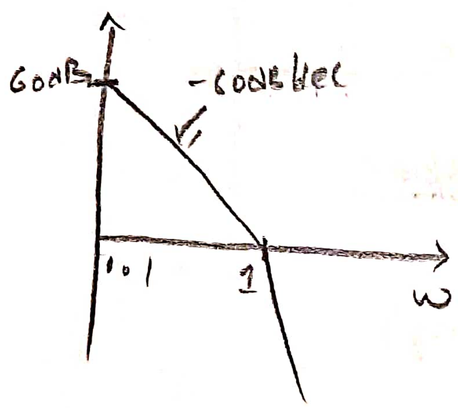
$G(j\omega) = \dfrac{1}{(j\omega)^3\,(j\omega+1)}$

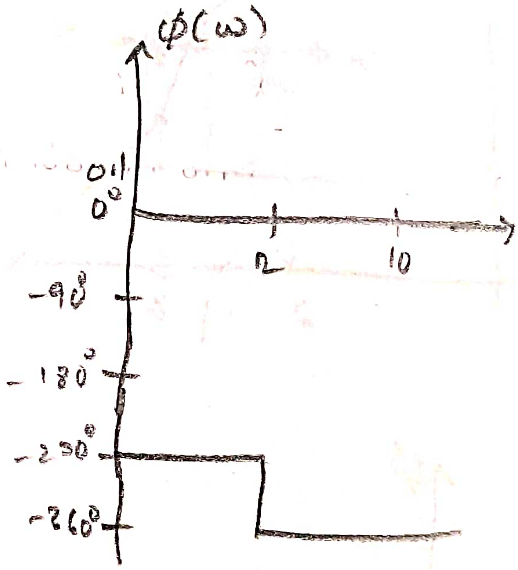
@ $\omega = 0.01 \Rightarrow G(j\omega) \rightarrow \dfrac{1}{(j\omega)^3}$

$20\log_{10} |G(j\omega)| = 0 - 20\log(\omega^3)$

$= -20 \times \underbrace{3\log(0.1)}_{-1}$

$= 60\,dB$





(e) $G(s) = \dfrac{10(s^2+0.4s+1)}{s(s^2+0.8s+9)} = \dfrac{\frac{10}{9}(s^2+0.4s+1)}{s\left((s/3)^2+\frac{0.8}{9}s+1\right)}$

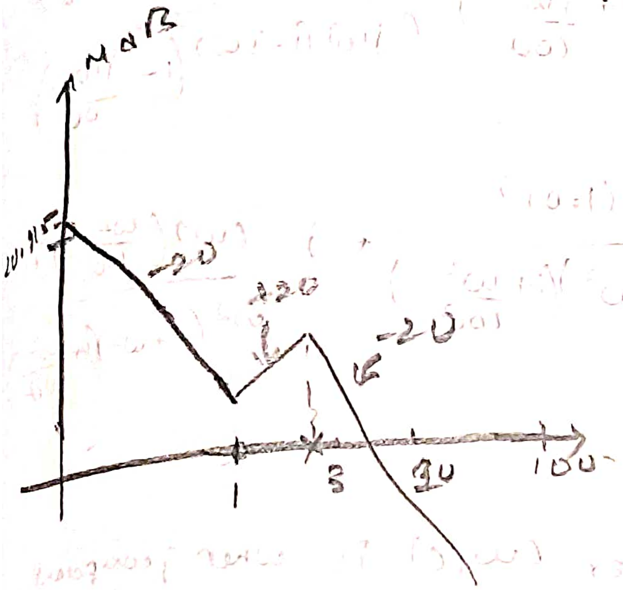
$G(j\omega) = \dfrac{(10/9)\left((j\omega)^2+0.4\,j\omega+1\right)}{(j\omega)\left(\left(\frac{j\omega}{3}\right)^2+\frac{0.8j\omega}{9}+1\right)}$

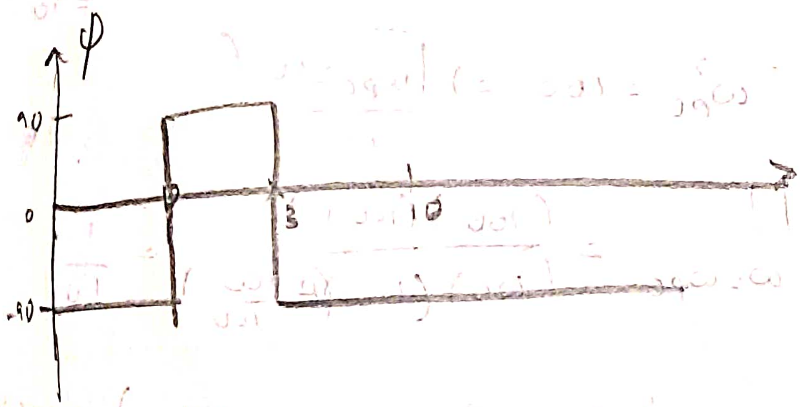
starts at

$\cancel{20 + 20\log_{10}\left(\frac{10}{9}\right) = 0.915\,dB}$ +20

$= 20.91\,dB$

break @ $\omega_1 = 1$ & $\omega_2 = \sqrt{9} = 3$, pole @ origin :
+40 -40





(4) → pole @ origin

→ pole @ $s=-1$
$s = -100$

$$G(s) = \frac{k}{s(1+s)\left(1+\frac{s}{100}\right)}$$

-20 dB line crosses origin at $\omega=1$ so $k=1$

$$G(s) = \frac{1}{s(1+s)\left(1+\frac{s}{100}\right)}$$ (minimum phase TF)

$$G(j\omega) = \frac{1}{j\omega(1+j\omega)\left(1+\frac{j\omega}{100}\right)}$$

for min phase

$$G(j\omega) = \frac{1}{(j\omega)(1+j\omega)\left(1+\frac{j\omega}{100}\right)} \frac{(j\omega)(1-j\omega)\left(1-\frac{j\omega}{100}\right)}{(j\omega)(1-j\omega)\left(1-\frac{j\omega}{100}\right)}$$

$$= \frac{-\omega^2(1.01)}{\omega^2(1+\omega^2)\left(1+\frac{\omega^2}{100^2}\right)} + j\frac{(\omega)\left(\frac{\omega^2}{100}-1\right)}{\omega^2(1+\omega^2)\left(1+\frac{\omega^2}{100^2}\right)}$$

GM & PM

Phase crossover (ω_{pc}) is when j component $\Rightarrow 0$.

$$\omega_{pc}^2 = 100 \Rightarrow \boxed{\omega_{pc} = 10}$$

$$|G(j\omega)|\Big|_{\omega=\omega_{pc}} = \frac{(100)(1.01)}{(100)(1.01)\left(1+\frac{100}{100}\right)} = \frac{1}{101}$$

$$GM = 20\log_{10}\left(\frac{1}{|G(j\omega)|}\right) = 20\log_{10}\left(\frac{1}{\frac{1}{101}}\right) = 40.06\,dB$$

$$|G(j\omega)| = \frac{100}{\sqrt{\omega^2(1+\omega^2)(100+\omega^2)}} = 1$$

$$\omega_{gc} = 0.7861$$

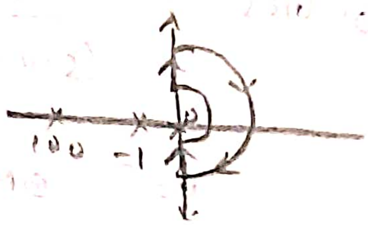
$$PM = 180 + \angle G(j\omega_{gc})$$

$$= 180 - \tan^{-1}\left(\frac{\omega_{gc}}{0}\right) - \tan^{-1}\left(\frac{\omega_{gc}}{1}\right) - \tan^{-1}\left(\frac{\omega_{gc}}{100}\right)$$

$$PM = 51.333^\circ.$$

Nyquist plot

$$G(j\omega)\Big|_{\omega=0} = \frac{-10100}{(1)(100\times100)}$$



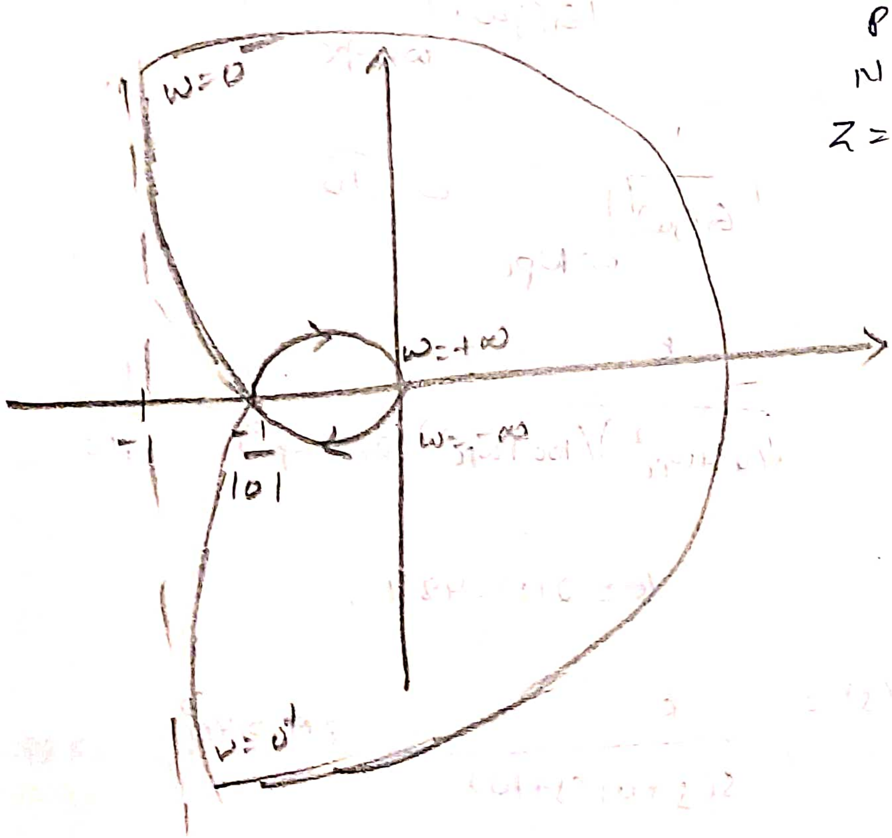
$$= -1.01 - j\infty$$

$$G(j\omega)\Big|_{\omega=\infty} = 0+j0$$

$$\angle G(j\omega)\Big|_{\omega\to\infty} = -90-\tan^{-1}(\omega s c)-\tan^{-1}\left(\frac{\omega s c}{100}\right)\Big|_{\omega=\infty}$$

$$= -90-90-90$$

$$= -270 \sim 90^{\circ}$$



$P=0$

$N=0$

$Z=N+P=0$

stable.

(5)(a) $G(s) = \dfrac{K}{(s+4)(s+10)(s+15)}$

$K = ?$ $GM = 10\,dB$

$$\angle G(j\omega)\Big|_{\omega=\omega_{pc}} = -180^\circ$$

$$\tan^{-1}\left(\frac{\omega_{pc}}{4}\right) + \tan^{-1}\left(\frac{\omega_{pc}}{10}\right) + \tan^{-1}\left(\frac{\omega_{pc}}{15}\right) = 180^\circ$$

$\omega_R = 15.8$ rad/s.

$$GM = 20\log_{10}\left(\frac{1}{|G(j\omega)|_{\omega=\omega_{pc}}}\right) = 10$$

$$\frac{1}{|G(j\omega)|_{\omega=\omega_{pc}}} = \sqrt{10}$$

$$\frac{K}{\sqrt{(4^2+\omega_{pc}^2)(100+\omega_{pc}^2)(225+\omega_{pc}^2)}} = \frac{1}{\sqrt{10}}$$

$K = 2113.489$.

(b) $G(s) = \dfrac{K}{s(s+4)(s+10)}$ $PM = 40^\circ$

→ The bode plot of uncompensated system i.e ($K = 1$)

has $PM = 89.5^\circ$

→ for PM of 40° → $\angle G(j\omega)\Big|_{\omega=\omega_{pc}} = -140^\circ$

@ $\omega = \omega_{pc}$ → $|G(j\omega)|_{\omega_{pc}} = -43\,dB$

so gain must be so the magnitude plot must be increased by 43 dB

$$\text{so } 20 \log_{10} k = 43$$

$$k = 10^{43/20} = 141.25$$

(c) $$G(s) = \frac{k(s+2)}{s(s+4)(s+6)(s+10)} \qquad \omega_{gc} = 1 \text{ rad/s}$$

$$|G(j\omega)|_{\omega=\omega_{gc}=1} = \frac{|2+j|}{|j|\,|j+4|\,|j+6|\,|j+10|}$$

$$20 \log_{10} |G(j\omega)|_{\omega=\omega_{gc}=1} = -41.1 \text{ dB}$$

$$k = 1$$

so we want to increase gain / move up the plot by 41.1 dB

$$20 \log_{10} k = 41.1$$

$$k = 10^{41.1/20} = 113.501$$

(d) $$G(s) = \frac{\overbrace{10\, e^{-0.16s}}^{G'}}{s(s+20)} \qquad PM = 40^\circ$$

find ω_0 at which $\angle G'(j\omega)\big|_{\omega=\omega_0} = -140^\circ$

$$-0.6\,\omega_0 \times 53.4 - 90 - \tan^{-1}\left(\frac{\omega_0}{20}\right) = -140^\circ$$

$$\Rightarrow \omega_0 = 1.35 \text{ rad/s}$$

$$|G(j\omega)|_{\omega=\omega_0} = \frac{1}{\omega_0\sqrt{\omega_0^2+400}} =)-28.7\,dB$$

$$20\log_{10} k = 28.7$$

$$k = 10^{\frac{28.7}{20}}$$

$$k = 27.227.$$

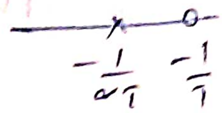
(6) $$G(s) = \frac{k}{s(0.4s+1)(s+1)}$$

lead compensator (s+1) PM = 45°, GM > 8 dB,

$K_v = 4s^{-1}$

First let the lead compensator be of form

$$G_c'(s) = \frac{k\left(s+\frac{1}{T}\right)}{\left(s+\frac{1}{\alpha T}\right)} \quad \frac{1}{\alpha}$$



$\frac{1}{\alpha} > 1$

$\alpha < 1$

$$G_c'(j\omega) = \frac{k(1+j\omega T)}{(1+j\omega\alpha T)}$$

$$K_v = \lim_{s\to 0} s\,G(s)\,G'(s) = k = 4$$

$\therefore k = 4$

Gain compensated system's bode plots were drawn.

$PM = 17.7^\circ$, $GM = 8.79\ dB$

$PM_{req} = 45^\circ$

$PM_{add} = 45 - 17.7 = 27.3$

$\phi_m = 27.3 + 10^\circ$

$= 37.3^\circ$

$\Rightarrow \quad \phi_m = \sin^{-1}\left(\frac{1-\alpha}{1+\alpha}\right)$

$$\alpha = \frac{1 - \sin\phi_m}{1 + \sin\phi_m} = 0.21$$

Gain of lead compensator @ max phase contribution is

$= 20\log_{10}\left(\frac{1}{\sqrt{\alpha}}\right) = 6.7778\ dB$

→ New gain crossover freq is where gain of uncompensated system $= -6.7778\ dB$

$\rightarrow \omega_{gc}$ new $= 2.81$ rad/s.

$$\frac{1}{\sqrt{\alpha}\,T} = 2.81$$

$$\boxed{T = 0.7766}$$

$$G_c'(s) = \frac{k(s\tau+1)}{(\alpha s\tau+1)}$$

$$G_c'(s) = \frac{4(0.27665s+1)}{(0.163095s+1)}$$

(7) $$G(s) = \frac{k(s+2)}{s(s+5)(s+15)}$$

lead compenator

$\% M_p = 15\%$, $t_s = 0.1s$, $k_v = 1000$

$\% M_p = 15\% \rightarrow \xi = 0.517 \Rightarrow PM = 53.17^\circ$

$t_s = 0.1s \rightarrow \omega_n = \frac{4}{t_s \xi} = 77.3793$

$\Rightarrow \omega_{gc} = 59.909$ rad/s.

$$G_c'(s) = \frac{k(s\tau+1)}{(s\alpha\tau+1)}$$

$k_v = \lim_{s \to 0} GG' = \frac{k(2)}{5(15)} = 1000$

$k = 10714.28$

$\phi_m = 53.17 - \underset{\text{gain comp}}{PM} + \underset{\text{correction}}{10^\circ}$

$= 53.13 - 7.18 + 10$

$= 55.99$

$$\alpha = \frac{1-\sin\phi_m}{1+\sin\phi_m}$$

$$\boxed{\alpha = 0.11195}$$

$$|G_c'(j\omega)|\Big|_{\omega=\omega_m} = 20\log\left(\frac{1}{\sqrt{\alpha}}\right) = 9.5096\ dB$$

ω_m = freq at which gain of uncompensated system -9.5096 dB

$\omega_m = 179$ rad/s.

$$\omega_m = \frac{1}{\sqrt{\alpha}\,T} = 179 \text{ rad/s}$$

$$T = 0.0166\,s$$

$$G_c'(s) = 10714.28\ \frac{(s(0.166)+1)}{(0.001869s+1)}$$

(8) $$G(s) = \frac{(s+10)(s+11)}{s(s+3)(s+6)(s+9)}$$

% $M_p = 15\%$ $\rightarrow$ $\xi = 0.517$

$\Rightarrow PM = 53.17$

$$G_c'(s) = \frac{K(sT+1)}{(s\beta T+1)} \qquad \beta > 1$$

→ first choosing k

$$\lim_{s\to 0} s\,G\,G_c' = \frac{K(10)(11)}{3\times6\times9} = 1000$$

$$k = 1472.73$$

PM uncompensated $= -4^\circ$

PM req $= 53.17 + 10 = 63.17$

$$180 + \angle G(j\omega)\Big|_{\omega = \omega_0} = 63.17$$

$\Rightarrow \omega_0 = 1.21$ rad/s

$$|G(j\omega)|\Big|_{\omega = \omega_0 = 1.21 \text{ rad/s}} = 57.5 \text{ dB.}$$

$$20 \log\left(\frac{1}{\beta}\right) = -57.5$$

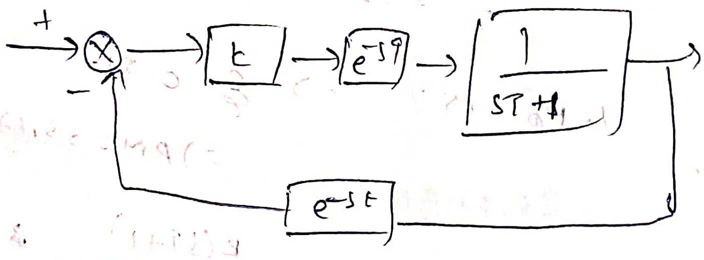
$$\beta = 749.8$$

$$\frac{1}{T} = (0.1)(1.21)$$

$\Rightarrow T = 8.264$ s

$$G_c'(s) = 1472.73 \frac{(8.2645s + 1)}{(6197s + 1)}$$

(1)



$T = 1.28$ s, $\tau = 0.25$ s.

(a) $k = ?$ $PM = 30^\circ$

$$G(s) = \frac{k e^{-2s\tau}}{(sT + 1)}$$

$$G'(s) = \frac{e^{-2s\tau}}{s\tau+1} = \frac{e^{-2.56s}}{(0.28s+1)}$$

for PM of 30°, phase of system $= -150°$

this occurs @ $\omega = 0.93$ rad/s.

at this ω, gain $= -0.231$ dB

∴ so the magnitude plot must be moved up by 0.231 dB ie

$$20\log_{10} k = 0.231\,\text{dB}$$

$$k = 10^{\frac{0.231}{20}}$$

$$\boxed{k = 1.027}$$

$$k_p = \lim_{s\to 0} G(s) = k = 1.027$$

$$\%\ \text{error} = \frac{1}{1+k_p}\times 100 = \frac{1}{1+1.027}\times 100$$

$$= 49.33\%.$$

(b) $\%\ e_{ss} = 5\%$

PM $= 50°$

lag compensator $\longrightarrow G_c(s) = \frac{k'(s\tau+1)}{(s\beta\tau+1)}$ $\beta > 1$

$$\frac{1}{1+k_{p\,new}} = 0.05$$

$$\boxed{k_p\ new = 20.1\text{?} = 19}$$

$$\lim_{s\to 0} Gc'G(s) = 19$$

$$(k')(k) = 19$$

$$k' = \frac{19}{1.022} = 18.55$$

→ the gain adjustment destabilise system

→ req. PM = 50°

this occurs at phase of −130°

Adding a correction factor of 10°,

look for ω_0 where phase = −120°

$$\omega_0 = 0.744 \text{ rad/s}$$

at this ω_0 gain = 25.4dB

→ Thus the lag compensator must introduce a gain of −25.4 dB

@ ω_0 that is its high asymptote

$$20 \log_{10}\left(\frac{1}{\beta}\right) = -25.4$$

$$\rightarrow \boxed{\beta = 18.6209}$$

→ taking high corner frequency

as $0.1\omega_0$

i.e. $\frac{1}{T} = (0.1)(0.744)$

$$\boxed{T = 13.459}$$

$$G_c'(s) = 18.500 \frac{(13.459s+1)}{(2506s+1)}$$